Computer algebra independent integration tests

1-Algebraic-functions/1.2-Trinomial-products/1.2.1-Quadratic/1.2.1.6-g+h-x-^m-a+b-x+c-x^2-^p-d+e-x+f-x^2-^q

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Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [143]. This is test number [37].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.3 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12.1 on windows 10.
3. Maple 2021.1 (64 bit) on windows 10.
4. Maxima 5.44 on Linux. (via sagemath 9.3)
5. Fricas 1.3.7 on Linux (via sagemath 9.3)
6. Giac/Xcas 1.7 on Linux. (via sagemath 9.3)
7. Sympy 1.8 under Python 3.8.8 using Anaconda distribution on Ubuntu.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 under windows 10 (64 bit)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100.00 (143)	% 0.00 (0)
Mathematica	% 99.30 (142)	% 0.70 (1)
Maple	% 98.60 (141)	% 1.40 (2)
Maxima	% 10.49 (15)	% 89.51 (128)
Fricas	% 46.15 (66)	% 53.85 (77)
Sympy	% 7.69 (11)	% 92.31 (132)
Giac	% 34.97 (50)	% 65.03 (93)
Mupad	% 13.29 (19)	% 86.71 (124)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

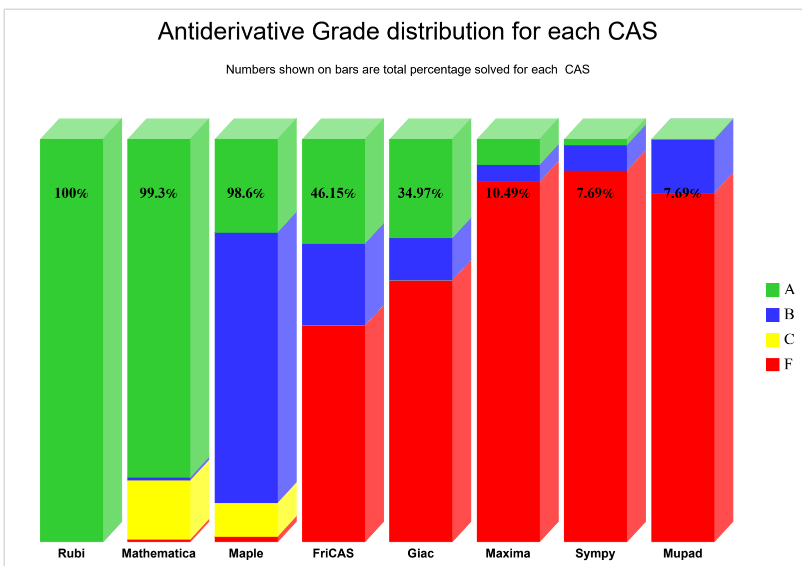
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

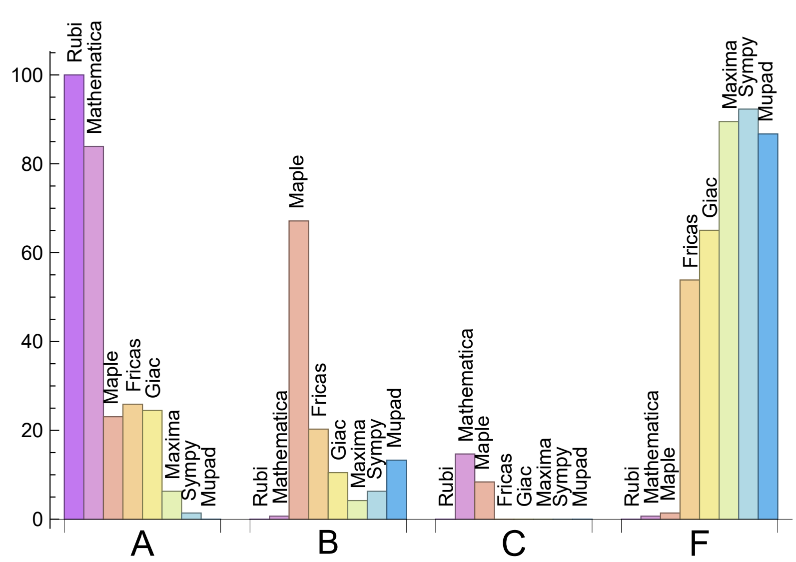
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.00	0.00	0.00	0.00
Mathematica	83.92	0.70	14.69	0.70
Maple	23.08	67.13	8.39	1.40
Maxima	6.29	4.20	0.00	89.51
Fricas	25.87	20.28	0.00	53.85
Sympy	1.40	6.29	0.00	92.31
Giac	24.48	10.49	0.00	65.03
Mupad	0.00	13.29	0.00	86.71

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	1	100.00 %	0.00 %	0.00 %
Maple	2	100.00 %	0.00 %	0.00 %
Maxima	128	52.34 %	0.00 %	47.66 %
Fricas	77	1.30 %	98.70 %	0.00 %
Sympy	132	87.88 %	12.12 %	0.00 %
Giac	93	22.58 %	18.28 %	59.14 %
Mupad	124	99.19 %	0.81 %	0.00 %

Table 1.4: Time and leaf size performance for each CAS

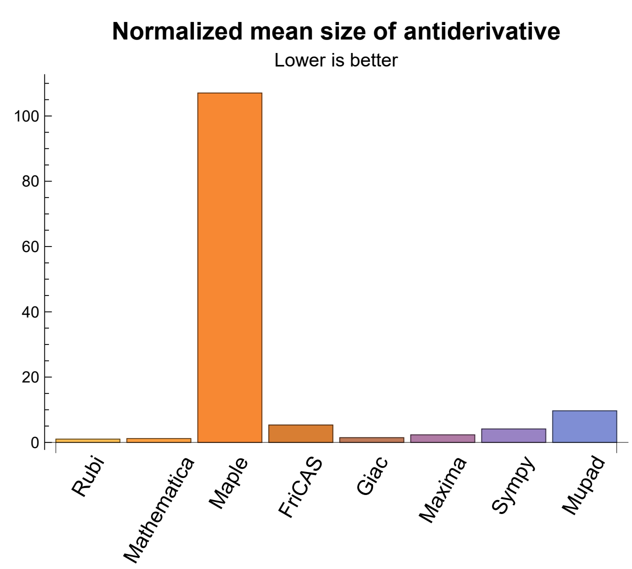
1.3 Performance

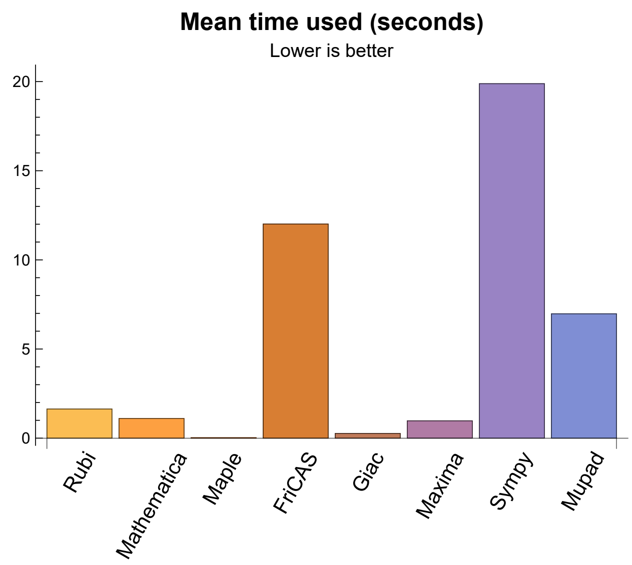
The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	1.64	333.90	1.00	302.00	1.00
Mathematica	1.10	347.54	1.17	290.50	0.97
Maple	0.03	52015.57	107.04	1172.00	3.85
Maxima	0.97	388.27	2.32	220.00	1.11
Fricas	12.01	1356.59	5.33	337.00	2.16
Sympy	19.88	971.36	4.13	680.00	4.09
Giac	0.27	259.92	1.44	146.00	1.21
Mupad	6.98	7822.16	9.68	187.00	1.25

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used columns from the above table.





1.4 list of integrals that has no closed form antiderivative

{}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {25, 28, 142}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 14 such integrals out of total 705, or about 2 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. The exception message will indicate of the error is due to the interactive question being asked or not.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
```

```
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy and Giac antiderivatives is determined using the following function, thanks to user `slelievre` at https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which is called directly from Python, the following code is used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative, Maple was used to determine the leaf size of Mupad output by post processing.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

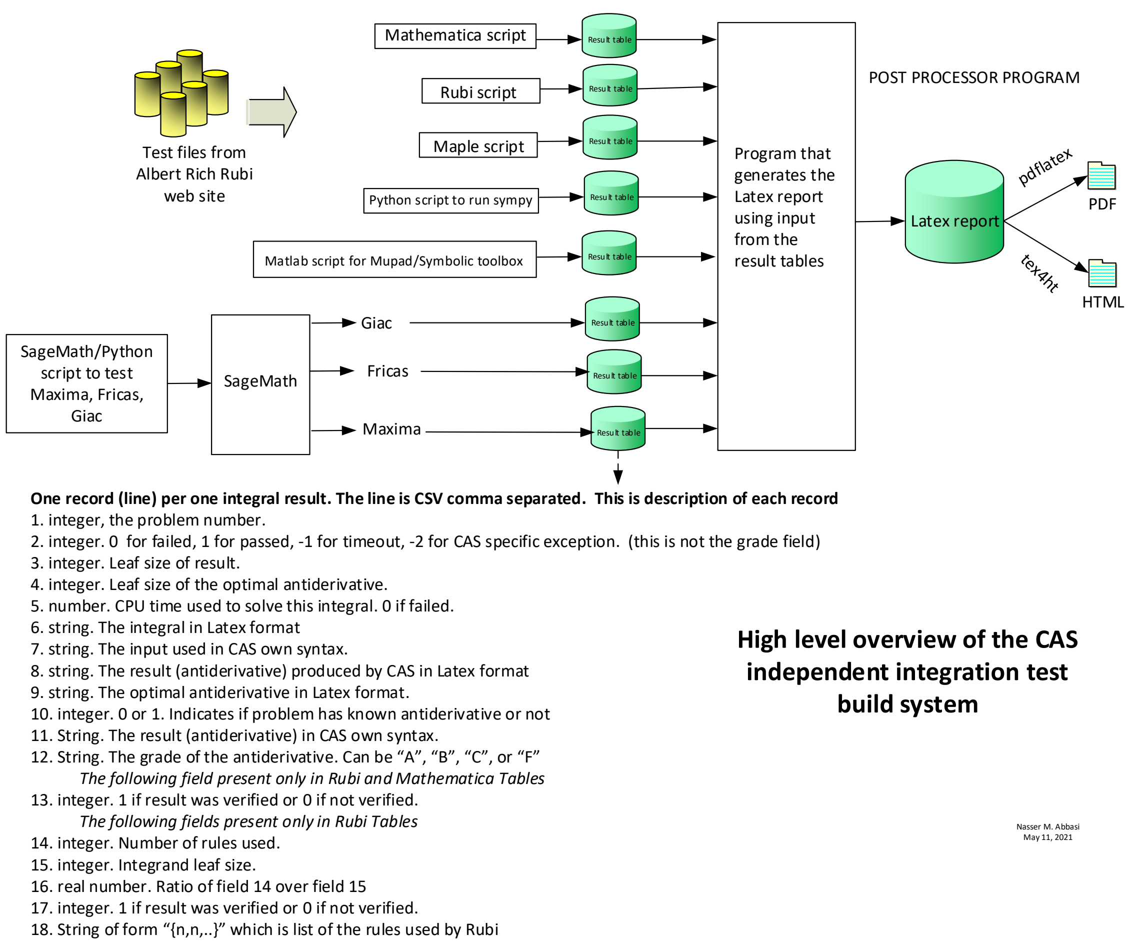
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 36, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 133, 134, 135, 136, 137, 138, 139, 140, 141 }

B grade: { 35 }

C grade: { 11, 12, 31, 32, 33, 34, 37, 38, 62, 63, 75, 76, 93, 126, 127, 128, 129, 130, 131, 132, 142 }

F grade: { 143 }

2.1.3 Maple

A grade: { 1, 2, 3, 10, 22, 28, 31, 32, 33, 34, 36, 38, 39, 92, 94, 95, 96, 99, 100, 101, 126, 127, 128, 129, 130, 131, 132, 133, 134, 138, 139, 140, 141 }

B grade: { 4, 5, 6, 7, 8, 9, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 23, 24, 25, 26, 27, 29, 30, 35, 37, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 93, 97, 98, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 135, 136, 137 }

C grade: { 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51 }

F grade: { 142, 143 }

2.1.4 Maxima

A grade: { 1, 2, 3, 10, 92, 133, 134, 138, 139 }

B grade: { 25, 26, 27, 28, 29, 30 }

C grade: { }

F grade: { 4, 5, 6, 7, 8, 9, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 135, 136, 137, 140, 141, 142, 143 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 10, 13, 14, 33, 36, 38, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 92, 126, 127, 128, 129, 130, 131, 132, 133, 134, 136, 137, 138, 139, 140, 141 }

B grade: { 7, 11, 17, 18, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 34, 37, 55, 66, 67, 81, 82, 93, 97, 98, 99, 116, 117, 135 }

C grade: { }

F grade: { 4, 5, 6, 8, 9, 12, 15, 16, 19, 20, 21, 35, 39, 52, 53, 54, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 83, 84, 85, 86, 87, 88, 89, 90, 91, 94, 95, 96, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 118, 119, 120, 121, 122, 123, 124, 125, 142, 143 }

2.1.6 Sympy

A grade: { 33, 139 }

B grade: { 1, 2, 3, 13, 14, 17, 18, 31, 138 }

C grade: { }

F grade: { 4, 5, 6, 7, 8, 9, 10, 11, 12, 15, 16, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 32, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 140, 141, 142, 143 }

2.1.7 Giac

A grade: { 1, 2, 3, 4, 10, 13, 14, 15, 17, 18, 28, 29, 30, 36, 40, 41, 42, 43, 44, 45, 46, 47, 48, 92, 126, 127, 129, 131, 133, 134, 137, 138, 139, 140, 141 }

B grade: { 5, 11, 16, 31, 32, 33, 34, 37, 38, 51, 128, 130, 132, 135, 136 }

C grade: { }

F grade: { 6, 7, 8, 9, 12, 19, 20, 21, 22, 23, 24, 25, 26, 27, 35, 39, 49, 50, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 142, 143 }

2.1.8 Mupad

A grade: { }

B grade: { 1, 2, 3, 4, 5, 13, 14, 16, 17, 18, 31, 33, 36, 133, 134, 138, 139, 140, 141 }

C grade: { }

F grade: { 6, 7, 8, 9, 10, 11, 12, 15, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 32, 34, 35, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 135, 136, 137, 142, 143 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	86	133	84	200	333	87	97
normalized size	1	1.00	0.91	1.41	0.89	2.13	3.54	0.93	1.03
time (sec)	N/A	0.113	0.072	0.031	0.968	0.860	1.854	0.151	3.436

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	228	228	204	373	220	500	933	263	253
normalized size	1	1.00	0.89	1.64	0.96	2.19	4.09	1.15	1.11
time (sec)	N/A	0.330	0.204	0.012	0.979	0.688	7.873	0.159	0.249

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	441	441	422	822	471	1014	1962	623	552
normalized size	1	1.00	0.96	1.86	1.07	2.30	4.45	1.41	1.25
time (sec)	N/A	0.621	0.459	0.008	0.989	0.710	23.623	0.164	3.792

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	274	274	212	745	0	0	0	266	3888
normalized size	1	1.00	0.77	2.72	0.00	0.00	0.00	0.97	14.19
time (sec)	N/A	0.282	0.368	0.016	0.000	0.000	0.000	0.168	38.323

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	596	596	523	9311	0	0	0	1313	23006
normalized size	1	1.00	0.88	15.62	0.00	0.00	0.00	2.20	38.60
time (sec)	N/A	1.772	1.842	0.055	0.000	0.000	0.000	0.205	7.534

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	331	331	322	3358	0	0	0	0	-1
normalized size	1	1.00	0.97	10.15	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.601	0.801	0.085	0.000	0.000	0.000	0.000	0.000

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	249	249	249	714	0	6113	0	0	-1
normalized size	1	1.00	1.00	2.87	0.00	24.55	0.00	0.00	-0.00
time (sec)	N/A	0.204	0.240	0.024	0.000	147.137	0.000	0.000	0.000

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	381	380	440	2758	0	0	0	0	-1
normalized size	1	1.00	1.15	7.24	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.798	0.650	0.026	0.000	0.000	0.000	0.000	0.000

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	797	796	674	6422	0	0	0	0	-1
normalized size	1	1.00	0.85	8.06	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.870	4.060	0.031	0.000	0.000	0.000	0.000	0.000

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	49	46	65	46	0	48	-1
normalized size	1	1.00	1.04	0.98	1.38	0.98	0.00	1.02	-0.02
time (sec)	N/A	0.034	0.008	0.018	0.973	1.120	0.000	0.246	0.000

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	78	637	0	758	0	457	-1
normalized size	1	1.00	0.67	5.44	0.00	6.48	0.00	3.91	-0.01
time (sec)	N/A	0.169	0.033	0.176	0.000	1.005	0.000	0.376	0.000

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	484	484	136	6871419	0	0	0	0	-1
normalized size	1	1.00	0.28	14197.15	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	23.581	0.083	0.515	0.000	0.000	0.000	0.000	0.000

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	182	175	510	0	583	1260	191	273
normalized size	1	0.99	0.95	2.77	0.00	3.17	6.85	1.04	1.48
time (sec)	N/A	0.346	0.211	0.008	0.000	1.387	16.651	0.169	3.852

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	542	542	535	1672	0	1837	4663	738	893
normalized size	1	1.00	0.99	3.08	0.00	3.39	8.60	1.36	1.65
time (sec)	N/A	1.102	0.604	0.012	0.000	1.957	145.638	0.246	4.854

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	406	398	267	1698	0	0	0	416	-1
normalized size	1	0.98	0.66	4.18	0.00	0.00	0.00	1.02	-0.00
time (sec)	N/A	0.478	0.462	0.013	0.000	0.000	0.000	0.169	0.000

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1075	1067	952	51470	0	0	0	3226	118429
normalized size	1	0.99	0.89	47.88	0.00	0.00	0.00	3.00	110.17
time (sec)	N/A	4.177	6.700	0.054	0.000	0.000	0.000	0.451	30.314

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	131	340	0	1150	709	219	395
normalized size	1	1.00	0.94	2.43	0.00	8.21	5.06	1.56	2.82
time (sec)	N/A	0.131	0.143	0.006	0.000	1.419	2.437	0.172	0.422

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	131	340	0	1130	680	207	375
normalized size	1	1.00	0.94	2.43	0.00	8.07	4.86	1.48	2.68
time (sec)	N/A	0.104	0.027	0.006	0.000	0.886	2.259	0.170	4.020

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	617	615	517	16209	0	0	0	0	-1
normalized size	1	1.00	0.84	26.27	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	8.998	2.086	0.051	0.000	0.000	0.000	0.000	0.000

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1092	1092	1627	59465	0	0	0	0	-1
normalized size	1	1.00	1.49	54.46	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	18.867	6.564	0.032	0.000	0.000	0.000	0.000	0.000

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	416	416	393	2269	0	0	0	0	-1
normalized size	1	1.00	0.94	5.45	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.702	4.175	0.044	0.000	0.000	0.000	0.000	0.000

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	780	780	254	784	0	6861	0	0	-1
normalized size	1	1.00	0.33	1.01	0.00	8.80	0.00	0.00	-0.00
time (sec)	N/A	5.162	0.409	0.039	0.000	134.783	0.000	0.000	0.000

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	302	302	283	1771	0	8977	0	0	-1
normalized size	1	1.00	0.94	5.86	0.00	29.73	0.00	0.00	-0.00
time (sec)	N/A	0.843	0.430	0.033	0.000	124.907	0.000	0.000	0.000

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	154	608	0	1515	0	0	-1
normalized size	1	1.00	1.52	6.02	0.00	15.00	0.00	0.00	-0.01
time (sec)	N/A	0.127	0.174	0.021	0.000	1.199	0.000	0.000	0.000

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	140	324	361	322	0	0	-1
normalized size	1	1.00	1.01	2.33	2.60	2.32	0.00	0.00	-0.01
time (sec)	N/A	0.220	0.301	0.068	1.074	0.928	0.000	0.000	0.000

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	167	760	678	344	0	0	-1
normalized size	1	1.00	1.01	4.58	4.08	2.07	0.00	0.00	-0.01
time (sec)	N/A	0.221	0.378	0.027	1.112	0.899	0.000	0.000	0.000

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	193	185	1560	1276	439	0	0	-1
normalized size	1	1.00	0.96	8.08	6.61	2.27	0.00	0.00	-0.01
time (sec)	N/A	0.266	0.596	0.020	1.263	0.990	0.000	0.000	0.000

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	148	186	363	245	0	93	-1
normalized size	1	1.00	0.98	1.23	2.40	1.62	0.00	0.62	-0.01
time (sec)	N/A	0.229	0.356	0.051	1.069	1.228	0.000	0.477	0.000

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	172	466	668	365	0	112	-1
normalized size	1	1.00	0.99	2.68	3.84	2.10	0.00	0.64	-0.01
time (sec)	N/A	0.255	0.570	0.019	1.099	0.974	0.000	0.499	0.000

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	197	190	878	1276	435	0	121	-1
normalized size	1	1.00	0.96	4.46	6.48	2.21	0.00	0.61	-0.01
time (sec)	N/A	0.303	0.710	0.017	1.241	1.018	0.000	0.541	0.000

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	79	14	0	49	36	31	13
normalized size	1	1.00	5.27	0.93	0.00	3.27	2.40	2.07	0.87
time (sec)	N/A	0.017	0.045	0.020	0.000	0.870	6.999	0.347	3.763

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	101	40	0	106	0	108	-1
normalized size	1	1.00	2.30	0.91	0.00	2.41	0.00	2.45	-0.02
time (sec)	N/A	0.052	0.055	0.012	0.000	0.992	0.000	0.374	0.000

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	90	20	0	34	68	39	19
normalized size	1	1.00	3.75	0.83	0.00	1.42	2.83	1.62	0.79
time (sec)	N/A	0.019	0.063	0.018	0.000	1.097	6.813	0.355	3.781

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	114	45	0	307	0	133	-1
normalized size	1	1.00	2.04	0.80	0.00	5.48	0.00	2.38	-0.02
time (sec)	N/A	0.050	0.061	0.012	0.000	1.098	0.000	0.319	0.000

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F(-1)	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	249	249	767	3606	0	0	0	0	-1
normalized size	1	1.00	3.08	14.48	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.910	1.489	0.053	0.000	0.000	0.000	0.000	0.000

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	46	48	0	85	0	81	49
normalized size	1	1.00	0.96	1.00	0.00	1.77	0.00	1.69	1.02
time (sec)	N/A	0.050	0.185	0.005	0.000	1.973	0.000	0.266	3.753

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	165	94	0	56	0	98	-1
normalized size	1	1.00	9.71	5.53	0.00	3.29	0.00	5.76	-0.06
time (sec)	N/A	0.022	0.288	0.017	0.000	1.344	0.000	0.231	0.000

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	150	123	0	132	0	163	-1
normalized size	1	1.00	1.74	1.43	0.00	1.53	0.00	1.90	-0.01
time (sec)	N/A	0.185	0.109	0.013	0.000	1.022	0.000	0.275	0.000

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	108	121	0	0	0	0	-1
normalized size	1	1.00	0.79	0.89	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.114	0.091	0.027	0.000	119.134	0.000	0.000	0.000

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	212	212	129	103	0	175	0	117	-1
normalized size	1	1.00	0.61	0.49	0.00	0.83	0.00	0.55	-0.00
time (sec)	N/A	0.116	0.151	0.048	0.000	0.937	0.000	0.253	0.000

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	117	83	0	157	0	98	-1
normalized size	1	1.00	0.73	0.52	0.00	0.98	0.00	0.61	-0.01
time (sec)	N/A	0.067	0.116	0.010	0.000	1.000	0.000	0.213	0.000

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	85	65	0	128	0	79	-1
normalized size	1	1.00	0.57	0.44	0.00	0.86	0.00	0.53	-0.01
time (sec)	N/A	0.057	0.063	0.008	0.000	0.785	0.000	0.271	0.000

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	139	94	0	341	0	102	-1
normalized size	1	1.00	0.87	0.59	0.00	2.13	0.00	0.64	-0.01
time (sec)	N/A	0.119	0.143	0.009	0.000	0.993	0.000	0.247	0.000

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	118	118	0	333	0	126	-1
normalized size	1	1.00	0.76	0.76	0.00	2.13	0.00	0.81	-0.01
time (sec)	N/A	0.113	0.281	0.013	0.000	1.047	0.000	0.260	0.000

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	126	141	0	377	0	199	-1
normalized size	1	1.00	0.78	0.88	0.00	2.34	0.00	1.24	-0.01
time (sec)	N/A	0.116	0.116	0.015	0.000	0.758	0.000	0.275	0.000

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	317	317	198	530	0	517	0	368	-1
normalized size	1	1.00	0.62	1.67	0.00	1.63	0.00	1.16	-0.00
time (sec)	N/A	0.333	0.275	0.018	0.000	1.033	0.000	0.311	0.000

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	227	227	147	381	0	391	0	268	-1
normalized size	1	1.00	0.65	1.68	0.00	1.72	0.00	1.18	-0.00
time (sec)	N/A	0.127	0.131	0.013	0.000	0.902	0.000	0.283	0.000

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	134	257	0	287	0	185	-1
normalized size	1	1.00	0.68	1.30	0.00	1.45	0.00	0.93	-0.01
time (sec)	N/A	0.101	0.127	0.011	0.000	0.935	0.000	0.302	0.000

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	211	149	214	0	651	0	0	-1
normalized size	1	1.00	0.71	1.01	0.00	3.09	0.00	0.00	-0.00
time (sec)	N/A	0.221	0.198	0.011	0.000	2.544	0.000	0.000	0.000

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	155	249	0	647	0	0	-1
normalized size	1	1.00	0.77	1.23	0.00	3.20	0.00	0.00	-0.00
time (sec)	N/A	0.194	0.189	0.013	0.000	1.402	0.000	0.000	0.000

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	215	215	161	358	0	693	0	450	-1
normalized size	1	1.00	0.75	1.67	0.00	3.22	0.00	2.09	-0.00
time (sec)	N/A	0.183	0.218	0.016	0.000	1.813	0.000	0.413	0.000

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	452	452	516	7739	0	0	0	0	-1
normalized size	1	1.00	1.14	17.12	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.967	2.500	0.039	0.000	0.000	0.000	0.000	0.000

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	395	395	422	5581	0	0	0	0	-1
normalized size	1	1.00	1.07	14.13	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.931	1.524	0.015	0.000	0.000	0.000	0.000	0.000

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	298	298	282	3249	0	0	0	0	-1
normalized size	1	1.00	0.95	10.90	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.376	0.363	0.013	0.000	0.000	0.000	0.000	0.000

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	358	358	314	3544	0	2266	0	0	-1
normalized size	1	1.00	0.88	9.90	0.00	6.33	0.00	0.00	-0.00
time (sec)	N/A	1.314	0.651	0.019	0.000	83.145	0.000	0.000	0.000

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	382	382	569	3703	0	0	0	0	-1
normalized size	1	1.00	1.49	9.69	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.417	3.033	0.025	0.000	0.000	0.000	0.000	0.000

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	507	507	642	3993	0	0	0	0	-1
normalized size	1	1.00	1.27	7.88	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.880	2.607	0.019	0.000	0.000	0.000	0.000	0.000

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	795	795	793	19148	0	0	0	0	-1
normalized size	1	1.00	1.00	24.09	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	4.264	3.449	0.030	0.000	0.000	0.000	0.000	0.000

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	553	553	755	14709	0	0	0	0	-1
normalized size	1	1.00	1.37	26.60	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.431	2.011	0.015	0.000	0.000	0.000	0.000	0.000

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	484	482	603	8954	0	0	0	0	-1
normalized size	1	1.00	1.25	18.50	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	4.240	1.073	0.014	0.000	0.000	0.000	0.000	0.000

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	496	496	746	9728	0	0	0	0	-1
normalized size	1	1.00	1.50	19.61	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.569	1.575	0.018	0.000	0.000	0.000	0.000	0.000

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	604	604	885	9912	0	0	0	0	-1
normalized size	1	1.00	1.47	16.41	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.809	4.159	0.020	0.000	0.000	0.000	0.000	0.000

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	668	668	904	10298	0	0	0	0	-1
normalized size	1	1.00	1.35	15.42	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	3.465	3.223	0.019	0.000	0.000	0.000	0.000	0.000

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	380	380	378	2397	0	0	0	0	-1
normalized size	1	1.00	0.99	6.31	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.170	1.333	0.029	0.000	0.000	0.000	0.000	0.000

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	344	344	334	1796	0	0	0	0	-1
normalized size	1	1.00	0.97	5.22	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.541	0.701	0.015	0.000	0.000	0.000	0.000	0.000

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	294	294	275	1172	0	5085	0	0	-1
normalized size	1	1.00	0.94	3.99	0.00	17.30	0.00	0.00	-0.00
time (sec)	N/A	0.235	0.320	0.013	0.000	3.458	0.000	0.000	0.000

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	266	266	247	589	0	5073	0	0	-1
normalized size	1	1.00	0.93	2.21	0.00	19.07	0.00	0.00	-0.00
time (sec)	N/A	0.151	0.291	0.013	0.000	3.446	0.000	0.000	0.000

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	330	330	319	681	0	0	0	0	-1
normalized size	1	1.00	0.97	2.06	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.825	0.742	0.017	0.000	0.000	0.000	0.000	0.000

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	367	367	356	736	0	0	0	0	-1
normalized size	1	1.00	0.97	2.01	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.199	0.838	0.018	0.000	0.000	0.000	0.000	0.000

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	457	457	460	911	0	0	0	0	-1
normalized size	1	1.00	1.01	1.99	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.863	1.569	0.018	0.000	0.000	0.000	0.000	0.000

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	499	499	577	6124	0	0	0	0	-1
normalized size	1	1.00	1.16	12.27	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.107	2.696	0.032	0.000	0.000	0.000	0.000	0.000

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	410	410	509	4752	0	0	0	0	-1
normalized size	1	1.00	1.24	11.59	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.709	2.073	0.019	0.000	0.000	0.000	0.000	0.000

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	411	411	457	3000	0	0	0	0	-1
normalized size	1	1.00	1.11	7.30	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.826	0.806	0.015	0.000	0.000	0.000	0.000	0.000

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	416	416	320	1713	0	0	0	0	-1
normalized size	1	1.00	0.77	4.12	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.617	2.061	0.015	0.000	0.000	0.000	0.000	0.000

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	526	526	497	1945	0	0	0	0	-1
normalized size	1	1.00	0.94	3.70	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.183	3.497	0.017	0.000	0.000	0.000	0.000	0.000

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	618	618	557	2046	0	0	0	0	-1
normalized size	1	1.00	0.90	3.31	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.280	3.604	0.019	0.000	0.000	0.000	0.000	0.000

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	392	392	327	1817	0	0	0	0	-1
normalized size	1	1.00	0.83	4.64	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.952	0.889	0.018	0.000	0.000	0.000	0.000	0.000

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	316	316	302	1810	0	0	0	0	-1
normalized size	1	1.00	0.96	5.73	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.489	0.485	0.017	0.000	0.000	0.000	0.000	0.000

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	282	282	272	1667	0	0	0	0	-1
normalized size	1	1.00	0.96	5.91	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.295	0.294	0.014	0.000	0.000	0.000	0.000	0.000

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	266	266	253	1669	0	0	0	0	-1
normalized size	1	1.00	0.95	6.27	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.226	0.166	0.014	0.000	0.000	0.000	0.000	0.000

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	267	267	255	1764	0	1253	0	0	-1
normalized size	1	1.00	0.96	6.61	0.00	4.69	0.00	0.00	-0.00
time (sec)	N/A	0.778	0.295	0.018	0.000	26.876	0.000	0.000	0.000

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	286	286	275	1819	0	1094	0	0	-1
normalized size	1	1.00	0.96	6.36	0.00	3.83	0.00	0.00	-0.00
time (sec)	N/A	0.706	0.431	0.020	0.000	38.759	0.000	0.000	0.000

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	353	353	316	1953	0	0	0	0	-1
normalized size	1	1.00	0.90	5.53	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.879	0.522	0.017	0.000	0.000	0.000	0.000	0.000

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	501	501	447	4884	0	0	0	0	-1
normalized size	1	1.00	0.89	9.75	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.412	1.493	0.032	0.000	0.000	0.000	0.000	0.000

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	417	417	395	4900	0	0	0	0	-1
normalized size	1	1.00	0.95	11.75	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.015	1.017	0.018	0.000	0.000	0.000	0.000	0.000

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	349	349	330	4567	0	0	0	0	-1
normalized size	1	1.00	0.95	13.09	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.522	0.805	0.015	0.000	0.000	0.000	0.000	0.000

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	315	315	298	4574	0	0	0	0	-1
normalized size	1	1.00	0.95	14.52	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.517	0.812	0.015	0.000	0.000	0.000	0.000	0.000

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	469	469	755	4765	0	0	0	0	-1
normalized size	1	1.00	1.61	10.16	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.274	0.505	0.018	0.000	0.000	0.000	0.000	0.000

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	463	463	765	4799	0	0	0	0	-1
normalized size	1	1.00	1.65	10.37	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.201	0.638	0.018	0.000	0.000	0.000	0.000	0.000

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	614	614	303	5056	0	0	0	0	-1
normalized size	1	1.00	0.49	8.23	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.436	0.970	0.020	0.000	0.000	0.000	0.000	0.000

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	181	1346	0	0	0	0	-1
normalized size	1	1.00	0.96	7.12	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.285	0.583	0.018	0.000	0.000	0.000	0.000	0.000

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	75	102	83	70	0	73	-1
normalized size	1	1.00	1.00	1.36	1.11	0.93	0.00	0.97	-0.01
time (sec)	N/A	0.058	0.038	0.015	0.975	1.386	0.000	0.195	0.000

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	120	789	0	777	0	0	-1
normalized size	1	1.00	0.92	6.07	0.00	5.98	0.00	0.00	-0.01
time (sec)	N/A	0.160	0.161	0.137	0.000	1.281	0.000	0.000	0.000

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	369	369	300	516	0	0	0	0	-1
normalized size	1	1.00	0.81	1.40	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.809	1.709	0.027	0.000	0.000	0.000	0.000	0.000

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	287	287	325	410	0	0	0	0	-1
normalized size	1	1.00	1.13	1.43	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.633	1.158	0.018	0.000	0.000	0.000	0.000	0.000

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	266	266	250	399	0	0	0	0	-1
normalized size	1	1.00	0.94	1.50	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.216	0.425	0.015	0.000	0.000	0.000	0.000	0.000

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	220	220	211	354	0	2753	0	0	-1
normalized size	1	1.00	0.96	1.61	0.00	12.51	0.00	0.00	-0.00
time (sec)	N/A	0.130	0.173	0.015	0.000	2.398	0.000	0.000	0.000

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	220	220	209	358	0	2641	0	0	-1
normalized size	1	1.00	0.95	1.63	0.00	12.00	0.00	0.00	-0.00
time (sec)	N/A	0.118	0.096	0.015	0.000	2.147	0.000	0.000	0.000

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	267	267	252	391	0	5995	0	0	-1
normalized size	1	1.00	0.94	1.46	0.00	22.45	0.00	0.00	-0.00
time (sec)	N/A	0.663	0.443	0.016	0.000	149.604	0.000	0.000	0.000

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	291	291	325	427	0	0	0	0	-1
normalized size	1	1.00	1.12	1.47	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.657	1.033	0.017	0.000	0.000	0.000	0.000	0.000

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	376	376	314	519	0	0	0	0	-1
normalized size	1	1.00	0.84	1.38	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.734	2.009	0.022	0.000	0.000	0.000	0.000	0.000

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	466	466	562	1648	0	0	0	0	-1
normalized size	1	1.00	1.21	3.54	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.347	1.405	0.022	0.000	0.000	0.000	0.000	0.000

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	341	341	414	1480	0	0	0	0	-1
normalized size	1	1.00	1.21	4.34	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.041	1.197	0.019	0.000	0.000	0.000	0.000	0.000

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	297	297	352	1427	0	0	0	0	-1
normalized size	1	1.00	1.19	4.80	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.454	0.402	0.019	0.000	0.000	0.000	0.000	0.000

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	299	299	356	1360	0	0	0	0	-1
normalized size	1	1.00	1.19	4.55	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.400	0.353	0.016	0.000	0.000	0.000	0.000	0.000

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	310	310	360	1376	0	0	0	0	-1
normalized size	1	1.00	1.16	4.44	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.411	0.399	0.016	0.000	0.000	0.000	0.000	0.000

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	394	394	436	1518	0	0	0	0	-1
normalized size	1	1.00	1.11	3.85	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.182	0.866	0.019	0.000	0.000	0.000	0.000	0.000

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	454	454	488	1656	0	0	0	0	-1
normalized size	1	1.00	1.07	3.65	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.193	1.193	0.019	0.000	0.000	0.000	0.000	0.000

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	761	761	552	14815	0	0	0	0	-1
normalized size	1	1.00	0.73	19.47	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	3.135	2.312	0.019	0.000	0.000	0.000	0.000	0.000

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	549	549	496	10138	0	0	0	0	-1
normalized size	1	1.00	0.90	18.47	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	7.028	1.776	0.015	0.000	0.000	0.000	0.000	0.000

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	431	431	417	6019	0	0	0	0	-1
normalized size	1	1.00	0.97	13.97	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.650	0.738	0.015	0.000	0.000	0.000	0.000	0.000

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	523	521	454	6460	0	0	0	0	-1
normalized size	1	1.00	0.87	12.35	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	3.701	1.309	0.019	0.000	0.000	0.000	0.000	0.000

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	736	736	520	6765	0	0	0	0	-1
normalized size	1	1.00	0.71	9.19	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	3.476	1.683	0.019	0.000	0.000	0.000	0.000	0.000

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	545	545	550	3131	0	0	0	0	-1
normalized size	1	1.00	1.01	5.74	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	3.722	2.201	0.032	0.000	0.000	0.000	0.000	0.000

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	463	463	468	2321	0	0	0	0	-1
normalized size	1	1.00	1.01	5.01	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	3.436	0.947	0.017	0.000	0.000	0.000	0.000	0.000

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	402	402	407	1516	0	11311	0	0	-1
normalized size	1	1.00	1.01	3.77	0.00	28.14	0.00	0.00	-0.00
time (sec)	N/A	0.963	0.974	0.014	0.000	11.374	0.000	0.000	0.000

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	374	374	376	761	0	11287	0	0	-1
normalized size	1	1.00	1.01	2.03	0.00	30.18	0.00	0.00	-0.00
time (sec)	N/A	0.314	0.789	0.014	0.000	10.296	0.000	0.000	0.000

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	451	451	450	859	0	0	0	0	-1
normalized size	1	1.00	1.00	1.90	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.632	2.414	0.019	0.000	0.000	0.000	0.000	0.000

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	543	543	533	983	0	0	0	0	-1
normalized size	1	1.00	0.98	1.81	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	4.592	1.408	0.017	0.000	0.000	0.000	0.000	0.000

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	679	679	669	1296	0	0	0	0	-1
normalized size	1	1.00	0.99	1.91	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	11.226	2.110	0.020	0.000	0.000	0.000	0.000	0.000

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	779	779	1066	14651	0	0	0	0	-1
normalized size	1	1.00	1.37	18.81	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	14.170	2.624	0.035	0.000	0.000	0.000	0.000	0.000

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	609	609	1097	11341	0	0	0	0	-1
normalized size	1	1.00	1.80	18.62	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	5.842	6.585	0.021	0.000	0.000	0.000	0.000	0.000

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	609	609	770	7163	0	0	0	0	-1
normalized size	1	1.00	1.26	11.76	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	5.643	5.684	0.017	0.000	0.000	0.000	0.000	0.000

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	666	666	700	4099	0	0	0	0	-1
normalized size	1	1.00	1.05	6.15	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.746	5.042	0.018	0.000	0.000	0.000	0.000	0.000

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	816	814	1121	4594	0	0	0	0	-1
normalized size	1	1.00	1.37	5.63	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	15.916	6.560	0.036	0.000	0.000	0.000	0.000	0.000

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	210	159	0	178	0	188	-1
normalized size	1	1.00	1.50	1.14	0.00	1.27	0.00	1.34	-0.01
time (sec)	N/A	0.501	0.527	0.019	0.000	0.986	0.000	0.203	0.000

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	192	144	0	175	0	185	-1
normalized size	1	1.00	1.67	1.25	0.00	1.52	0.00	1.61	-0.01
time (sec)	N/A	0.420	0.438	0.015	0.000	1.504	0.000	0.247	0.000

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	159	130	0	161	0	171	-1
normalized size	1	1.00	1.62	1.33	0.00	1.64	0.00	1.74	-0.01
time (sec)	N/A	0.198	0.187	0.009	0.000	0.924	0.000	0.226	0.000

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	69	174	92	0	50	0	68	-1
normalized size	1	1.01	2.56	1.35	0.00	0.74	0.00	1.00	-0.01
time (sec)	N/A	0.062	0.168	0.011	0.000	0.876	0.000	0.185	0.000

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	150	121	0	132	0	165	-1
normalized size	1	1.00	1.58	1.27	0.00	1.39	0.00	1.74	-0.01
time (sec)	N/A	0.111	0.100	0.011	0.000	0.952	0.000	0.188	0.000

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	200	152	0	170	0	199	-1
normalized size	1	1.00	1.54	1.17	0.00	1.31	0.00	1.53	-0.01
time (sec)	N/A	0.417	0.437	0.019	0.000	0.665	0.000	0.201	0.000

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	225	169	0	194	0	269	-1
normalized size	1	1.00	1.49	1.12	0.00	1.28	0.00	1.78	-0.01
time (sec)	N/A	0.454	0.405	0.020	0.000	0.731	0.000	0.483	0.000

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	87	147	155	88	0	85	187
normalized size	1	1.00	0.58	0.99	1.04	0.59	0.00	0.57	1.26
time (sec)	N/A	0.093	0.191	0.019	1.001	0.582	0.000	0.297	5.085

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	72	96	104	73	0	70	136
normalized size	1	1.00	0.70	0.93	1.01	0.71	0.00	0.68	1.32
time (sec)	N/A	0.042	0.033	0.006	0.981	0.587	0.000	0.204	4.690

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	37	163	0	53	0	63	-1
normalized size	1	1.00	1.32	5.82	0.00	1.89	0.00	2.25	-0.04
time (sec)	N/A	0.048	0.098	0.016	0.000	0.700	0.000	0.272	0.000

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	114	245	0	126	0	159	-1
normalized size	1	1.00	1.36	2.92	0.00	1.50	0.00	1.89	-0.01
time (sec)	N/A	0.078	0.238	0.023	0.000	0.721	0.000	0.272	0.000

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	131	306	0	186	0	232	-1
normalized size	1	1.00	0.94	2.20	0.00	1.34	0.00	1.67	-0.01
time (sec)	N/A	0.117	0.367	0.021	0.000	0.638	0.000	0.260	0.000

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	13	16	11	11	31	11	15
normalized size	1	1.00	0.87	1.07	0.73	0.73	2.07	0.73	1.00
time (sec)	N/A	0.004	0.007	0.004	0.434	0.608	0.311	0.153	3.723

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	13	14	9	11	10	11	13
normalized size	1	1.00	0.81	0.88	0.56	0.69	0.62	0.69	0.81
time (sec)	N/A	0.006	0.003	0.003	0.428	0.561	4.252	0.150	3.664

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	13	20	0	11	0	11	15
normalized size	1	1.00	0.87	1.33	0.00	0.73	0.00	0.73	1.00
time (sec)	N/A	0.028	0.006	0.004	0.000	0.411	0.000	0.160	3.675

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	13	18	0	11	0	11	13
normalized size	1	1.00	0.87	1.20	0.00	0.73	0.00	0.73	0.87
time (sec)	N/A	0.058	0.004	0.004	0.000	0.424	0.000	0.201	3.607

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	242	242	268	0	0	0	0	0	-1
normalized size	1	1.00	1.11	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.093	0.560	0.135	0.000	0.000	0.000	0.000	0.000

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F(-1)	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	488	488	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.360	0.535	0.449	0.000	0.000	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [63] had the largest ratio of [.5556]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	5	4	1.00	25	0.160
2	A	5	4	1.00	27	0.148
3	A	5	4	1.00	27	0.148
4	A	8	8	1.00	27	0.296
5	A	9	9	1.00	27	0.333
6	A	9	6	1.00	30	0.200
7	A	5	3	1.00	30	0.100
8	A	6	4	1.00	30	0.133
9	A	7	5	1.00	30	0.167
10	A	5	4	1.00	23	0.174
11	A	5	4	1.00	23	0.174
12	A	5	4	1.00	30	0.133
13	A	6	5	0.99	28	0.179
14	A	6	5	1.00	30	0.167
15	A	9	5	0.98	30	0.167
16	A	10	6	0.99	30	0.200
17	A	5	5	1.00	34	0.147
18	A	5	5	1.00	34	0.147
19	A	9	6	1.00	32	0.188
20	A	10	7	1.00	32	0.219
21	A	5	3	1.00	32	0.094
22	A	5	3	1.00	29	0.103
23	A	5	3	1.00	29	0.103
24	A	6	6	1.00	26	0.231
25	A	5	4	1.00	30	0.133
26	A	7	6	1.00	30	0.200
27	A	7	6	1.00	30	0.200

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
28	A	5	3	1.00	30	0.100
29	A	6	4	1.00	30	0.133
30	A	7	5	1.00	30	0.167
31	A	2	2	1.00	26	0.077
32	A	5	5	1.00	26	0.192
33	A	2	2	1.00	24	0.083
34	A	5	5	1.00	20	0.250
35	A	6	5	1.00	36	0.139
36	A	2	2	1.00	36	0.056
37	A	2	2	1.00	32	0.062
38	A	13	9	1.00	32	0.281
39	A	5	5	1.00	38	0.132
40	A	6	6	1.00	35	0.171
41	A	5	5	1.00	33	0.152
42	A	5	5	1.00	32	0.156
43	A	8	8	1.00	35	0.229
44	A	8	8	1.00	35	0.229
45	A	8	8	1.00	35	0.229
46	A	6	6	1.00	38	0.158
47	A	5	5	1.00	36	0.139
48	A	5	5	1.00	35	0.143
49	A	7	6	1.00	38	0.158
50	A	7	6	1.00	38	0.158
51	A	7	6	1.00	38	0.158
52	A	9	6	1.00	27	0.222
53	A	9	6	1.00	25	0.240
54	A	8	5	1.00	24	0.208
55	A	12	9	1.00	27	0.333
56	A	18	12	1.00	27	0.444
57	A	22	13	1.00	27	0.482
58	A	10	7	1.00	27	0.259
59	A	10	7	1.00	25	0.280
60	A	9	6	1.00	24	0.250
61	A	17	11	1.00	27	0.407
62	A	21	14	1.00	27	0.518
63	A	26	15	1.00	27	0.556

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
64	A	10	6	1.00	27	0.222
65	A	8	5	1.00	27	0.185
66	A	5	3	1.00	25	0.120
67	A	5	3	1.00	24	0.125
68	A	10	7	1.00	27	0.259
69	A	11	8	1.00	27	0.296
70	A	15	9	1.00	27	0.333
71	A	10	7	1.00	27	0.259
72	A	6	4	1.00	27	0.148
73	A	6	4	1.00	25	0.160
74	A	6	4	1.00	24	0.167
75	A	12	9	1.00	27	0.333
76	A	14	11	1.00	27	0.407
77	A	15	9	1.00	28	0.321
78	A	9	6	1.00	28	0.214
79	A	9	6	1.00	26	0.231
80	A	8	5	1.00	25	0.200
81	A	17	9	1.00	28	0.321
82	A	16	8	1.00	28	0.286
83	A	20	10	1.00	28	0.357
84	A	17	10	1.00	28	0.357
85	A	10	7	1.00	28	0.250
86	A	10	7	1.00	26	0.269
87	A	9	6	1.00	25	0.240
88	A	19	11	1.00	28	0.393
89	A	18	10	1.00	28	0.357
90	A	26	13	1.00	28	0.464
91	A	9	6	1.00	24	0.250
92	A	8	6	1.00	22	0.273
93	A	10	9	1.00	17	0.529
94	A	13	7	1.00	28	0.250
95	A	10	6	1.00	28	0.214
96	A	8	5	1.00	28	0.179
97	A	5	3	1.00	26	0.115
98	A	5	3	1.00	25	0.120
99	A	9	4	1.00	28	0.143

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
100	A	10	5	1.00	28	0.179
101	A	13	6	1.00	28	0.214
102	A	13	9	1.00	28	0.321
103	A	9	6	1.00	28	0.214
104	A	6	4	1.00	28	0.143
105	A	6	4	1.00	26	0.154
106	A	6	4	1.00	25	0.160
107	A	12	7	1.00	28	0.250
108	A	12	7	1.00	28	0.250
109	A	9	6	1.00	30	0.200
110	A	9	6	1.00	28	0.214
111	A	8	5	1.00	27	0.185
112	A	17	9	1.00	30	0.300
113	A	23	10	1.00	30	0.333
114	A	12	6	1.00	30	0.200
115	A	8	5	1.00	30	0.167
116	A	5	3	1.00	28	0.107
117	A	5	3	1.00	27	0.111
118	A	9	4	1.00	30	0.133
119	A	12	5	1.00	30	0.167
120	A	16	7	1.00	30	0.233
121	A	10	7	1.00	30	0.233
122	A	6	4	1.00	30	0.133
123	A	6	4	1.00	28	0.143
124	A	6	4	1.00	27	0.148
125	A	12	7	1.00	30	0.233
126	A	24	14	1.00	30	0.467
127	A	20	13	1.00	30	0.433
128	A	16	12	1.00	30	0.400
129	A	6	4	1.01	28	0.143
130	A	10	8	1.00	27	0.296
131	A	17	11	1.00	30	0.367
132	A	20	12	1.00	30	0.400
133	A	8	6	1.00	34	0.176
134	A	6	5	1.00	30	0.167
135	A	3	3	1.00	34	0.088

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
136	A	5	5	1.00	34	0.147
137	A	7	7	1.00	34	0.206
138	A	1	1	1.00	17	0.059
139	A	1	1	1.00	15	0.067
140	A	2	2	1.00	23	0.087
141	A	3	3	1.00	21	0.143
142	A	2	2	1.00	40	0.050
143	A	2	2	1.00	104	0.019

Chapter 3

Listing of integrals

3.1 $\int \frac{(A+Bx)\left(a+bx+cx^2\right)}{d+fx^2}\,dx$

Optimal. Leaf size=94

$$-\frac{\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{d}}\right)(-aAf+Acd+bBd)}{\sqrt{d}f^{3/2}}-\frac{\log\left(d+fx^2\right)(-aBf-Abf+Bcd)}{2f^2}+\frac{x(Ac+bB)}{f}+\frac{Bcx^2}{2f}$$

```
[Out] (A*c+B*b)*x/f+1/2*B*c*x^2/f-1/2*(-A*b*f-B*a*f+B*c*d)*ln(f*x^2+d)/f^2-(-A*a*
f+A*c*d+B*b*d)*arctan(x*f^(1/2)/d^(1/2))/f^(3/2)/d^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {1629, 635, 205, 260}

$$-\frac{\log\left(d+fx^2\right)(-aBf-Abf+Bcd)}{2f^2}-\frac{\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{d}}\right)(-aAf+Acd+bBd)}{\sqrt{d}f^{3/2}}+\frac{x(Ac+bB)}{f}+\frac{Bcx^2}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x)*(a + b*x + c*x^2))/(d + f*x^2),x]
```

```
[Out] ((b*B + A*c)*x)/f + (B*c*x^2)/(2*f) - ((b*B*d + A*c*d - a*A*f)*ArcTan[(Sqrt
[f]*x)/Sqrt[d]])/(Sqrt[d]*f^(3/2)) - ((B*c*d - A*b*f - a*B*f)*Log[d + f*x^2
])/(2*f^2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)}{d+fx^2}\,dx = \int \left(\frac{bB+Ac}{f}+\frac{Bcx}{f}-\frac{bBd+Acd-aAf+(Bcd-Abf-aBf)x}{f\left(d+fx^2\right)}\right)dx$$

$$= \frac{(bB+Ac)x}{f}+\frac{Bcx^2}{2f}-\frac{\int \frac{bBd+Acd-aAf+(Bcd-Abf-aBf)x}{d+fx^2}\,dx}{f}$$

$$= \frac{(bB+Ac)x}{f}+\frac{Bcx^2}{2f}-\frac{(bBd+Acd-aAf)\int \frac{1}{d+fx^2}\,dx}{f}-\frac{(Bcd-Abf-aBf)\int \frac{}{d}}{f}$$

$$= \frac{(bB+Ac)x}{f}+\frac{Bcx^2}{2f}-\frac{(bBd+Acd-aAf)\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{d}}\right)}{\sqrt{d}\,f^{3/2}}-\frac{(Bcd-Abf-aBf)\,\mathrm{lo}}{2f^2}$$

Mathematica [A] time = 0.07, size = 86, normalized size = 0.91

$$\frac{\log\left(d+fx^2\right)(aBf+Abf-Bcd)-\frac{2\sqrt{f}\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{d}}\right)(-aAf+Acd+bBd)}{\sqrt{d}}+fx(2Ac+2bB+Bcx)}{2f^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x)*(a + b*x + c*x^2))/(d + f*x^2),x]
```

```
[Out] (f*x*(2*b*B + 2*A*c + B*c*x) - (2*Sqrt[f]*(b*B*d + A*c*d - a*A*f)*ArcTan[(S
qrt[f]*x)/Sqrt[d]])/Sqrt[d] + (-(B*c*d) + A*b*f + a*B*f)*Log[d + f*x^2])/(2
*f^2)
```

fricas [A] time = 0.86, size = 200, normalized size = 2.13

$$\left[\frac{Bcdfx^2+2(Bb+Ac)dfx-\left(Aaf-(Bb+Ac)d\right)\sqrt{-df}\,\log\left(\frac{fx^2-2\sqrt{-df}\,x-d}{fx^2+d}\right)-\left(Bcd^2-(Ba+Ab)df\right)\log\left(fx^2+\right.}{2df^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(c*x^2+b*x+a)/(f*x^2+d),x, algorithm="fricas")
```

```
[Out] [1/2*(B*c*d*f*x^2 + 2*(B*b + A*c)*d*f*x - (A*a*f - (B*b + A*c)*d)*sqrt(-d*f
)*log((f*x^2 - 2*sqrt(-d*f)*x - d)/(f*x^2 + d)) - (B*c*d^2 - (B*a + A*b)*d*
f)*log(f*x^2 + d))/(d*f^2), 1/2*(B*c*d*f*x^2 + 2*(B*b + A*c)*d*f*x + 2*(A*a
*f - (B*b + A*c)*d)*sqrt(d*f)*arctan(sqrt(d*f)*x/d) - (B*c*d^2 - (B*a + A*b
)*d*f)*log(f*x^2 + d))/(d*f^2)]
```

giac [A] time = 0.15, size = 87, normalized size = 0.93

$$-\frac{\left(Bbd+Acd-Aaf\right)\arctan\left(\frac{fx}{\sqrt{df}}\right)}{\sqrt{df}\,f}-\frac{\left(Bcd-Baf-Abf\right)\log\left(fx^2+d\right)}{2\,f^2}+\frac{Bcfx^2+2\,Bbfx+2\,Acfx}{2\,f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(c*x^2+b*x+a)/(f*x^2+d),x, algorithm="giac")
```

```
[Out] -(B*b*d + A*c*d - A*a*f)*arctan(f*x/sqrt(d*f))/(sqrt(d*f)*f) - 1/2*(B*c*d -
 B*a*f - A*b*f)*log(f*x^2 + d)/f^2 + 1/2*(B*c*f*x^2 + 2*B*b*f*x + 2*A*c*f*x
)/f^2
```

maple [A] time = 0.03, size = 133, normalized size = 1.41

$$\frac{Aa\arctan\left(\frac{fx}{\sqrt{df}}\right)}{\sqrt{df}}-\frac{Acd\arctan\left(\frac{fx}{\sqrt{df}}\right)}{\sqrt{df}\ f}-\frac{Bbd\arctan\left(\frac{fx}{\sqrt{df}}\right)}{\sqrt{df}\ f}+\frac{Bc\,x^2}{2f}+\frac{Ab\ln\left(f\,x^2+d\right)}{2f}+\frac{Acx}{f}+\frac{Ba\ln\left(f\,x^2+d\right)}{2f}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x+A)*(c*x^2+b*x+a)/(f*x^2+d),x)
```

```
[Out] 1/2*B*c*x^2/f+1/f*A*c*x+1/f*b*B*x+1/2/f*ln(f*x^2+d)*A*b+1/2/f*ln(f*x^2+d)*B
*a-1/2/f^2*ln(f*x^2+d)*B*c*d+1/(d*f)^(1/2)*arctan(x*f/(d*f)^(1/2))*A*a-1/f/
(d*f)^(1/2)*arctan(x*f/(d*f)^(1/2))*A*c*d-1/f/(d*f)^(1/2)*arctan(x*f/(d*f)^
(1/2))*B*b*d
```

maxima [A] time = 0.97, size = 84, normalized size = 0.89

$$\frac{\left(Aaf-(Bb+Ac)d\right)\arctan\left(\frac{fx}{\sqrt{df}}\right)}{\sqrt{df}\,f}+\frac{Bcx^2+2\,(Bb+Ac)x}{2\,f}-\frac{\left(Bcd-(Ba+Ab)f\right)\log\left(fx^2+d\right)}{2\,f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(c*x^2+b*x+a)/(f*x^2+d),x, algorithm="maxima")
```

```
[Out] (A*a*f - (B*b + A*c)*d)*arctan(f*x/sqrt(d*f))/(sqrt(d*f)*f) + 1/2*(B*c*x^2
+ 2*(B*b + A*c)*x)/f - 1/2*(B*c*d - (B*a + A*b)*f)*log(f*x^2 + d)/f^2
```

mupad [B] time = 3.44, size = 97, normalized size = 1.03

$$\frac{x\,\left(A\,c+B\,b\right)}{f}-\frac{\mathrm{atan}\left(\frac{\sqrt{f}\,x}{\sqrt{d}}\right)\,\left(A\,c\,d-A\,a\,f+B\,b\,d\right)}{\sqrt{d}\,f^{3/2}}+\frac{B\,c\,x^2}{2\,f}+\frac{\ln\left(f\,x^2+d\right)\,\left(4\,A\,b\,d\,f^3+4\,B\,a\,d\,f^3-4\,B\,c\,d^2\,f^2\right)}{8\,d\,f^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x)*(a + b*x + c*x^2))/(d + f*x^2),x)
```

```
[Out] (x*(A*c + B*b))/f - (atan((f^(1/2)*x)/d^(1/2))*(A*c*d - A*a*f + B*b*d))/(d^
(1/2)*f^(3/2)) + (B*c*x^2)/(2*f) + (log(d + f*x^2)*(4*A*b*d*f^3 + 4*B*a*d*f
^3 - 4*B*c*d^2*f^2))/(8*d*f^4)
```

sympy [B] time = 1.85, size = 333, normalized size = 3.54

$$\frac{Bcx^2}{2f}+x\left(\frac{Ac}{f}+\frac{Bb}{f}\right)+\left(\frac{Abf+Baf-Bcd}{2f^2}-\frac{\sqrt{-df^5}\,\left(Aaf-Acd-Bbd\right)}{2df^4}\right)\log\left(x+\frac{-Abdf-Badf+Bcd^2+2\ldots}{Aaf\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(c*x**2+b*x+a)/(f*x**2+d),x)
```

```
[Out] B*c*x**2/(2*f) + x*(A*c/f + B*b/f) + ((A*b*f + B*a*f - B*c*d)/(2*f**2) - sq
rt(-d*f**5)*(A*a*f - A*c*d - B*b*d)/(2*d*f**4))*log(x + (-A*b*d*f - B*a*d*f
 + B*c*d**2 + 2*d*f**2*((A*b*f + B*a*f - B*c*d)/(2*f**2) - sqrt(-d*f**5)*(A
*a*f - A*c*d - B*b*d)/(2*d*f**4)))/(A*a*f**2 - A*c*d*f - B*b*d*f)) + ((A*b*
```

```
f + B*a*f - B*c*d)/(2*f**2) + sqrt(-d*f**5)*(A*a*f - A*c*d - B*b*d)/(2*d*f*
*4))*log(x + (-A*b*d*f - B*a*d*f + B*c*d**2 + 2*d*f**2*((A*b*f + B*a*f - B*
c*d)/(2*f**2) + sqrt(-d*f**5)*(A*a*f - A*c*d - B*b*d)/(2*d*f**4)))/(A*a*f**
2 - A*c*d*f - B*b*d*f))
```

3.2 $\int \frac{(A+Bx)\left(a+bx+cx^2\right)^2}{d+fx^2}\,dx$

Optimal. Leaf size=228

$$-\frac{\log\left(d+fx^2\right)\left(2Abf(cd-af)-B\left(-f\left(b^2d-a^2f\right)-2acdf+c^2d^2\right)\right)}{2f^3}+\frac{x^2\left(2Abcf-B\left(-2acf+b^2(-f)+c^2d\right.\right.}{2f^2}$$

```
[Out] (A*b^2*f-A*c*(-2*a*f+c*d)-b*B*(-2*a*f+2*c*d))*x/f^2+1/2*(2*A*b*c*f-B*(-2*a*
c*f-b^2*f+c^2*d))*x^2/f^2+1/3*c*(A*c+2*B*b)*x^3/f+1/4*B*c^2*x^4/f-1/2*(2*A*
b*f*(-a*f+c*d)-B*(c^2*d^2-2*a*c*d*f-f*(-a^2*f+b^2*d)))*ln(f*x^2+d)/f^3-(A*b
^2*d*f-2*b*B*d*(-a*f+c*d)-A*(-a*f+c*d)^2)*arctan(x*f^(1/2)/d^(1/2))/f^(5/2)
/d^(1/2)
```

Rubi [A] time = 0.33, antiderivative size = 228, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {1012, 635, 205, 260}

$$-\frac{\log\left(d+fx^2\right)\left(2Abf(cd-af)-B\left(-f\left(b^2d-a^2f\right)-2acdf+c^2d^2\right)\right)}{2f^3}+\frac{x^2\left(2Abcf-B\left(-2acf+b^2(-f)+c^2d\right.\right.}{2f^2}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x)*(a + b*x + c*x^2)^2)/(d + f*x^2),x]
```

```
[Out] ((A*b^2*f - A*c*(c*d - 2*a*f) - b*B*(2*c*d - 2*a*f))*x)/f^2 + ((2*A*b*c*f -
 B*(c^2*d - b^2*f - 2*a*c*f))*x^2)/(2*f^2) + (c*(2*b*B + A*c)*x^3)/(3*f) +
(B*c^2*x^4)/(4*f) - ((A*b^2*d*f - 2*b*B*d*(c*d - a*f) - A*(c*d - a*f)^2)*Ar
cTan[(Sqrt[f]*x)/Sqrt[d]])/(Sqrt[d]*f^(5/2)) - ((2*A*b*f*(c*d - a*f) - B*(c
^2*d^2 - 2*a*c*d*f - f*(b^2*d - a^2*f)))*Log[d + f*x^2])/(2*f^3)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 1012

```
Int[((g_.) + (h_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f
_.)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(a + c*x^2)^p*(d + e*x +
 f*x^2)^q*(g + h*x), x], x] /; FreeQ[{a, c, d, e, f, g, h}, x] && NeQ[e^2 -
 4*d*f, 0] && IntegersQ[p, q] && (GtQ[p, 0] || GtQ[q, 0])
```

Rubi steps

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^2}{d+fx^2}dx = \int \left(\frac{Ab^2f-Ac(cd-2af)-bB(2cd-2af)}{f^2} + \frac{\left(2Abcf-B\left(c^2d-b^2f-2acf\right)\right)}{f^2}\right.$$

$$= \frac{\left(Ab^2f-Ac(cd-2af)-bB(2cd-2af)\right)x}{f^2} + \frac{\left(2Abcf-B\left(c^2d-b^2f-2acf\right)\right)}{2f^2}$$

$$= \frac{\left(Ab^2f-Ac(cd-2af)-bB(2cd-2af)\right)x}{f^2} + \frac{\left(2Abcf-B\left(c^2d-b^2f-2acf\right)\right)}{2f^2}$$

$$= \frac{\left(Ab^2f-Ac(cd-2af)-bB(2cd-2af)\right)x}{f^2} + \frac{\left(2Abcf-B\left(c^2d-b^2f-2acf\right)\right)}{2f^2}$$

Mathematica [A] time = 0.20, size = 204, normalized size = 0.89

$$\frac{6\log\left(d+fx^2\right)\left(B\left(a^2f^2-2acdf+b^2(-d)f+c^2d^2\right)+2Abf(af-cd)\right)+fx\left(4Ac\left(6af-3cd+cfx^2\right)+4bB\left(6a\right.\right.}{12f^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x)*(a + b*x + c*x^2)^2)/(d + f*x^2),x]
```

```
[Out] ((-(A*b^2*d*f) + 2*b*B*d*(c*d - a*f) + A*(c*d - a*f)^2)*ArcTan[(Sqrt[f]*x)/
Sqrt[d]])/(Sqrt[d]*f^(5/2)) + (f*x*(12*A*b*c*f*x + 6*b^2*f*(2*A + B*x) + 3*
B*c*x*(-2*c*d + 4*a*f + c*f*x^2) + 4*A*c*(-3*c*d + 6*a*f + c*f*x^2) + 4*b*B
*(-6*c*d + 6*a*f + 2*c*f*x^2)) + 6*(2*A*b*f*(-(c*d) + a*f) + B*(c^2*d^2 - b
^2*d*f - 2*a*c*d*f + a^2*f^2))*Log[d + f*x^2])/(12*f^3)
```

fricas [A] time = 0.69, size = 500, normalized size = 2.19

$$\left[\frac{3\,Bc^2df^2x^4+4\left(2\,Bbc+Ac^2\right)df^2x^3-6\left(Bc^2d^2f-\left(Bb^2+2\,(Ba+Ab)c\right)df^2\right)x^2-6\left(Aa^2f^2+\left(2\,Bbc+Ac^2\right)d^2-\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(c*x^2+b*x+a)^2/(f*x^2+d),x, algorithm="fricas")
```

```
[Out] [1/12*(3*B*c^2*d*f^2*x^4 + 4*(2*B*b*c + A*c^2)*d*f^2*x^3 - 6*(B*c^2*d^2*f -
 (B*b^2 + 2*(B*a + A*b)*c)*d*f^2)*x^2 - 6*(A*a^2*f^2 + (2*B*b*c + A*c^2)*d^
2 - (2*B*a*b + A*b^2 + 2*A*a*c)*d*f)*sqrt(-d*f)*log((f*x^2 - 2*sqrt(-d*f)*x
 - d)/(f*x^2 + d)) - 12*((2*B*b*c + A*c^2)*d^2*f - (2*B*a*b + A*b^2 + 2*A*a
*c)*d*f^2)*x + 6*(B*c^2*d^3 - (B*b^2 + 2*(B*a + A*b)*c)*d^2*f + (B*a^2 + 2*
A*a*b)*d*f^2)*log(f*x^2 + d))/(d*f^3), 1/12*(3*B*c^2*d*f^2*x^4 + 4*(2*B*b*c
 + A*c^2)*d*f^2*x^3 - 6*(B*c^2*d^2*f - (B*b^2 + 2*(B*a + A*b)*c)*d*f^2)*x^2
 + 12*(A*a^2*f^2 + (2*B*b*c + A*c^2)*d^2 - (2*B*a*b + A*b^2 + 2*A*a*c)*d*f)
*sqrt(d*f)*arctan(sqrt(d*f)*x/d) - 12*((2*B*b*c + A*c^2)*d^2*f - (2*B*a*b +
 A*b^2 + 2*A*a*c)*d*f^2)*x + 6*(B*c^2*d^3 - (B*b^2 + 2*(B*a + A*b)*c)*d^2*f
 + (B*a^2 + 2*A*a*b)*d*f^2)*log(f*x^2 + d))/(d*f^3)]
```

giac [A] time = 0.16, size = 263, normalized size = 1.15

$$\frac{\left(2\,Bbcd^2+Ac^2d^2-2\,Babdf-Ab^2df-2\,Aacdf+Aa^2f^2\right)\arctan\left(\frac{fx}{\sqrt{df}}\right)}{\sqrt{df}\,f^2}+\frac{\left(Bc^2d^2-Bb^2df-2\,Bacdf-2\,Abcdf\right.}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(c*x^2+b*x+a)^2/(f*x^2+d),x, algorithm="giac")
```

```
[Out] (2*B*b*c*d^2 + A*c^2*d^2 - 2*B*a*b*d*f - A*b^2*d*f - 2*A*a*c*d*f + A*a^2*f^
2)*arctan(f*x/sqrt(d*f))/(sqrt(d*f)*f^2) + 1/2*(B*c^2*d^2 - B*b^2*d*f - 2*B
*a*c*d*f - 2*A*b*c*d*f + B*a^2*f^2 + 2*A*a*b*f^2)*log(f*x^2 + d)/f^3 + 1/12
*(3*B*c^2*f^3*x^4 + 8*B*b*c*f^3*x^3 + 4*A*c^2*f^3*x^3 - 6*B*c^2*d*f^2*x^2 +
 6*B*b^2*f^3*x^2 + 12*B*a*c*f^3*x^2 + 12*A*b*c*f^3*x^2 - 24*B*b*c*d*f^2*x -
 12*A*c^2*d*f^2*x + 24*B*a*b*f^3*x + 12*A*b^2*f^3*x + 24*A*a*c*f^3*x)/f^4
```

maple [A] time = 0.01, size = 373, normalized size = 1.64

$$\frac{B c^2 x^4}{4f}+\frac{A c^2 x^3}{3f}+\frac{2Bbc x^3}{3f}+\frac{A a^2 \arctan\left(\frac{fx}{\sqrt{df}}\right)}{\sqrt{df}}-\frac{2Aacd \arctan\left(\frac{fx}{\sqrt{df}}\right)}{\sqrt{df}\, f}-\frac{A b^2 d \arctan\left(\frac{fx}{\sqrt{df}}\right)}{\sqrt{df}\, f}+\frac{Abc x^2}{f}+\frac{A c^2 d^2 \text{ar}}{\sqrt{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x+A)*(c*x^2+b*x+a)^2/(f*x^2+d),x)
```

```
[Out] 1/4*B*c^2*x^4/f+1/3/f*A*x^3*c^2+2/3/f*B*x^3*b*c+1/f*A*x^2*b*c+1/f*B*x^2*a*c
+1/2/f*B*x^2*b^2-1/2/f^2*B*x^2*c^2*d+2/f*A*a*c*x+1/f*A*b^2*x-1/f^2*A*c^2*d*
x+2/f*B*a*b*x-2/f^2*B*b*c*d*x+1/f*ln(f*x^2+d)*A*a*b-1/f^2*ln(f*x^2+d)*A*b*c
*d+1/2/f*ln(f*x^2+d)*B*a^2-1/f^2*ln(f*x^2+d)*B*a*c*d-1/2/f^2*ln(f*x^2+d)*B*
b^2*d+1/2/f^3*ln(f*x^2+d)*B*c^2*d^2+1/(d*f)^(1/2)*arctan(1/(d*f)^(1/2)*f*x)
*A*a^2-2/f/(d*f)^(1/2)*arctan(1/(d*f)^(1/2)*f*x)*A*a*c*d-1/f/(d*f)^(1/2)*ar
ctan(1/(d*f)^(1/2)*f*x)*A*b^2*d+1/f^2/(d*f)^(1/2)*arctan(1/(d*f)^(1/2)*f*x)
*A*c^2*d^2-2/f/(d*f)^(1/2)*arctan(1/(d*f)^(1/2)*f*x)*B*a*b*d+2/f^2/(d*f)^(1
/2)*arctan(1/(d*f)^(1/2)*f*x)*B*b*c*d^2
```

maxima [A] time = 0.98, size = 220, normalized size = 0.96

$$\frac{\left(Aa^2f^2+\left(2\,Bbc+Ac^2\right)d^2-\left(2\,Bab+Ab^2+2\,Aac\right)df\right)\arctan\left(\frac{fx}{\sqrt{df}}\right)}{\sqrt{df}\,f^2}+\frac{3\,Bc^2fx^4+4\left(2\,Bbc+Ac^2\right)fx^3-6\,(B}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(c*x^2+b*x+a)^2/(f*x^2+d),x, algorithm="maxima")
```

```
[Out] (A*a^2*f^2 + (2*B*b*c + A*c^2)*d^2 - (2*B*a*b + A*b^2 + 2*A*a*c)*d*f)*arcta
n(f*x/sqrt(d*f))/(sqrt(d*f)*f^2) + 1/12*(3*B*c^2*f*x^4 + 4*(2*B*b*c + A*c^2
)*f*x^3 - 6*(B*c^2*d - (B*b^2 + 2*(B*a + A*b)*c)*f)*x^2 - 12*((2*B*b*c + A*
c^2)*d - (2*B*a*b + A*b^2 + 2*A*a*c)*f)*x)/f^2 + 1/2*(B*c^2*d^2 - (B*b^2 +
2*(B*a + A*b)*c)*d*f + (B*a^2 + 2*A*a*b)*f^2)*log(f*x^2 + d)/f^3
```

mupad [B] time = 0.25, size = 253, normalized size = 1.11

$$x\left(\frac{A\,b^2+2\,B\,a\,b+2\,A\,a\,c}{f}-\frac{d\,\left(A\,c^2+2\,B\,b\,c\right)}{f^2}\right)+x^2\,\left(\frac{B\,b^2+2\,A\,c\,b+2\,B\,a\,c}{2\,f}-\frac{B\,c^2\,d}{2\,f^2}\right)+\frac{x^3\,\left(A\,c^2+2\,B\,b\,c\right)}{3\,f}+\frac{}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x)*(a + b*x + c*x^2)^2)/(d + f*x^2),x)
```

```
[Out] x*((A*b^2 + 2*A*a*c + 2*B*a*b)/f - (d*(A*c^2 + 2*B*b*c))/f^2) + x^2*((B*b^2
 + 2*A*b*c + 2*B*a*c)/(2*f) - (B*c^2*d)/(2*f^2)) + (x^3*(A*c^2 + 2*B*b*c))/
(3*f) + (B*c^2*x^4)/(4*f) + (atan((f^(1/2)*x)/d^(1/2))*(A*a^2*f^2 + A*c^2*d
^2 + 2*B*b*c*d^2 - A*b^2*d*f - 2*A*a*c*d*f - 2*B*a*b*d*f))/(d^(1/2)*f^(5/2)
) + (log(d + f*x^2)*(4*B*a^2*d*f^5 - 4*B*b^2*d^2*f^4 + 4*B*c^2*d^3*f^3 + 8*
A*a*b*d*f^5 - 8*A*b*c*d^2*f^4 - 8*B*a*c*d^2*f^4))/(8*d*f^6)
```

sympy [B] time = 7.87, size = 933, normalized size = 4.09

$$\frac{Bc^2x^4}{4f}+x^3\left(\frac{Ac^2}{3f}+\frac{2Bbc}{3f}\right)+x^2\left(\frac{Abc}{f}+\frac{Bac}{f}+\frac{Bb^2}{2f}-\frac{Bc^2d}{2f^2}\right)+x\left(\frac{2Aac}{f}+\frac{Ab^2}{f}-\frac{Ac^2d}{f^2}+\frac{2Bab}{f}-\frac{2Bbcd}{f^2}\right)+\left(\frac{2Aabf}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(c*x**2+b*x+a)**2/(f*x**2+d),x)
```

```
[Out] B*c**2*x**4/(4*f) + x**3*(A*c**2/(3*f) + 2*B*b*c/(3*f)) + x**2*(A*b*c/f + B
*a*c/f + B*b**2/(2*f) - B*c**2*d/(2*f**2)) + x*(2*A*a*c/f + A*b**2/f - A*c*
*2*d/f**2 + 2*B*a*b/f - 2*B*b*c*d/f**2) + ((2*A*a*b*f**2 - 2*A*b*c*d*f + B*
a**2*f**2 - 2*B*a*c*d*f - B*b**2*d*f + B*c**2*d**2)/(2*f**3) - sqrt(-d*f**7
)*(A*a**2*f**2 - 2*A*a*c*d*f - A*b**2*d*f + A*c**2*d**2 - 2*B*a*b*d*f + 2*B
*b*c*d**2)/(2*d*f**6))*log(x + (-2*A*a*b*d*f**2 + 2*A*b*c*d**2*f - B*a**2*d
*f**2 + 2*B*a*c*d**2*f + B*b**2*d**2*f - B*c**2*d**3 + 2*d*f**3*((2*A*a*b*f
**2 - 2*A*b*c*d*f + B*a**2*f**2 - 2*B*a*c*d*f - B*b**2*d*f + B*c**2*d**2)/(
2*f**3) - sqrt(-d*f**7)*(A*a**2*f**2 - 2*A*a*c*d*f - A*b**2*d*f + A*c**2*d*
*2 - 2*B*a*b*d*f + 2*B*b*c*d**2)/(2*d*f**6)))/(A*a**2*f**3 - 2*A*a*c*d*f**2
 - A*b**2*d*f**2 + A*c**2*d**2*f - 2*B*a*b*d*f**2 + 2*B*b*c*d**2*f)) + ((2*
A*a*b*f**2 - 2*A*b*c*d*f + B*a**2*f**2 - 2*B*a*c*d*f - B*b**2*d*f + B*c**2*
d**2)/(2*f**3) + sqrt(-d*f**7)*(A*a**2*f**2 - 2*A*a*c*d*f - A*b**2*d*f + A*
c**2*d**2 - 2*B*a*b*d*f + 2*B*b*c*d**2)/(2*d*f**6))*log(x + (-2*A*a*b*d*f**
2 + 2*A*b*c*d**2*f - B*a**2*d*f**2 + 2*B*a*c*d**2*f + B*b**2*d**2*f - B*c**
2*d**3 + 2*d*f**3*((2*A*a*b*f**2 - 2*A*b*c*d*f + B*a**2*f**2 - 2*B*a*c*d*f
- B*b**2*d*f + B*c**2*d**2)/(2*f**3) + sqrt(-d*f**7)*(A*a**2*f**2 - 2*A*a*c
*d*f - A*b**2*d*f + A*c**2*d**2 - 2*B*a*b*d*f + 2*B*b*c*d**2)/(2*d*f**6)))/
(A*a**2*f**3 - 2*A*a*c*d*f**2 - A*b**2*d*f**2 + A*c**2*d**2*f - 2*B*a*b*d*f
**2 + 2*B*b*c*d**2*f))
```

3.3 $\int \frac{(A+Bx)\left(a+bx+cx^2\right)^3}{d+fx^2}\,dx$

Optimal. Leaf size=441

$$\frac{\log\left(d+fx^2\right)\left(Abf\left(-f\left(b^2d-3a^2f\right)-6acdf+3c^2d^2\right)-B(cd-af)\left(-f\left(3b^2d-a^2f\right)-2acdf+c^2d^2\right)\right)}{2f^4}-\frac{x^2\left(\right.}{}$$

```
[Out] -(b^3*B*d*f+3*A*b^2*f*(-a*f+c*d)-3*b*B*(-a*f+c*d)^2-A*c*(3*a^2*f^2-3*a*c*d*
f+c^2*d^2))*x/f^3-1/2*(A*b*f*(-6*a*c*f-b^2*f+3*c^2*d)-B*(c^3*d^2-3*a*c^2*d*
f+3*a*b^2*f^2-3*c*f*(-a^2*f+b^2*d)))*x^2/f^3+1/3*(b^3*B*f+3*A*b^2*c*f-A*c^2
*(-3*a*f+c*d)-3*b*B*c*(-2*a*f+c*d))*x^3/f^2+1/4*c*(3*A*b*c*f-B*(-3*a*c*f-3*
b^2*f+c^2*d))*x^4/f^2+1/5*c^2*(A*c+3*B*b)*x^5/f+1/6*B*c^3*x^6/f+1/2*(A*b*f*
(3*c^2*d^2-6*a*c*d*f-f*(-3*a^2*f+b^2*d))-B*(-a*f+c*d)*(c^2*d^2-2*a*c*d*f-f*
(-a^2*f+3*b^2*d)))*ln(f*x^2+d)/f^4+(b^3*B*d^2*f+3*A*b^2*d*f*(-a*f+c*d)-3*b*
B*d*(-a*f+c*d)^2-A*(-a*f+c*d)^3)*arctan(x*f^(1/2)/d^(1/2))/f^(7/2)/d^(1/2)
```

Rubi [A] time = 0.62, antiderivative size = 441, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {1012, 635, 205, 260}

$$-\frac{x^2\left(Abf\left(-6acf+b^2(-f)+3c^2d\right)-B\left(-3cf\left(b^2d-a^2f\right)+3ab^2f^2-3ac^2df+c^3d^2\right)\right)}{2f^3}+\log\left(d+fx^2\right)\left(Abf\left(\right.\right.$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x)*(a + b*x + c*x^2)^3)/(d + f*x^2),x]
```

```
[Out] -(((b^3*B*d*f + 3*A*b^2*f*(c*d - a*f) - 3*b*B*(c*d - a*f)^2 - A*c*(c^2*d^2
- 3*a*c*d*f + 3*a^2*f^2))*x)/f^3) - ((A*b*f*(3*c^2*d - b^2*f - 6*a*c*f) - B
*(c^3*d^2 - 3*a*c^2*d*f + 3*a*b^2*f^2 - 3*c*f*(b^2*d - a^2*f)))*x^2)/(2*f^3
) + ((b^3*B*f + 3*A*b^2*c*f - A*c^2*(c*d - 3*a*f) - 3*b*B*c*(c*d - 2*a*f))*
x^3)/(3*f^2) + (c*(3*A*b*c*f - B*(c^2*d - 3*b^2*f - 3*a*c*f))*x^4)/(4*f^2)
+ (c^2*(3*b*B + A*c)*x^5)/(5*f) + (B*c^3*x^6)/(6*f) + ((b^3*B*d^2*f + 3*A*b
^2*d*f*(c*d - a*f) - 3*b*B*d*(c*d - a*f)^2 - A*(c*d - a*f)^3)*ArcTan[(Sqrt[
f]*x)/Sqrt[d]])/(Sqrt[d]*f^(7/2)) + ((A*b*f*(3*c^2*d^2 - 6*a*c*d*f - f*(b^2
*d - 3*a^2*f)) - B*(c*d - a*f)*(c^2*d^2 - 2*a*c*d*f - f*(3*b^2*d - a^2*f)))
*Log[d + f*x^2])/(2*f^4)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 1012

```
Int[((g_.) + (h_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f
_.)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(a + c*x^2)^p*(d + e*x +
```

```
f*x^2)^q*(g + h*x), x], x] /; FreeQ[{a, c, d, e, f, g, h}, x] && NeQ[e^2 -
4*d*f, 0] && IntegersQ[p, q] && (GtQ[p, 0] || GtQ[q, 0])
```

Rubi steps

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^3}{d+fx^2}\,dx = \int \left(-\frac{b^3Bdf+3Ab^2f(cd-af)-3bB(cd-af)^2-Ac\left(c^2d^2-3acdf+3a^2f^2\right)}{f^3}\right.$$

$$= -\frac{\left(b^3Bdf+3Ab^2f(cd-af)-3bB(cd-af)^2-Ac\left(c^2d^2-3acdf+3a^2f^2\right)\right)x}{f^3}$$

$$= -\frac{\left(b^3Bdf+3Ab^2f(cd-af)-3bB(cd-af)^2-Ac\left(c^2d^2-3acdf+3a^2f^2\right)\right)x}{f^3}$$

$$= -\frac{\left(b^3Bdf+3Ab^2f(cd-af)-3bB(cd-af)^2-Ac\left(c^2d^2-3acdf+3a^2f^2\right)\right)x}{f^3}$$

Mathematica [A] time = 0.46, size = 422, normalized size = 0.96

$$\frac{fx\left(3b\left(4B\left(15a^2f^2+10acf\left(fx^2-3d\right)+c^2\left(15d^2-5dfx^2+3f^2x^4\right)\right)+15Acfx\left(4af-2cd+cfx^2\right)\right)+c\left(4A\left(45\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x)*(a + b*x + c*x^2)^3)/(d + f*x^2),x]
```

```
[Out] ((b^3*B*d^2*f + 3*A*b^2*d*f*(c*d - a*f) - 3*b*B*d*(c*d - a*f)^2 - A*(c*d -
a*f)^3)*ArcTan[(Sqrt[f]*x)/Sqrt[d]])/(Sqrt[d]*f^(7/2)) + (f*x*(10*b^3*f*(-6
*B*d + 3*A*f*x + 2*B*f*x^2) + 15*b^2*f*(3*B*x*(-2*c*d + 2*a*f + c*f*x^2) +
4*A*(-3*c*d + 3*a*f + c*f*x^2)) + 3*b*(15*A*c*f*x*(-2*c*d + 4*a*f + c*f*x^2
) + 4*B*(15*a^2*f^2 + 10*a*c*f*(-3*d + f*x^2) + c^2*(15*d^2 - 5*d*f*x^2 + 3
*f^2*x^4))) + c*(5*B*x*(18*a^2*f^2 + 9*a*c*f*(-2*d + f*x^2) + c^2*(6*d^2 -
3*d*f*x^2 + 2*f^2*x^4)) + 4*A*(45*a^2*f^2 + 15*a*c*f*(-3*d + f*x^2) + c^2*(
15*d^2 - 5*d*f*x^2 + 3*f^2*x^4)))) - 30*(A*b*f*(-3*c^2*d^2 + b^2*d*f + 6*a*
c*d*f - 3*a^2*f^2) + B*(c*d - a*f)*(c^2*d^2 - 3*b^2*d*f - 2*a*c*d*f + a^2*f
^2))*Log[d + f*x^2])/(60*f^4)
```

fricas [A] time = 0.71, size = 1014, normalized size = 2.30

$$\left[\frac{10\,Bc^3df^3x^6+12\left(3\,Bbc^2+Ac^3\right)df^3x^5-15\left(Bc^3d^2f^2-3\left(Bb^2c+(Ba+Ab)c^2\right)df^3\right)x^4-20\left(\left(3\,Bbc^2+Ac^3\right)d^2f\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(c*x^2+b*x+a)^3/(f*x^2+d),x, algorithm="fricas")
```

```
[Out] [1/60*(10*B*c^3*d*f^3*x^6 + 12*(3*B*b*c^2 + A*c^3)*d*f^3*x^5 - 15*(B*c^3*d^
2*f^2 - 3*(B*b^2*c + (B*a + A*b)*c^2)*d*f^3)*x^4 - 20*((3*B*b*c^2 + A*c^3)*
d^2*f^2 - (B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b^2)*c)*d*f^3)*x^3 + 30*(B*c^
3*d^3*f - 3*(B*b^2*c + (B*a + A*b)*c^2)*d^2*f^2 + (3*B*a*b^2 + A*b^3 + 3*(B
*a^2 + 2*A*a*b)*c)*d*f^3)*x^2 - 30*(A*a^3*f^3 - (3*B*b*c^2 + A*c^3)*d^3 + (
B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b^2)*c)*d^2*f - 3*(B*a^2*b + A*a*b^2 + A
*a^2*c)*d*f^2)*sqrt(-d*f)*log((f*x^2 - 2*sqrt(-d*f)*x - d)/(f*x^2 + d)) + 6
0*((3*B*b*c^2 + A*c^3)*d^3*f - (B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b^2)*c)*
```

```
d^2*f^2 + 3*(B*a^2*b + A*a*b^2 + A*a^2*c)*d*f^3)*x - 30*(B*c^3*d^4 - 3*(B*b
^2*c + (B*a + A*b)*c^2)*d^3*f + (3*B*a*b^2 + A*b^3 + 3*(B*a^2 + 2*A*a*b)*c)
*d^2*f^2 - (B*a^3 + 3*A*a^2*b)*d*f^3)*log(f*x^2 + d))/(d*f^4), 1/60*(10*B*c
^3*d*f^3*x^6 + 12*(3*B*b*c^2 + A*c^3)*d*f^3*x^5 - 15*(B*c^3*d^2*f^2 - 3*(B*
b^2*c + (B*a + A*b)*c^2)*d*f^3)*x^4 - 20*((3*B*b*c^2 + A*c^3)*d^2*f^2 - (B*
b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b^2)*c)*d*f^3)*x^3 + 30*(B*c^3*d^3*f - 3*(
B*b^2*c + (B*a + A*b)*c^2)*d^2*f^2 + (3*B*a*b^2 + A*b^3 + 3*(B*a^2 + 2*A*a*
b)*c)*d*f^3)*x^2 + 60*(A*a^3*f^3 - (3*B*b*c^2 + A*c^3)*d^3 + (B*b^3 + 3*A*a
*c^2 + 3*(2*B*a*b + A*b^2)*c)*d^2*f - 3*(B*a^2*b + A*a*b^2 + A*a^2*c)*d*f^2
)*sqrt(d*f)*arctan(sqrt(d*f)*x/d) + 60*((3*B*b*c^2 + A*c^3)*d^3*f - (B*b^3
+ 3*A*a*c^2 + 3*(2*B*a*b + A*b^2)*c)*d^2*f^2 + 3*(B*a^2*b + A*a*b^2 + A*a^2
*c)*d*f^3)*x - 30*(B*c^3*d^4 - 3*(B*b^2*c + (B*a + A*b)*c^2)*d^3*f + (3*B*a
*b^2 + A*b^3 + 3*(B*a^2 + 2*A*a*b)*c)*d^2*f^2 - (B*a^3 + 3*A*a^2*b)*d*f^3)*
log(f*x^2 + d))/(d*f^4)]
```

giac [A] time = 0.16, size = 623, normalized size = 1.41

$$-\frac{\left(3\,Bbc^2d^3 + Ac^3d^3 - Bb^3d^2f - 6\,Babcd^2f - 3\,Ab^2cd^2f - 3\,Aac^2d^2f + 3\,Ba^2bdf^2 + 3\,Aab^2df^2 + 3\,Aa^2cdf^2 \cdots\right.}{\sqrt{df}\,f^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(c*x^2+b*x+a)^3/(f*x^2+d),x, algorithm="giac")
```

```
[Out] -(3*B*b*c^2*d^3 + A*c^3*d^3 - B*b^3*d^2*f - 6*B*a*b*c*d^2*f - 3*A*b^2*c*d^2
*f - 3*A*a*c^2*d^2*f + 3*B*a^2*b*d*f^2 + 3*A*a*b^2*d*f^2 + 3*A*a^2*c*d*f^2
- A*a^3*f^3)*arctan(f*x/sqrt(d*f))/(sqrt(d*f)*f^3) - 1/2*(B*c^3*d^3 - 3*B*b
^2*c*d^2*f - 3*B*a*c^2*d^2*f - 3*A*b*c^2*d^2*f + 3*B*a*b^2*d*f^2 + A*b^3*d*
f^2 + 3*B*a^2*c*d*f^2 + 6*A*a*b*c*d*f^2 - B*a^3*f^3 - 3*A*a^2*b*f^3)*log(f*
x^2 + d)/f^4 + 1/60*(10*B*c^3*f^5*x^6 + 36*B*b*c^2*f^5*x^5 + 12*A*c^3*f^5*x
^5 - 15*B*c^3*d*f^4*x^4 + 45*B*b^2*c*f^5*x^4 + 45*B*a*c^2*f^5*x^4 + 45*A*b*
c^2*f^5*x^4 - 60*B*b*c^2*d*f^4*x^3 - 20*A*c^3*d*f^4*x^3 + 20*B*b^3*f^5*x^3
+ 120*B*a*b*c*f^5*x^3 + 60*A*b^2*c*f^5*x^3 + 60*A*a*c^2*f^5*x^3 + 30*B*c^3*
d^2*f^3*x^2 - 90*B*b^2*c*d*f^4*x^2 - 90*B*a*c^2*d*f^4*x^2 - 90*A*b*c^2*d*f^
4*x^2 + 90*B*a*b^2*f^5*x^2 + 30*A*b^3*f^5*x^2 + 90*B*a^2*c*f^5*x^2 + 180*A*
a*b*c*f^5*x^2 + 180*B*b*c^2*d^2*f^3*x + 60*A*c^3*d^2*f^3*x - 60*B*b^3*d*f^4
*x - 360*B*a*b*c*d*f^4*x - 180*A*b^2*c*d*f^4*x - 180*A*a*c^2*d*f^4*x + 180*
B*a^2*b*f^5*x + 180*A*a*b^2*f^5*x + 180*A*a^2*c*f^5*x)/f^6
```

maple [A] time = 0.01, size = 822, normalized size = 1.86

$$\frac{B\,c^3x^6}{6f}+\frac{A\,c^3x^5}{5f}+\frac{3Bb\,c^2x^5}{5f}+\frac{3Ab\,c^2x^4}{4f}+\frac{3Ba\,c^2x^4}{4f}+\frac{3B\,b^2c\,x^4}{4f}-\frac{B\,c^3d\,x^4}{4f^2}+\frac{Aa\,c^2x^3}{f}+\frac{A\,b^2c\,x^3}{f}-\frac{A\,c^3d\,x^3}{3f^2}+\frac{2Babc\,x^3}{f}\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x+A)*(c*x^2+b*x+a)^3/(f*x^2+d),x)
```

```
[Out] 6/f^2/(d*f)^(1/2)*arctan(1/(d*f)^(1/2)*f*x)*B*a*b*c*d^2-1/f^2*b^3*B*d*x+3/f
*A*a*b^2*x+1/6*B*c^3*x^6/f-3/2/f^2*ln(f*x^2+d)*B*a^2*c*d-3/2/f^2*ln(f*x^2+d
)*B*a*b^2*d+3/2/f^3*ln(f*x^2+d)*B*a*c^2*d^2+3/2/f^3*ln(f*x^2+d)*B*b^2*c*d^2
-1/f^3/(d*f)^(1/2)*arctan(1/(d*f)^(1/2)*f*x)*A*c^3*d^3+1/f^2/(d*f)^(1/2)*ar
ctan(1/(d*f)^(1/2)*f*x)*b^3*B*d^2+3/f^3*B*b*c^2*d^2*x+2/f*B*x^3*a*b*c-1/f^2
*B*x^3*b*c^2*d+3/f*A*x^2*a*b*c-3/2/f^2*A*x^2*b*c^2*d-3/2/f^2*B*x^2*a*c^2*d-
3/2/f^2*B*x^2*b^2*c*d-3/f^2*A*a*c^2*d*x-3/f^2*A*b^2*c*d*x+3/2/f^3*ln(f*x^2+
d)*A*b*c^2*d^2+1/2/f*ln(f*x^2+d)*B*a^3+1/(d*f)^(1/2)*arctan(1/(d*f)^(1/2)*f
*x)*A*a^3+1/3/f*B*x^3*b^3+1/2/f*A*x^2*b^3+1/5/f*A*x^5*c^3-3/f^2*ln(f*x^2+d)
*A*a*b*c*d-3/f/(d*f)^(1/2)*arctan(1/(d*f)^(1/2)*f*x)*A*a^2*c*d-3/f/(d*f)^(1
/2)*arctan(1/(d*f)^(1/2)*f*x)*A*a*b^2*d+3/f^2/(d*f)^(1/2)*arctan(1/(d*f)^(1
```

```
/2)*f*x)*A*a*c^2*d^2+3/f^2/(d*f)^(1/2)*arctan(1/(d*f)^(1/2)*f*x)*A*b^2*c*d^
2-3/f/(d*f)^(1/2)*arctan(1/(d*f)^(1/2)*f*x)*B*a^2*b*d-3/f^3/(d*f)^(1/2)*arc
tan(1/(d*f)^(1/2)*f*x)*B*b*c^2*d^3-6/f^2*B*a*b*c*d*x+3/5/f*B*x^5*b*c^2+3/4/
f*A*x^4*b*c^2+1/f*A*x^3*a*c^2+3/2/f*ln(f*x^2+d)*A*a^2*b-1/2/f^2*ln(f*x^2+d)
*A*b^3*d-1/2/f^4*ln(f*x^2+d)*B*c^3*d^3+1/f*A*x^3*b^2*c+1/2/f^3*B*x^2*c^3*d^
2+1/f^3*A*c^3*d^2*x+3/f*B*a^2*b*x-1/3/f^2*A*x^3*c^3*d+3/2/f*B*x^2*a^2*c+3/2
/f*B*x^2*a*b^2+3/f*A*a^2*c*x+3/4/f*B*x^4*a*c^2+3/4/f*B*x^4*b^2*c-1/4/f^2*B*
x^4*c^3*d
```

maxima [A] time = 0.99, size = 471, normalized size = 1.07

$$\frac{\left(Aa^3f^3 - \left(3\,Bbc^2 + Ac^3\right)d^3 + \left(Bb^3 + 3\,Aac^2 + 3\left(2\,Bab + Ab^2\right)c\right)d^2f - 3\left(Ba^2b + Aab^2 + Aa^2c\right)df^2\right)\arctan\left(\frac{fx}{\sqrt{df}}\right)}{\sqrt{df}\,f^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(c*x^2+b*x+a)^3/(f*x^2+d),x, algorithm="maxima")
```

```
[Out] (A*a^3*f^3 - (3*B*b*c^2 + A*c^3)*d^3 + (B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*
b^2)*c)*d^2*f - 3*(B*a^2*b + A*a*b^2 + A*a^2*c)*d*f^2)*arctan(f*x/sqrt(d*f)
)/(sqrt(d*f)*f^3) + 1/60*(10*B*c^3*f^2*x^6 + 12*(3*B*b*c^2 + A*c^3)*f^2*x^5
 - 15*(B*c^3*d*f - 3*(B*b^2*c + (B*a + A*b)*c^2)*f^2)*x^4 - 20*((3*B*b*c^2
+ A*c^3)*d*f - (B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b^2)*c)*f^2)*x^3 + 30*(B
*c^3*d^2 - 3*(B*b^2*c + (B*a + A*b)*c^2)*d*f + (3*B*a*b^2 + A*b^3 + 3*(B*a^
2 + 2*A*a*b)*c)*f^2)*x^2 + 60*((3*B*b*c^2 + A*c^3)*d^2 - (B*b^3 + 3*A*a*c^2
 + 3*(2*B*a*b + A*b^2)*c)*d*f + 3*(B*a^2*b + A*a*b^2 + A*a^2*c)*f^2)*x)/f^3
 - 1/2*(B*c^3*d^3 - 3*(B*b^2*c + (B*a + A*b)*c^2)*d^2*f + (3*B*a*b^2 + A*b^
3 + 3*(B*a^2 + 2*A*a*b)*c)*d*f^2 - (B*a^3 + 3*A*a^2*b)*f^3)*log(f*x^2 + d)/
f^4
```

mupad [B] time = 3.79, size = 552, normalized size = 1.25

$$x^2\left(\frac{3\,B\,c\,a^2+3\,B\,a\,b^2+6\,A\,c\,a\,b+A\,b^3}{2\,f}-\frac{d\,\left(\frac{3\,B\,b^2\,c+3\,A\,b\,c^2+3\,B\,a\,c^2}{f}-\frac{B\,c^3\,d}{f^2}\right)}{2\,f}\right)+x\left(\frac{3\,B\,a^2\,b+3\,A\,c\,a^2+3\,A\,a\,b^2}{f}-\frac{d}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x)*(a + b*x + c*x^2)^3)/(d + f*x^2),x)
```

```
[Out] x^2*((A*b^3 + 3*B*a*b^2 + 3*B*a^2*c + 6*A*a*b*c)/(2*f) - (d*((3*A*b*c^2 + 3
*B*a*c^2 + 3*B*b^2*c)/f - (B*c^3*d)/f^2))/(2*f)) + x*((3*A*a*b^2 + 3*A*a^2*
c + 3*B*a^2*b)/f - (d*((B*b^3 + 3*A*a*c^2 + 3*A*b^2*c + 6*B*a*b*c)/f - (d*(
A*c^3 + 3*B*b*c^2))/f^2))/f) + x^3*((B*b^3 + 3*A*a*c^2 + 3*A*b^2*c + 6*B*a*
b*c)/(3*f) - (d*(A*c^3 + 3*B*b*c^2))/(3*f^2)) + x^4*((3*A*b*c^2 + 3*B*a*c^2
 + 3*B*b^2*c)/(4*f) - (B*c^3*d)/(4*f^2)) + (x^5*(A*c^3 + 3*B*b*c^2))/(5*f)
+ (B*c^3*x^6)/(6*f) + (log(d + f*x^2)*(4*B*a^3*d*f^7 - 4*A*b^3*d^2*f^6 - 4*
B*c^3*d^4*f^4 - 12*B*a*b^2*d^2*f^6 + 12*A*b*c^2*d^3*f^5 + 12*B*a*c^2*d^3*f^
5 - 12*B*a^2*c*d^2*f^6 + 12*B*b^2*c*d^3*f^5 + 12*A*a^2*b*d*f^7 - 24*A*a*b*c
*d^2*f^6))/(8*d*f^8) + (atan((f^(1/2)*x)/d^(1/2))*(A*a^3*f^3 - A*c^3*d^3 -
3*B*b*c^2*d^3 + B*b^3*d^2*f - 3*A*a*b^2*d*f^2 + 3*A*a*c^2*d^2*f - 3*A*a^2*c
*d*f^2 - 3*B*a^2*b*d*f^2 + 3*A*b^2*c*d^2*f + 6*B*a*b*c*d^2*f))/(d^(1/2)*f^(
7/2))
```

sympy [B] time = 23.62, size = 1962, normalized size = 4.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(c*x**2+b*x+a)**3/(f*x**2+d),x)
```

```
[Out] B*c**3*x**6/(6*f) + x**5*(A*c**3/(5*f) + 3*B*b*c**2/(5*f)) + x**4*(3*A*b*c*
*2/(4*f) + 3*B*a*c**2/(4*f) + 3*B*b**2*c/(4*f) - B*c**3*d/(4*f**2)) + x**3*
(A*a*c**2/f + A*b**2*c/f - A*c**3*d/(3*f**2) + 2*B*a*b*c/f + B*b**3/(3*f) -
 B*b*c**2*d/f**2) + x**2*(3*A*a*b*c/f + A*b**3/(2*f) - 3*A*b*c**2*d/(2*f**2
) + 3*B*a**2*c/(2*f) + 3*B*a*b**2/(2*f) - 3*B*a*c**2*d/(2*f**2) - 3*B*b**2*
c*d/(2*f**2) + B*c**3*d**2/(2*f**3)) + x*(3*A*a**2*c/f + 3*A*a*b**2/f - 3*A
*a*c**2*d/f**2 - 3*A*b**2*c*d/f**2 + A*c**3*d**2/f**3 + 3*B*a**2*b/f - 6*B*
a*b*c*d/f**2 - B*b**3*d/f**2 + 3*B*b*c**2*d**2/f**3) + ((3*A*a**2*b*f**3 -
6*A*a*b*c*d*f**2 - A*b**3*d*f**2 + 3*A*b*c**2*d**2*f + B*a**3*f**3 - 3*B*a*
*2*c*d*f**2 - 3*B*a*b**2*d*f**2 + 3*B*a*c**2*d**2*f + 3*B*b**2*c*d**2*f - B
*c**3*d**3)/(2*f**4) - sqrt(-d*f**9)*(A*a**3*f**3 - 3*A*a**2*c*d*f**2 - 3*A
*a*b**2*d*f**2 + 3*A*a*c**2*d**2*f + 3*A*b**2*c*d**2*f - A*c**3*d**3 - 3*B*
a**2*b*d*f**2 + 6*B*a*b*c*d**2*f + B*b**3*d**2*f - 3*B*b*c**2*d**3)/(2*d*f*
*8))*log(x + (-3*A*a**2*b*d*f**3 + 6*A*a*b*c*d**2*f**2 + A*b**3*d**2*f**2 -
 3*A*b*c**2*d**3*f - B*a**3*d*f**3 + 3*B*a**2*c*d**2*f**2 + 3*B*a*b**2*d**2
*f**2 - 3*B*a*c**2*d**3*f - 3*B*b**2*c*d**3*f + B*c**3*d**4 + 2*d*f**4*((3*
A*a**2*b*f**3 - 6*A*a*b*c*d*f**2 - A*b**3*d*f**2 + 3*A*b*c**2*d**2*f + B*a*
*3*f**3 - 3*B*a**2*c*d*f**2 - 3*B*a*b**2*d*f**2 + 3*B*a*c**2*d**2*f + 3*B*b
**2*c*d**2*f - B*c**3*d**3)/(2*f**4) - sqrt(-d*f**9)*(A*a**3*f**3 - 3*A*a**
2*c*d*f**2 - 3*A*a*b**2*d*f**2 + 3*A*a*c**2*d**2*f + 3*A*b**2*c*d**2*f - A*
c**3*d**3 - 3*B*a**2*b*d*f**2 + 6*B*a*b*c*d**2*f + B*b**3*d**2*f - 3*B*b*c*
*2*d**3)/(2*d*f**8)))/(A*a**3*f**4 - 3*A*a**2*c*d*f**3 - 3*A*a*b**2*d*f**3
+ 3*A*a*c**2*d**2*f**2 + 3*A*b**2*c*d**2*f**2 - A*c**3*d**3*f - 3*B*a**2*b*
d*f**3 + 6*B*a*b*c*d**2*f**2 + B*b**3*d**2*f**2 - 3*B*b*c**2*d**3*f)) + ((3
*A*a**2*b*f**3 - 6*A*a*b*c*d*f**2 - A*b**3*d*f**2 + 3*A*b*c**2*d**2*f + B*a
**3*f**3 - 3*B*a**2*c*d*f**2 - 3*B*a*b**2*d*f**2 + 3*B*a*c**2*d**2*f + 3*B*
b**2*c*d**2*f - B*c**3*d**3)/(2*f**4) + sqrt(-d*f**9)*(A*a**3*f**3 - 3*A*a*
*2*c*d*f**2 - 3*A*a*b**2*d*f**2 + 3*A*a*c**2*d**2*f + 3*A*b**2*c*d**2*f - A
*c**3*d**3 - 3*B*a**2*b*d*f**2 + 6*B*a*b*c*d**2*f + B*b**3*d**2*f - 3*B*b*c
**2*d**3)/(2*d*f**8))*log(x + (-3*A*a**2*b*d*f**3 + 6*A*a*b*c*d**2*f**2 + A
*b**3*d**2*f**2 - 3*A*b*c**2*d**3*f - B*a**3*d*f**3 + 3*B*a**2*c*d**2*f**2
+ 3*B*a*b**2*d**2*f**2 - 3*B*a*c**2*d**3*f - 3*B*b**2*c*d**3*f + B*c**3*d**
4 + 2*d*f**4*((3*A*a**2*b*f**3 - 6*A*a*b*c*d*f**2 - A*b**3*d*f**2 + 3*A*b*c
**2*d**2*f + B*a**3*f**3 - 3*B*a**2*c*d*f**2 - 3*B*a*b**2*d*f**2 + 3*B*a*c*
*2*d**2*f + 3*B*b**2*c*d**2*f - B*c**3*d**3)/(2*f**4) + sqrt(-d*f**9)*(A*a*
*3*f**3 - 3*A*a**2*c*d*f**2 - 3*A*a*b**2*d*f**2 + 3*A*a*c**2*d**2*f + 3*A*b
**2*c*d**2*f - A*c**3*d**3 - 3*B*a**2*b*d*f**2 + 6*B*a*b*c*d**2*f + B*b**3*
d**2*f - 3*B*b*c**2*d**3)/(2*d*f**8)))/(A*a**3*f**4 - 3*A*a**2*c*d*f**3 - 3
*A*a*b**2*d*f**3 + 3*A*a*c**2*d**2*f**2 + 3*A*b**2*c*d**2*f**2 - A*c**3*d**
3*f - 3*B*a**2*b*d*f**3 + 6*B*a*b*c*d**2*f**2 + B*b**3*d**2*f**2 - 3*B*b*c*
*2*d**3*f))
```

3.4 $\int \frac{A+Bx}{(a+bx+cx^2)(d+fx^2)} \, dx$

Optimal. Leaf size=274

$$\frac{\log\left(a+bx+cx^2\right)(-aBf+Abf+Bcd)}{2\left(f\left(a^2f+b^2d\right)-2acdf+c^2d^2\right)}-\frac{\log\left(d+fx^2\right)(-aBf+Abf+Bcd)}{2\left(f\left(a^2f+b^2d\right)-2acdf+c^2d^2\right)}+\frac{\sqrt{f}\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{d}}\right)(aAf-Acd+b\ldots}{\sqrt{d}\left(f\left(a^2f+b^2d\right)-2acdf+c^2\ldots\right.}$$

```
[Out] 1/2*(A*b*f-B*a*f+B*c*d)*ln(c*x^2+b*x+a)/(c^2*d^2-2*a*c*d*f+f*(a^2*f+b^2*d))
-1/2*(A*b*f-B*a*f+B*c*d)*ln(f*x^2+d)/(c^2*d^2-2*a*c*d*f+f*(a^2*f+b^2*d))-(A
*b^2*f+2*A*c*(-a*f+c*d)-b*B*(a*f+c*d))*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2)
)/(c^2*d^2-2*a*c*d*f+f*(a^2*f+b^2*d))/(-4*a*c+b^2)^(1/2)+(A*a*f-A*c*d+B*b*d
)*arctan(x*f^(1/2)/d^(1/2))*f^(1/2)/(c^2*d^2-2*a*c*d*f+f*(a^2*f+b^2*d))/d^(
1/2)
```

Rubi [A] time = 0.28, antiderivative size = 274, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {1023, 634, 618, 206, 628, 635, 205, 260}

$$\frac{\log\left(a+bx+cx^2\right)(-aBf+Abf+Bcd)}{2\left(f\left(a^2f+b^2d\right)-2acdf+c^2d^2\right)}-\frac{\log\left(d+fx^2\right)(-aBf+Abf+Bcd)}{2\left(f\left(a^2f+b^2d\right)-2acdf+c^2d^2\right)}+\frac{\sqrt{f}\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{d}}\right)(aAf-Acd+b\ldots}{\sqrt{d}\left(f\left(a^2f+b^2d\right)-2acdf+c^2\ldots\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x)/((a + b*x + c*x^2)*(d + f*x^2)),x]
```

```
[Out] (Sqrt[f]*(b*B*d - A*c*d + a*A*f)*ArcTan[(Sqrt[f]*x)/Sqrt[d]])/(Sqrt[d]*(c^2
*d^2 - 2*a*c*d*f + f*(b^2*d + a^2*f))) - ((A*b^2*f + 2*A*c*(c*d - a*f) - b*
B*(c*d + a*f))*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(Sqrt[b^2 - 4*a*c]*(
c^2*d^2 - 2*a*c*d*f + f*(b^2*d + a^2*f))) + ((B*c*d + A*b*f - a*B*f)*Log[a
+ b*x + c*x^2])/(2*(c^2*d^2 - 2*a*c*d*f + f*(b^2*d + a^2*f))) - ((B*c*d + A
*b*f - a*B*f)*Log[d + f*x^2])/(2*(c^2*d^2 - 2*a*c*d*f + f*(b^2*d + a^2*f)))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 1023

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*((d_) + (f_.)*
(x_)^2)), x_Symbol] :> With[{q = Simplify[c^2*d^2 + b^2*d*f - 2*a*c*d*f + a
^2*f^2]}, Dist[1/q, Int[Simp[g*c^2*d + g*b^2*f - a*b*h*f - a*g*c*f + c*(h*c
*d + g*b*f - a*h*f)*x, x]/(a + b*x + c*x^2), x], x] + Dist[1/q, Int[Simp[b*
h*d*f - g*c*d*f + a*g*f^2 - f*(h*c*d + g*b*f - a*h*f)*x, x]/(d + f*x^2), x]
, x] /; NeQ[q, 0]] /; FreeQ[{a, b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0
]
```

Rubi steps

$$\int \frac{A+Bx}{\left(a+bx+cx^2\right)\left(d+fx^2\right)}\,dx = \frac{\int \frac{-abBf+A\left(c^2d+b^2f-acf\right)+c(Bcd+Abf-aBf)x}{a+bx+cx^2}\,dx}{c^2d^2-2acdf+f\left(b^2d+a^2f\right)} + \frac{\int \frac{f(bBd-Acd+aAf)-f(Bcd+Abf-aBf)x}{d+fx^2}}{c^2d^2-2acdf+f\left(b^2d+a^2f\right)}$$

$$= \frac{(f(bBd-Acd+aAf))\int \frac{1}{d+fx^2}\,dx}{c^2d^2-2acdf+f\left(b^2d+a^2f\right)} + \frac{(Bcd+Abf-aBf)\int \frac{b+2cx}{a+bx+cx^2}\,dx}{2\left(c^2d^2-2acdf+f\left(b^2d+a^2f\right)\right)} - \frac{(f(}{c}$$

$$= \frac{\sqrt{f}\,(bBd-Acd+aAf)\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{d}}\right)}{\sqrt{d}\left(c^2d^2-2acdf+f\left(b^2d+a^2f\right)\right)} + \frac{(Bcd+Abf-aBf)\log\left(a+bx+cx\right.}{2\left(c^2d^2-2acdf+f\left(b^2d+a^2f\right)\right)}$$

$$= \frac{\sqrt{f}\,(bBd-Acd+aAf)\tan^{-1}\left(\frac{\sqrt{f}\,x}{\sqrt{d}}\right)}{\sqrt{d}\left(c^2d^2-2acdf+f\left(b^2d+a^2f\right)\right)} - \frac{\left(Ab^2f+2Ac(cd-af)-bB(cd+af\right.}{\sqrt{b^2-4ac}\left(c^2d^2-2acdf+f\right.}$$

Mathematica [A] time = 0.37, size = 212, normalized size = 0.77

$$\frac{\sqrt{d}\left(\sqrt{4ac-b^2}\,(-aBf+Abf+Bcd)\left(\log(a+x(b+cx))-\log\left(d+fx^2\right)\right)+2\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)\left(2Ac(cd-af)\right.\right.}{2\sqrt{d}\sqrt{4ac-b^2}\left(f\left(a^2f+b^2d\right)-2acdf+\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x)/((a + b*x + c*x^2)*(d + f*x^2)),x]
```

```
[Out] (2*Sqrt[-b^2 + 4*a*c]*Sqrt[f]*(b*B*d - A*c*d + a*A*f)*ArcTan[(Sqrt[f]*x)/Sq
rt[d]] + Sqrt[d]*(2*(A*b^2*f + 2*A*c*(c*d - a*f) - b*B*(c*d + a*f))*ArcTan[
(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]] + Sqrt[-b^2 + 4*a*c]*(B*c*d + A*b*f - a*B*f
)*(-Log[d + f*x^2] + Log[a + x*(b + c*x)])))/(2*Sqrt[-b^2 + 4*a*c]*Sqrt[d]*
(c^2*d^2 - 2*a*c*d*f + f*(b^2*d + a^2*f)))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(c*x^2+b*x+a)/(f*x^2+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.17, size = 266, normalized size = 0.97

$$\frac{\left(Bcd - Baf + Abf\right)\log\left(cx^2 + bx + a\right)}{2\left(c^2d^2 + b^2df - 2\,acdf + a^2f^2\right)} - \frac{\left(Bcd - Baf + Abf\right)\log\left(fx^2 + d\right)}{2\left(c^2d^2 + b^2df - 2\,acdf + a^2f^2\right)} + \frac{\left(Bbdf - Acdf + Aaf^2\right)\arctan\left(\frac{fx}{\sqrt{df}}\right)}{\left(c^2d^2 + b^2df - 2\,acdf + a^2f^2\right)\sqrt{df}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(c*x^2+b*x+a)/(f*x^2+d),x, algorithm="giac")
```

```
[Out] 1/2*(B*c*d - B*a*f + A*b*f)*log(c*x^2 + b*x + a)/(c^2*d^2 + b^2*d*f - 2*a*c
*d*f + a^2*f^2) - 1/2*(B*c*d - B*a*f + A*b*f)*log(f*x^2 + d)/(c^2*d^2 + b^2
*d*f - 2*a*c*d*f + a^2*f^2) + (B*b*d*f - A*c*d*f + A*a*f^2)*arctan(f*x/sqrt
(d*f))/((c^2*d^2 + b^2*d*f - 2*a*c*d*f + a^2*f^2)*sqrt(d*f)) - (B*b*c*d - 2
*A*c^2*d + B*a*b*f - A*b^2*f + 2*A*a*c*f)*arctan((2*c*x + b)/sqrt(-b^2 + 4*
a*c))/((c^2*d^2 + b^2*d*f - 2*a*c*d*f + a^2*f^2)*sqrt(-b^2 + 4*a*c))
```

maple [B] time = 0.02, size = 745, normalized size = 2.72

$$-\frac{2Aacf\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\left(a^2f^2 - 2acdf + b^2df + c^2d^2\right)\sqrt{4ac-b^2}} + \frac{Aaf^2\arctan\left(\frac{fx}{\sqrt{df}}\right)}{\left(a^2f^2 - 2acdf + b^2df + c^2d^2\right)\sqrt{df}} + \frac{Ab^2f\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\left(a^2f^2 - 2acdf + b^2df + c^2d^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x+A)/(c*x^2+b*x+a)/(f*x^2+d),x)
```

```
[Out] -1/2*f/(a^2*f^2-2*a*c*d*f+b^2*d*f+c^2*d^2)*ln(f*x^2+d)*A*b+1/2*f/(a^2*f^2-2
*a*c*d*f+b^2*d*f+c^2*d^2)*ln(f*x^2+d)*B*a-1/2/(a^2*f^2-2*a*c*d*f+b^2*d*f+c^
2*d^2)*ln(f*x^2+d)*B*c*d+f^2/(a^2*f^2-2*a*c*d*f+b^2*d*f+c^2*d^2)/(d*f)^(1/2
)*arctan(1/(d*f)^(1/2)*f*x)*A*a-f/(a^2*f^2-2*a*c*d*f+b^2*d*f+c^2*d^2)/(d*f)
^(1/2)*arctan(1/(d*f)^(1/2)*f*x)*A*c*d+f/(a^2*f^2-2*a*c*d*f+b^2*d*f+c^2*d^2
)/(d*f)^(1/2)*arctan(1/(d*f)^(1/2)*f*x)*B*b*d+1/2/(a^2*f^2-2*a*c*d*f+b^2*d*
f+c^2*d^2)*ln(c*x^2+b*x+a)*A*b*f-1/2/(a^2*f^2-2*a*c*d*f+b^2*d*f+c^2*d^2)*ln
(c*x^2+b*x+a)*B*a*f+1/2/(a^2*f^2-2*a*c*d*f+b^2*d*f+c^2*d^2)*c*ln(c*x^2+b*x+
a)*B*d-2/(a^2*f^2-2*a*c*d*f+b^2*d*f+c^2*d^2)/(4*a*c-b^2)^(1/2)*arctan((2*c*
x+b)/(4*a*c-b^2)^(1/2))*A*a*c*f+1/(a^2*f^2-2*a*c*d*f+b^2*d*f+c^2*d^2)/(4*a*
c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*A*b^2*f+2/(a^2*f^2-2*a*c*d
*f+b^2*d*f+c^2*d^2)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*A
*c^2*d-1/(a^2*f^2-2*a*c*d*f+b^2*d*f+c^2*d^2)/(4*a*c-b^2)^(1/2)*arctan((2*c*
x+b)/(4*a*c-b^2)^(1/2))*B*a*b*f-1/(a^2*f^2-2*a*c*d*f+b^2*d*f+c^2*d^2)/(4*a*
c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*B*b*c*d
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(c*x^2+b*x+a)/(f*x^2+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 38.32, size = 3888, normalized size = 14.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x)/((d + f*x^2)*(a + b*x + c*x^2)),x)
```

```
[Out] (log(B^3*c^2*f^2*x + (((B*c*d^2)/2 + (A*b*d*f)/2 - (B*a*d*f)/2 - (A*a*f*(-d
*f)^(1/2))/2 + (A*c*d*(-d*f)^(1/2))/2 - (B*b*d*(-d*f)^(1/2))/2)*((((B*c*d^2
)/2 + (A*b*d*f)/2 - (B*a*d*f)/2 - (A*a*f*(-d*f)^(1/2))/2 + (A*c*d*(-d*f)^(1
/2))/2 - (B*b*d*(-d*f)^(1/2))/2)*(4*A*a^2*c^2*f^4 + 4*A*c^4*d^2*f^2 - c*f^2
*x*(3*A*b^3*f^2 + 4*B*c^3*d^2 - B*a*b^2*f^2 + 4*B*a^2*c*f^2 - 12*A*a*b*c*f^
2 + 12*A*b*c^2*d*f - 8*B*a*c^2*d*f - 3*B*b^2*c*d*f) - 3*A*b^2*c^2*d*f^3 - 4
*B*b*c^3*d^2*f^2 - A*a*b^2*c*f^4 - 8*A*a*c^3*d*f^3 + B*b^3*c*d*f^3 + (2*c*f
^2*((B*c*d^2)/2 + (A*b*d*f)/2 - (B*a*d*f)/2 - (A*a*f*(-d*f)^(1/2))/2 + (A*c
*d*(-d*f)^(1/2))/2 - (B*b*d*(-d*f)^(1/2))/2)*(2*b*c^3*d^3 + 4*c^4*d^3*x - a
^2*b^2*f^3*x + 4*a*b^3*d*f^2 - 2*b^3*c*d^2*f + 4*a^3*c*f^3*x + 3*b^4*d*f^2*
x + 12*a*b*c^2*d^2*f - 14*a^2*b*c*d*f^2 - 4*a*c^3*d^2*f*x - 4*a^2*c^2*d*f^2
*x + 3*b^2*c^2*d^2*f*x - 10*a*b^2*c*d*f^2*x))/(d*(a^2*f^2 + c^2*d^2 + b^2*d
*f - 2*a*c*d*f)) + 4*B*a*b*c^2*d*f^3))/(d*(a^2*f^2 + c^2*d^2 + b^2*d*f - 2*
a*c*d*f)) - c*f^2*x*(2*B^2*c^2*d - 4*A^2*c^2*f - B^2*b^2*f + 2*B^2*a*c*f +
2*A*B*b*c*f) + A^2*b*c^2*f^3 + B^2*b*c^2*d*f^2 - 4*A*B*a*c^2*f^3 + A*B*b^2*
c*f^3 - 4*A*B*c^3*d*f^2))/(d*(a^2*f^2 + c^2*d^2 + b^2*d*f - 2*a*c*d*f)) + A
*B^2*c^2*f^2)*(f*((B*a*d)/2 - (A*b*d)/2 + (A*a*(-d*f)^(1/2))/2) - (B*c*d^2)
/2 - (A*c*d*(-d*f)^(1/2))/2 + (B*b*d*(-d*f)^(1/2))/2))/(c^2*d^3 + a^2*d*f^2
 + b^2*d^2*f - 2*a*c*d^2*f) - (log(B^3*c^2*f^2*x + (((B*c*d^2)/2 + (A*b*d*f
)/2 - (B*a*d*f)/2 + (A*a*f*(-d*f)^(1/2))/2 - (A*c*d*(-d*f)^(1/2))/2 + (B*b*
d*(-d*f)^(1/2))/2)*((((B*c*d^2)/2 + (A*b*d*f)/2 - (B*a*d*f)/2 + (A*a*f*(-d*
f)^(1/2))/2 - (A*c*d*(-d*f)^(1/2))/2 + (B*b*d*(-d*f)^(1/2))/2)*(4*A*a^2*c^2
*f^4 + 4*A*c^4*d^2*f^2 - c*f^2*x*(3*A*b^3*f^2 + 4*B*c^3*d^2 - B*a*b^2*f^2 +
 4*B*a^2*c*f^2 - 12*A*a*b*c*f^2 + 12*A*b*c^2*d*f - 8*B*a*c^2*d*f - 3*B*b^2*
c*d*f) - 3*A*b^2*c^2*d*f^3 - 4*B*b*c^3*d^2*f^2 - A*a*b^2*c*f^4 - 8*A*a*c^3*
d*f^3 + B*b^3*c*d*f^3 + (2*c*f^2*((B*c*d^2)/2 + (A*b*d*f)/2 - (B*a*d*f)/2 +
 (A*a*f*(-d*f)^(1/2))/2 - (A*c*d*(-d*f)^(1/2))/2 + (B*b*d*(-d*f)^(1/2))/2)*
(2*b*c^3*d^3 + 4*c^4*d^3*x - a^2*b^2*f^3*x + 4*a*b^3*d*f^2 - 2*b^3*c*d^2*f
+ 4*a^3*c*f^3*x + 3*b^4*d*f^2*x + 12*a*b*c^2*d^2*f - 14*a^2*b*c*d*f^2 - 4*a
*c^3*d^2*f*x - 4*a^2*c^2*d*f^2*x + 3*b^2*c^2*d^2*f*x - 10*a*b^2*c*d*f^2*x))
/(d*(a^2*f^2 + c^2*d^2 + b^2*d*f - 2*a*c*d*f)) + 4*B*a*b*c^2*d*f^3))/(d*(a^
2*f^2 + c^2*d^2 + b^2*d*f - 2*a*c*d*f)) - c*f^2*x*(2*B^2*c^2*d - 4*A^2*c^2*
f - B^2*b^2*f + 2*B^2*a*c*f + 2*A*B*b*c*f) + A^2*b*c^2*f^3 + B^2*b*c^2*d*f^
2 - 4*A*B*a*c^2*f^3 + A*B*b^2*c*f^3 - 4*A*B*c^3*d*f^2))/(d*(a^2*f^2 + c^2*d
^2 + b^2*d*f - 2*a*c*d*f)) + A*B^2*c^2*f^2)*(f*((A*b*d)/2 - (B*a*d)/2 + (A*
a*(-d*f)^(1/2))/2) + (B*c*d^2)/2 - (A*c*d*(-d*f)^(1/2))/2 + (B*b*d*(-d*f)^(
1/2))/2))/(c^2*d^3 + a^2*d*f^2 + b^2*d^2*f - 2*a*c*d^2*f) - (log(B^3*c^2*f^
2*x + A*B^2*c^2*f^2 - ((((A*f*(b^2 - 4*a*c)^(3/2) + 2*A*b*f*(4*a*c - b^2) -
 2*B*a*f*(4*a*c - b^2) + 2*B*c*d*(4*a*c - b^2) + 4*A*c^2*d*(b^2 - 4*a*c)^(1
/2) + A*b^2*f*(b^2 - 4*a*c)^(1/2) - 2*B*a*b*f*(b^2 - 4*a*c)^(1/2) - 2*B*b*c
*d*(b^2 - 4*a*c)^(1/2))*(c*f^2*x*(3*A*b^3*f^2 + 4*B*c^3*d^2 - B*a*b^2*f^2 +
 4*B*a^2*c*f^2 - 12*A*a*b*c*f^2 + 12*A*b*c^2*d*f - 8*B*a*c^2*d*f - 3*B*b^2*
c*d*f) - 4*A*c^4*d^2*f^2 - 4*A*a^2*c^2*f^4 + 3*A*b^2*c^2*d*f^3 + 4*B*b*c^3*
d^2*f^2 + A*a*b^2*c*f^4 + 8*A*a*c^3*d*f^3 - B*b^3*c*d*f^3 - 4*B*a*b*c^2*d*f
^3 + (c*f^2*(A*f*(b^2 - 4*a*c)^(3/2) + 2*A*b*f*(4*a*c - b^2) - 2*B*a*f*(4*a
*c - b^2) + 2*B*c*d*(4*a*c - b^2) + 4*A*c^2*d*(b^2 - 4*a*c)^(1/2) + A*b^2*f
*(b^2 - 4*a*c)^(1/2) - 2*B*a*b*f*(b^2 - 4*a*c)^(1/2) - 2*B*b*c*d*(b^2 - 4*a
*c)^(1/2))*(2*b*c^3*d^3 + 4*c^4*d^3*x - a^2*b^2*f^3*x + 4*a*b^3*d*f^2 - 2*b
^3*c*d^2*f + 4*a^3*c*f^3*x + 3*b^4*d*f^2*x + 12*a*b*c^2*d^2*f - 14*a^2*b*c*
d*f^2 - 4*a*c^3*d^2*f*x - 4*a^2*c^2*d*f^2*x + 3*b^2*c^2*d^2*f*x - 10*a*b^2*
```

```
c*d*f^2*x))/(2*(4*a*c - b^2)*(a^2*f^2 + c^2*d^2 + b^2*d*f - 2*a*c*d*f))))/(
4*(4*a*c - b^2)*(a^2*f^2 + c^2*d^2 + b^2*d*f - 2*a*c*d*f)) - c*f^2*x*(2*B^2
*c^2*d - 4*A^2*c^2*f - B^2*b^2*f + 2*B^2*a*c*f + 2*A*B*b*c*f) + A^2*b*c^2*f
^3 + B^2*b*c^2*d*f^2 - 4*A*B*a*c^2*f^3 + A*B*b^2*c*f^3 - 4*A*B*c^3*d*f^2)*(
A*f*(b^2 - 4*a*c)^(3/2) + 2*A*b*f*(4*a*c - b^2) - 2*B*a*f*(4*a*c - b^2) + 2
*B*c*d*(4*a*c - b^2) + 4*A*c^2*d*(b^2 - 4*a*c)^(1/2) + A*b^2*f*(b^2 - 4*a*c
)^(1/2) - 2*B*a*b*f*(b^2 - 4*a*c)^(1/2) - 2*B*b*c*d*(b^2 - 4*a*c)^(1/2)))/(
4*(4*a*c - b^2)*(a^2*f^2 + c^2*d^2 + b^2*d*f - 2*a*c*d*f)))*(A*f*(b^2 - 4*a
*c)^(3/2) + 2*A*b*f*(4*a*c - b^2) - 2*B*a*f*(4*a*c - b^2) + 2*B*c*d*(4*a*c
 - b^2) + 4*A*c^2*d*(b^2 - 4*a*c)^(1/2) + A*b^2*f*(b^2 - 4*a*c)^(1/2) - 2*B*
a*b*f*(b^2 - 4*a*c)^(1/2) - 2*B*b*c*d*(b^2 - 4*a*c)^(1/2)))/(b^2*(4*a^2*f^2
 + 4*c^2*d^2 + 4*b^2*d*f - 24*a*c*d*f) - 4*a*c*(4*a^2*f^2 + 4*c^2*d^2 - 8*a
*c*d*f)) + (log(B^3*c^2*f^2*x + A*B^2*c^2*f^2 + ((((A*f*(b^2 - 4*a*c)^(3/2)
 - 2*A*b*f*(4*a*c - b^2) + 2*B*a*f*(4*a*c - b^2) - 2*B*c*d*(4*a*c - b^2) +
4*A*c^2*d*(b^2 - 4*a*c)^(1/2) + A*b^2*f*(b^2 - 4*a*c)^(1/2) - 2*B*a*b*f*(b^
2 - 4*a*c)^(1/2) - 2*B*b*c*d*(b^2 - 4*a*c)^(1/2))*(4*A*a^2*c^2*f^4 + 4*A*c^
4*d^2*f^2 - c*f^2*x*(3*A*b^3*f^2 + 4*B*c^3*d^2 - B*a*b^2*f^2 + 4*B*a^2*c*f^
2 - 12*A*a*b*c*f^2 + 12*A*b*c^2*d*f - 8*B*a*c^2*d*f - 3*B*b^2*c*d*f) - 3*A*
b^2*c^2*d*f^3 - 4*B*b*c^3*d^2*f^2 - A*a*b^2*c*f^4 - 8*A*a*c^3*d*f^3 + B*b^3
*c*d*f^3 + 4*B*a*b*c^2*d*f^3 + (c*f^2*(A*f*(b^2 - 4*a*c)^(3/2) - 2*A*b*f*(4
*a*c - b^2) + 2*B*a*f*(4*a*c - b^2) - 2*B*c*d*(4*a*c - b^2) + 4*A*c^2*d*(b^
2 - 4*a*c)^(1/2) + A*b^2*f*(b^2 - 4*a*c)^(1/2) - 2*B*a*b*f*(b^2 - 4*a*c)^(1
/2) - 2*B*b*c*d*(b^2 - 4*a*c)^(1/2))*(2*b*c^3*d^3 + 4*c^4*d^3*x - a^2*b^2*f
^3*x + 4*a*b^3*d*f^2 - 2*b^3*c*d^2*f + 4*a^3*c*f^3*x + 3*b^4*d*f^2*x + 12*a
*b*c^2*d^2*f - 14*a^2*b*c*d*f^2 - 4*a*c^3*d^2*f*x - 4*a^2*c^2*d*f^2*x + 3*b
^2*c^2*d^2*f*x - 10*a*b^2*c*d*f^2*x))/(2*(4*a*c - b^2)*(a^2*f^2 + c^2*d^2 +
 b^2*d*f - 2*a*c*d*f))))/(4*(4*a*c - b^2)*(a^2*f^2 + c^2*d^2 + b^2*d*f - 2*
a*c*d*f)) - c*f^2*x*(2*B^2*c^2*d - 4*A^2*c^2*f - B^2*b^2*f + 2*B^2*a*c*f +
2*A*B*b*c*f) + A^2*b*c^2*f^3 + B^2*b*c^2*d*f^2 - 4*A*B*a*c^2*f^3 + A*B*b^2*
c*f^3 - 4*A*B*c^3*d*f^2)*(A*f*(b^2 - 4*a*c)^(3/2) - 2*A*b*f*(4*a*c - b^2) +
 2*B*a*f*(4*a*c - b^2) - 2*B*c*d*(4*a*c - b^2) + 4*A*c^2*d*(b^2 - 4*a*c)^(1
/2) + A*b^2*f*(b^2 - 4*a*c)^(1/2) - 2*B*a*b*f*(b^2 - 4*a*c)^(1/2) - 2*B*b*c
*d*(b^2 - 4*a*c)^(1/2)))/(4*(4*a*c - b^2)*(a^2*f^2 + c^2*d^2 + b^2*d*f - 2*
a*c*d*f)))*(A*f*(b^2 - 4*a*c)^(3/2) - 2*A*b*f*(4*a*c - b^2) + 2*B*a*f*(4*a*
c - b^2) - 2*B*c*d*(4*a*c - b^2) + 4*A*c^2*d*(b^2 - 4*a*c)^(1/2) + A*b^2*f*
(b^2 - 4*a*c)^(1/2) - 2*B*a*b*f*(b^2 - 4*a*c)^(1/2) - 2*B*b*c*d*(b^2 - 4*a*
c)^(1/2)))/(b^2*(4*a^2*f^2 + 4*c^2*d^2 + 4*b^2*d*f - 24*a*c*d*f) - 4*a*c*(4
*a^2*f^2 + 4*c^2*d^2 - 8*a*c*d*f))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(c*x**2+b*x+a)/(f*x**2+d),x)
```

```
[Out] Timed out
```

3.5 $\int \frac{A+Bx}{\left(a+bx+cx^2\right)^2\left(d+fx^2\right)}\,dx$

Optimal. Leaf size=596

$$-\frac{f^{3/2}\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{d}}\right)\left(-A(cd-af)^2+2bBd(cd-af)+Ab^2df\right)}{\sqrt{d}\left(f\left(a^2f+b^2d\right)-2acdf+c^2d^2\right)^2}-\frac{f\log\left(a+bx+cx^2\right)\left(B\left(-f\left(b^2d-a^2f\right)-2acd\right.\right.}{2\left(f\left(a^2f+b^2d\right)-2acdf+\right.}$$

```
[Out] (A*b*c*(a*f+c*d)-(A*b-B*a)*(-2*a*c*f+b^2*f+2*c^2*d)-c*(A*b^2*f+2*A*c*(-a*f+
c*d)-b*B*(a*f+c*d))*x)/(-4*a*c+b^2)/(b^2*d*f+(-a*f+c*d)^2)/(c*x^2+b*x+a)-(b
^5*B*d*f^2-2*A*b^4*f^2*(-a*f+c*d)-4*A*c^2*(-3*a*f+c*d)*(-a*f+c*d)^2+b^3*B*f
*(-a^2*f^2-4*a*c*d*f+5*c^2*d^2)-4*A*b^2*c*f*(3*a^2*f^2-3*a*c*d*f+2*c^2*d^2)
+2*b*B*c*(3*a^3*f^3+3*a^2*c*d*f^2-7*a*c^2*d^2*f+c^3*d^3))*arctanh((2*c*x+b)
/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(3/2)/(c^2*d^2-2*a*c*d*f+f*(a^2*f+b^2*d))
^2-1/2*f*(2*A*b*f*(-a*f+c*d)+B*(c^2*d^2-2*a*c*d*f-f*(-a^2*f+b^2*d)))*ln(c*x
^2+b*x+a)/(c^2*d^2-2*a*c*d*f+f*(a^2*f+b^2*d))^2+1/2*f*(2*A*b*f*(-a*f+c*d)+B
*(c^2*d^2-2*a*c*d*f-f*(-a^2*f+b^2*d)))*ln(f*x^2+d)/(c^2*d^2-2*a*c*d*f+f*(a^
2*f+b^2*d))^2-f^(3/2)*(A*b^2*d*f+2*b*B*d*(-a*f+c*d)-A*(-a*f+c*d)^2)*arctan(
x*f^(1/2)/d^(1/2))/(c^2*d^2-2*a*c*d*f+f*(a^2*f+b^2*d))^2/d^(1/2)
```

Rubi [A] time = 1.77, antiderivative size = 596, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1018, 1074, 634, 618, 206, 628, 635, 205, 260}

$$-\frac{f^{3/2}\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{d}}\right)\left(-A(cd-af)^2+2bBd(cd-af)+Ab^2df\right)}{\sqrt{d}\left(f\left(a^2f+b^2d\right)-2acdf+c^2d^2\right)^2}-\frac{\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\left(-4Ab^2cf\left(3a^2f^2-3acdf+\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x)/((a + b*x + c*x^2)^2*(d + f*x^2)),x]
```

```
[Out] (A*b*c*(c*d + a*f) - (A*b - a*B)*(2*c^2*d + b^2*f - 2*a*c*f) - c*(A*b^2*f +
 2*A*c*(c*d - a*f) - b*B*(c*d + a*f))*x)/((b^2 - 4*a*c)*(b^2*d*f + (c*d - a
*f)^2)*(a + b*x + c*x^2)) - (f^(3/2)*(A*b^2*d*f + 2*b*B*d*(c*d - a*f) - A*(
c*d - a*f)^2)*ArcTan[(Sqrt[f]*x)/Sqrt[d]])/(Sqrt[d]*(c^2*d^2 - 2*a*c*d*f +
f*(b^2*d + a^2*f))^2) - ((b^5*B*d*f^2 - 2*A*b^4*f^2*(c*d - a*f) - 4*A*c^2*(
c*d - 3*a*f)*(c*d - a*f)^2 + b^3*B*f*(5*c^2*d^2 - 4*a*c*d*f - a^2*f^2) - 4*
A*b^2*c*f*(2*c^2*d^2 - 3*a*c*d*f + 3*a^2*f^2) + 2*b*B*c*(c^3*d^3 - 7*a*c^2*
d^2*f + 3*a^2*c*d*f^2 + 3*a^3*f^3))*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])
/((b^2 - 4*a*c)^(3/2)*(c^2*d^2 - 2*a*c*d*f + f*(b^2*d + a^2*f))^2) - (f*(2*
A*b*f*(c*d - a*f) + B*(c^2*d^2 - 2*a*c*d*f - f*(b^2*d - a^2*f)))*Log[a + b*
x + c*x^2])/(2*(c^2*d^2 - 2*a*c*d*f + f*(b^2*d + a^2*f))^2) + (f*(2*A*b*f*(
c*d - a*f) + B*(c^2*d^2 - 2*a*c*d*f - f*(b^2*d - a^2*f)))*Log[d + f*x^2])/(
2*(c^2*d^2 - 2*a*c*d*f + f*(b^2*d + a^2*f))^2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 1018

```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (f
_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((a + b*x + c*x^2)^(p + 1)*(d + f*x^2)^
(q + 1)*((g*c)*(-(b*(c*d + a*f))) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(2*a*f
)) + c*(g*(2*c^2*d + b^2*f - c*(2*a*f)) - h*(b*c*d + a*b*f))*x))/((b^2 - 4*
a*c)*(b^2*d*f + (c*d - a*f)^2)*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*(b^2*d*
f + (c*d - a*f)^2)*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + f*x^2)^q*Si
mp[(b*h - 2*g*c)*((c*d - a*f)^2 - (b*d)*(-(b*f)))*(p + 1) + (b^2*(g*f) - b*
(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(a*f*(p + 1) - c*d*(p + 2)) - (2*f*(
(g*c)*(-(b*(c*d + a*f))) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(2*a*f)))*(p +
q + 2) - (b^2*(g*f) - b*(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(b*f*(p + 1)
))*x - c*f*(b^2*(g*f) - b*(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(2*p + 2*q
 + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, g, h, q}, x] && NeQ[b^2 - 4*
a*c, 0] && LtQ[p, -1] && NeQ[b^2*d*f + (c*d - a*f)^2, 0] &&  !( !IntegerQ[p
] && ILtQ[q, -1])
```

Rule 1074

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*((d_) + (f_.)*(x_)^2)), x_Symbol] :> With[{q = c^2*d^2 + b^2*d*f - 2*a*c*d
*f + a^2*f^2}, Dist[1/q, Int[(A*c^2*d - a*c*C*d + A*b^2*f - a*b*B*f - a*A*c
*f + a^2*C*f + c*(B*c*d - b*C*d + A*b*f - a*B*f)*x)/(a + b*x + c*x^2), x],
x] + Dist[1/q, Int[(c*C*d^2 + b*B*d*f - A*c*d*f - a*C*d*f + a*A*f^2 - f*(B*
c*d - b*C*d + A*b*f - a*B*f)*x)/(d + f*x^2), x], x] /; NeQ[q, 0]] /; FreeQ[
{a, b, c, d, f, A, B, C}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{A+Bx}{\left(a+bx+cx^2\right)^2\left(d+fx^2\right)}\,dx = \frac{Abc(cd+af)-(Ab-aB)\left(2c^2d+b^2f-2acf\right)-c\left(Ab^2f+2Ac(cd-af)-\right.}{\left(b^2-4ac\right)\left(b^2df+(cd-af)^2\right)\left(a+bx+cx^2\right)}$$

$$= \frac{Abc(cd+af)-(Ab-aB)\left(2c^2d+b^2f-2acf\right)-c\left(Ab^2f+2Ac(cd-af)-\right.}{\left(b^2-4ac\right)\left(b^2df+(cd-af)^2\right)\left(a+bx+cx^2\right)}$$

$$= \frac{Abc(cd+af)-(Ab-aB)\left(2c^2d+b^2f-2acf\right)-c\left(Ab^2f+2Ac(cd-af)-\right.}{\left(b^2-4ac\right)\left(b^2df+(cd-af)^2\right)\left(a+bx+cx^2\right)}$$

$$= \frac{Abc(cd+af)-(Ab-aB)\left(2c^2d+b^2f-2acf\right)-c\left(Ab^2f+2Ac(cd-af)-\right.}{\left(b^2-4ac\right)\left(b^2df+(cd-af)^2\right)\left(a+bx+cx^2\right)}$$

$$= \frac{Abc(cd+af)-(Ab-aB)\left(2c^2d+b^2f-2acf\right)-c\left(Ab^2f+2Ac(cd-af)-\right.}{\left(b^2-4ac\right)\left(b^2df+(cd-af)^2\right)\left(a+bx+cx^2\right)}$$

Mathematica [A] time = 1.84, size = 523, normalized size = 0.88

$$\frac{f\log\left(d+fx^2\right)\left(B\left(f\left(a^2f-b^2d\right)-2acdf+c^2d^2\right)+2Abf(cd-af)\right)+f\log(a+x(b+cx))\left(B\left(f\left(b^2d-a^2f\right)\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x)/((a + b*x + c*x^2)^2*(d + f*x^2)),x]
```

```
[Out] ((-2*(c^2*d^2 - 2*a*c*d*f + f*(b^2*d + a^2*f))*(A*(b^3*f + b*c*(c*d - 3*a*f
) + b^2*c*f*x + 2*c^2*(c*d - a*f)*x) + B*(2*a^2*c*f - b*c^2*d*x - a*(2*c^2*
d + b^2*f + b*c*f*x))))/((b^2 - 4*a*c)*(a + x*(b + c*x))) + (2*f^(3/2)*(-(A
*b^2*d*f) + A*(c*d - a*f)^2 + 2*b*B*d*(-(c*d) + a*f))*ArcTan[(Sqrt[f]*x)/Sq
rt[d]])/Sqrt[d] - (2*(b^5*B*d*f^2 - 4*A*c^2*(c*d - 3*a*f)*(c*d - a*f)^2 + 2
*A*b^4*f^2*(-(c*d) + a*f) - b^3*B*f*(-5*c^2*d^2 + 4*a*c*d*f + a^2*f^2) - 4*
A*b^2*c*f*(2*c^2*d^2 - 3*a*c*d*f + 3*a^2*f^2) + 2*b*B*c*(c^3*d^3 - 7*a*c^2*
d^2*f + 3*a^2*c*d*f^2 + 3*a^3*f^3))*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])
/(-b^2 + 4*a*c)^(3/2) + f*(2*A*b*f*(c*d - a*f) + B*(c^2*d^2 - 2*a*c*d*f + f
*(-(b^2*d) + a^2*f)))*Log[d + f*x^2] + f*(2*A*b*f*(-(c*d) + a*f) + B*(-(c^2
*d^2) + 2*a*c*d*f + f*(b^2*d - a^2*f)))*Log[a + x*(b + c*x)])/(2*(c^2*d^2 -
 2*a*c*d*f + f*(b^2*d + a^2*f))^2)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(c*x^2+b*x+a)^2/(f*x^2+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 0.21, size = 1313, normalized size = 2.20

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(c*x^2+b*x+a)^2/(f*x^2+d),x, algorithm="giac")
```

```
[Out] -1/2*(B*c^2*d^2*f - B*b^2*d*f^2 - 2*B*a*c*d*f^2 + 2*A*b*c*d*f^2 + B*a^2*f^3
 - 2*A*a*b*f^3)*log(c*x^2 + b*x + a)/(c^4*d^4 + 2*b^2*c^2*d^3*f - 4*a*c^3*d
^3*f + b^4*d^2*f^2 - 4*a*b^2*c*d^2*f^2 + 6*a^2*c^2*d^2*f^2 + 2*a^2*b^2*d*f^
3 - 4*a^3*c*d*f^3 + a^4*f^4) + 1/2*(B*c^2*d^2*f - B*b^2*d*f^2 - 2*B*a*c*d*f
^2 + 2*A*b*c*d*f^2 + B*a^2*f^3 - 2*A*a*b*f^3)*log(f*x^2 + d)/(c^4*d^4 + 2*b
^2*c^2*d^3*f - 4*a*c^3*d^3*f + b^4*d^2*f^2 - 4*a*b^2*c*d^2*f^2 + 6*a^2*c^2*
d^2*f^2 + 2*a^2*b^2*d*f^3 - 4*a^3*c*d*f^3 + a^4*f^4) - (2*B*b*c*d^2*f^2 - A
*c^2*d^2*f^2 - 2*B*a*b*d*f^3 + A*b^2*d*f^3 + 2*A*a*c*d*f^3 - A*a^2*f^4)*arc
tan(f*x/sqrt(d*f))/((c^4*d^4 + 2*b^2*c^2*d^3*f - 4*a*c^3*d^3*f + b^4*d^2*f^
2 - 4*a*b^2*c*d^2*f^2 + 6*a^2*c^2*d^2*f^2 + 2*a^2*b^2*d*f^3 - 4*a^3*c*d*f^3
 + a^4*f^4)*sqrt(d*f)) + (2*B*b*c^4*d^3 - 4*A*c^5*d^3 + 5*B*b^3*c^2*d^2*f -
 14*B*a*b*c^3*d^2*f - 8*A*b^2*c^3*d^2*f + 20*A*a*c^4*d^2*f + B*b^5*d*f^2 -
4*B*a*b^3*c*d*f^2 - 2*A*b^4*c*d*f^2 + 6*B*a^2*b*c^2*d*f^2 + 12*A*a*b^2*c^2*
d*f^2 - 28*A*a^2*c^3*d*f^2 - B*a^2*b^3*f^3 + 2*A*a*b^4*f^3 + 6*B*a^3*b*c*f^
3 - 12*A*a^2*b^2*c*f^3 + 12*A*a^3*c^2*f^3)*arctan((2*c*x + b)/sqrt(-b^2 + 4
*a*c))/((b^2*c^4*d^4 - 4*a*c^5*d^4 + 2*b^4*c^2*d^3*f - 12*a*b^2*c^3*d^3*f +
 16*a^2*c^4*d^3*f + b^6*d^2*f^2 - 8*a*b^4*c*d^2*f^2 + 22*a^2*b^2*c^2*d^2*f^
2 - 24*a^3*c^3*d^2*f^2 + 2*a^2*b^4*d*f^3 - 12*a^3*b^2*c*d*f^3 + 16*a^4*c^2*
d*f^3 + a^4*b^2*f^4 - 4*a^5*c*f^4)*sqrt(-b^2 + 4*a*c)) + (2*B*a*c^4*d^3 - A
*b*c^4*d^3 + 3*B*a*b^2*c^2*d^2*f - 2*A*b^3*c^2*d^2*f - 6*B*a^2*c^3*d^2*f +
5*A*a*b*c^3*d^2*f + B*a*b^4*d*f^2 - A*b^5*d*f^2 - 4*B*a^2*b^2*c*d*f^2 + 5*A
*a*b^3*c*d*f^2 + 6*B*a^3*c^2*d*f^2 - 7*A*a^2*b*c^2*d*f^2 + B*a^3*b^2*f^3 -
A*a^2*b^3*f^3 - 2*B*a^4*c*f^3 + 3*A*a^3*b*c*f^3 + (B*b*c^4*d^3 - 2*A*c^5*d^
3 + B*b^3*c^2*d^2*f - B*a*b*c^3*d^2*f - 3*A*b^2*c^3*d^2*f + 6*A*a*c^4*d^2*f
 + B*a*b^3*c*d*f^2 - A*b^4*c*d*f^2 - B*a^2*b*c^2*d*f^2 + 4*A*a*b^2*c^2*d*f^
2 - 6*A*a^2*c^3*d*f^2 + B*a^3*b*c*f^3 - A*a^2*b^2*c*f^3 + 2*A*a^3*c^2*f^3)*
x)/((c^2*d^2 + b^2*d*f - 2*a*c*d*f + a^2*f^2)^2*(c*x^2 + b*x + a)*(b^2 - 4*
a*c))
```

maple [B] time = 0.06, size = 9311, normalized size = 15.62

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x+A)/(c*x^2+b*x+a)^2/(f*x^2+d),x)
```

```
[Out] result too large to display
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(c*x^2+b*x+a)^2/(f*x^2+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 7.53, size = 23006, normalized size = 38.60

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x)/((d + f*x^2)*(a + b*x + c*x^2)^2),x)
```

```
[Out] ((A*b^3*f + A*b*c^2*d - 2*B*a*c^2*d - B*a*b^2*f + 2*B*a^2*c*f - 3*A*a*b*c*f
)/((4*a*c - b^2)*(a^2*f^2 + c^2*d^2 + b^2*d*f - 2*a*c*d*f)) - (x*(2*A*a*c^2
```

```
*f - 2*A*c^3*d + B*b*c^2*d - A*b^2*c*f + B*a*b*c*f))/((4*a*c - b^2)*(a^2*f^
2 + c^2*d^2 + b^2*d*f - 2*a*c*d*f)))/(a + b*x + c*x^2) + symsum(log((x*(4*A
^3*b^3*c^4*f^6 + 16*B^3*a^3*c^4*f^6 - 3*B^3*a^2*b^2*c^3*f^6 + B^3*b^2*c^5*d
^2*f^4 - 16*A^3*a*b*c^5*f^6 + 20*A^2*B*a^2*c^5*f^6 - 3*A^2*B*b^4*c^3*f^6 +
4*A^2*B*c^7*d^2*f^4 - 16*B^3*a^2*c^5*d*f^5 + 6*B^3*a*b^2*c^4*d*f^5 - 24*A^2
*B*a*c^6*d*f^5 + 6*A*B^2*a*b^3*c^3*f^6 - 28*A*B^2*a^2*b*c^4*f^6 + 8*A^2*B*a
*b^2*c^4*f^6 - 4*A*B^2*b*c^6*d^2*f^4 - 6*A*B^2*b^3*c^4*d*f^5 + 8*A^2*B*b^2*
c^5*d*f^5 + 16*A*B^2*a*b*c^5*d*f^5))/(16*a^2*c^6*d^4 + a^4*b^4*f^4 + b^4*c^
4*d^4 + 16*a^6*c^2*f^4 + b^8*d^2*f^2 - 8*a*b^2*c^5*d^4 - 8*a^5*b^2*c*f^4 +
2*a^2*b^6*d*f^3 - 64*a^3*c^5*d^3*f - 64*a^5*c^3*d*f^3 + 2*b^6*c^2*d^3*f + 9
6*a^4*c^4*d^2*f^2 + 54*a^2*b^4*c^2*d^2*f^2 - 112*a^3*b^2*c^3*d^2*f^2 - 20*a
*b^4*c^3*d^3*f - 12*a*b^6*c*d^2*f^2 - 20*a^3*b^4*c*d*f^3 + 64*a^2*b^2*c^4*d
^3*f + 64*a^4*b^2*c^2*d*f^3) - root(2560*a^3*b^2*c^9*d^8*f*z^4 - 1152*a^2*b
^4*c^8*d^8*f*z^4 + 384*a^5*b^8*c*d^3*f^6*z^4 + 384*a*b^8*c^5*d^7*f^2*z^4 +
288*a^3*b^10*c*d^4*f^5*z^4 + 288*a*b^10*c^3*d^6*f^3*z^4 + 224*a^7*b^6*c*d^2
*f^7*z^4 - 192*a^10*b^2*c^2*d*f^8*z^4 + 224*a*b^6*c^7*d^8*f*z^4 + 80*a*b^12
*c*d^5*f^4*z^4 + 48*a^9*b^4*c*d*f^8*z^4 - 33920*a^6*b^2*c^6*d^5*f^4*z^4 + 2
7936*a^5*b^4*c^5*d^5*f^4*z^4 + 26112*a^7*b^2*c^5*d^4*f^5*z^4 + 26112*a^5*b^
2*c^7*d^6*f^3*z^4 - 20352*a^6*b^4*c^4*d^4*f^5*z^4 - 20352*a^4*b^4*c^6*d^6*f
^3*z^4 - 13080*a^4*b^6*c^4*d^5*f^4*z^4 - 11520*a^8*b^2*c^4*d^3*f^6*z^4 - 11
520*a^4*b^2*c^8*d^7*f^2*z^4 + 8736*a^5*b^6*c^3*d^4*f^5*z^4 + 8736*a^3*b^6*c
^5*d^6*f^3*z^4 + 7488*a^7*b^4*c^3*d^3*f^6*z^4 + 7488*a^3*b^4*c^7*d^7*f^2*z^
4 + 3840*a^3*b^8*c^3*d^5*f^4*z^4 + 2560*a^9*b^2*c^3*d^2*f^7*z^4 - 2416*a^6*
b^6*c^2*d^3*f^6*z^4 - 2416*a^2*b^6*c^6*d^7*f^2*z^4 - 2160*a^4*b^8*c^2*d^4*f
^5*z^4 - 2160*a^2*b^8*c^4*d^6*f^3*z^4 - 1152*a^8*b^4*c^2*d^2*f^7*z^4 - 720*
a^2*b^10*c^2*d^5*f^4*z^4 - 16*b^8*c^6*d^8*f*z^4 - 2048*a^4*c^10*d^8*f*z^4 +
 256*a^11*c^3*d*f^8*z^4 - 4*a^8*b^6*d*f^8*z^4 + 48*a*b^4*c^9*d^9*z^4 - 24*b
^10*c^4*d^7*f^2*z^4 - 16*b^12*c^2*d^6*f^3*z^4 + 17920*a^7*c^7*d^5*f^4*z^4 -
 14336*a^8*c^6*d^4*f^5*z^4 - 14336*a^6*c^8*d^6*f^3*z^4 + 7168*a^9*c^5*d^3*f
^6*z^4 + 7168*a^5*c^9*d^7*f^2*z^4 - 2048*a^10*c^4*d^2*f^7*z^4 - 24*a^4*b^10
*d^3*f^6*z^4 - 16*a^6*b^8*d^2*f^7*z^4 - 16*a^2*b^12*d^4*f^5*z^4 - 192*a^2*b
^2*c^10*d^9*z^4 - 4*b^14*d^5*f^4*z^4 - 4*b^6*c^8*d^9*z^4 + 256*a^3*c^11*d^9
*z^4 + 912*A*B*a^6*b*c^3*d*f^6*z^2 + 192*A*B*a^4*b^5*c*d*f^6*z^2 + 920*A*B*
a^4*b^3*c^3*d^2*f^5*z^2 - 480*A*B*a^2*b^5*c^3*d^3*f^4*z^2 - 336*A*B*a^2*b^3
*c^5*d^4*f^3*z^2 - 272*A*B*a^3*b^3*c^4*d^3*f^4*z^2 + 240*A*B*a^3*b^5*c^2*d^
2*f^5*z^2 + 192*A*B*a*b*c^8*d^6*f*z^2 - 2496*A*B*a^5*b*c^4*d^2*f^5*z^2 + 18
72*A*B*a^4*b*c^5*d^3*f^4*z^2 - 744*A*B*a^5*b^3*c^2*d*f^6*z^2 - 720*A*B*a^2*
b*c^7*d^5*f^2*z^2 + 504*A*B*a*b^3*c^6*d^5*f^2*z^2 + 256*A*B*a^3*b*c^6*d^4*f
^3*z^2 + 168*A*B*a*b^7*c^2*d^3*f^4*z^2 - 144*A*B*a^2*b^7*c*d^2*f^5*z^2 + 14
4*A*B*a*b^5*c^4*d^4*f^3*z^2 - 56*B^2*a*b^2*c^7*d^6*f*z^2 - 36*B^2*a^5*b^4*c
*d*f^6*z^2 - 16*B^2*a*b^8*c*d^3*f^4*z^2 - 164*A^2*a^3*b^6*c*d*f^6*z^2 - 16*
A^2*a*b^8*c*d^2*f^5*z^2 - 96*A*B*b^5*c^5*d^5*f^2*z^2 - 24*A*B*b^7*c^3*d^4*f
^3*z^2 - 580*B^2*a^4*b^2*c^4*d^3*f^4*z^2 + 536*B^2*a^3*b^4*c^3*d^3*f^4*z^2
- 348*B^2*a^4*b^4*c^2*d^2*f^5*z^2 + 316*B^2*a^2*b^2*c^6*d^5*f^2*z^2 + 200*B
^2*a^5*b^2*c^3*d^2*f^5*z^2 - 120*B^2*a^2*b^4*c^4*d^4*f^3*z^2 - 66*B^2*a^2*b
^6*c^2*d^3*f^4*z^2 - 16*B^2*a^3*b^2*c^5*d^4*f^3*z^2 + 1952*A^2*a^4*b^2*c^4*
d^2*f^5*z^2 - 1792*A^2*a^3*b^2*c^5*d^3*f^4*z^2 - 1272*A^2*a^3*b^4*c^3*d^2*f
^5*z^2 + 976*A^2*a^2*b^2*c^6*d^4*f^3*z^2 + 960*A^2*a^2*b^4*c^4*d^3*f^4*z^2
+ 282*A^2*a^2*b^6*c^2*d^2*f^5*z^2 - 72*A*B*b^3*c^7*d^6*f*z^2 - 16*A*B*b^9*c
*d^3*f^4*z^2 - 16*A*B*a^3*b^7*d*f^6*z^2 + 16*A*B*a*b^9*d^2*f^5*z^2 - 180*B^
2*a*b^4*c^5*d^5*f^2*z^2 + 132*B^2*a^6*b^2*c^2*d*f^6*z^2 + 108*B^2*a^3*b^6*c
*d^2*f^5*z^2 + 20*B^2*a*b^6*c^3*d^4*f^3*z^2 - 736*A^2*a^5*b^2*c^3*d*f^6*z^2
 + 624*A^2*a^4*b^4*c^2*d*f^6*z^2 - 416*A^2*a*b^2*c^7*d^5*f^2*z^2 - 276*A^2*
a*b^4*c^5*d^4*f^3*z^2 - 196*A^2*a*b^6*c^3*d^3*f^4*z^2 + 31*B^2*b^6*c^4*d^5*
f^2*z^2 + 2*B^2*b^8*c^2*d^4*f^3*z^2 - 768*B^2*a^5*c^5*d^3*f^4*z^2 + 512*B^2
*a^6*c^4*d^2*f^5*z^2 + 512*B^2*a^4*c^6*d^4*f^3*z^2 - 128*B^2*a^3*c^7*d^5*f^
2*z^2 + 80*A^2*b^4*c^6*d^5*f^2*z^2 + 31*A^2*b^6*c^4*d^4*f^3*z^2 + 14*A^2*b^
8*c^2*d^3*f^4*z^2 - 1152*A^2*a^3*c^7*d^4*f^3*z^2 + 1008*A^2*a^4*c^6*d^3*f^4
*z^2 + 624*A^2*a^2*c^8*d^5*f^2*z^2 - 288*A^2*a^5*c^5*d^2*f^5*z^2 - 10*B^2*a
```

^2*b^8*d^2*f^5*z^2 - 48*A^2*a^6*b^2*c^2*f^7*z^2 - 16*A*B*b*c^9*d^7*z^2 + 20
*B^2*b^4*c^6*d^6*f*z^2 - 128*B^2*a^7*c^3*d*f^6*z^2 + 64*A^2*b^2*c^8*d^6*f*z
^2 - 112*A^2*a^6*c^4*d*f^6*z^2 + 3*B^2*a^4*b^6*d*f^6*z^2 + 14*A^2*a^2*b^8*d
*f^6*z^2 + 12*A^2*a^5*b^4*c*f^7*z^2 - 160*A^2*a*c^9*d^6*f*z^2 + 3*B^2*b^10*
d^3*f^4*z^2 - A^2*b^10*d^2*f^5*z^2 + 64*A^2*a^7*c^3*f^7*z^2 + 4*B^2*b^2*c^8
*d^7*z^2 - A^2*a^4*b^6*f^7*z^2 + 16*A^2*c^10*d^7*z^2 - 160*A*B^2*a*b*c^6*d^
4*f^2*z + 112*A*B^2*a^4*b*c^3*d*f^5*z - 24*A*B^2*a^2*b^5*c*d*f^5*z + 480*A^
2*B*a^2*b^2*c^4*d^2*f^4*z - 176*A*B^2*a^2*b^3*c^3*d^2*f^4*z - 10*A^2*B*a*b^
6*c*d*f^5*z + 384*A*B^2*a^2*b*c^5*d^3*f^3*z - 352*A*B^2*a^3*b*c^4*d^2*f^4*z
 - 288*A^2*B*a*b^2*c^5*d^3*f^3*z - 160*A^2*B*a^3*b^2*c^3*d*f^5*z - 148*A^2*
B*a*b^4*c^3*d^2*f^4*z + 112*A*B^2*a*b^3*c^4*d^3*f^3*z + 72*A^2*B*a^2*b^4*c^
2*d*f^5*z + 72*A*B^2*a*b^5*c^2*d^2*f^4*z + 48*A*B^2*a^3*b^3*c^2*d*f^5*z + 4
8*B^3*a*b^2*c^5*d^4*f^2*z - 36*B^3*a^4*b^2*c^2*d*f^5*z - 4*B^3*a*b^4*c^3*d^
3*f^3*z - 480*A^3*a^2*b*c^5*d^2*f^4*z - 160*A^3*a^2*b^3*c^3*d*f^5*z + 128*A
^3*a*b^3*c^4*d^2*f^4*z + 112*A^2*B*b^4*c^4*d^3*f^3*z - 64*A*B^2*b^5*c^3*d^3
*f^3*z + 16*A^2*B*b^2*c^6*d^4*f^2*z + 16*A*B^2*b^3*c^5*d^4*f^2*z - A^2*B*b^
6*c^2*d^2*f^4*z + 448*A^2*B*a^3*c^5*d^2*f^4*z - 352*A^2*B*a^2*c^6*d^3*f^3*z
 - 48*A^2*B*a^4*b^2*c^2*f^6*z + 12*B^3*a^3*b^4*c*d*f^5*z - 10*B^3*a*b^6*c*d
^2*f^4*z + 416*A^3*a^3*b*c^4*d*f^5*z + 224*A^3*a*b*c^6*d^3*f^3*z + 24*A^3*a
*b^5*c^2*d*f^5*z - 2*A*B^2*b^7*c*d^2*f^4*z - 272*A^2*B*a^4*c^4*d*f^5*z + 12
8*A^2*B*a*c^7*d^4*f^2*z + 12*A^2*B*a^3*b^4*c*f^6*z - 120*B^3*a^2*b^2*c^4*d^
3*f^3*z + 112*B^3*a^3*b^2*c^3*d^2*f^4*z + 16*A*B^2*b*c^7*d^5*f*z + 2*A*B^2*
a*b^7*d*f^5*z - 2*A^3*b^7*c*d*f^5*z - 16*A^2*B*c^8*d^5*f*z + 11*B^3*b^6*c^2
*d^3*f^3*z - 8*B^3*b^4*c^4*d^4*f^2*z - 64*A^3*b^3*c^5*d^3*f^3*z + 96*A^3*a^
3*b^3*c^2*f^6*z - 4*B^3*b^2*c^6*d^5*f*z - 32*A^3*b*c^7*d^4*f^2*z - B^3*a^2*
b^6*d*f^5*z - 128*A^3*a^4*b*c^3*f^6*z - 24*A^3*a^2*b^5*c*f^6*z + 64*A^2*B*a
^5*c^3*f^6*z - A^2*B*a^2*b^6*f^6*z + A^2*B*b^8*d*f^5*z + 2*A^3*a*b^7*f^6*z
+ B^3*b^8*d^2*f^4*z + 32*A^3*B*a*b*c^4*d*f^4 - 18*A^2*B^2*a*b^2*c^3*d*f^4 +
 32*A*B^3*a*b*c^4*d^2*f^3 - 28*A*B^3*a^2*b*c^3*d*f^4 + 6*A*B^3*a*b^3*c^2*d*
f^4 - 10*A^3*B*b^3*c^3*d*f^4 - 4*A^3*B*b*c^5*d^2*f^3 - 4*A*B^3*b*c^5*d^3*f^
2 - 28*A^3*B*a^2*b*c^3*f^5 + 6*A^3*B*a*b^3*c^2*f^5 + 9*A^2*B^2*b^2*c^4*d^2*
f^3 - 3*A^2*B^2*a^2*b^2*c^2*f^5 - 10*B^4*a*b^2*c^3*d^2*f^3 - 3*B^4*a^2*b^2*
c^2*d*f^4 - 10*A*B^3*b^3*c^3*d^2*f^3 + 3*A^2*B^2*b^4*c^2*d*f^4 + 36*A^2*B^2
*a^2*c^4*d*f^4 - 24*A^2*B^2*a*c^5*d^2*f^3 + 4*A^2*B^2*c^6*d^3*f^2 + 16*A^2*
B^2*a^3*c^3*f^5 + 16*B^4*a^3*c^3*d*f^4 + 8*A^4*b^2*c^4*d*f^4 - 8*A^4*a*b^2*
c^3*f^5 - 24*A^4*a*c^5*d*f^4 + 3*B^4*b^4*c^2*d^2*f^3 + 4*A^4*c^6*d^2*f^3 +
36*A^4*a^2*c^4*f^5 + B^4*b^2*c^4*d^3*f^2, z, k)*(root(2560*a^3*b^2*c^9*d^8*
f*z^4 - 1152*a^2*b^4*c^8*d^8*f*z^4 + 384*a^5*b^8*c*d^3*f^6*z^4 + 384*a*b^8*
c^5*d^7*f^2*z^4 + 288*a^3*b^10*c*d^4*f^5*z^4 + 288*a*b^10*c^3*d^6*f^3*z^4 +
 224*a^7*b^6*c*d^2*f^7*z^4 - 192*a^10*b^2*c^2*d*f^8*z^4 + 224*a*b^6*c^7*d^8
*f*z^4 + 80*a*b^12*c*d^5*f^4*z^4 + 48*a^9*b^4*c*d*f^8*z^4 - 33920*a^6*b^2*c
^6*d^5*f^4*z^4 + 27936*a^5*b^4*c^5*d^5*f^4*z^4 + 26112*a^7*b^2*c^5*d^4*f^5*
z^4 + 26112*a^5*b^2*c^7*d^6*f^3*z^4 - 20352*a^6*b^4*c^4*d^4*f^5*z^4 - 20352
*a^4*b^4*c^6*d^6*f^3*z^4 - 13080*a^4*b^6*c^4*d^5*f^4*z^4 - 11520*a^8*b^2*c^
4*d^3*f^6*z^4 - 11520*a^4*b^2*c^8*d^7*f^2*z^4 + 8736*a^5*b^6*c^3*d^4*f^5*z^
4 + 8736*a^3*b^6*c^5*d^6*f^3*z^4 + 7488*a^7*b^4*c^3*d^3*f^6*z^4 + 7488*a^3*
b^4*c^7*d^7*f^2*z^4 + 3840*a^3*b^8*c^3*d^5*f^4*z^4 + 2560*a^9*b^2*c^3*d^2*f
^7*z^4 - 2416*a^6*b^6*c^2*d^3*f^6*z^4 - 2416*a^2*b^6*c^6*d^7*f^2*z^4 - 2160
*a^4*b^8*c^2*d^4*f^5*z^4 - 2160*a^2*b^8*c^4*d^6*f^3*z^4 - 1152*a^8*b^4*c^2*
d^2*f^7*z^4 - 720*a^2*b^10*c^2*d^5*f^4*z^4 - 16*b^8*c^6*d^8*f*z^4 - 2048*a^
4*c^10*d^8*f*z^4 + 256*a^11*c^3*d*f^8*z^4 - 4*a^8*b^6*d*f^8*z^4 + 48*a*b^4*
c^9*d^9*z^4 - 24*b^10*c^4*d^7*f^2*z^4 - 16*b^12*c^2*d^6*f^3*z^4 + 17920*a^7
*c^7*d^5*f^4*z^4 - 14336*a^8*c^6*d^4*f^5*z^4 - 14336*a^6*c^8*d^6*f^3*z^4 +
7168*a^9*c^5*d^3*f^6*z^4 + 7168*a^5*c^9*d^7*f^2*z^4 - 2048*a^10*c^4*d^2*f^7
*z^4 - 24*a^4*b^10*d^3*f^6*z^4 - 16*a^6*b^8*d^2*f^7*z^4 - 16*a^2*b^12*d^4*f
^5*z^4 - 192*a^2*b^2*c^10*d^9*z^4 - 4*b^14*d^5*f^4*z^4 - 4*b^6*c^8*d^9*z^4
+ 256*a^3*c^11*d^9*z^4 + 912*A*B*a^6*b*c^3*d*f^6*z^2 + 192*A*B*a^4*b^5*c*d*
f^6*z^2 + 920*A*B*a^4*b^3*c^3*d^2*f^5*z^2 - 480*A*B*a^2*b^5*c^3*d^3*f^4*z^2
 - 336*A*B*a^2*b^3*c^5*d^4*f^3*z^2 - 272*A*B*a^3*b^3*c^4*d^3*f^4*z^2 + 240*

A*B*a^3*b^5*c^2*d^2*f^5*z^2 + 192*A*B*a*b*c^8*d^6*f*z^2 - 2496*A*B*a^5*b*c^
4*d^2*f^5*z^2 + 1872*A*B*a^4*b*c^5*d^3*f^4*z^2 - 744*A*B*a^5*b^3*c^2*d*f^6*
z^2 - 720*A*B*a^2*b*c^7*d^5*f^2*z^2 + 504*A*B*a*b^3*c^6*d^5*f^2*z^2 + 256*A
*B*a^3*b*c^6*d^4*f^3*z^2 + 168*A*B*a*b^7*c^2*d^3*f^4*z^2 - 144*A*B*a^2*b^7*
c*d^2*f^5*z^2 + 144*A*B*a*b^5*c^4*d^4*f^3*z^2 - 56*B^2*a*b^2*c^7*d^6*f*z^2
- 36*B^2*a^5*b^4*c*d*f^6*z^2 - 16*B^2*a*b^8*c*d^3*f^4*z^2 - 164*A^2*a^3*b^6
*c*d*f^6*z^2 - 16*A^2*a*b^8*c*d^2*f^5*z^2 - 96*A*B*b^5*c^5*d^5*f^2*z^2 - 24
*A*B*b^7*c^3*d^4*f^3*z^2 - 580*B^2*a^4*b^2*c^4*d^3*f^4*z^2 + 536*B^2*a^3*b^
4*c^3*d^3*f^4*z^2 - 348*B^2*a^4*b^4*c^2*d^2*f^5*z^2 + 316*B^2*a^2*b^2*c^6*d
^5*f^2*z^2 + 200*B^2*a^5*b^2*c^3*d^2*f^5*z^2 - 120*B^2*a^2*b^4*c^4*d^4*f^3*
z^2 - 66*B^2*a^2*b^6*c^2*d^3*f^4*z^2 - 16*B^2*a^3*b^2*c^5*d^4*f^3*z^2 + 195
2*A^2*a^4*b^2*c^4*d^2*f^5*z^2 - 1792*A^2*a^3*b^2*c^5*d^3*f^4*z^2 - 1272*A^2
*a^3*b^4*c^3*d^2*f^5*z^2 + 976*A^2*a^2*b^2*c^6*d^4*f^3*z^2 + 960*A^2*a^2*b^
4*c^4*d^3*f^4*z^2 + 282*A^2*a^2*b^6*c^2*d^2*f^5*z^2 - 72*A*B*b^3*c^7*d^6*f*
z^2 - 16*A*B*b^9*c*d^3*f^4*z^2 - 16*A*B*a^3*b^7*d*f^6*z^2 + 16*A*B*a*b^9*d^
2*f^5*z^2 - 180*B^2*a*b^4*c^5*d^5*f^2*z^2 + 132*B^2*a^6*b^2*c^2*d*f^6*z^2 +
108*B^2*a^3*b^6*c*d^2*f^5*z^2 + 20*B^2*a*b^6*c^3*d^4*f^3*z^2 - 736*A^2*a^5
*b^2*c^3*d*f^6*z^2 + 624*A^2*a^4*b^4*c^2*d*f^6*z^2 - 416*A^2*a*b^2*c^7*d^5*
f^2*z^2 - 276*A^2*a*b^4*c^5*d^4*f^3*z^2 - 196*A^2*a*b^6*c^3*d^3*f^4*z^2 + 3
1*B^2*b^6*c^4*d^5*f^2*z^2 + 2*B^2*b^8*c^2*d^4*f^3*z^2 - 768*B^2*a^5*c^5*d^3
*f^4*z^2 + 512*B^2*a^6*c^4*d^2*f^5*z^2 + 512*B^2*a^4*c^6*d^4*f^3*z^2 - 128*
B^2*a^3*c^7*d^5*f^2*z^2 + 80*A^2*b^4*c^6*d^5*f^2*z^2 + 31*A^2*b^6*c^4*d^4*f
^3*z^2 + 14*A^2*b^8*c^2*d^3*f^4*z^2 - 1152*A^2*a^3*c^7*d^4*f^3*z^2 + 1008*A
^2*a^4*c^6*d^3*f^4*z^2 + 624*A^2*a^2*c^8*d^5*f^2*z^2 - 288*A^2*a^5*c^5*d^2*
f^5*z^2 - 10*B^2*a^2*b^8*d^2*f^5*z^2 - 48*A^2*a^6*b^2*c^2*f^7*z^2 - 16*A*B*
b*c^9*d^7*z^2 + 20*B^2*b^4*c^6*d^6*f*z^2 - 128*B^2*a^7*c^3*d*f^6*z^2 + 64*A
^2*b^2*c^8*d^6*f*z^2 - 112*A^2*a^6*c^4*d*f^6*z^2 + 3*B^2*a^4*b^6*d*f^6*z^2
+ 14*A^2*a^2*b^8*d*f^6*z^2 + 12*A^2*a^5*b^4*c*f^7*z^2 - 160*A^2*a*c^9*d^6*f
*z^2 + 3*B^2*b^10*d^3*f^4*z^2 - A^2*b^10*d^2*f^5*z^2 + 64*A^2*a^7*c^3*f^7*z
^2 + 4*B^2*b^2*c^8*d^7*z^2 - A^2*a^4*b^6*f^7*z^2 + 16*A^2*c^10*d^7*z^2 - 16
0*A*B^2*a*b*c^6*d^4*f^2*z + 112*A*B^2*a^4*b*c^3*d*f^5*z - 24*A*B^2*a^2*b^5*
c*d*f^5*z + 480*A^2*B*a^2*b^2*c^4*d^2*f^4*z - 176*A*B^2*a^2*b^3*c^3*d^2*f^4
*z - 10*A^2*B*a*b^6*c*d*f^5*z + 384*A*B^2*a^2*b*c^5*d^3*f^3*z - 352*A*B^2*a
^3*b*c^4*d^2*f^4*z - 288*A^2*B*a*b^2*c^5*d^3*f^3*z - 160*A^2*B*a^3*b^2*c^3*
d*f^5*z - 148*A^2*B*a*b^4*c^3*d^2*f^4*z + 112*A*B^2*a*b^3*c^4*d^3*f^3*z + 7
2*A^2*B*a^2*b^4*c^2*d*f^5*z + 72*A*B^2*a*b^5*c^2*d^2*f^4*z + 48*A*B^2*a^3*b
^3*c^2*d*f^5*z + 48*B^3*a*b^2*c^5*d^4*f^2*z - 36*B^3*a^4*b^2*c^2*d*f^5*z -
4*B^3*a*b^4*c^3*d^3*f^3*z - 480*A^3*a^2*b*c^5*d^2*f^4*z - 160*A^3*a^2*b^3*c
^3*d*f^5*z + 128*A^3*a*b^3*c^4*d^2*f^4*z + 112*A^2*B*b^4*c^4*d^3*f^3*z - 64
*A*B^2*b^5*c^3*d^3*f^3*z + 16*A^2*B*b^2*c^6*d^4*f^2*z + 16*A*B^2*b^3*c^5*d^
4*f^2*z - A^2*B*b^6*c^2*d^2*f^4*z + 448*A^2*B*a^3*c^5*d^2*f^4*z - 352*A^2*B
*a^2*c^6*d^3*f^3*z - 48*A^2*B*a^4*b^2*c^2*f^6*z + 12*B^3*a^3*b^4*c*d*f^5*z
- 10*B^3*a*b^6*c*d^2*f^4*z + 416*A^3*a^3*b*c^4*d*f^5*z + 224*A^3*a*b*c^6*d^
3*f^3*z + 24*A^3*a*b^5*c^2*d*f^5*z - 2*A*B^2*b^7*c*d^2*f^4*z - 272*A^2*B*a^
4*c^4*d*f^5*z + 128*A^2*B*a*c^7*d^4*f^2*z + 12*A^2*B*a^3*b^4*c*f^6*z - 120*
B^3*a^2*b^2*c^4*d^3*f^3*z + 112*B^3*a^3*b^2*c^3*d^2*f^4*z + 16*A*B^2*b*c^7*
d^5*f*z + 2*A*B^2*a*b^7*d*f^5*z - 2*A^3*b^7*c*d*f^5*z - 16*A^2*B*c^8*d^5*f*
z + 11*B^3*b^6*c^2*d^3*f^3*z - 8*B^3*b^4*c^4*d^4*f^2*z - 64*A^3*b^3*c^5*d^3
*f^3*z + 96*A^3*a^3*b^3*c^2*f^6*z - 4*B^3*b^2*c^6*d^5*f*z - 32*A^3*b*c^7*d^
4*f^2*z - B^3*a^2*b^6*d*f^5*z - 128*A^3*a^4*b*c^3*f^6*z - 24*A^3*a^2*b^5*c*
f^6*z + 64*A^2*B*a^5*c^3*f^6*z - A^2*B*a^2*b^6*f^6*z + A^2*B*b^8*d*f^5*z +
2*A^3*a*b^7*f^6*z + B^3*b^8*d^2*f^4*z + 32*A^3*B*a*b*c^4*d*f^4 - 18*A^2*B^2
*a*b^2*c^3*d*f^4 + 32*A*B^3*a*b*c^4*d^2*f^3 - 28*A*B^3*a^2*b*c^3*d*f^4 + 6*
A*B^3*a*b^3*c^2*d*f^4 - 10*A^3*B*b^3*c^3*d*f^4 - 4*A^3*B*b*c^5*d^2*f^3 - 4*
A*B^3*b*c^5*d^3*f^2 - 28*A^3*B*a^2*b*c^3*f^5 + 6*A^3*B*a*b^3*c^2*f^5 + 9*A^
2*B^2*b^2*c^4*d^2*f^3 - 3*A^2*B^2*a^2*b^2*c^2*f^5 - 10*B^4*a*b^2*c^3*d^2*f^
3 - 3*B^4*a^2*b^2*c^2*d*f^4 - 10*A*B^3*b^3*c^3*d^2*f^3 + 3*A^2*B^2*b^4*c^2*
d*f^4 + 36*A^2*B^2*a^2*c^4*d*f^4 - 24*A^2*B^2*a*c^5*d^2*f^3 + 4*A^2*B^2*c^6
*d^3*f^2 + 16*A^2*B^2*a^3*c^3*f^5 + 16*B^4*a^3*c^3*d*f^4 + 8*A^4*b^2*c^4*d*

f^4 - 8*A^4*a*b^2*c^3*f^5 - 24*A^4*a*c^5*d*f^4 + 3*B^4*b^4*c^2*d^2*f^3 + 4*
A^4*c^6*d^2*f^3 + 36*A^4*a^2*c^4*f^5 + B^4*b^2*c^4*d^3*f^2, z, k)*(root(256
0*a^3*b^2*c^9*d^8*f*z^4 - 1152*a^2*b^4*c^8*d^8*f*z^4 + 384*a^5*b^8*c*d^3*f^
6*z^4 + 384*a*b^8*c^5*d^7*f^2*z^4 + 288*a^3*b^10*c*d^4*f^5*z^4 + 288*a*b^10
*c^3*d^6*f^3*z^4 + 224*a^7*b^6*c*d^2*f^7*z^4 - 192*a^10*b^2*c^2*d*f^8*z^4 +
224*a*b^6*c^7*d^8*f*z^4 + 80*a*b^12*c*d^5*f^4*z^4 + 48*a^9*b^4*c*d*f^8*z^4
- 33920*a^6*b^2*c^6*d^5*f^4*z^4 + 27936*a^5*b^4*c^5*d^5*f^4*z^4 + 26112*a^
7*b^2*c^5*d^4*f^5*z^4 + 26112*a^5*b^2*c^7*d^6*f^3*z^4 - 20352*a^6*b^4*c^4*d
^4*f^5*z^4 - 20352*a^4*b^4*c^6*d^6*f^3*z^4 - 13080*a^4*b^6*c^4*d^5*f^4*z^4
- 11520*a^8*b^2*c^4*d^3*f^6*z^4 - 11520*a^4*b^2*c^8*d^7*f^2*z^4 + 8736*a^5*
b^6*c^3*d^4*f^5*z^4 + 8736*a^3*b^6*c^5*d^6*f^3*z^4 + 7488*a^7*b^4*c^3*d^3*f
^6*z^4 + 7488*a^3*b^4*c^7*d^7*f^2*z^4 + 3840*a^3*b^8*c^3*d^5*f^4*z^4 + 2560
*a^9*b^2*c^3*d^2*f^7*z^4 - 2416*a^6*b^6*c^2*d^3*f^6*z^4 - 2416*a^2*b^6*c^6*
d^7*f^2*z^4 - 2160*a^4*b^8*c^2*d^4*f^5*z^4 - 2160*a^2*b^8*c^4*d^6*f^3*z^4 -
1152*a^8*b^4*c^2*d^2*f^7*z^4 - 720*a^2*b^10*c^2*d^5*f^4*z^4 - 16*b^8*c^6*d
^8*f*z^4 - 2048*a^4*c^10*d^8*f*z^4 + 256*a^11*c^3*d*f^8*z^4 - 4*a^8*b^6*d*f
^8*z^4 + 48*a*b^4*c^9*d^9*z^4 - 24*b^10*c^4*d^7*f^2*z^4 - 16*b^12*c^2*d^6*f
^3*z^4 + 17920*a^7*c^7*d^5*f^4*z^4 - 14336*a^8*c^6*d^4*f^5*z^4 - 14336*a^6*
c^8*d^6*f^3*z^4 + 7168*a^9*c^5*d^3*f^6*z^4 + 7168*a^5*c^9*d^7*f^2*z^4 - 204
8*a^10*c^4*d^2*f^7*z^4 - 24*a^4*b^10*d^3*f^6*z^4 - 16*a^6*b^8*d^2*f^7*z^4 -
16*a^2*b^12*d^4*f^5*z^4 - 192*a^2*b^2*c^10*d^9*z^4 - 4*b^14*d^5*f^4*z^4 -
4*b^6*c^8*d^9*z^4 + 256*a^3*c^11*d^9*z^4 + 912*A*B*a^6*b*c^3*d*f^6*z^2 + 19
2*A*B*a^4*b^5*c*d*f^6*z^2 + 920*A*B*a^4*b^3*c^3*d^2*f^5*z^2 - 480*A*B*a^2*b
^5*c^3*d^3*f^4*z^2 - 336*A*B*a^2*b^3*c^5*d^4*f^3*z^2 - 272*A*B*a^3*b^3*c^4*
d^3*f^4*z^2 + 240*A*B*a^3*b^5*c^2*d^2*f^5*z^2 + 192*A*B*a*b*c^8*d^6*f*z^2 -
2496*A*B*a^5*b*c^4*d^2*f^5*z^2 + 1872*A*B*a^4*b*c^5*d^3*f^4*z^2 - 744*A*B*
a^5*b^3*c^2*d*f^6*z^2 - 720*A*B*a^2*b*c^7*d^5*f^2*z^2 + 504*A*B*a*b^3*c^6*d
^5*f^2*z^2 + 256*A*B*a^3*b*c^6*d^4*f^3*z^2 + 168*A*B*a*b^7*c^2*d^3*f^4*z^2
- 144*A*B*a^2*b^7*c*d^2*f^5*z^2 + 144*A*B*a*b^5*c^4*d^4*f^3*z^2 - 56*B^2*a*
b^2*c^7*d^6*f*z^2 - 36*B^2*a^5*b^4*c*d*f^6*z^2 - 16*B^2*a*b^8*c*d^3*f^4*z^2
- 164*A^2*a^3*b^6*c*d*f^6*z^2 - 16*A^2*a*b^8*c*d^2*f^5*z^2 - 96*A*B*b^5*c^
5*d^5*f^2*z^2 - 24*A*B*b^7*c^3*d^4*f^3*z^2 - 580*B^2*a^4*b^2*c^4*d^3*f^4*z^
2 + 536*B^2*a^3*b^4*c^3*d^3*f^4*z^2 - 348*B^2*a^4*b^4*c^2*d^2*f^5*z^2 + 316
*B^2*a^2*b^2*c^6*d^5*f^2*z^2 + 200*B^2*a^5*b^2*c^3*d^2*f^5*z^2 - 120*B^2*a^
2*b^4*c^4*d^4*f^3*z^2 - 66*B^2*a^2*b^6*c^2*d^3*f^4*z^2 - 16*B^2*a^3*b^2*c^5
*d^4*f^3*z^2 + 1952*A^2*a^4*b^2*c^4*d^2*f^5*z^2 - 1792*A^2*a^3*b^2*c^5*d^3*
f^4*z^2 - 1272*A^2*a^3*b^4*c^3*d^2*f^5*z^2 + 976*A^2*a^2*b^2*c^6*d^4*f^3*z^
2 + 960*A^2*a^2*b^4*c^4*d^3*f^4*z^2 + 282*A^2*a^2*b^6*c^2*d^2*f^5*z^2 - 72*
A*B*b^3*c^7*d^6*f*z^2 - 16*A*B*b^9*c*d^3*f^4*z^2 - 16*A*B*a^3*b^7*d*f^6*z^2
+ 16*A*B*a*b^9*d^2*f^5*z^2 - 180*B^2*a*b^4*c^5*d^5*f^2*z^2 + 132*B^2*a^6*b
^2*c^2*d*f^6*z^2 + 108*B^2*a^3*b^6*c*d^2*f^5*z^2 + 20*B^2*a*b^6*c^3*d^4*f^3
*z^2 - 736*A^2*a^5*b^2*c^3*d*f^6*z^2 + 624*A^2*a^4*b^4*c^2*d*f^6*z^2 - 416*
A^2*a*b^2*c^7*d^5*f^2*z^2 - 276*A^2*a*b^4*c^5*d^4*f^3*z^2 - 196*A^2*a*b^6*c
^3*d^3*f^4*z^2 + 31*B^2*b^6*c^4*d^5*f^2*z^2 + 2*B^2*b^8*c^2*d^4*f^3*z^2 - 7
68*B^2*a^5*c^5*d^3*f^4*z^2 + 512*B^2*a^6*c^4*d^2*f^5*z^2 + 512*B^2*a^4*c^6*
d^4*f^3*z^2 - 128*B^2*a^3*c^7*d^5*f^2*z^2 + 80*A^2*b^4*c^6*d^5*f^2*z^2 + 31
*A^2*b^6*c^4*d^4*f^3*z^2 + 14*A^2*b^8*c^2*d^3*f^4*z^2 - 1152*A^2*a^3*c^7*d^
4*f^3*z^2 + 1008*A^2*a^4*c^6*d^3*f^4*z^2 + 624*A^2*a^2*c^8*d^5*f^2*z^2 - 28
8*A^2*a^5*c^5*d^2*f^5*z^2 - 10*B^2*a^2*b^8*d^2*f^5*z^2 - 48*A^2*a^6*b^2*c^2
*f^7*z^2 - 16*A*B*b*c^9*d^7*z^2 + 20*B^2*b^4*c^6*d^6*f*z^2 - 128*B^2*a^7*c^
3*d*f^6*z^2 + 64*A^2*b^2*c^8*d^6*f*z^2 - 112*A^2*a^6*c^4*d*f^6*z^2 + 3*B^2*
a^4*b^6*d*f^6*z^2 + 14*A^2*a^2*b^8*d*f^6*z^2 + 12*A^2*a^5*b^4*c*f^7*z^2 - 1
60*A^2*a*c^9*d^6*f*z^2 + 3*B^2*b^10*d^3*f^4*z^2 - A^2*b^10*d^2*f^5*z^2 + 64
*A^2*a^7*c^3*f^7*z^2 + 4*B^2*b^2*c^8*d^7*z^2 - A^2*a^4*b^6*f^7*z^2 + 16*A^2
*c^10*d^7*z^2 - 160*A*B^2*a*b*c^6*d^4*f^2*z + 112*A*B^2*a^4*b*c^3*d*f^5*z -
24*A*B^2*a^2*b^5*c*d*f^5*z + 480*A^2*B*a^2*b^2*c^4*d^2*f^4*z - 176*A*B^2*a
^2*b^3*c^3*d^2*f^4*z - 10*A^2*B*a*b^6*c*d*f^5*z + 384*A*B^2*a^2*b*c^5*d^3*f
^3*z - 352*A*B^2*a^3*b*c^4*d^2*f^4*z - 288*A^2*B*a*b^2*c^5*d^3*f^3*z - 160*
A^2*B*a^3*b^2*c^3*d*f^5*z - 148*A^2*B*a*b^4*c^3*d^2*f^4*z + 112*A*B^2*a*b^3

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*c^4*d^3*f^3*z + 72*A^2*B*a^2*b^4*c^2*d*f^5*z + 72*A*B^2*a*b^5*c^2*d^2*f^4*
z + 48*A*B^2*a^3*b^3*c^2*d*f^5*z + 48*B^3*a*b^2*c^5*d^4*f^2*z - 36*B^3*a^4*
b^2*c^2*d*f^5*z - 4*B^3*a*b^4*c^3*d^3*f^3*z - 480*A^3*a^2*b*c^5*d^2*f^4*z -
 160*A^3*a^2*b^3*c^3*d*f^5*z + 128*A^3*a*b^3*c^4*d^2*f^4*z + 112*A^2*B*b^4*
c^4*d^3*f^3*z - 64*A*B^2*b^5*c^3*d^3*f^3*z + 16*A^2*B*b^2*c^6*d^4*f^2*z + 1
6*A*B^2*b^3*c^5*d^4*f^2*z - A^2*B*b^6*c^2*d^2*f^4*z + 448*A^2*B*a^3*c^5*d^2
*f^4*z - 352*A^2*B*a^2*c^6*d^3*f^3*z - 48*A^2*B*a^4*b^2*c^2*f^6*z + 12*B^3*
a^3*b^4*c*d*f^5*z - 10*B^3*a*b^6*c*d^2*f^4*z + 416*A^3*a^3*b*c^4*d*f^5*z +
224*A^3*a*b*c^6*d^3*f^3*z + 24*A^3*a*b^5*c^2*d*f^5*z - 2*A*B^2*b^7*c*d^2*f^
4*z - 272*A^2*B*a^4*c^4*d*f^5*z + 128*A^2*B*a*c^7*d^4*f^2*z + 12*A^2*B*a^3*
b^4*c*f^6*z - 120*B^3*a^2*b^2*c^4*d^3*f^3*z + 112*B^3*a^3*b^2*c^3*d^2*f^4*z
 + 16*A*B^2*b*c^7*d^5*f*z + 2*A*B^2*a*b^7*d*f^5*z - 2*A^3*b^7*c*d*f^5*z - 1
6*A^2*B*c^8*d^5*f*z + 11*B^3*b^6*c^2*d^3*f^3*z - 8*B^3*b^4*c^4*d^4*f^2*z -
64*A^3*b^3*c^5*d^3*f^3*z + 96*A^3*a^3*b^3*c^2*f^6*z - 4*B^3*b^2*c^6*d^5*f*z
 - 32*A^3*b*c^7*d^4*f^2*z - B^3*a^2*b^6*d*f^5*z - 128*A^3*a^4*b*c^3*f^6*z -
 24*A^3*a^2*b^5*c*f^6*z + 64*A^2*B*a^5*c^3*f^6*z - A^2*B*a^2*b^6*f^6*z + A^
2*B*b^8*d*f^5*z + 2*A^3*a*b^7*f^6*z + B^3*b^8*d^2*f^4*z + 32*A^3*B*a*b*c^4*
d*f^4 - 18*A^2*B^2*a*b^2*c^3*d*f^4 + 32*A*B^3*a*b*c^4*d^2*f^3 - 28*A*B^3*a^
2*b*c^3*d*f^4 + 6*A*B^3*a*b^3*c^2*d*f^4 - 10*A^3*B*b^3*c^3*d*f^4 - 4*A^3*B*
b*c^5*d^2*f^3 - 4*A*B^3*b*c^5*d^3*f^2 - 28*A^3*B*a^2*b*c^3*f^5 + 6*A^3*B*a*
b^3*c^2*f^5 + 9*A^2*B^2*b^2*c^4*d^2*f^3 - 3*A^2*B^2*a^2*b^2*c^2*f^5 - 10*B^
4*a*b^2*c^3*d^2*f^3 - 3*B^4*a^2*b^2*c^2*d*f^4 - 10*A*B^3*b^3*c^3*d^2*f^3 +
3*A^2*B^2*b^4*c^2*d*f^4 + 36*A^2*B^2*a^2*c^4*d*f^4 - 24*A^2*B^2*a*c^5*d^2*f
^3 + 4*A^2*B^2*c^6*d^3*f^2 + 16*A^2*B^2*a^3*c^3*f^5 + 16*B^4*a^3*c^3*d*f^4
+ 8*A^4*b^2*c^4*d*f^4 - 8*A^4*a*b^2*c^3*f^5 - 24*A^4*a*c^5*d*f^4 + 3*B^4*b^
4*c^2*d^2*f^3 + 4*A^4*c^6*d^2*f^3 + 36*A^4*a^2*c^4*f^5 + B^4*b^2*c^4*d^3*f^
2, z, k)*((4*b^5*c^8*d^7*f^2 + 4*b^7*c^6*d^6*f^3 - 4*b^9*c^4*d^5*f^4 - 4*b^
11*c^2*d^4*f^5 - 612*a^2*b^5*c^6*d^5*f^4 - 712*a^2*b^7*c^4*d^4*f^5 - 132*a^
2*b^9*c^2*d^3*f^6 + 1696*a^3*b^3*c^7*d^5*f^4 + 2736*a^3*b^5*c^5*d^4*f^5 + 8
96*a^3*b^7*c^3*d^3*f^6 - 5120*a^4*b^3*c^6*d^4*f^5 - 3140*a^4*b^5*c^4*d^3*f^
6 - 220*a^4*b^7*c^2*d^2*f^7 + 5664*a^5*b^3*c^5*d^3*f^6 + 1128*a^5*b^5*c^3*d
^2*f^7 - 2560*a^6*b^3*c^4*d^2*f^7 + 8*a*b^11*c*d^3*f^6 + 8*a^5*b^7*c*d*f^8
- 448*a^8*b*c^4*d*f^8 - 32*a*b^3*c^9*d^7*f^2 - 24*a*b^5*c^7*d^6*f^3 + 88*a*
b^7*c^5*d^5*f^4 + 88*a*b^9*c^3*d^4*f^5 + 64*a^2*b*c^10*d^7*f^2 + 128*a^3*b*
c^9*d^6*f^3 + 16*a^3*b^9*c*d^2*f^7 - 1600*a^4*b*c^8*d^5*f^4 + 3840*a^5*b*c^
7*d^4*f^5 - 4160*a^6*b*c^6*d^3*f^6 - 92*a^6*b^5*c^2*d*f^8 + 2176*a^7*b*c^5*
d^2*f^7 + 352*a^7*b^3*c^3*d*f^8)/(16*a^2*c^6*d^4 + a^4*b^4*f^4 + b^4*c^4*d^
4 + 16*a^6*c^2*f^4 + b^8*d^2*f^2 - 8*a*b^2*c^5*d^4 - 8*a^5*b^2*c*f^4 + 2*a^
2*b^6*d*f^3 - 64*a^3*c^5*d^3*f - 64*a^5*c^3*d*f^3 + 2*b^6*c^2*d^3*f + 96*a^
4*c^4*d^2*f^2 + 54*a^2*b^4*c^2*d^2*f^2 - 112*a^3*b^2*c^3*d^2*f^2 - 20*a*b^4
*c^3*d^3*f - 12*a*b^6*c*d^2*f^2 - 20*a^3*b^4*c*d*f^3 + 64*a^2*b^2*c^4*d^3*f
 + 64*a^4*b^2*c^2*d*f^3) + (x*(128*a^9*c^4*f^9 - 2*a^6*b^6*c*f^9 - 640*a^8*
c^5*d*f^8 + 6*b^12*c*d^3*f^6 + 24*a^7*b^4*c^2*f^9 - 96*a^8*b^2*c^3*f^9 + 12
8*a^2*c^11*d^7*f^2 - 640*a^3*c^10*d^6*f^3 + 1152*a^4*c^9*d^5*f^4 - 640*a^5*
c^8*d^4*f^5 - 640*a^6*c^7*d^3*f^6 + 1152*a^7*c^6*d^2*f^7 + 8*b^4*c^9*d^7*f^
2 + 22*b^6*c^7*d^6*f^3 + 26*b^8*c^5*d^5*f^4 + 18*b^10*c^3*d^4*f^5 + 672*a^2
*b^2*c^9*d^6*f^3 + 1224*a^2*b^4*c^7*d^5*f^4 + 1202*a^2*b^6*c^5*d^4*f^5 + 56
4*a^2*b^8*c^3*d^3*f^6 - 2048*a^3*b^2*c^8*d^5*f^4 - 2744*a^3*b^4*c^6*d^4*f^5
 - 1736*a^3*b^6*c^4*d^3*f^6 - 128*a^3*b^8*c^2*d^2*f^7 + 2656*a^4*b^2*c^7*d^
4*f^5 + 2648*a^4*b^4*c^5*d^3*f^6 + 570*a^4*b^6*c^3*d^2*f^7 - 1344*a^5*b^2*c
^6*d^3*f^6 - 904*a^5*b^4*c^4*d^2*f^7 - 160*a^6*b^2*c^5*d^2*f^7 + 2*a^4*b^8*
c*d*f^8 - 64*a*b^2*c^10*d^7*f^2 - 216*a*b^4*c^8*d^6*f^3 - 300*a*b^6*c^6*d^5
*f^4 - 240*a*b^8*c^4*d^4*f^5 - 92*a*b^10*c^2*d^3*f^6 + 10*a^2*b^10*c*d^2*f^
7 - 12*a^5*b^6*c^2*d*f^8 - 40*a^6*b^4*c^3*d*f^8 + 384*a^7*b^2*c^4*d*f^8))/(
16*a^2*c^6*d^4 + a^4*b^4*f^4 + b^4*c^4*d^4 + 16*a^6*c^2*f^4 + b^8*d^2*f^2 -
 8*a*b^2*c^5*d^4 - 8*a^5*b^2*c*f^4 + 2*a^2*b^6*d*f^3 - 64*a^3*c^5*d^3*f - 6
4*a^5*c^3*d*f^3 + 2*b^6*c^2*d^3*f + 96*a^4*c^4*d^2*f^2 + 54*a^2*b^4*c^2*d^2
*f^2 - 112*a^3*b^2*c^3*d^2*f^2 - 20*a*b^4*c^3*d^3*f - 12*a*b^6*c*d^2*f^2 -
20*a^3*b^4*c*d*f^3 + 64*a^2*b^2*c^4*d^3*f + 64*a^4*b^2*c^2*d*f^3)) + (A*a^4
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*b^6*c*f^8 - 64*A*a^7*c^4*f^8 - 32*A*a*c^10*d^6*f^2 + 352*A*a^6*c^5*d*f^7 +
 A*b^10*c*d^2*f^6 - 12*A*a^5*b^4*c^2*f^8 + 48*A*a^6*b^2*c^3*f^8 + 224*A*a^2
*c^9*d^5*f^3 - 640*A*a^3*c^8*d^4*f^4 + 960*A*a^4*c^7*d^3*f^5 - 800*A*a^5*c^
6*d^2*f^6 + 8*A*b^2*c^9*d^6*f^2 + 16*A*b^4*c^7*d^5*f^3 + A*b^6*c^5*d^4*f^4
 - 6*A*b^8*c^3*d^3*f^5 - 4*B*b^3*c^8*d^6*f^2 - 12*B*b^5*c^6*d^5*f^3 - 4*B*b^
7*c^4*d^4*f^4 + 4*B*b^9*c^2*d^3*f^5 - 120*A*a*b^2*c^8*d^5*f^3 - 60*A*a*b^4*
c^6*d^4*f^4 + 36*A*a*b^6*c^4*d^3*f^5 - 8*A*a*b^8*c^2*d^2*f^6 - 20*A*a^3*b^6
*c^2*d*f^7 + 80*A*a^4*b^4*c^3*d*f^7 - 216*A*a^5*b^2*c^4*d*f^7 + 92*B*a*b^3*
c^7*d^5*f^3 + 72*B*a*b^5*c^5*d^4*f^4 - 20*B*a*b^7*c^3*d^3*f^5 - 176*B*a^2*b
*c^8*d^5*f^3 + 544*B*a^3*b*c^7*d^4*f^4 - 736*B*a^4*b*c^6*d^3*f^5 - 4*B*a^4*
b^5*c^2*d*f^7 + 464*B*a^5*b*c^5*d^2*f^6 + 44*B*a^5*b^3*c^3*d*f^7 + 384*A*a^
2*b^2*c^7*d^4*f^4 + 32*A*a^2*b^4*c^5*d^3*f^5 + 14*A*a^2*b^6*c^3*d^2*f^6 - 5
60*A*a^3*b^2*c^6*d^3*f^5 - 56*A*a^3*b^4*c^4*d^2*f^6 + 456*A*a^4*b^2*c^5*d^2
*f^6 - 360*B*a^2*b^3*c^6*d^4*f^4 - 64*B*a^2*b^5*c^4*d^3*f^5 + 504*B*a^3*b^3
*c^5*d^3*f^5 + 40*B*a^3*b^5*c^3*d^2*f^6 - 276*B*a^4*b^3*c^4*d^2*f^6 + 2*A*a
^2*b^8*c*d*f^7 + 16*B*a*b*c^9*d^6*f^2 - 112*B*a^6*b*c^4*d*f^7)/(16*a^2*c^6*
d^4 + a^4*b^4*f^4 + b^4*c^4*d^4 + 16*a^6*c^2*f^4 + b^8*d^2*f^2 - 8*a*b^2*c^
5*d^4 - 8*a^5*b^2*c*f^4 + 2*a^2*b^6*d*f^3 - 64*a^3*c^5*d^3*f - 64*a^5*c^3*d
*f^3 + 2*b^6*c^2*d^3*f + 96*a^4*c^4*d^2*f^2 + 54*a^2*b^4*c^2*d^2*f^2 - 112*
a^3*b^2*c^3*d^2*f^2 - 20*a*b^4*c^3*d^3*f - 12*a*b^6*c*d^2*f^2 - 20*a^3*b^4*
c*d*f^3 + 64*a^2*b^2*c^4*d^3*f + 64*a^4*b^2*c^2*d*f^3) + (x*(64*B*a^7*c^4*f
^8 + 4*A*a^3*b^7*c*f^8 - 256*A*a^6*b*c^4*f^8 - B*a^4*b^6*c*f^8 - 320*B*a^6*
c^5*d*f^7 + 3*B*b^10*c*d^2*f^6 - 48*A*a^4*b^5*c^2*f^8 + 192*A*a^5*b^3*c^3*f
^8 + 12*B*a^5*b^4*c^2*f^8 - 48*B*a^6*b^2*c^3*f^8 - 16*A*b^3*c^8*d^5*f^3 - 4
8*A*b^5*c^6*d^4*f^4 - 36*A*b^7*c^4*d^3*f^5 - 4*A*b^9*c^2*d^2*f^6 - 64*B*a^2
*c^9*d^5*f^3 + 320*B*a^3*c^8*d^4*f^4 - 640*B*a^4*c^7*d^3*f^5 + 640*B*a^5*c^
6*d^2*f^6 + 4*B*b^4*c^7*d^5*f^3 + 23*B*b^6*c^5*d^4*f^4 + 22*B*b^8*c^3*d^3*f
^5 + 320*A*a*b^3*c^7*d^4*f^4 + 352*A*a*b^5*c^5*d^3*f^5 + 76*A*a*b^7*c^3*d^2
*f^6 - 512*A*a^2*b*c^8*d^4*f^4 - 60*A*a^2*b^7*c^2*d*f^7 + 1408*A*a^3*b*c^7*
d^3*f^5 + 352*A*a^3*b^5*c^3*d*f^7 - 1792*A*a^4*b*c^6*d^2*f^6 - 976*A*a^4*b^
3*c^4*d*f^7 - 132*B*a*b^4*c^6*d^4*f^4 - 196*B*a*b^6*c^4*d^3*f^5 - 40*B*a*b^
8*c^2*d^2*f^6 - 20*B*a^3*b^6*c^2*d*f^7 + 52*B*a^4*b^4*c^3*d*f^7 + 64*B*a^5*
b^2*c^4*d*f^7 + 4*A*a*b^9*c*d*f^7 - 1184*A*a^2*b^3*c^6*d^3*f^5 - 544*A*a^2*
b^5*c^4*d^2*f^6 + 1664*A*a^3*b^3*c^5*d^2*f^6 + 80*B*a^2*b^2*c^7*d^4*f^4 + 5
20*B*a^2*b^4*c^5*d^3*f^5 + 210*B*a^2*b^6*c^3*d^2*f^6 - 192*B*a^3*b^2*c^6*d^
3*f^5 - 456*B*a^3*b^4*c^4*d^2*f^6 + 96*B*a^4*b^2*c^5*d^2*f^6 + 64*A*a*b*c^9
*d^5*f^3 + 1088*A*a^5*b*c^5*d*f^7 + 2*B*a^2*b^8*c*d*f^7))/(16*a^2*c^6*d^4 +
 a^4*b^4*f^4 + b^4*c^4*d^4 + 16*a^6*c^2*f^4 + b^8*d^2*f^2 - 8*a*b^2*c^5*d^4
 - 8*a^5*b^2*c*f^4 + 2*a^2*b^6*d*f^3 - 64*a^3*c^5*d^3*f - 64*a^5*c^3*d*f^3
+ 2*b^6*c^2*d^3*f + 96*a^4*c^4*d^2*f^2 + 54*a^2*b^4*c^2*d^2*f^2 - 112*a^3*b
^2*c^3*d^2*f^2 - 20*a*b^4*c^3*d^3*f - 12*a*b^6*c*d^2*f^2 - 20*a^3*b^4*c*d*f
^3 + 64*a^2*b^2*c^4*d^3*f + 64*a^4*b^2*c^2*d*f^3)) + (13*A^2*a^2*b^5*c^2*f^
7 - 56*A^2*a^3*b^3*c^3*f^7 + 16*A^2*b^3*c^6*d^3*f^4 + A^2*b^5*c^4*d^2*f^5 +
 8*B^2*b^3*c^6*d^4*f^3 + 9*B^2*b^5*c^4*d^3*f^4 - 64*A*B*a^5*c^4*f^7 - A^2*a
*b^7*c*f^7 + 80*A^2*a^4*b*c^4*f^7 + 16*A^2*b*c^8*d^4*f^3 + 2*A^2*b^7*c^2*d*
f^6 - 48*A^2*a*b*c^7*d^3*f^4 - 22*A^2*a*b^5*c^3*d*f^6 - 48*A^2*a^3*b*c^5*d*
f^6 - 16*B^2*a*b*c^7*d^4*f^3 - 64*B^2*a^4*b*c^4*d*f^6 - A*B*b^8*c*d*f^6 - 8
*A^2*a*b^3*c^5*d^2*f^5 + 64*A^2*a^2*b^3*c^4*d*f^6 - 56*B^2*a*b^3*c^5*d^3*f^
4 + 2*B^2*a*b^5*c^3*d^2*f^5 + 96*B^2*a^2*b*c^6*d^3*f^4 - 11*B^2*a^2*b^5*c^2
*d*f^6 - 16*B^2*a^3*b*c^5*d^2*f^5 + 40*B^2*a^3*b^3*c^3*d*f^6 + A*B*a^2*b^6*
c*f^7 + 32*A*B*a*c^8*d^4*f^3 + 32*A*B*a^4*c^5*d*f^6 + B^2*a*b^7*c*d*f^6 - 8
*B^2*a^2*b^3*c^4*d^2*f^5 - 12*A*B*a^3*b^4*c^2*f^7 + 48*A*B*a^4*b^2*c^3*f^7
- 160*A*B*a^2*c^7*d^3*f^4 + 160*A*B*a^3*c^6*d^2*f^5 - 24*A*B*b^2*c^7*d^4*f^
3 - 24*A*B*b^4*c^5*d^3*f^4 + A*B*b^6*c^3*d^2*f^5 + 120*A*B*a*b^2*c^6*d^3*f^
4 - 4*A*B*a*b^4*c^4*d^2*f^5 - 24*A*B*a^2*b^4*c^3*d*f^6 + 8*A*B*a^3*b^2*c^4*
d*f^6 - 24*A*B*a^2*b^2*c^5*d^2*f^5 + 10*A*B*a*b^6*c^2*d*f^6)/(16*a^2*c^6*d^
4 + a^4*b^4*f^4 + b^4*c^4*d^4 + 16*a^6*c^2*f^4 + b^8*d^2*f^2 - 8*a*b^2*c^5*
d^4 - 8*a^5*b^2*c*f^4 + 2*a^2*b^6*d*f^3 - 64*a^3*c^5*d^3*f - 64*a^5*c^3*d*f
^3 + 2*b^6*c^2*d^3*f + 96*a^4*c^4*d^2*f^2 + 54*a^2*b^4*c^2*d^2*f^2 - 112*a^
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3*b^2*c^3*d^2*f^2 - 20*a*b^4*c^3*d^3*f - 12*a*b^6*c*d^2*f^2 - 20*a^3*b^4*c*
d*f^3 + 64*a^2*b^2*c^4*d^3*f + 64*a^4*b^2*c^2*d*f^3) + (x*(104*A^2*a^4*c^5*
f^7 - 32*B^2*a^5*c^4*f^7 + 8*A^2*c^9*d^4*f^3 + A^2*b^8*c*f^7 + 50*A^2*a^2*b
^4*c^3*f^7 - 96*A^2*a^3*b^2*c^4*f^7 - 12*B^2*a^3*b^4*c^2*f^7 + 42*B^2*a^4*b
^2*c^3*f^7 + 208*A^2*a^2*c^7*d^2*f^5 + 8*A^2*b^2*c^7*d^3*f^4 + 18*A^2*b^4*c
^5*d^2*f^5 - 32*B^2*a^2*c^7*d^3*f^4 + 32*B^2*a^3*c^6*d^2*f^5 + 2*B^2*b^2*c^
7*d^4*f^3 - 6*B^2*b^4*c^5*d^3*f^4 + 9*B^2*b^6*c^3*d^2*f^5 - 12*A^2*a*b^6*c^
2*f^7 + B^2*a^2*b^6*c*f^7 - 64*A^2*a*c^8*d^3*f^4 - 256*A^2*a^3*c^6*d*f^6 +
2*A^2*b^6*c^3*d*f^6 + 32*B^2*a^4*c^5*d*f^6 - 36*A^2*a*b^4*c^4*d*f^6 - 2*B^2
*a*b^6*c^2*d*f^6 - 2*A*B*a*b^7*c*f^7 - 144*A^2*a*b^2*c^6*d^2*f^5 + 168*A^2*
a^2*b^2*c^5*d*f^6 + 24*B^2*a*b^2*c^6*d^3*f^4 - 64*B^2*a*b^4*c^4*d^2*f^5 + 2
6*B^2*a^2*b^4*c^3*d*f^6 - 88*B^2*a^3*b^2*c^4*d*f^6 + 72*A*B*a^4*b*c^4*f^7 -
 8*A*B*b*c^8*d^4*f^3 + 2*A*B*b^7*c^2*d*f^6 + 84*B^2*a^2*b^2*c^5*d^2*f^5 + 2
4*A*B*a^2*b^5*c^2*f^7 - 84*A*B*a^3*b^3*c^3*f^7 + 4*A*B*b^3*c^6*d^3*f^4 - 20
*A*B*b^5*c^4*d^2*f^5 + 148*A*B*a*b^3*c^5*d^2*f^5 - 192*A*B*a^2*b*c^6*d^2*f^
5 - 4*A*B*a^2*b^3*c^4*d*f^6 + 16*A*B*a*b*c^7*d^3*f^4 - 12*A*B*a*b^5*c^3*d*f
^6 + 112*A*B*a^3*b*c^5*d*f^6))/(16*a^2*c^6*d^4 + a^4*b^4*f^4 + b^4*c^4*d^4
+ 16*a^6*c^2*f^4 + b^8*d^2*f^2 - 8*a*b^2*c^5*d^4 - 8*a^5*b^2*c*f^4 + 2*a^2*
b^6*d*f^3 - 64*a^3*c^5*d^3*f - 64*a^5*c^3*d*f^3 + 2*b^6*c^2*d^3*f + 96*a^4*
c^4*d^2*f^2 + 54*a^2*b^4*c^2*d^2*f^2 - 112*a^3*b^2*c^3*d^2*f^2 - 20*a*b^4*c
^3*d^3*f - 12*a*b^6*c*d^2*f^2 - 20*a^3*b^4*c*d*f^3 + 64*a^2*b^2*c^4*d^3*f +
 64*a^4*b^2*c^2*d*f^3)) - (16*A^3*a*c^6*d*f^5 - 4*A^3*c^7*d^2*f^4 - B^3*b^3
*c^4*d^2*f^4 - 12*A^3*a^2*c^5*f^6 - 16*A*B^2*a^3*c^4*f^6 + 2*A^3*a*b^2*c^4*
f^6 - 6*A^3*b^2*c^5*d*f^5 + 4*B^3*a*b*c^5*d^2*f^4 + 3*B^3*a*b^3*c^3*d*f^5 -
 12*B^3*a^2*b*c^4*d*f^5 + 3*A*B^2*a^2*b^2*c^3*f^6 + A*B^2*b^2*c^5*d^2*f^4 -
 3*A^2*B*a*b^3*c^3*f^6 + 16*A^2*B*a^2*b*c^4*f^6 - 8*A*B^2*a*c^6*d^2*f^4 + 2
4*A*B^2*a^2*c^5*d*f^5 - 3*A*B^2*b^4*c^3*d*f^5 + 4*A^2*B*b*c^6*d^2*f^4 + 9*A
^2*B*b^3*c^4*d*f^5 + 4*A*B^2*a*b^2*c^4*d*f^5 - 28*A^2*B*a*b*c^5*d*f^5)/(16*
a^2*c^6*d^4 + a^4*b^4*f^4 + b^4*c^4*d^4 + 16*a^6*c^2*f^4 + b^8*d^2*f^2 - 8*
a*b^2*c^5*d^4 - 8*a^5*b^2*c*f^4 + 2*a^2*b^6*d*f^3 - 64*a^3*c^5*d^3*f - 64*a
^5*c^3*d*f^3 + 2*b^6*c^2*d^3*f + 96*a^4*c^4*d^2*f^2 + 54*a^2*b^4*c^2*d^2*f^
2 - 112*a^3*b^2*c^3*d^2*f^2 - 20*a*b^4*c^3*d^3*f - 12*a*b^6*c*d^2*f^2 - 20*
a^3*b^4*c*d*f^3 + 64*a^2*b^2*c^4*d^3*f + 64*a^4*b^2*c^2*d*f^3))*root(2560*a
^3*b^2*c^9*d^8*f*z^4 - 1152*a^2*b^4*c^8*d^8*f*z^4 + 384*a^5*b^8*c*d^3*f^6*z
^4 + 384*a*b^8*c^5*d^7*f^2*z^4 + 288*a^3*b^10*c*d^4*f^5*z^4 + 288*a*b^10*c^
3*d^6*f^3*z^4 + 224*a^7*b^6*c*d^2*f^7*z^4 - 192*a^10*b^2*c^2*d*f^8*z^4 + 22
4*a*b^6*c^7*d^8*f*z^4 + 80*a*b^12*c*d^5*f^4*z^4 + 48*a^9*b^4*c*d*f^8*z^4 -
33920*a^6*b^2*c^6*d^5*f^4*z^4 + 27936*a^5*b^4*c^5*d^5*f^4*z^4 + 26112*a^7*b
^2*c^5*d^4*f^5*z^4 + 26112*a^5*b^2*c^7*d^6*f^3*z^4 - 20352*a^6*b^4*c^4*d^4*
f^5*z^4 - 20352*a^4*b^4*c^6*d^6*f^3*z^4 - 13080*a^4*b^6*c^4*d^5*f^4*z^4 - 1
1520*a^8*b^2*c^4*d^3*f^6*z^4 - 11520*a^4*b^2*c^8*d^7*f^2*z^4 + 8736*a^5*b^6
*c^3*d^4*f^5*z^4 + 8736*a^3*b^6*c^5*d^6*f^3*z^4 + 7488*a^7*b^4*c^3*d^3*f^6*
z^4 + 7488*a^3*b^4*c^7*d^7*f^2*z^4 + 3840*a^3*b^8*c^3*d^5*f^4*z^4 + 2560*a^
9*b^2*c^3*d^2*f^7*z^4 - 2416*a^6*b^6*c^2*d^3*f^6*z^4 - 2416*a^2*b^6*c^6*d^7
*f^2*z^4 - 2160*a^4*b^8*c^2*d^4*f^5*z^4 - 2160*a^2*b^8*c^4*d^6*f^3*z^4 - 11
52*a^8*b^4*c^2*d^2*f^7*z^4 - 720*a^2*b^10*c^2*d^5*f^4*z^4 - 16*b^8*c^6*d^8*
f*z^4 - 2048*a^4*c^10*d^8*f*z^4 + 256*a^11*c^3*d*f^8*z^4 - 4*a^8*b^6*d*f^8*
z^4 + 48*a*b^4*c^9*d^9*z^4 - 24*b^10*c^4*d^7*f^2*z^4 - 16*b^12*c^2*d^6*f^3*
z^4 + 17920*a^7*c^7*d^5*f^4*z^4 - 14336*a^8*c^6*d^4*f^5*z^4 - 14336*a^6*c^8
*d^6*f^3*z^4 + 7168*a^9*c^5*d^3*f^6*z^4 + 7168*a^5*c^9*d^7*f^2*z^4 - 2048*a
^10*c^4*d^2*f^7*z^4 - 24*a^4*b^10*d^3*f^6*z^4 - 16*a^6*b^8*d^2*f^7*z^4 - 16
*a^2*b^12*d^4*f^5*z^4 - 192*a^2*b^2*c^10*d^9*z^4 - 4*b^14*d^5*f^4*z^4 - 4*b
^6*c^8*d^9*z^4 + 256*a^3*c^11*d^9*z^4 + 912*A*B*a^6*b*c^3*d*f^6*z^2 + 192*A
*B*a^4*b^5*c*d*f^6*z^2 + 920*A*B*a^4*b^3*c^3*d^2*f^5*z^2 - 480*A*B*a^2*b^5*
c^3*d^3*f^4*z^2 - 336*A*B*a^2*b^3*c^5*d^4*f^3*z^2 - 272*A*B*a^3*b^3*c^4*d^3
*f^4*z^2 + 240*A*B*a^3*b^5*c^2*d^2*f^5*z^2 + 192*A*B*a*b*c^8*d^6*f*z^2 - 24
96*A*B*a^5*b*c^4*d^2*f^5*z^2 + 1872*A*B*a^4*b*c^5*d^3*f^4*z^2 - 744*A*B*a^5
*b^3*c^2*d*f^6*z^2 - 720*A*B*a^2*b*c^7*d^5*f^2*z^2 + 504*A*B*a*b^3*c^6*d^5*
f^2*z^2 + 256*A*B*a^3*b*c^6*d^4*f^3*z^2 + 168*A*B*a*b^7*c^2*d^3*f^4*z^2 - 1
```

```
44*A*B*a^2*b^7*c*d^2*f^5*z^2 + 144*A*B*a*b^5*c^4*d^4*f^3*z^2 - 56*B^2*a*b^2
*c^7*d^6*f*z^2 - 36*B^2*a^5*b^4*c*d*f^6*z^2 - 16*B^2*a*b^8*c*d^3*f^4*z^2 -
164*A^2*a^3*b^6*c*d*f^6*z^2 - 16*A^2*a*b^8*c*d^2*f^5*z^2 - 96*A*B*b^5*c^5*d
^5*f^2*z^2 - 24*A*B*b^7*c^3*d^4*f^3*z^2 - 580*B^2*a^4*b^2*c^4*d^3*f^4*z^2 +
 536*B^2*a^3*b^4*c^3*d^3*f^4*z^2 - 348*B^2*a^4*b^4*c^2*d^2*f^5*z^2 + 316*B^
2*a^2*b^2*c^6*d^5*f^2*z^2 + 200*B^2*a^5*b^2*c^3*d^2*f^5*z^2 - 120*B^2*a^2*b
^4*c^4*d^4*f^3*z^2 - 66*B^2*a^2*b^6*c^2*d^3*f^4*z^2 - 16*B^2*a^3*b^2*c^5*d^
4*f^3*z^2 + 1952*A^2*a^4*b^2*c^4*d^2*f^5*z^2 - 1792*A^2*a^3*b^2*c^5*d^3*f^4
*z^2 - 1272*A^2*a^3*b^4*c^3*d^2*f^5*z^2 + 976*A^2*a^2*b^2*c^6*d^4*f^3*z^2 +
 960*A^2*a^2*b^4*c^4*d^3*f^4*z^2 + 282*A^2*a^2*b^6*c^2*d^2*f^5*z^2 - 72*A*B
*b^3*c^7*d^6*f*z^2 - 16*A*B*b^9*c*d^3*f^4*z^2 - 16*A*B*a^3*b^7*d*f^6*z^2 +
16*A*B*a*b^9*d^2*f^5*z^2 - 180*B^2*a*b^4*c^5*d^5*f^2*z^2 + 132*B^2*a^6*b^2*
c^2*d*f^6*z^2 + 108*B^2*a^3*b^6*c*d^2*f^5*z^2 + 20*B^2*a*b^6*c^3*d^4*f^3*z^
2 - 736*A^2*a^5*b^2*c^3*d*f^6*z^2 + 624*A^2*a^4*b^4*c^2*d*f^6*z^2 - 416*A^2
*a*b^2*c^7*d^5*f^2*z^2 - 276*A^2*a*b^4*c^5*d^4*f^3*z^2 - 196*A^2*a*b^6*c^3*
d^3*f^4*z^2 + 31*B^2*b^6*c^4*d^5*f^2*z^2 + 2*B^2*b^8*c^2*d^4*f^3*z^2 - 768*
B^2*a^5*c^5*d^3*f^4*z^2 + 512*B^2*a^6*c^4*d^2*f^5*z^2 + 512*B^2*a^4*c^6*d^4
*f^3*z^2 - 128*B^2*a^3*c^7*d^5*f^2*z^2 + 80*A^2*b^4*c^6*d^5*f^2*z^2 + 31*A^
2*b^6*c^4*d^4*f^3*z^2 + 14*A^2*b^8*c^2*d^3*f^4*z^2 - 1152*A^2*a^3*c^7*d^4*f
^3*z^2 + 1008*A^2*a^4*c^6*d^3*f^4*z^2 + 624*A^2*a^2*c^8*d^5*f^2*z^2 - 288*A
^2*a^5*c^5*d^2*f^5*z^2 - 10*B^2*a^2*b^8*d^2*f^5*z^2 - 48*A^2*a^6*b^2*c^2*f^
7*z^2 - 16*A*B*b*c^9*d^7*z^2 + 20*B^2*b^4*c^6*d^6*f*z^2 - 128*B^2*a^7*c^3*d
*f^6*z^2 + 64*A^2*b^2*c^8*d^6*f*z^2 - 112*A^2*a^6*c^4*d*f^6*z^2 + 3*B^2*a^4
*b^6*d*f^6*z^2 + 14*A^2*a^2*b^8*d*f^6*z^2 + 12*A^2*a^5*b^4*c*f^7*z^2 - 160*
A^2*a*c^9*d^6*f*z^2 + 3*B^2*b^10*d^3*f^4*z^2 - A^2*b^10*d^2*f^5*z^2 + 64*A^
2*a^7*c^3*f^7*z^2 + 4*B^2*b^2*c^8*d^7*z^2 - A^2*a^4*b^6*f^7*z^2 + 16*A^2*c^
10*d^7*z^2 - 160*A*B^2*a*b*c^6*d^4*f^2*z + 112*A*B^2*a^4*b*c^3*d*f^5*z - 24
*A*B^2*a^2*b^5*c*d*f^5*z + 480*A^2*B*a^2*b^2*c^4*d^2*f^4*z - 176*A*B^2*a^2*
b^3*c^3*d^2*f^4*z - 10*A^2*B*a*b^6*c*d*f^5*z + 384*A*B^2*a^2*b*c^5*d^3*f^3*
z - 352*A*B^2*a^3*b*c^4*d^2*f^4*z - 288*A^2*B*a*b^2*c^5*d^3*f^3*z - 160*A^2
*B*a^3*b^2*c^3*d*f^5*z - 148*A^2*B*a*b^4*c^3*d^2*f^4*z + 112*A*B^2*a*b^3*c^
4*d^3*f^3*z + 72*A^2*B*a^2*b^4*c^2*d*f^5*z + 72*A*B^2*a*b^5*c^2*d^2*f^4*z +
 48*A*B^2*a^3*b^3*c^2*d*f^5*z + 48*B^3*a*b^2*c^5*d^4*f^2*z - 36*B^3*a^4*b^2
*c^2*d*f^5*z - 4*B^3*a*b^4*c^3*d^3*f^3*z - 480*A^3*a^2*b*c^5*d^2*f^4*z - 16
0*A^3*a^2*b^3*c^3*d*f^5*z + 128*A^3*a*b^3*c^4*d^2*f^4*z + 112*A^2*B*b^4*c^4
*d^3*f^3*z - 64*A*B^2*b^5*c^3*d^3*f^3*z + 16*A^2*B*b^2*c^6*d^4*f^2*z + 16*A
*B^2*b^3*c^5*d^4*f^2*z - A^2*B*b^6*c^2*d^2*f^4*z + 448*A^2*B*a^3*c^5*d^2*f^
4*z - 352*A^2*B*a^2*c^6*d^3*f^3*z - 48*A^2*B*a^4*b^2*c^2*f^6*z + 12*B^3*a^3
*b^4*c*d*f^5*z - 10*B^3*a*b^6*c*d^2*f^4*z + 416*A^3*a^3*b*c^4*d*f^5*z + 224
*A^3*a*b*c^6*d^3*f^3*z + 24*A^3*a*b^5*c^2*d*f^5*z - 2*A*B^2*b^7*c*d^2*f^4*z
 - 272*A^2*B*a^4*c^4*d*f^5*z + 128*A^2*B*a*c^7*d^4*f^2*z + 12*A^2*B*a^3*b^4
*c*f^6*z - 120*B^3*a^2*b^2*c^4*d^3*f^3*z + 112*B^3*a^3*b^2*c^3*d^2*f^4*z +
16*A*B^2*b*c^7*d^5*f*z + 2*A*B^2*a*b^7*d*f^5*z - 2*A^3*b^7*c*d*f^5*z - 16*A
^2*B*c^8*d^5*f*z + 11*B^3*b^6*c^2*d^3*f^3*z - 8*B^3*b^4*c^4*d^4*f^2*z - 64*
A^3*b^3*c^5*d^3*f^3*z + 96*A^3*a^3*b^3*c^2*f^6*z - 4*B^3*b^2*c^6*d^5*f*z -
32*A^3*b*c^7*d^4*f^2*z - B^3*a^2*b^6*d*f^5*z - 128*A^3*a^4*b*c^3*f^6*z - 24
*A^3*a^2*b^5*c*f^6*z + 64*A^2*B*a^5*c^3*f^6*z - A^2*B*a^2*b^6*f^6*z + A^2*B
*b^8*d*f^5*z + 2*A^3*a*b^7*f^6*z + B^3*b^8*d^2*f^4*z + 32*A^3*B*a*b*c^4*d*f
^4 - 18*A^2*B^2*a*b^2*c^3*d*f^4 + 32*A*B^3*a*b*c^4*d^2*f^3 - 28*A*B^3*a^2*b
*c^3*d*f^4 + 6*A*B^3*a*b^3*c^2*d*f^4 - 10*A^3*B*b^3*c^3*d*f^4 - 4*A^3*B*b*c
^5*d^2*f^3 - 4*A*B^3*b*c^5*d^3*f^2 - 28*A^3*B*a^2*b*c^3*f^5 + 6*A^3*B*a*b^3
*c^2*f^5 + 9*A^2*B^2*b^2*c^4*d^2*f^3 - 3*A^2*B^2*a^2*b^2*c^2*f^5 - 10*B^4*a
*b^2*c^3*d^2*f^3 - 3*B^4*a^2*b^2*c^2*d*f^4 - 10*A*B^3*b^3*c^3*d^2*f^3 + 3*A
^2*B^2*b^4*c^2*d*f^4 + 36*A^2*B^2*a^2*c^4*d*f^4 - 24*A^2*B^2*a*c^5*d^2*f^3
+ 4*A^2*B^2*c^6*d^3*f^2 + 16*A^2*B^2*a^3*c^3*f^5 + 16*B^4*a^3*c^3*d*f^4 + 8
*A^4*b^2*c^4*d*f^4 - 8*A^4*a*b^2*c^3*f^5 - 24*A^4*a*c^5*d*f^4 + 3*B^4*b^4*c
^2*d^2*f^3 + 4*A^4*c^6*d^2*f^3 + 36*A^4*a^2*c^4*f^5 + B^4*b^2*c^4*d^3*f^2,
z, k), k, 1, 4)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(c*x**2+b*x+a)**2/(f*x**2+d),x)
```

```
[Out] Timed out
```

3.6 $\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{d-fx^2}\,dx$

Optimal. Leaf size=331

$$-\frac{\left(B\sqrt{d}-A\sqrt{f}\right)\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}\,\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\,\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2\sqrt{d}\,f^{3/2}}+\frac{\left(A\sqrt{f}+B\sqrt{d}\right)\sqrt{af+b\sqrt{d}\,\sqrt{}}}{}$$

```
[Out] -1/2*(2*A*c+B*b)*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/f/c^(1/
2)-B*(c*x^2+b*x+a)^(1/2)/f-1/2*arctanh(1/2*(b*d^(1/2)-2*a*f^(1/2)+x*(2*c*d^
(1/2)-b*f^(1/2)))/(c*x^2+b*x+a)^(1/2)/(c*d+a*f-b*d^(1/2)*f^(1/2))^(1/2))*(B
*d^(1/2)-A*f^(1/2))*(c*d+a*f-b*d^(1/2)*f^(1/2))^(1/2)/f^(3/2)/d^(1/2)+1/2*a
rctanh(1/2*(b*d^(1/2)+2*a*f^(1/2)+x*(2*c*d^(1/2)+b*f^(1/2)))/(c*x^2+b*x+a)^
(1/2)/(c*d+a*f+b*d^(1/2)*f^(1/2))^(1/2))*(B*d^(1/2)+A*f^(1/2))*(c*d+a*f+b*d
^(1/2)*f^(1/2))^(1/2)/f^(3/2)/d^(1/2)
```

Rubi [A] time = 0.60, antiderivative size = 331, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1021, 1078, 621, 206, 1033, 724}

$$-\frac{\left(B\sqrt{d}-A\sqrt{f}\right)\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}\,\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\,\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2\sqrt{d}\,f^{3/2}}+\frac{\left(A\sqrt{f}+B\sqrt{d}\right)\sqrt{af+b\sqrt{d}\,\sqrt{}}}{}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x)*Sqrt[a + b*x + c*x^2])/(d - f*x^2),x]
```

```
[Out] -((B*Sqrt[a + b*x + c*x^2])/f) - ((b*B + 2*A*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt
[c]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[c]*f) - ((B*Sqrt[d] - A*Sqrt[f])*Sqrt[
c*d - b*Sqrt[d]*Sqrt[f] + a*f]*ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt
[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c
*x^2])])/(2*Sqrt[d]*f^(3/2)) + ((B*Sqrt[d] + A*Sqrt[f])*Sqrt[c*d + b*Sqrt[d
]*Sqrt[f] + a*f]*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f
])*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*Sq
rt[d]*f^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1021

```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (f
_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(h*(a + b*x + c*x^2)^p*(d + f*x^2)^(q +
 1))/(2*f*(p + q + 1)), x] - Dist[1/(2*f*(p + q + 1)), Int[(a + b*x + c*x^2
)^(p - 1)*(d + f*x^2)^q*Simp[h*p*(b*d) + a*(-2*g*f)*(p + q + 1) + (2*h*p*(c
*d - a*f) + b*(-2*g*f)*(p + q + 1))*x + (h*p*(-(b*f)) + c*(-2*g*f)*(p + q +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, g, h, q}, x] && NeQ[b^2 - 4*a
*c, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0]
```

Rule 1033

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q
), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q
), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f
, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]
```

Rule 1078

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
 (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqrt[d + e*x + f
*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + B*c*x)/((a + c*x^2)*Sqrt[d + e*x
 + f*x^2]), x], x] /; FreeQ[{a, c, d, e, f, A, B, C}, x] && NeQ[e^2 - 4*d*f
, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{d-fx^2}\,dx &= -\frac{B\sqrt{a+bx+cx^2}}{f}+\frac{\int \frac{\frac{1}{2}(bBd+2aAf)+(Bcd+Abf+aBf)x+\frac{1}{2}(bB+2Ac)fx^2}{\sqrt{a+bx+cx^2}\left(d-fx^2\right)}\,dx}{f}\\
&= -\frac{B\sqrt{a+bx+cx^2}}{f}-\frac{\int \frac{-\frac{1}{2}(bB+2Ac)df-\frac{1}{2}f(bBd+2aAf)-f(Bcd+Abf+aBf)x}{\sqrt{a+bx+cx^2}\left(d-fx^2\right)}\,dx}{f^2}-\frac{(bB+\ldots}{\ldots}\\
&= -\frac{B\sqrt{a+bx+cx^2}}{f}-\frac{(bB+2Ac)\,\mathrm{Subst}\left(\int \frac{1}{4c-x^2}\,dx,x,\frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{f}+\frac{\left(\left(B\sqrt{d}-\ldots\right.\right.}{\ldots}\\
&= -\frac{B\sqrt{a+bx+cx^2}}{f}-\frac{(bB+2Ac)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}f}-\frac{\left(\left(B\sqrt{d}-A\sqrt{f}\right)\left(\ldots\right.\right.}{\ldots}\\
&= -\frac{B\sqrt{a+bx+cx^2}}{f}-\frac{(bB+2Ac)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}f}-\frac{\left(B\sqrt{d}-A\sqrt{f}\right)\sqrt{\ldots}}{\ldots}\end{aligned}$$

Mathematica [A] time = 0.80, size = 322, normalized size = 0.97

$$\frac{\left(A\sqrt{f}+B\sqrt{d}\right)\sqrt{af+b\sqrt{d}\sqrt{f}+cd}\,\tanh^{-1}\left(\frac{2a\sqrt{f}+b\sqrt{d}+b\sqrt{f}x+2c\sqrt{d}x}{2\sqrt{a+x(b+cx)}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)-\left(B\sqrt{d}-A\sqrt{f}\right)\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{\ldots}}}{2\sqrt{d}f^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x)*Sqrt[a + b*x + c*x^2])/(d - f*x^2),x]

[Out] -1/2*((b*B + 2*A*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])
/(Sqrt[c]*f) + (-2*B*Sqrt[d]*Sqrt[f]*Sqrt[a + x*(b + c*x)] + (B*Sqrt[d] + A
```

```
*Sqrt[f])*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt
[f] + 2*c*Sqrt[d]*x + b*Sqrt[f]*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*S
qrt[a + x*(b + c*x)])] - (B*Sqrt[d] - A*Sqrt[f])*Sqrt[c*d - b*Sqrt[d]*Sqrt[
f] + a*f]*ArcTanh[(-2*a*Sqrt[f] + 2*c*Sqrt[d]*x + b*(Sqrt[d] - Sqrt[f]*x))/
(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])])/(2*Sqrt[d]*
f^(3/2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [B] time = 0.08, size = 3358, normalized size = 10.15

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x+A)*(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x)
```

```
[Out] 1/2/(d*f)^(1/2)*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^
(1/2)/f)+1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2)*A-1/2/f*((x+(d*f)^(1/2)/f)^2*c
+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+a*f+c*d))
^(1/2)*B-1/2/f*ln((1/2/f*(-2*c*(d*f)^(1/2)+b*f)+c*(x+(d*f)^(1/2)/f))/c^(1/2
)+((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(
-b*(d*f)^(1/2)+a*f+c*d))^(1/2))*c^(1/2)*A+1/2*(d*f)^(1/2)/f^2*ln((1/2/f*(-2
*c*(d*f)^(1/2)+b*f)+c*(x+(d*f)^(1/2)/f))/c^(1/2)+((x+(d*f)^(1/2)/f)^2*c+1/f
*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/
2))*c^(1/2)*B+1/4/(d*f)^(1/2)*ln((1/2/f*(-2*c*(d*f)^(1/2)+b*f)+c*(x+(d*f)^(
1/2)/f))/c^(1/2)+((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)
^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2))/c^(1/2)*b*A-1/4/f*ln((1/2/f*
(-2*c*(d*f)^(1/2)+b*f)+c*(x+(d*f)^(1/2)/f))/c^(1/2)+((x+(d*f)^(1/2)/f)^2*c+
1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+a*f+c*d))^
(1/2))/c^(1/2)*b*B+1/2/f/(1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2)*ln((2/f*(-b*(
d*f)^(1/2)+a*f+c*d)+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+2*(1/f*(-b
*(d*f)^(1/2)+a*f+c*d))^(1/2)*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b
*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2))/(x+(d*f)^(1/2)/f
))*b*A-1/2*(d*f)^(1/2)/f^2/(1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2)*ln((2/f*(-b
*(d*f)^(1/2)+a*f+c*d)+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+2*(1/f*(
-b*(d*f)^(1/2)+a*f+c*d))^(1/2)*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)
+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2))/(x+(d*f)^(1/2)
/f))*b*B-1/2/(d*f)^(1/2)/(1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2)*ln((2/f*(-b*(
d*f)^(1/2)+a*f+c*d)+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+2*(1/f*(-b
*(d*f)^(1/2)+a*f+c*d))^(1/2)*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b
*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2))/(x+(d*f)^(1/2)/f
))*a*A+1/2/f/(1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2)*ln((2/f*(-b*(d*f)^(1/2)+a
```

```
*f+c*d)+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+2*(1/f*(-b*(d*f)^(1/2)
+a*f+c*d))^(1/2)*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)
^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2))/(x+(d*f)^(1/2)/f))*a*B-1/2/(
d*f)^(1/2)/f/(1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2)*ln((2/f*(-b*(d*f)^(1/2)+a
*f+c*d)+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+2*(1/f*(-b*(d*f)^(1/2)
+a*f+c*d))^(1/2)*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)
^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2))/(x+(d*f)^(1/2)/f))*c*d*A+1/2
/f^2/(1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2)*ln((2/f*(-b*(d*f)^(1/2)+a*f+c*d)+
1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+2*(1/f*(-b*(d*f)^(1/2)+a*f+c*d
))^(1/2)*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f
)+1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2))/(x+(d*f)^(1/2)/f))*c*d*B-1/2/(d*f)^(
1/2)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d
*f)^(1/2)+a*f+c*d)/f)^(1/2)*A-1/2/f*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)
+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)*B-1/2/f*ln((1/2*
(2*c*(d*f)^(1/2)+b*f)/f+c*(x-(d*f)^(1/2)/f))/c^(1/2)+((x-(d*f)^(1/2)/f)^2*c
+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)
)*c^(1/2)*A-1/2*(d*f)^(1/2)/f^2*ln((1/2*(2*c*(d*f)^(1/2)+b*f)/f+c*(x-(d*f)^
(1/2)/f))/c^(1/2)+((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(
1/2)/f)+(b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2))*c^(1/2)*B-1/4/(d*f)^(1/2)*ln((1/2
*(2*c*(d*f)^(1/2)+b*f)/f+c*(x-(d*f)^(1/2)/f))/c^(1/2)+((x-(d*f)^(1/2)/f)^2*
c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2
))/c^(1/2)*b*A-1/4/f*ln((1/2*(2*c*(d*f)^(1/2)+b*f)/f+c*(x-(d*f)^(1/2)/f))/c
^(1/2)+((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*
(d*f)^(1/2)+a*f+c*d)/f)^(1/2))/c^(1/2)*b*B+1/2/f/((b*(d*f)^(1/2)+a*f+c*d)/f
)^(1/2)*ln((2*(b*(d*f)^(1/2)+a*f+c*d)/f+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1
/2)/f)+2*((b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f
)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2))/(x-(d*f)
^(1/2)/f))*b*A+1/2*(d*f)^(1/2)/f^2/((b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)*ln((2*
(b*(d*f)^(1/2)+a*f+c*d)/f+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+2*((b*(
d*f)^(1/2)+a*f+c*d)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f
*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2))/(x-(d*f)^(1/2)/f))*b*B
+1/2/(d*f)^(1/2)/((b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)*ln((2*(b*(d*f)^(1/2)+a*f
+c*d)/f+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+2*((b*(d*f)^(1/2)+a*f+c*d
)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)
+(b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2))/(x-(d*f)^(1/2)/f))*a*A+1/2/f/((b*(d*f)^(
1/2)+a*f+c*d)/f)^(1/2)*ln((2*(b*(d*f)^(1/2)+a*f+c*d)/f+(2*c*(d*f)^(1/2)+b*f
)/f*(x-(d*f)^(1/2)/f)+2*((b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)*((x-(d*f)^(1/2)/f
)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+a*f+c*d)/f)^
(1/2))/(x-(d*f)^(1/2)/f))*a*B+1/2/(d*f)^(1/2)/f/((b*(d*f)^(1/2)+a*f+c*d)/f)
^(1/2)*ln((2*(b*(d*f)^(1/2)+a*f+c*d)/f+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/
2)/f)+2*((b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)
^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2))/(x-(d*f)^
(1/2)/f))*c*d*A+1/2/f^2/((b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)*ln((2*(b*(d*f)^(1
/2)+a*f+c*d)/f+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+2*((b*(d*f)^(1/2)+
a*f+c*d)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(
1/2)/f)+(b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2))/(x-(d*f)^(1/2)/f))*c*d*B
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(((c*sqrt(4*d*f))/(2*f^2)>0)', see `
assume?` for more details)Is ((c*sqrt(4*d*f))/(2*f^2)    +b/(2*f))    ^2
 -(c*((b*sqrt(4*d*f))                  /(2*f)                 +(c*d)/f+a))
     /f^2 positive, negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\,x)\,\sqrt{c\,x^2+b\,x+a}}{d-f\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x)*(a + b*x + c*x^2)^(1/2))/(d - f*x^2),x)
[Out] int(((A + B*x)*(a + b*x + c*x^2)^(1/2))/(d - f*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{A\sqrt{a+bx+cx^2}}{-d+fx^2}\,dx - \int \frac{Bx\sqrt{a+bx+cx^2}}{-d+fx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(c*x**2+b*x+a)**(1/2)/(-f*x**2+d),x)
[Out] -Integral(A*sqrt(a + b*x + c*x**2)/(-d + f*x**2), x) - Integral(B*x*sqrt(a
+ b*x + c*x**2)/(-d + f*x**2), x)
```

3.7 $\int \frac{A+Bx}{\sqrt{a+bx+cx^2}\left(d-fx^2\right)}\,dx$

Optimal. Leaf size=249

$$\frac{\left(\frac{A\sqrt{f}}{\sqrt{d}}+B\right)\tanh^{-1}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2\sqrt{f}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}-\frac{\left(B-\frac{A\sqrt{f}}{\sqrt{d}}\right)\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2\sqrt{f}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}$$

```
[Out] -1/2*arctanh(1/2*(b*d^(1/2)-2*a*f^(1/2)+x*(2*c*d^(1/2)-b*f^(1/2)))/(c*x^2+b
*x+a)^(1/2)/(c*d+a*f-b*d^(1/2)*f^(1/2))^(1/2))*(B-A*f^(1/2)/d^(1/2))/f^(1/2
)/(c*d+a*f-b*d^(1/2)*f^(1/2))^(1/2)+1/2*arctanh(1/2*(b*d^(1/2)+2*a*f^(1/2)+
x*(2*c*d^(1/2)+b*f^(1/2)))/(c*x^2+b*x+a)^(1/2)/(c*d+a*f+b*d^(1/2)*f^(1/2))^
(1/2))*(B+A*f^(1/2)/d^(1/2))/f^(1/2)/(c*d+a*f+b*d^(1/2)*f^(1/2))^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 249, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1033, 724, 206}

$$\frac{\left(\frac{A\sqrt{f}}{\sqrt{d}}+B\right)\tanh^{-1}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2\sqrt{f}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}-\frac{\left(B-\frac{A\sqrt{f}}{\sqrt{d}}\right)\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2\sqrt{f}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x)/(Sqrt[a + b*x + c*x^2]*(d - f*x^2)),x]
```

```
[Out] -((B - (A*Sqrt[f])/Sqrt[d])*ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d]
 - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^
2])])/(2*Sqrt[f]*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]) + ((B + (A*Sqrt[f])/S
qrt[d])*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*
Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[f]*Sqr
t[c*d + b*Sqrt[d]*Sqrt[f] + a*f])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1033

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q
), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q
), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f
, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]
```

Rubi steps

$$\int \frac{A+Bx}{\sqrt{a+bx+cx^2}\left(d-fx^2\right)}dx = \frac{1}{2}\left(B-\frac{A\sqrt{f}}{\sqrt{d}}\right)\int \frac{1}{\left(-\sqrt{d}\sqrt{f}-fx\right)\sqrt{a+bx+cx^2}}dx + \frac{1}{2}\left(B+\frac{A\sqrt{f}}{\sqrt{d}}\right)\int \frac{}{\left(\sqrt{d}\right.}$$

$$= \left(-B-\frac{A\sqrt{f}}{\sqrt{d}}\right)\text{Subst}\left(\int \frac{1}{4cdf+4b\sqrt{d}f^{3/2}+4af^2-x^2}dx, x, \frac{-b\sqrt{d}\sqrt{f}-2af}{\sqrt{a+}}\right.$$

$$= -\frac{\left(B-\frac{A\sqrt{f}}{\sqrt{d}}\right)\tanh^{-1}\left(\frac{b\sqrt{d}-2a\sqrt{f}+\left(2c\sqrt{d}-b\sqrt{f}\right)x}{2\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{f}\sqrt{cd-b\sqrt{d}\sqrt{f}+af}} + \frac{\left(B+\frac{A\sqrt{f}}{\sqrt{d}}\right)\tanh^{-1}\left(\frac{b\sqrt{d}+2a}{2\sqrt{cd+b}}\right.}{2\sqrt{f}\sqrt{cd+b\sqrt{d}}}$$

Mathematica [A] time = 0.24, size = 249, normalized size = 1.00

$$\frac{-\frac{\left(B\sqrt{d}-A\sqrt{f}\right)\tanh^{-1}\left(\frac{-2a\sqrt{f}+b\left(\sqrt{d}-\sqrt{f}x\right)+2c\sqrt{d}x}{2\sqrt{a+x(b+cx)}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}-\frac{\left(A\sqrt{f}+B\sqrt{d}\right)\tanh^{-1}\left(\frac{-2\left(a\sqrt{f}+c\sqrt{d}x\right)-b\left(\sqrt{d}+\sqrt{f}x\right)}{2\sqrt{a+x(b+cx)}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}}{2\sqrt{d}\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x)/(Sqrt[a + b*x + c*x^2]*(d - f*x^2)),x]
```

```
[Out] (-(((B*Sqrt[d] - A*Sqrt[f])*ArcTanh[(-2*a*Sqrt[f] + 2*c*Sqrt[d]*x + b*(Sqrt
[d] - Sqrt[f]*x))/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*
x)])])/Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]) - ((B*Sqrt[d] + A*Sqrt[f])*ArcT
anh[(-2*(a*Sqrt[f] + c*Sqrt[d]*x) - b*(Sqrt[d] + Sqrt[f]*x))/(2*Sqrt[c*d +
b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])])/Sqrt[c*d + b*Sqrt[d]*Sqrt
[f] + a*f])/(2*Sqrt[d]*Sqrt[f])
```

fricas [B] time = 147.14, size = 6113, normalized size = 24.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt((B^2*c*d^2 + A^2*a*f^2 + (B^2*a - 2*A*B*b + A^2*c)*d*f + (c^2*d^3*
f + a^2*d*f^3 - (b^2 - 2*a*c)*d^2*f^2)*sqrt(((B^4*b^2 - 4*A*B^3*b*c + 4*A^2
*B^2*c^2)*d^2 - 2*(2*A*B^3*a*b - A^2*B^2*b^2 - 2*(2*A^2*B^2*a - A^3*B*b)*c)
*d*f + (4*A^2*B^2*a^2 - 4*A^3*B*a*b + A^4*b^2)*f^2)/(c^4*d^5*f + a^4*d*f^5
- 2*(b^2*c^2 - 2*a*c^3)*d^4*f^2 + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3*f^3 - 2
*(a^2*b^2 - 2*a^3*c)*d^2*f^4)))/(c^2*d^3*f + a^2*d*f^3 - (b^2 - 2*a*c)*d^2*
f^2))*log(-((B^4*b^2 - 2*A*B^3*b*c)*d^2 - 2*(A*B^3*a*b - A^3*B*b*c)*d*f + (
2*A^3*B*a*b - A^4*b^2)*f^2 + 2*((2*A^3*B*a - A^4*b)*c*f^2 + (B^4*b*c - 2*A*
B^3*c^2)*d^2 - 2*(A*B^3*a*c - A^3*B*c^2)*d*f)*x + 2*((B^3*b^2 - 3*A*B^2*b*c
 + 2*A^2*B*c^2)*d^2*f - (3*A*B^2*a*b - A^2*B*b^2 - (4*A^2*B*a - A^3*b)*c)*d
*f^2 + (2*A^2*B*a^2 - A^3*a*b)*f^3 - (B*c^3*d^4*f - (B*b^2*c - (3*B*a - A*b
)*c^2)*d^3*f^2 - (B*a*b^2 - A*b^3 - (3*B*a^2 - 2*A*a*b)*c)*d^2*f^3 + (B*a^3
 - A*a^2*b)*d*f^4)*sqrt(((B^4*b^2 - 4*A*B^3*b*c + 4*A^2*B^2*c^2)*d^2 - 2*(2
*A*B^3*a*b - A^2*B^2*b^2 - 2*(2*A^2*B^2*a - A^3*B*b)*c)*d*f + (4*A^2*B^2*a^
2 - 4*A^3*B*a*b + A^4*b^2)*f^2)/(c^4*d^5*f + a^4*d*f^5 - 2*(b^2*c^2 - 2*a*c
^3)*d^4*f^2 + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3*f^3 - 2*(a^2*b^2 - 2*a^3*c)
*d^2*f^4)))*sqrt(c*x^2 + b*x + a)*sqrt((B^2*c*d^2 + A^2*a*f^2 + (B^2*a - 2*
A*B*b + A^2*c)*d*f + (c^2*d^3*f + a^2*d*f^3 - (b^2 - 2*a*c)*d^2*f^2)*sqrt((
(B^4*b^2 - 4*A*B^3*b*c + 4*A^2*B^2*c^2)*d^2 - 2*(2*A*B^3*a*b - A^2*B^2*b^2
- 2*(2*A^2*B^2*a - A^3*B*b)*c)*d*f + (4*A^2*B^2*a^2 - 4*A^3*B*a*b + A^4*b^2
```

```
)*f^2)/(c^4*d^5*f + a^4*d*f^5 - 2*(b^2*c^2 - 2*a*c^3)*d^4*f^2 + (b^4 - 4*a*
b^2*c + 6*a^2*c^2)*d^3*f^3 - 2*(a^2*b^2 - 2*a^3*c)*d^2*f^4)))/(c^2*d^3*f +
a^2*d*f^3 - (b^2 - 2*a*c)*d^2*f^2)) - (2*B^2*a*c^2*d^3*f - 2*A^2*a^3*f^4 -
2*(B^2*a*b^2 - 2*B^2*a^2*c + A^2*a*c^2)*d^2*f^2 + 2*(B^2*a^3 + A^2*a*b^2 -
2*A^2*a^2*c)*d*f^3 + (B^2*b*c^2*d^3*f - A^2*a^2*b*f^4 - (B^2*b^3 - 2*B^2*a*
b*c + A^2*b*c^2)*d^2*f^2 + (B^2*a^2*b + A^2*b^3 - 2*A^2*a*b*c)*d*f^3)*x)*sq
rt(((B^4*b^2 - 4*A*B^3*b*c + 4*A^2*B^2*c^2)*d^2 - 2*(2*A*B^3*a*b - A^2*B^2*
b^2 - 2*(2*A^2*B^2*a - A^3*B*b)*c)*d*f + (4*A^2*B^2*a^2 - 4*A^3*B*a*b + A^4
*b^2)*f^2)/(c^4*d^5*f + a^4*d*f^5 - 2*(b^2*c^2 - 2*a*c^3)*d^4*f^2 + (b^4 -
4*a*b^2*c + 6*a^2*c^2)*d^3*f^3 - 2*(a^2*b^2 - 2*a^3*c)*d^2*f^4)))/x) - 1/4*
sqrt((B^2*c*d^2 + A^2*a*f^2 + (B^2*a - 2*A*B*b + A^2*c)*d*f + (c^2*d^3*f +
a^2*d*f^3 - (b^2 - 2*a*c)*d^2*f^2)*sqrt(((B^4*b^2 - 4*A*B^3*b*c + 4*A^2*B^2
*c^2)*d^2 - 2*(2*A*B^3*a*b - A^2*B^2*b^2 - 2*(2*A^2*B^2*a - A^3*B*b)*c)*d*f
 + (4*A^2*B^2*a^2 - 4*A^3*B*a*b + A^4*b^2)*f^2)/(c^4*d^5*f + a^4*d*f^5 - 2*
(b^2*c^2 - 2*a*c^3)*d^4*f^2 + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3*f^3 - 2*(a^
2*b^2 - 2*a^3*c)*d^2*f^4)))/(c^2*d^3*f + a^2*d*f^3 - (b^2 - 2*a*c)*d^2*f^2)
)*log(-((B^4*b^2 - 2*A*B^3*b*c)*d^2 - 2*(A*B^3*a*b - A^3*B*b*c)*d*f + (2*A^
3*B*a*b - A^4*b^2)*f^2 + 2*((2*A^3*B*a - A^4*b)*c*f^2 + (B^4*b*c - 2*A*B^3*
c^2)*d^2 - 2*(A*B^3*a*c - A^3*B*c^2)*d*f)*x - 2*((B^3*b^2 - 3*A*B^2*b*c + 2
*A^2*B*c^2)*d^2*f - (3*A*B^2*a*b - A^2*B*b^2 - (4*A^2*B*a - A^3*b)*c)*d*f^2
 + (2*A^2*B*a^2 - A^3*a*b)*f^3 - (B*c^3*d^4*f - (B*b^2*c - (3*B*a - A*b)*c^
2)*d^3*f^2 - (B*a*b^2 - A*b^3 - (3*B*a^2 - 2*A*a*b)*c)*d^2*f^3 + (B*a^3 - A
*a^2*b)*d*f^4)*sqrt(((B^4*b^2 - 4*A*B^3*b*c + 4*A^2*B^2*c^2)*d^2 - 2*(2*A*B
^3*a*b - A^2*B^2*b^2 - 2*(2*A^2*B^2*a - A^3*B*b)*c)*d*f + (4*A^2*B^2*a^2 -
4*A^3*B*a*b + A^4*b^2)*f^2)/(c^4*d^5*f + a^4*d*f^5 - 2*(b^2*c^2 - 2*a*c^3)*
d^4*f^2 + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3*f^3 - 2*(a^2*b^2 - 2*a^3*c)*d^2
*f^4)))*sqrt(c*x^2 + b*x + a)*sqrt((B^2*c*d^2 + A^2*a*f^2 + (B^2*a - 2*A*B*
b + A^2*c)*d*f + (c^2*d^3*f + a^2*d*f^3 - (b^2 - 2*a*c)*d^2*f^2)*sqrt(((B^4
*b^2 - 4*A*B^3*b*c + 4*A^2*B^2*c^2)*d^2 - 2*(2*A*B^3*a*b - A^2*B^2*b^2 - 2*
(2*A^2*B^2*a - A^3*B*b)*c)*d*f + (4*A^2*B^2*a^2 - 4*A^3*B*a*b + A^4*b^2)*f^
2)/(c^4*d^5*f + a^4*d*f^5 - 2*(b^2*c^2 - 2*a*c^3)*d^4*f^2 + (b^4 - 4*a*b^2*
c + 6*a^2*c^2)*d^3*f^3 - 2*(a^2*b^2 - 2*a^3*c)*d^2*f^4)))/(c^2*d^3*f + a^2*
d*f^3 - (b^2 - 2*a*c)*d^2*f^2)) - (2*B^2*a*c^2*d^3*f - 2*A^2*a^3*f^4 - 2*(B
^2*a*b^2 - 2*B^2*a^2*c + A^2*a*c^2)*d^2*f^2 + 2*(B^2*a^3 + A^2*a*b^2 - 2*A^
2*a^2*c)*d*f^3 + (B^2*b*c^2*d^3*f - A^2*a^2*b*f^4 - (B^2*b^3 - 2*B^2*a*b*c
+ A^2*b*c^2)*d^2*f^2 + (B^2*a^2*b + A^2*b^3 - 2*A^2*a*b*c)*d*f^3)*x)*sqrt((
(B^4*b^2 - 4*A*B^3*b*c + 4*A^2*B^2*c^2)*d^2 - 2*(2*A*B^3*a*b - A^2*B^2*b^2
- 2*(2*A^2*B^2*a - A^3*B*b)*c)*d*f + (4*A^2*B^2*a^2 - 4*A^3*B*a*b + A^4*b^2
)*f^2)/(c^4*d^5*f + a^4*d*f^5 - 2*(b^2*c^2 - 2*a*c^3)*d^4*f^2 + (b^4 - 4*a*
b^2*c + 6*a^2*c^2)*d^3*f^3 - 2*(a^2*b^2 - 2*a^3*c)*d^2*f^4)))/x) + 1/4*sqrt
((B^2*c*d^2 + A^2*a*f^2 + (B^2*a - 2*A*B*b + A^2*c)*d*f - (c^2*d^3*f + a^2*
d*f^3 - (b^2 - 2*a*c)*d^2*f^2)*sqrt(((B^4*b^2 - 4*A*B^3*b*c + 4*A^2*B^2*c^2
)*d^2 - 2*(2*A*B^3*a*b - A^2*B^2*b^2 - 2*(2*A^2*B^2*a - A^3*B*b)*c)*d*f + (
4*A^2*B^2*a^2 - 4*A^3*B*a*b + A^4*b^2)*f^2)/(c^4*d^5*f + a^4*d*f^5 - 2*(b^2
*c^2 - 2*a*c^3)*d^4*f^2 + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3*f^3 - 2*(a^2*b^
2 - 2*a^3*c)*d^2*f^4)))/(c^2*d^3*f + a^2*d*f^3 - (b^2 - 2*a*c)*d^2*f^2))*lo
g(-((B^4*b^2 - 2*A*B^3*b*c)*d^2 - 2*(A*B^3*a*b - A^3*B*b*c)*d*f + (2*A^3*B*
a*b - A^4*b^2)*f^2 + 2*((2*A^3*B*a - A^4*b)*c*f^2 + (B^4*b*c - 2*A*B^3*c^2)
*d^2 - 2*(A*B^3*a*c - A^3*B*c^2)*d*f)*x + 2*((B^3*b^2 - 3*A*B^2*b*c + 2*A^2
*B*c^2)*d^2*f - (3*A*B^2*a*b - A^2*B*b^2 - (4*A^2*B*a - A^3*b)*c)*d*f^2 + (
2*A^2*B*a^2 - A^3*a*b)*f^3 + (B*c^3*d^4*f - (B*b^2*c - (3*B*a - A*b)*c^2)*d
^3*f^2 - (B*a*b^2 - A*b^3 - (3*B*a^2 - 2*A*a*b)*c)*d^2*f^3 + (B*a^3 - A*a^2
*b)*d*f^4)*sqrt(((B^4*b^2 - 4*A*B^3*b*c + 4*A^2*B^2*c^2)*d^2 - 2*(2*A*B^3*a
*b - A^2*B^2*b^2 - 2*(2*A^2*B^2*a - A^3*B*b)*c)*d*f + (4*A^2*B^2*a^2 - 4*A^
3*B*a*b + A^4*b^2)*f^2)/(c^4*d^5*f + a^4*d*f^5 - 2*(b^2*c^2 - 2*a*c^3)*d^4*
f^2 + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3*f^3 - 2*(a^2*b^2 - 2*a^3*c)*d^2*f^4
)))*sqrt(c*x^2 + b*x + a)*sqrt((B^2*c*d^2 + A^2*a*f^2 + (B^2*a - 2*A*B*b +
A^2*c)*d*f - (c^2*d^3*f + a^2*d*f^3 - (b^2 - 2*a*c)*d^2*f^2)*sqrt(((B^4*b^2
 - 4*A*B^3*b*c + 4*A^2*B^2*c^2)*d^2 - 2*(2*A*B^3*a*b - A^2*B^2*b^2 - 2*(2*A
```

```
^2*B^2*a - A^3*B*b)*c)*d*f + (4*A^2*B^2*a^2 - 4*A^3*B*a*b + A^4*b^2)*f^2)/(
c^4*d^5*f + a^4*d*f^5 - 2*(b^2*c^2 - 2*a*c^3)*d^4*f^2 + (b^4 - 4*a*b^2*c +
6*a^2*c^2)*d^3*f^3 - 2*(a^2*b^2 - 2*a^3*c)*d^2*f^4)))/(c^2*d^3*f + a^2*d*f^
3 - (b^2 - 2*a*c)*d^2*f^2)) + (2*B^2*a*c^2*d^3*f - 2*A^2*a^3*f^4 - 2*(B^2*a
*b^2 - 2*B^2*a^2*c + A^2*a*c^2)*d^2*f^2 + 2*(B^2*a^3 + A^2*a*b^2 - 2*A^2*a^
2*c)*d*f^3 + (B^2*b*c^2*d^3*f - A^2*a^2*b*f^4 - (B^2*b^3 - 2*B^2*a*b*c + A^
2*b*c^2)*d^2*f^2 + (B^2*a^2*b + A^2*b^3 - 2*A^2*a*b*c)*d*f^3)*x)*sqrt(((B^4
*b^2 - 4*A*B^3*b*c + 4*A^2*B^2*c^2)*d^2 - 2*(2*A*B^3*a*b - A^2*B^2*b^2 - 2*
(2*A^2*B^2*a - A^3*B*b)*c)*d*f + (4*A^2*B^2*a^2 - 4*A^3*B*a*b + A^4*b^2)*f^
2)/(c^4*d^5*f + a^4*d*f^5 - 2*(b^2*c^2 - 2*a*c^3)*d^4*f^2 + (b^4 - 4*a*b^2*
c + 6*a^2*c^2)*d^3*f^3 - 2*(a^2*b^2 - 2*a^3*c)*d^2*f^4)))/x) - 1/4*sqrt((B^
2*c*d^2 + A^2*a*f^2 + (B^2*a - 2*A*B*b + A^2*c)*d*f - (c^2*d^3*f + a^2*d*f^
3 - (b^2 - 2*a*c)*d^2*f^2)*sqrt(((B^4*b^2 - 4*A*B^3*b*c + 4*A^2*B^2*c^2)*d^
2 - 2*(2*A*B^3*a*b - A^2*B^2*b^2 - 2*(2*A^2*B^2*a - A^3*B*b)*c)*d*f + (4*A^
2*B^2*a^2 - 4*A^3*B*a*b + A^4*b^2)*f^2)/(c^4*d^5*f + a^4*d*f^5 - 2*(b^2*c^2
 - 2*a*c^3)*d^4*f^2 + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3*f^3 - 2*(a^2*b^2 -
2*a^3*c)*d^2*f^4)))/(c^2*d^3*f + a^2*d*f^3 - (b^2 - 2*a*c)*d^2*f^2))*log(-(
(B^4*b^2 - 2*A*B^3*b*c)*d^2 - 2*(A*B^3*a*b - A^3*B*b*c)*d*f + (2*A^3*B*a*b
- A^4*b^2)*f^2 + 2*((2*A^3*B*a - A^4*b)*c*f^2 + (B^4*b*c - 2*A*B^3*c^2)*d^2
 - 2*(A*B^3*a*c - A^3*B*c^2)*d*f)*x - 2*((B^3*b^2 - 3*A*B^2*b*c + 2*A^2*B*c
^2)*d^2*f - (3*A*B^2*a*b - A^2*B*b^2 - (4*A^2*B*a - A^3*b)*c)*d*f^2 + (2*A^
2*B*a^2 - A^3*a*b)*f^3 + (B*c^3*d^4*f - (B*b^2*c - (3*B*a - A*b)*c^2)*d^3*f
^2 - (B*a*b^2 - A*b^3 - (3*B*a^2 - 2*A*a*b)*c)*d^2*f^3 + (B*a^3 - A*a^2*b)*
d*f^4)*sqrt(((B^4*b^2 - 4*A*B^3*b*c + 4*A^2*B^2*c^2)*d^2 - 2*(2*A*B^3*a*b -
 A^2*B^2*b^2 - 2*(2*A^2*B^2*a - A^3*B*b)*c)*d*f + (4*A^2*B^2*a^2 - 4*A^3*B*
a*b + A^4*b^2)*f^2)/(c^4*d^5*f + a^4*d*f^5 - 2*(b^2*c^2 - 2*a*c^3)*d^4*f^2
+ (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3*f^3 - 2*(a^2*b^2 - 2*a^3*c)*d^2*f^4)))*
sqrt(c*x^2 + b*x + a)*sqrt((B^2*c*d^2 + A^2*a*f^2 + (B^2*a - 2*A*B*b + A^2*
c)*d*f - (c^2*d^3*f + a^2*d*f^3 - (b^2 - 2*a*c)*d^2*f^2)*sqrt(((B^4*b^2 - 4
*A*B^3*b*c + 4*A^2*B^2*c^2)*d^2 - 2*(2*A*B^3*a*b - A^2*B^2*b^2 - 2*(2*A^2*B
^2*a - A^3*B*b)*c)*d*f + (4*A^2*B^2*a^2 - 4*A^3*B*a*b + A^4*b^2)*f^2)/(c^4*
d^5*f + a^4*d*f^5 - 2*(b^2*c^2 - 2*a*c^3)*d^4*f^2 + (b^4 - 4*a*b^2*c + 6*a^
2*c^2)*d^3*f^3 - 2*(a^2*b^2 - 2*a^3*c)*d^2*f^4)))/(c^2*d^3*f + a^2*d*f^3 -
(b^2 - 2*a*c)*d^2*f^2)) + (2*B^2*a*c^2*d^3*f - 2*A^2*a^3*f^4 - 2*(B^2*a*b^2
 - 2*B^2*a^2*c + A^2*a*c^2)*d^2*f^2 + 2*(B^2*a^3 + A^2*a*b^2 - 2*A^2*a^2*c)
*d*f^3 + (B^2*b*c^2*d^3*f - A^2*a^2*b*f^4 - (B^2*b^3 - 2*B^2*a*b*c + A^2*b*
c^2)*d^2*f^2 + (B^2*a^2*b + A^2*b^3 - 2*A^2*a*b*c)*d*f^3)*x)*sqrt(((B^4*b^2
 - 4*A*B^3*b*c + 4*A^2*B^2*c^2)*d^2 - 2*(2*A*B^3*a*b - A^2*B^2*b^2 - 2*(2*A
^2*B^2*a - A^3*B*b)*c)*d*f + (4*A^2*B^2*a^2 - 4*A^3*B*a*b + A^4*b^2)*f^2)/(
c^4*d^5*f + a^4*d*f^5 - 2*(b^2*c^2 - 2*a*c^3)*d^4*f^2 + (b^4 - 4*a*b^2*c +
6*a^2*c^2)*d^3*f^3 - 2*(a^2*b^2 - 2*a^3*c)*d^2*f^4)))/x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument ValueConj Error: Bad Argument Typ
e
```

maple [B] time = 0.02, size = 714, normalized size = 2.87

$$-\frac{A\ln\left(\frac{\frac{2af+2cd-2\sqrt{df}\,b}{f}+\frac{\left(bf-2\sqrt{df}\,c\right)\left(x+\frac{\sqrt{df}}{f}\right)}{f}+2\sqrt{\frac{af+cd-\sqrt{df}\,b}{f}}\sqrt{\left(x+\frac{\sqrt{df}}{f}\right)^2c+\frac{\left(bf-2\sqrt{df}\,c\right)\left(x+\frac{\sqrt{df}}{f}\right)}{f}+\frac{af+cd-\sqrt{df}\,b}{f}}}{x+\frac{\sqrt{df}}{f}}\right)}{2\sqrt{df}\,\sqrt{\frac{af+cd-\sqrt{df}\,b}{f}}}+\frac{A\ln\left(\frac{2af+2cd+2\sqrt{df}}{f}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x+A)/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x)
```

```
[Out] -1/2/(d*f)^(1/2)/(1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2)*ln((2/f*(-b*(d*f)^(1/
2)+a*f+c*d)+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+2*(1/f*(-b*(d*f)^(
1/2)+a*f+c*d))^(1/2)*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(
d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2))/(x+(d*f)^(1/2)/f))*A+1/2
/f/(1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2)*ln((2/f*(-b*(d*f)^(1/2)+a*f+c*d)+1/
f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+2*(1/f*(-b*(d*f)^(1/2)+a*f+c*d))
^(1/2)*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+
1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2))/(x+(d*f)^(1/2)/f))*B+1/2/(d*f)^(1/2)/(
(b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)*ln((2*(b*(d*f)^(1/2)+a*f+c*d)/f+(2*c*(d*f)
^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+2*((b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)*((x-(d*
f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+a*
f+c*d)/f)^(1/2))/(x-(d*f)^(1/2)/f))*A+1/2/f/((b*(d*f)^(1/2)+a*f+c*d)/f)^(1/
2)*ln((2*(b*(d*f)^(1/2)+a*f+c*d)/f+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f
)+2*((b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/
2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2))/(x-(d*f)^(1/2
)/f))*B
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(((c*sqrt(4*d*f))/(2*f^2)>0)', see `
assume?` for more details)Is ((c*sqrt(4*d*f))/(2*f^2)    +b/(2*f))    ^2
 -(c*((b*sqrt(4*d*f))                  /(2*f)                +(c*d)/f+a))
     /f^2 zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+B\,x}{\left(d-f\,x^2\right)\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x)/((d - f*x^2)*(a + b*x + c*x^2)^(1/2)),x)
```

```
[Out] int((A + B*x)/((d - f*x^2)*(a + b*x + c*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{A}{-d\sqrt{a+bx+cx^2}+fx^2\sqrt{a+bx+cx^2}}\,dx-\int \frac{Bx}{-d\sqrt{a+bx+cx^2}+fx^2\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(c*x**2+b*x+a)**(1/2)/(-f*x**2+d),x)
```

```
[Out] -Integral(A/(-d*sqrt(a + b*x + c*x**2) + f*x**2*sqrt(a + b*x + c*x**2)), x)
 - Integral(B*x/(-d*sqrt(a + b*x + c*x**2) + f*x**2*sqrt(a + b*x + c*x**2))
, x)
```

3.8 $\int \frac{A+Bx}{\left(a+bx+cx^2\right)^{3/2}\left(d-fx^2\right)}\,dx$

Optimal. Leaf size=381

$$-\frac{2\left(A\left(b^3f-bc(3af+cd)\right)+cx\left(-2Ac(af+cd)+bB(cd-af)+Ab^2f\right)+aB\left(2acf+b^2(-f)+2c^2d\right)\right)}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left(b^2df-(af+cd)^2\right)}-\frac{\sqrt{f}}{}$$

```
[Out] -1/2*arctanh(1/2*(b*d^(1/2)-2*a*f^(1/2)+x*(2*c*d^(1/2)-b*f^(1/2)))/(c*x^2+b
*x+a)^(1/2)/(c*d+a*f-b*d^(1/2)*f^(1/2))^(1/2))*f^(1/2)*(B*d^(1/2)-A*f^(1/2)
)/d^(1/2)/(c*d+a*f-b*d^(1/2)*f^(1/2))^(3/2)+1/2*arctanh(1/2*(b*d^(1/2)+2*a*
f^(1/2)+x*(2*c*d^(1/2)+b*f^(1/2)))/(c*x^2+b*x+a)^(1/2)/(c*d+a*f+b*d^(1/2)*f
^(1/2))^(1/2))*f^(1/2)*(B*d^(1/2)+A*f^(1/2))/d^(1/2)/(c*d+a*f+b*d^(1/2)*f^(
1/2))^(3/2)-2*(a*B*(2*a*c*f-b^2*f+2*c^2*d)+A*(b^3*f-b*c*(3*a*f+c*d))+c*(A*b
^2*f+b*B*(-a*f+c*d)-2*A*c*(a*f+c*d))*x)/(-4*a*c+b^2)/(b^2*d*f-(a*f+c*d)^2)/
(c*x^2+b*x+a)^(1/2)
```

Rubi [A] time = 0.80, antiderivative size = 380, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {1018, 1033, 724, 206}

$$-\frac{2\left(cx\left(-2Ac(af+cd)+bB(cd-af)+Ab^2f\right)-Abc(3af+cd)+aB\left(2acf+b^2(-f)+2c^2d\right)+Ab^3f\right)}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left(b^2df-(af+cd)^2\right)}-\frac{\sqrt{f}\,(}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x)/((a + b*x + c*x^2)^(3/2)*(d - f*x^2)),x]
```

```
[Out] (-2*(A*b^3*f - A*b*c*(c*d + 3*a*f) + a*B*(2*c^2*d - b^2*f + 2*a*c*f) + c*(A
*b^2*f + b*B*(c*d - a*f) - 2*A*c*(c*d + a*f))*x))/((b^2 - 4*a*c)*(b^2*d*f -
 (c*d + a*f)^2)*Sqrt[a + b*x + c*x^2]) - ((B*Sqrt[d] - A*Sqrt[f])*Sqrt[f]*A
rcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d
- b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])]])/(2*Sqrt[d]*(c*d - b*Sqr
t[d]*Sqrt[f] + a*f)^(3/2)) + ((B*Sqrt[d] + A*Sqrt[f])*Sqrt[f]*ArcTanh[(b*Sq
rt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqrt[d]*
Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])]])/(2*Sqrt[d]*(c*d + b*Sqrt[d]*Sqrt[f]
 + a*f)^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1018

```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (f
_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((a + b*x + c*x^2)^(p + 1)*(d + f*x^2)^
(q + 1)*((g*c)*(-(b*(c*d + a*f))) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(2*a*f
```

```
)) + c*(g*(2*c^2*d + b^2*f - c*(2*a*f)) - h*(b*c*d + a*b*f))*x))/((b^2 - 4*
a*c)*(b^2*d*f + (c*d - a*f)^2)*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*(b^2*d*
f + (c*d - a*f)^2)*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + f*x^2)^q*Si
mp[(b*h - 2*g*c)*((c*d - a*f)^2 - (b*d)*(-(b*f)))*(p + 1) + (b^2*(g*f) - b*
(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(a*f*(p + 1) - c*d*(p + 2)) - (2*f*(
(g*c)*(-(b*(c*d + a*f))) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(2*a*f)))*(p +
q + 2) - (b^2*(g*f) - b*(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(b*f*(p + 1)
))*x - c*f*(b^2*(g*f) - b*(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(2*p + 2*q
 + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, g, h, q}, x] && NeQ[b^2 - 4*
a*c, 0] && LtQ[p, -1] && NeQ[b^2*d*f + (c*d - a*f)^2, 0] &&  !( !IntegerQ[p
] && ILtQ[q, -1])
```

Rule 1033

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q
), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q
), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f
, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]
```

Rubi steps

$$\int \frac{A+Bx}{\left(a+bx+cx^2\right)^{3/2}\left(d-fx^2\right)}dx = -\frac{2\left(Ab^3f-Abc(cd+3af)+aB\left(2c^2d-b^2f+2acf\right)+c\left(Ab^2f+bB(cd-a\right.\right.}{\left(b^2-4ac\right)\left(b^2df-(cd+af)^2\right)\sqrt{a+bx+cx^2}}$$

$$= -\frac{2\left(Ab^3f-Abc(cd+3af)+aB\left(2c^2d-b^2f+2acf\right)+c\left(Ab^2f+bB(cd-a\right.\right.}{\left(b^2-4ac\right)\left(b^2df-(cd+af)^2\right)\sqrt{a+bx+cx^2}}$$

$$= -\frac{2\left(Ab^3f-Abc(cd+3af)+aB\left(2c^2d-b^2f+2acf\right)+c\left(Ab^2f+bB(cd-a\right.\right.}{\left(b^2-4ac\right)\left(b^2df-(cd+af)^2\right)\sqrt{a+bx+cx^2}}$$

$$= -\frac{2\left(Ab^3f-Abc(cd+3af)+aB\left(2c^2d-b^2f+2acf\right)+c\left(Ab^2f+bB(cd-a\right.\right.}{\left(b^2-4ac\right)\left(b^2df-(cd+af)^2\right)\sqrt{a+bx+cx^2}}$$

Mathematica [A] time = 0.65, size = 440, normalized size = 1.15

$$\frac{2\left(\frac{B\left(2a^2cf+a\left(b^2(-f)-bcfx+2c^2d\right)+bc^2dx\right)+A\left(-bc(3af+cd)-2c^2x(af+cd)+b^3f+b^2cfx\right)}{\sqrt{a+x(b+cx)}}+\frac{\sqrt{f}\left(b^2-4ac\right)\left(A\sqrt{f}-B\sqrt{d}\right)\left(af+b\sqrt{d}\sqrt{f}+cd\right)\tanh^{-1}\left(\ldots\right.}{4\sqrt{d}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right.}{\left(b^2-4ac\right)\left((af+cd)^2-b\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x)/((a + b*x + c*x^2)^(3/2)*(d - f*x^2)),x]
```

```
[Out] (2*((A*(b^3*f - b*c*(c*d + 3*a*f) + b^2*c*f*x - 2*c^2*(c*d + a*f)*x) + B*(2
*a^2*c*f + b*c^2*d*x + a*(2*c^2*d - b^2*f - b*c*f*x)))/Sqrt[a + x*(b + c*x)
] + ((b^2 - 4*a*c)*(-(B*Sqrt[d]) + A*Sqrt[f])*Sqrt[f]*(c*d + b*Sqrt[d]*Sqrt
[f] + a*f)*ArcTanh[(-2*a*Sqrt[f] + 2*c*Sqrt[d]*x + b*(Sqrt[d] - Sqrt[f]*x))
/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])])/(4*Sqrt[d]
```

```
*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]) + ((-b^2 + 4*a*c)*(B*Sqrt[d] + A*Sqrt
[f])*Sqrt[f]*(c*d - b*Sqrt[d]*Sqrt[f] + a*f)*ArcTanh[(-2*(a*Sqrt[f] + c*Sqr
t[d]*x) - b*(Sqrt[d] + Sqrt[f]*x))/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*S
qrt[a + x*(b + c*x)])])/(4*Sqrt[d]*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f])))/(
(b^2 - 4*a*c)*(-(b^2*d*f) + (c*d + a*f)^2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument ValueEvaluation time: 10.66Conj E
rror: Bad Argument Type
```

maple [B] time = 0.03, size = 2758, normalized size = 7.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x+A)/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x)
```

```
[Out] 1/2/(d*f)^(1/2)*f/(a*f+c*d-(d*f)^(1/2)*b)/((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*
f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*A-1/2/(a*f
+c*d-(d*f)^(1/2)*b)/((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(
1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*B+2/(a*f+c*d-(d*f)^(1/2)*b)/(4*a
*c-b^2)/((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a
*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*x*c^2*A-2*(d*f)^(1/2)/f/(a*f+c*d-(d*f)^(1/2)
*b)/(4*a*c-b^2)/((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)
/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*x*c^2*B-1/(d*f)^(1/2)*f/(a*f+c*d-(d*
f)^(1/2)*b)/(4*a*c-b^2)/((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*
f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*x*b*c*A+1/(a*f+c*d-(d*f)^(1/
2)*b)/(4*a*c-b^2)/((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/
2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*x*b*c*B+1/(a*f+c*d-(d*f)^(1/2)*b)/
(4*a*c-b^2)/((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/
f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*b*c*A-(d*f)^(1/2)/f/(a*f+c*d-(d*f)^(1/2)
*b)/(4*a*c-b^2)/((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)
/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*b*c*B-1/2/(d*f)^(1/2)*f/(a*f+c*d-(d*
f)^(1/2)*b)/(4*a*c-b^2)/((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*
f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*b^2*A+1/2/(a*f+c*d-(d*f)^(1/
2)*b)/(4*a*c-b^2)/((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/
2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*b^2*B-1/2/(d*f)^(1/2)*f/(a*f+c*d-(
d*f)^(1/2)*b)/((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d-(d*f)^(1/2)*
b)/f+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+2*((a*f+c*d-(d*f)^(1/2)*b)/f
)^(1/2)*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a
*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))/(x+(d*f)^(1/2)/f))*A+1/2/(a*f+c*d-(d*f)^(1/
2)*b)/((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d-(d*f)^(1/2)*b)/f+(b*
```

```
f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+2*((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*
((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(
d*f)^(1/2)*b)/f)^(1/2))/(x+(d*f)^(1/2)/f))*B-1/2/(d*f)^(1/2)*f/(a*f+c*d+(d*
f)^(1/2)*b)/((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/
f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*A-1/2/(a*f+c*d+(d*f)^(1/2)*b)/((x-(d*f)^
(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)
*b)/f)^(1/2)*B+2/(a*f+c*d+(d*f)^(1/2)*b)/(4*a*c-b^2)/((x-(d*f)^(1/2)/f)^2*c
+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)
*x*c^2*A+2*(d*f)^(1/2)/f/(a*f+c*d+(d*f)^(1/2)*b)/(4*a*c-b^2)/((x-(d*f)^(1/2
)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/
f)^(1/2)*x*c^2*B+1/(d*f)^(1/2)*f/(a*f+c*d+(d*f)^(1/2)*b)/(4*a*c-b^2)/((x-(d
*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(
1/2)*b)/f)^(1/2)*x*b*c*A+1/(a*f+c*d+(d*f)^(1/2)*b)/(4*a*c-b^2)/((x-(d*f)^(1
/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b
)/f)^(1/2)*x*b*c*B+1/(a*f+c*d+(d*f)^(1/2)*b)/(4*a*c-b^2)/((x-(d*f)^(1/2)/f)
^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(
1/2)*b*c*A+(d*f)^(1/2)/f/(a*f+c*d+(d*f)^(1/2)*b)/(4*a*c-b^2)/((x-(d*f)^(1/2
)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/
f)^(1/2)*b*c*B+1/2/(d*f)^(1/2)*f/(a*f+c*d+(d*f)^(1/2)*b)/(4*a*c-b^2)/((x-(d
*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(
1/2)*b)/f)^(1/2)*b^2*A+1/2/(a*f+c*d+(d*f)^(1/2)*b)/(4*a*c-b^2)/((x-(d*f)^(1
/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b
)/f)^(1/2)*b^2*B+1/2/(d*f)^(1/2)*f/(a*f+c*d+(d*f)^(1/2)*b)/((a*f+c*d+(d*f)^
(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d+(d*f)^(1/2)*b)/f+(b*f+2*(d*f)^(1/2)*c)*(x-
(d*f)^(1/2)/f)/f+2*((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c
+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)
)/(x-(d*f)^(1/2)/f))*A+1/2/(a*f+c*d+(d*f)^(1/2)*b)/((a*f+c*d+(d*f)^(1/2)*b)
/f)^(1/2)*ln((2*(a*f+c*d+(d*f)^(1/2)*b)/f+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1
/2)/f)/f+2*((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*
(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))/(x-(d*
f)^(1/2)/f))*B
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(((c*sqrt(4*d*f))/(2*f^2)>0)', see `
assume?` for more details)Is ((c*sqrt(4*d*f))/(2*f^2)    +b/(2*f))    ^2
 -(c*((b*sqrt(4*d*f))                    /(2*f)                  +(c*d)/f+a))
     /f^2 zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+B\,x}{\left(d-f\,x^2\right)\,{\left(c\,x^2+b\,x+a\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x)/((d - f*x^2)*(a + b*x + c*x^2)^(3/2)),x)
```

```
[Out] int((A + B*x)/((d - f*x^2)*(a + b*x + c*x^2)^(3/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(c*x**2+b*x+a)**(3/2)/(-f*x**2+d),x)
```

```
[Out] Timed out
```

3.9 $\int \frac{A+Bx}{(a+bx+cx^2)^{5/2}(d-fx^2)}\,dx$

Optimal. Leaf size=797

$$-\frac{\left(B\sqrt{d}-A\sqrt{f}\right)\tanh^{-1}\left(\frac{-2\sqrt{f}\,a+\left(2c\sqrt{d}-b\sqrt{f}\right)x+b\sqrt{d}}{2\sqrt{-\sqrt{d}\sqrt{f}\,b+cd+af}\,\sqrt{cx^2+bx+a}}\right)f^{3/2}}{2\sqrt{d}\left(-\sqrt{d}\sqrt{f}\,b+cd+af\right)^{5/2}}+\frac{\left(\sqrt{f}\,A+B\sqrt{d}\right)\tanh^{-1}\left(\frac{2\sqrt{f}\,a+\left(\sqrt{f}\,b+2c\sqrt{d}\right)x+b\sqrt{d}}{2\sqrt{\sqrt{d}\sqrt{f}\,b+cd+af}\,\sqrt{cx^2+bx+a}}\right)f^{3/2}}{2\sqrt{d}\left(\sqrt{d}\sqrt{f}\,b+cd+af\right)^{5/2}}$$

```
[Out] -2/3*(a*B*(2*a*c*f-b^2*f+2*c^2*d)+A*(b^3*f-b*c*(3*a*f+c*d))+c*(A*b^2*f+b*B*
(-a*f+c*d)-2*A*c*(a*f+c*d))*x)/(-4*a*c+b^2)/(b^2*d*f-(a*f+c*d)^2)/(c*x^2+b*
x+a)^(3/2)-1/2*f^(3/2)*arctanh(1/2*(b*d^(1/2)-2*a*f^(1/2)+x*(2*c*d^(1/2)-b*
f^(1/2)))/(c*x^2+b*x+a)^(1/2)/(c*d+a*f-b*d^(1/2)*f^(1/2))^(1/2))*(B*d^(1/2)
-A*f^(1/2))/d^(1/2)/(c*d+a*f-b*d^(1/2)*f^(1/2))^(5/2)+1/2*f^(3/2)*arctanh(1
/2*(b*d^(1/2)+2*a*f^(1/2)+x*(2*c*d^(1/2)+b*f^(1/2)))/(c*x^2+b*x+a)^(1/2)/(c
*d+a*f+b*d^(1/2)*f^(1/2))^(1/2))*(B*d^(1/2)+A*f^(1/2))/d^(1/2)/(c*d+a*f+b*d
^(1/2)*f^(1/2))^(5/2)-2/3*(3*b^6*B*d*f^2+24*a^2*B*c^2*f*(a*f+c*d)^2-A*b^5*f
^2*(6*a*f+7*c*d)-b^4*B*f*(-3*a^2*f^2+14*a*c*d*f+7*c^2*d^2)+A*b^3*c*f*(43*a^
2*f^2+46*a*c*d*f+15*c^2*d^2)+2*b^2*B*c*(-11*a^3*f^3+4*a^2*c*d*f^2+5*a*c^2*d
^2*f+2*c^3*d^3)-4*A*b*c^2*(17*a^3*f^3+24*a^2*c*d*f^2+9*a*c^2*d^2*f+2*c^3*d^
3)+c*(3*b^5*B*d*f^2-2*A*b^4*f^2*(3*a*f+4*c*d)-8*A*c^2*(a*f+c*d)^2*(5*a*f+2*
c*d)-b^3*B*f*(-3*a^2*f^2+10*a*c*d*f+17*c^2*d^2)+2*A*b^2*c*f*(19*a^2*f^2+22*
a*c*d*f+15*c^2*d^2)+4*b*B*c*(-5*a^3*f^3+4*a^2*c*d*f^2+11*a*c^2*d^2*f+2*c^3*
d^3))*x)/(-4*a*c+b^2)^2/(c^2*d^2+2*a*c*d*f-f*(-a^2*f+b^2*d))^2/(c*x^2+b*x+a
)^(1/2)
```

Rubi [A] time = 1.87, antiderivative size = 796, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1018, 1064, 1033, 724, 206}

$$-\frac{\left(B\sqrt{d}-A\sqrt{f}\right)\tanh^{-1}\left(\frac{-2\sqrt{f}\,a+\left(2c\sqrt{d}-b\sqrt{f}\right)x+b\sqrt{d}}{2\sqrt{-\sqrt{d}\sqrt{f}\,b+cd+af}\,\sqrt{cx^2+bx+a}}\right)f^{3/2}}{2\sqrt{d}\left(-\sqrt{d}\sqrt{f}\,b+cd+af\right)^{5/2}}+\frac{\left(\sqrt{f}\,A+B\sqrt{d}\right)\tanh^{-1}\left(\frac{2\sqrt{f}\,a+\left(\sqrt{f}\,b+2c\sqrt{d}\right)x+b\sqrt{d}}{2\sqrt{\sqrt{d}\sqrt{f}\,b+cd+af}\,\sqrt{cx^2+bx+a}}\right)f^{3/2}}{2\sqrt{d}\left(\sqrt{d}\sqrt{f}\,b+cd+af\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x)/((a + b*x + c*x^2)^(5/2)*(d - f*x^2)),x]
```

```
[Out] (-2*(A*b^3*f - A*b*c*(c*d + 3*a*f) + a*B*(2*c^2*d - b^2*f + 2*a*c*f) + c*(A
*b^2*f + b*B*(c*d - a*f) - 2*A*c*(c*d + a*f))*x))/(3*(b^2 - 4*a*c)*(b^2*d*f
 - (c*d + a*f)^2)*(a + b*x + c*x^2)^(3/2)) - (2*(3*b^6*B*d*f^2 + 24*a^2*B*c
^2*f*(c*d + a*f)^2 - A*b^5*f^2*(7*c*d + 6*a*f) - b^4*B*f*(7*c^2*d^2 + 14*a*
c*d*f - 3*a^2*f^2) + A*b^3*c*f*(15*c^2*d^2 + 46*a*c*d*f + 43*a^2*f^2) + 2*b
^2*B*c*(2*c^3*d^3 + 5*a*c^2*d^2*f + 4*a^2*c*d*f^2 - 11*a^3*f^3) - 4*A*b*c^2
*(2*c^3*d^3 + 9*a*c^2*d^2*f + 24*a^2*c*d*f^2 + 17*a^3*f^3) + c*(3*b^5*B*d*f
^2 - 2*A*b^4*f^2*(4*c*d + 3*a*f) - 8*A*c^2*(c*d + a*f)^2*(2*c*d + 5*a*f) -
b^3*B*f*(17*c^2*d^2 + 10*a*c*d*f - 3*a^2*f^2) + 2*A*b^2*c*f*(15*c^2*d^2 + 2
2*a*c*d*f + 19*a^2*f^2) + 4*b*B*c*(2*c^3*d^3 + 11*a*c^2*d^2*f + 4*a^2*c*d*f
^2 - 5*a^3*f^3))*x))/(3*(b^2 - 4*a*c)^2*(c^2*d^2 + 2*a*c*d*f - f*(b^2*d - a
^2*f))^2*Sqrt[a + b*x + c*x^2]) - ((B*Sqrt[d] - A*Sqrt[f])*f^(3/2)*ArcTanh[
(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqr
t[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[d]*(c*d - b*Sqrt[d]*Sq
rt[f] + a*f)^(5/2)) + ((B*Sqrt[d] + A*Sqrt[f])*f^(3/2)*ArcTanh[(b*Sqrt[d] +
 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f]
 + a*f]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[d]*(c*d + b*Sqrt[d]*Sqrt[f] + a*f)
^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1018

```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (f
_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((a + b*x + c*x^2)^(p + 1)*(d + f*x^2)^
(q + 1)*((g*c)*(-(b*(c*d + a*f))) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(2*a*f
)) + c*(g*(2*c^2*d + b^2*f - c*(2*a*f)) - h*(b*c*d + a*b*f))*x))/((b^2 - 4*
a*c)*(b^2*d*f + (c*d - a*f)^2)*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*(b^2*d*
f + (c*d - a*f)^2)*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + f*x^2)^q*Si
mp[(b*h - 2*g*c)*((c*d - a*f)^2 - (b*d)*(-(b*f)))*(p + 1) + (b^2*(g*f) - b*
(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(a*f*(p + 1) - c*d*(p + 2)) - (2*f*(
(g*c)*(-(b*(c*d + a*f))) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(2*a*f)))*(p +
q + 2) - (b^2*(g*f) - b*(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(b*f*(p + 1)
))*x - c*f*(b^2*(g*f) - b*(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(2*p + 2*q
 + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, g, h, q}, x] && NeQ[b^2 - 4*
a*c, 0] && LtQ[p, -1] && NeQ[b^2*d*f + (c*d - a*f)^2, 0] &&  !( !IntegerQ[p
] && ILtQ[q, -1])
```

Rule 1033

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q
), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q
), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f
, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]
```

Rule 1064

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((a + b*x + c*x^2)^(p +
1)*(d + f*x^2)^(q + 1)*((A*c - a*C)*(-(b*(c*d + a*f))) + (A*b - a*B)*(2*c^2
*d + b^2*f - c*(2*a*f)) + c*(A*(2*c^2*d + b^2*f - c*(2*a*f)) - B*(b*c*d + a
*b*f) + C*(b^2*d - 2*a*(c*d - a*f)))*x))/((b^2 - 4*a*c)*(b^2*d*f + (c*d - a
*f)^2)*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*(b^2*d*f + (c*d - a*f)^2)*(p +
1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + f*x^2)^q*Simp[(b*B - 2*A*c - 2*a*C)
*((c*d - a*f)^2 - (b*d)*(-(b*f)))*(p + 1) + (b^2*(C*d + A*f) - b*(B*c*d + a
*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)
) - (2*f*((A*c - a*C)*(-(b*(c*d + a*f))) + (A*b - a*B)*(2*c^2*d + b^2*f - c
*(2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d + a*B*f) + 2*(A*c*(c*d
 - a*f) - a*(c*C*d - a*C*f)))*(b*f*(p + 1)))*x - c*f*(b^2*(C*d + A*f) - b*(
B*c*d + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - a*C*f)))*(2*p + 2*q + 5)*x
^2, x], x], x] /; FreeQ[{a, b, c, d, f, A, B, C, q}, x] && NeQ[b^2 - 4*a*c,
 0] && LtQ[p, -1] && NeQ[b^2*d*f + (c*d - a*f)^2, 0] &&  !( !IntegerQ[p] &&
 ILtQ[q, -1]) &&  !IGtQ[q, 0]
```

Rubi steps

$$\int \frac{A+Bx}{\left(a+bx+cx^2\right)^{5/2}\left(d-fx^2\right)}\,dx = -\frac{2\left(Ab^3f-Abc(cd+3af)+aB\left(2c^2d-b^2f+2acf\right)+c\left(Ab^2f+bB(cd-a\right.\right.}{3\left(b^2-4ac\right)\left(b^2df-(cd+af)^2\right)\left(a+bx+cx^2\right)^{3/2}}$$

$$= -\frac{2\left(Ab^3f-Abc(cd+3af)+aB\left(2c^2d-b^2f+2acf\right)+c\left(Ab^2f+bB(cd-a\right.\right.}{3\left(b^2-4ac\right)\left(b^2df-(cd+af)^2\right)\left(a+bx+cx^2\right)^{3/2}}$$

$$= -\frac{2\left(Ab^3f-Abc(cd+3af)+aB\left(2c^2d-b^2f+2acf\right)+c\left(Ab^2f+bB(cd-a\right.\right.}{3\left(b^2-4ac\right)\left(b^2df-(cd+af)^2\right)\left(a+bx+cx^2\right)^{3/2}}$$

$$= -\frac{2\left(Ab^3f-Abc(cd+3af)+aB\left(2c^2d-b^2f+2acf\right)+c\left(Ab^2f+bB(cd-a\right.\right.}{3\left(b^2-4ac\right)\left(b^2df-(cd+af)^2\right)\left(a+bx+cx^2\right)^{3/2}}$$

$$= -\frac{2\left(Ab^3f-Abc(cd+3af)+aB\left(2c^2d-b^2f+2acf\right)+c\left(Ab^2f+bB(cd-a\right.\right.}{3\left(b^2-4ac\right)\left(b^2df-(cd+af)^2\right)\left(a+bx+cx^2\right)^{3/2}}$$

Mathematica [A] time = 4.06, size = 674, normalized size = 0.85

$$2\left(\frac{B\left(2a^2cf+a\left(b^2(-f)-bcfx+2c^2d\right)+bc^2dx\right)+A\left(-bc(3af+cd)-2c^2x(af+cd)+b^3f+b^2cfx\right)}{(a+x(b+cx))^{3/2}}-\frac{3f\left(-b^2\left(B\left(-a^2f^2+2acdf+c^2d^2\right)+2aAcf^2x\right)+b^3f(Bcdx-A\right.}{\sqrt{a+x(b\cdots}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x)/((a + b*x + c*x^2)^(5/2)*(d - f*x^2)),x]
```

```
[Out] (2*((4*c*(-(A*b^2*f) + b*B*(-(c*d) + a*f) + 2*A*c*(c*d + a*f))*(b + 2*c*x))
/((b^2 - 4*a*c)*Sqrt[a + x*(b + c*x)]) - (3*f*(b^4*B*d*f + 2*c*(c*d + a*f)^
2*(-(a*B) + A*c*x) + b^3*f*(-(A*(c*d + 2*a*f)) + B*c*d*x) + b*c*(c*d + a*f)
*(A*c*d + 5*a*A*f - 3*B*c*d*x + a*B*f*x) - b^2*(B*(c^2*d^2 + 2*a*c*d*f - a^
2*f^2) + 2*a*A*c*f^2*x)))/((c^2*d^2 + 2*a*c*d*f + f*(-(b^2*d) + a^2*f))*Sqr
t[a + x*(b + c*x)]) + (A*(b^3*f - b*c*(c*d + 3*a*f) + b^2*c*f*x - 2*c^2*(c*
d + a*f)*x) + B*(2*a^2*c*f + b*c^2*d*x + a*(2*c^2*d - b^2*f - b*c*f*x)))/(a
 + x*(b + c*x))^(3/2) + (3*(b^2 - 4*a*c)*f^(3/2)*(((-(B*Sqrt[d]) + A*Sqrt[f
])*(c*d + b*Sqrt[d]*Sqrt[f] + a*f)^2*ArcTanh[(-2*a*Sqrt[f] + 2*c*Sqrt[d]*x
+ b*(Sqrt[d] - Sqrt[f]*x))/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a +
x*(b + c*x)])])/Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f] - ((B*Sqrt[d] + A*Sqrt[
f])*(c*d - b*Sqrt[d]*Sqrt[f] + a*f)^2*ArcTanh[(-2*(a*Sqrt[f] + c*Sqrt[d]*x)
 - b*(Sqrt[d] + Sqrt[f]*x))/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a +
 x*(b + c*x)])])/Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]))/(4*Sqrt[d]*(-(b^2*d*
f) + (c*d + a*f)^2))))/(3*(b^2 - 4*a*c)*(-(b^2*d*f) + (c*d + a*f)^2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(c*x^2+b*x+a)^(5/2)/(-f*x^2+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(c*x^2+b*x+a)^(5/2)/(-f*x^2+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument ValueEvaluation time: 5.84Conj Er
ror: Bad Argument Type
```

maple [B] time = 0.03, size = 6422, normalized size = 8.06

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x+A)/(c*x^2+b*x+a)^(5/2)/(-f*x^2+d),x)
```

```
[Out] result too large to display
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(c*x^2+b*x+a)^(5/2)/(-f*x^2+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(((c*sqrt(4*d*f))/(2*f^2)>0)', see `
assume?` for more details)Is ((c*sqrt(4*d*f))/(2*f^2)    +b/(2*f))    ^2
 -(c*((b*sqrt(4*d*f))                    /(2*f)                 +(c*d)/f+a))
    /f^2 zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+B\,x}{\left(d-f\,x^2\right)\,{\left(c\,x^2+b\,x+a\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x)/((d - f*x^2)*(a + b*x + c*x^2)^(5/2)),x)
```

```
[Out] int((A + B*x)/((d - f*x^2)*(a + b*x + c*x^2)^(5/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(c*x**2+b*x+a)**(5/2)/(-f*x**2+d),x)
```

```
[Out] Timed out
```

3.10 $\int \frac{1+2x}{(-1+x^2)\sqrt{-1+x+x^2}}\,dx$

Optimal. Leaf size=47

$$\frac{3}{2}\tanh^{-1}\left(\frac{1-3x}{2\sqrt{x^2+x-1}}\right)-\frac{1}{2}\tan^{-1}\left(\frac{x+3}{2\sqrt{x^2+x-1}}\right)$$

```
[Out] -1/2*arctan(1/2*(3+x)/(x^2+x-1)^(1/2))+3/2*arctanh(1/2*(1-3*x)/(x^2+x-1)^(1
/2))
```

Rubi [A] time = 0.03, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {1033, 724, 206, 204}

$$\frac{3}{2}\tanh^{-1}\left(\frac{1-3x}{2\sqrt{x^2+x-1}}\right)-\frac{1}{2}\tan^{-1}\left(\frac{x+3}{2\sqrt{x^2+x-1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x)/((-1 + x^2)*Sqrt[-1 + x + x^2]),x]
```

```
[Out] -ArcTan[(3 + x)/(2*Sqrt[-1 + x + x^2])]/2 + (3*ArcTanh[(1 - 3*x)/(2*Sqrt[-1
 + x + x^2])])/2
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1033

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q
), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q
), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f
, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]
```

Rubi steps

$$\int \frac{1+2x}{\left(-1+x^2\right)\sqrt{-1+x+x^2}}\,dx = \frac{1}{2}\int \frac{1}{(1+x)\sqrt{-1+x+x^2}}\,dx + \frac{3}{2}\int \frac{1}{(-1+x)\sqrt{-1+x+x^2}}\,dx$$
$$= -\left(3\,\text{Subst}\left(\int \frac{1}{4-x^2}\,dx, x, \frac{-1+3x}{\sqrt{-1+x+x^2}}\right)\right) - \text{Subst}\left(\int \frac{1}{-4-x^2}\,dx, x, \frac{\cdot}{\sqrt{-1}}\right.$$
$$= \frac{1}{2}\tan^{-1}\left(\frac{-3-x}{2\sqrt{-1+x+x^2}}\right) + \frac{3}{2}\tanh^{-1}\left(\frac{1-3x}{2\sqrt{-1+x+x^2}}\right)$$

Mathematica [A] time = 0.01, size = 49, normalized size = 1.04

$$\frac{1}{2}\tan^{-1}\left(\frac{-x-3}{2\sqrt{x^2+x-1}}\right) - \frac{3}{2}\tanh^{-1}\left(\frac{3x-1}{2\sqrt{x^2+x-1}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x)/((-1 + x^2)*Sqrt[-1 + x + x^2]),x]
```

```
[Out] ArcTan[(-3 - x)/(2*Sqrt[-1 + x + x^2])]/2 - (3*ArcTanh[(-1 + 3*x)/(2*Sqrt[-
1 + x + x^2])])/2
```

fricas [A] time = 1.12, size = 46, normalized size = 0.98

$$\arctan\left(-x+\sqrt{x^2+x-1}-1\right) - \frac{3}{2}\log\left(-x+\sqrt{x^2+x-1}+2\right) + \frac{3}{2}\log\left(-x+\sqrt{x^2+x-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)/(x^2-1)/(x^2+x-1)^(1/2),x, algorithm="fricas")
```

```
[Out] arctan(-x + sqrt(x^2 + x - 1) - 1) - 3/2*log(-x + sqrt(x^2 + x - 1) + 2) +
3/2*log(-x + sqrt(x^2 + x - 1))
```

giac [A] time = 0.25, size = 48, normalized size = 1.02

$$\arctan\left(-x+\sqrt{x^2+x-1}-1\right) - \frac{3}{2}\log\left(\left|-x+\sqrt{x^2+x-1}+2\right|\right) + \frac{3}{2}\log\left(\left|-x+\sqrt{x^2+x-1}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)/(x^2-1)/(x^2+x-1)^(1/2),x, algorithm="giac")
```

```
[Out] arctan(-x + sqrt(x^2 + x - 1) - 1) - 3/2*log(abs(-x + sqrt(x^2 + x - 1) + 2
)) + 3/2*log(abs(-x + sqrt(x^2 + x - 1)))
```

maple [A] time = 0.02, size = 46, normalized size = 0.98

$$-\frac{3\,\text{arctanh}\left(\frac{3x-1}{2\sqrt{3x+(x-1)^2-2}}\right)}{2} + \frac{\arctan\left(\frac{-x-3}{2\sqrt{-x+(x+1)^2-2}}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+2*x)/(x^2-1)/(x^2+x-1)^(1/2),x)
```

```
[Out] -3/2*arctanh(1/2*(-1+3*x)/((x-1)^2+3*x-2)^(1/2))+1/2*arctan(1/2*(-3-x)/((1+
x)^2-x-2)^(1/2))
```

maxima [A] time = 0.97, size = 65, normalized size = 1.38

$$-\frac{1}{2}\arcsin\left(\frac{2\sqrt{5}\,x}{5\,|2\,x+2|} + \frac{6\sqrt{5}}{5\,|2\,x+2|}\right) - \frac{3}{2}\log\left(\frac{2\sqrt{x^2+x-1}}{|2\,x-2|} + \frac{2}{|2\,x-2|} + \frac{3}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)/(x^2-1)/(x^2+x-1)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*arcsin(2/5*sqrt(5)*x/abs(2*x + 2) + 6/5*sqrt(5)/abs(2*x + 2)) - 3/2*lo
g(2*sqrt(x^2 + x - 1)/abs(2*x - 2) + 2/abs(2*x - 2) + 3/2)
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{2\,x+1}{\left(x^2-1\right)\,\sqrt{x^2+x-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x + 1)/((x^2 - 1)*(x + x^2 - 1)^(1/2)),x)
```

```
[Out] int((2*x + 1)/((x^2 - 1)*(x + x^2 - 1)^(1/2)), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2x+1}{(x-1)\,(x+1)\,\sqrt{x^2+x-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)/(x**2-1)/(x**2+x-1)**(1/2),x)
```

```
[Out] Integral((2*x + 1)/((x - 1)*(x + 1)*sqrt(x**2 + x - 1)), x)
```

3.11 $\int \frac{1+2x}{(1+x^2)\sqrt{-1+x+x^2}}\,dx$

Optimal. Leaf size=117

$$\sqrt{\frac{1}{2}\left(\sqrt{5}-2\right)}\tanh^{-1}\left(\frac{\sqrt{5}x-2\sqrt{5}+5}{\sqrt{10\left(\sqrt{5}-2\right)}\sqrt{x^2+x-1}}\right)-\sqrt{\frac{1}{2}\left(2+\sqrt{5}\right)}\tan^{-1}\left(\frac{-\sqrt{5}x+2\sqrt{5}+5}{\sqrt{10\left(2+\sqrt{5}\right)}\sqrt{x^2+x-1}}\right)$$

```
[Out] 1/2*arctanh((5-2*5^(1/2)+x*5^(1/2))/(x^2+x-1)^(1/2)/(-20+10*5^(1/2))^(1/2))
*(-4+2*5^(1/2))^(1/2)-1/2*arctan((5+2*5^(1/2)-x*5^(1/2))/(x^2+x-1)^(1/2)/(2
0+10*5^(1/2))^(1/2))*(4+2*5^(1/2))^(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {1036, 1030, 207, 203}

$$\sqrt{\frac{1}{2}\left(\sqrt{5}-2\right)}\tanh^{-1}\left(\frac{\sqrt{5}x-2\sqrt{5}+5}{\sqrt{10\left(\sqrt{5}-2\right)}\sqrt{x^2+x-1}}\right)-\sqrt{\frac{1}{2}\left(2+\sqrt{5}\right)}\tan^{-1}\left(\frac{-\sqrt{5}x+2\sqrt{5}+5}{\sqrt{10\left(2+\sqrt{5}\right)}\sqrt{x^2+x-1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x)/((1 + x^2)*Sqrt[-1 + x + x^2]),x]
```

```
[Out] -(Sqrt[(2 + Sqrt[5])/2]*ArcTan[(5 + 2*Sqrt[5] - Sqrt[5]*x)/(Sqrt[10*(2 + Sq
rt[5])]*Sqrt[-1 + x + x^2])]) + Sqrt[(-2 + Sqrt[5])/2]*ArcTanh[(5 - 2*Sqrt[
5] + Sqrt[5]*x)/(Sqrt[10*(-2 + Sqrt[5])]*Sqrt[-1 + x + x^2])]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 1030

```
Int[((g_) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f
_.)*(x_)^2]), x_Symbol] :> Dist[-2*a*g*h, Subst[Int[1/Simp[2*a^2*g*h*c + a*
e*x^2, x], x], x, Simp[a*h - g*c*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[
{a, c, d, e, f, g, h}, x] && EqQ[a*h^2*e + 2*g*h*(c*d - a*f) - g^2*c*e, 0]
```

Rule 1036

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 + a*c*e^2, 2]}, Dist
[1/(2*q), Int[Simp[-(a*h*e) - g*(c*d - a*f - q) + (h*(c*d - a*f + q) - g*c*
e)*x, x]/((a + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Si
mp[-(a*h*e) - g*(c*d - a*f + q) + (h*(c*d - a*f - q) - g*c*e)*x, x]/((a + c
*x^2)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f, g, h}, x] &&
 NeQ[e^2 - 4*d*f, 0] && NegQ[-(a*c)]
```

Rubi steps

$$\int \frac{1+2x}{\left(1+x^2\right)\sqrt{-1+x+x^2}}\,dx = -\frac{\int \frac{-\sqrt{5}+\left(-5-2\sqrt{5}\right)x}{\left(1+x^2\right)\sqrt{-1+x+x^2}}\,dx}{2\sqrt{5}} + \frac{\int \frac{\sqrt{5}+\left(-5+2\sqrt{5}\right)x}{\left(1+x^2\right)\sqrt{-1+x+x^2}}\,dx}{2\sqrt{5}}$$

$$= -\left(\left(-5+2\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{10\left(2-\sqrt{5}\right)+x^2}\,dx, x, \frac{-5+2\sqrt{5}-\sqrt{5}x}{\sqrt{-1+x+x^2}}\right)\right) + \left(5+2\sqrt{}\right.$$

$$= -\sqrt{\frac{1}{2}\left(2+\sqrt{5}\right)}\tan^{-1}\left(\frac{5+2\sqrt{5}-\sqrt{5}x}{\sqrt{10\left(2+\sqrt{5}\right)}\sqrt{-1+x+x^2}}\right) + \sqrt{\frac{1}{2}\left(-2+\sqrt{5}\right)}\tanh^{-1}$$

Mathematica [C] time = 0.03, size = 78, normalized size = 0.67

$$-\frac{1}{2}i\left(\sqrt{2+i}\tanh^{-1}\left(\frac{\sqrt{2+i}(x-i)}{2\sqrt{x^2+x-1}}\right) - \sqrt{2-i}\tanh^{-1}\left(\frac{\sqrt{2-i}(x+i)}{2\sqrt{x^2+x-1}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x)/((1 + x^2)*Sqrt[-1 + x + x^2]),x]
```

```
[Out] (-1/2*I)*(Sqrt[2 + I]*ArcTanh[(Sqrt[2 + I]*(-I + x))/(2*Sqrt[-1 + x + x^2])
] - Sqrt[2 - I]*ArcTanh[(Sqrt[2 - I]*(I + x))/(2*Sqrt[-1 + x + x^2])])
```

fricas [B] time = 1.01, size = 758, normalized size = 6.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)/(x^2+1)/(x^2+x-1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/20*5^(1/4)*sqrt(4*sqrt(5) + 10)*(2*sqrt(5) - 5)*log(2*x^2 - 2*sqrt(x^2 +
x - 1)*x + 1/5*(5^(1/4)*sqrt(x^2 + x - 1)*(2*sqrt(5) - 5) - 5^(1/4)*(sqrt(5
)*(2*x + 1) - 5*x))*sqrt(4*sqrt(5) + 10) + x + sqrt(5)) - 1/20*5^(1/4)*sqrt
(4*sqrt(5) + 10)*(2*sqrt(5) - 5)*log(2*x^2 - 2*sqrt(x^2 + x - 1)*x - 1/5*(5
^(1/4)*sqrt(x^2 + x - 1)*(2*sqrt(5) - 5) - 5^(1/4)*(sqrt(5)*(2*x + 1) - 5*x
))*sqrt(4*sqrt(5) + 10) + x + sqrt(5)) - 1/5*5^(3/4)*sqrt(4*sqrt(5) + 10)*a
rctan(2/55*sqrt(5)*(sqrt(5)*(2*x - 1) + 3*x + 4) + 1/275*sqrt(10*x^2 - 10*s
qrt(x^2 + x - 1)*x + (5^(1/4)*sqrt(x^2 + x - 1)*(2*sqrt(5) - 5) - 5^(1/4)*(
sqrt(5)*(2*x + 1) - 5*x))*sqrt(4*sqrt(5) + 10) + 5*x + 5*sqrt(5))*((5^(3/4)
*(2*sqrt(5) + 3) + 2*5^(1/4)*(4*sqrt(5) - 5))*sqrt(4*sqrt(5) + 10) + 2*sqrt
(5)*(3*sqrt(5) + 10) - 20*sqrt(5) + 80) - 2/55*sqrt(x^2 + x - 1)*(sqrt(5)*(
2*sqrt(5) + 3) + 8*sqrt(5) - 10) + 1/55*sqrt(5)*(16*x + 3) + 1/275*(5^(3/4)
*(sqrt(5)*(3*x + 4) + 10*x - 5) - sqrt(x^2 + x - 1)*(5^(3/4)*(3*sqrt(5) + 1
0) - 10*5^(1/4)*(sqrt(5) - 4)) - 10*5^(1/4)*(sqrt(5)*(x - 6) - 4*x + 13))*s
qrt(4*sqrt(5) + 10) - 4/11*x + 2/11) - 1/5*5^(3/4)*sqrt(4*sqrt(5) + 10)*arc
tan(-2/55*sqrt(5)*(sqrt(5)*(2*x - 1) + 3*x + 4) + 1/275*sqrt(10*x^2 - 10*sq
rt(x^2 + x - 1)*x - (5^(1/4)*sqrt(x^2 + x - 1)*(2*sqrt(5) - 5) - 5^(1/4)*(s
qrt(5)*(2*x + 1) - 5*x))*sqrt(4*sqrt(5) + 10) + 5*x + 5*sqrt(5))*((5^(3/4)*
(2*sqrt(5) + 3) + 2*5^(1/4)*(4*sqrt(5) - 5))*sqrt(4*sqrt(5) + 10) - 2*sqrt(
5)*(3*sqrt(5) + 10) + 20*sqrt(5) - 80) + 2/55*sqrt(x^2 + x - 1)*(sqrt(5)*(2
*sqrt(5) + 3) + 8*sqrt(5) - 10) - 1/55*sqrt(5)*(16*x + 3) + 1/275*(5^(3/4)*
(sqrt(5)*(3*x + 4) + 10*x - 5) - sqrt(x^2 + x - 1)*(5^(3/4)*(3*sqrt(5) + 10
) - 10*5^(1/4)*(sqrt(5) - 4)) - 10*5^(1/4)*(sqrt(5)*(x - 6) - 4*x + 13))*sq
rt(4*sqrt(5) + 10) + 4/11*x - 2/11)
```

giac [B] time = 0.38, size = 457, normalized size = 3.91

$$\frac{1}{4}\sqrt{2\sqrt{5}-4}\log\left(16\left(15\sqrt{5}\left(x-\sqrt{x^2+x-1}\right)+33x+5\sqrt{5}-33\sqrt{x^2+x-1}+2\sqrt{5\sqrt{5}+11}+11\right)^2+16$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)/(x^2+1)/(x^2+x-1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2*sqrt(5) - 4)*log(16*(15*sqrt(5)*(x - sqrt(x^2 + x - 1)) + 33*x +
 5*sqrt(5) - 33*sqrt(x^2 + x - 1) + 2*sqrt(5*sqrt(5) + 11) + 11)^2 + 16*(5*
sqrt(5)*(x - sqrt(x^2 + x - 1)) + 11*x - 5*sqrt(5)*sqrt(5*sqrt(5) + 11) - 1
5*sqrt(5) - 11*sqrt(x^2 + x - 1) - 11*sqrt(5*sqrt(5) + 11) - 33)^2) - 1/4*s
qrt(2*sqrt(5) - 4)*log(16*(15*sqrt(5)*(x - sqrt(x^2 + x - 1)) + 33*x + 5*sq
rt(5) - 33*sqrt(x^2 + x - 1) - 2*sqrt(5*sqrt(5) + 11) + 11)^2 + 16*(5*sqrt(
5)*(x - sqrt(x^2 + x - 1)) + 11*x + 5*sqrt(5)*sqrt(5*sqrt(5) + 11) - 15*sqr
t(5) - 11*sqrt(x^2 + x - 1) + 11*sqrt(5*sqrt(5) + 11) - 33)^2) + 1/2*sqrt(2
*sqrt(5) - 4)*(arctan(3) + arctan(1/10*(x - sqrt(x^2 + x - 1))*(sqrt(5)*sqr
t(5*sqrt(5) + 11) + 4*sqrt(5) - 5*sqrt(5*sqrt(5) + 11)) - 7/10*sqrt(5)*sqrt
(5*sqrt(5) + 11) + 1/5*sqrt(5) + 3/2*sqrt(5*sqrt(5) + 11)))/(sqrt(5) - 2) -
 1/2*sqrt(2*sqrt(5) - 4)*(arctan(3) + arctan(-1/10*(x - sqrt(x^2 + x - 1))*
(sqrt(5)*sqrt(5*sqrt(5) + 11) - 4*sqrt(5) - 5*sqrt(5*sqrt(5) + 11)) + 7/10*
sqrt(5)*sqrt(5*sqrt(5) + 11) + 1/5*sqrt(5) - 3/2*sqrt(5*sqrt(5) + 11)))/(sq
rt(5) - 2)
```

maple [B] time = 0.18, size = 637, normalized size = 5.44

$$\frac{\sqrt{\frac{10\left(x-\sqrt{5}-2\right)^2}{\left(-x-\sqrt{5}+2\right)^2}-\frac{5\sqrt{5}\left(x-\sqrt{5}-2\right)^2}{\left(-x-\sqrt{5}+2\right)^2}+10+5\sqrt{5}}\;\sqrt{5}\left(\operatorname{arctanh}\left(\frac{\sqrt{\frac{10\left(x-\sqrt{5}-2\right)^2}{\left(-x-\sqrt{5}+2\right)^2}-\frac{5\sqrt{5}\left(x-\sqrt{5}-2\right)^2}{\left(-x-\sqrt{5}+2\right)^2}+10+5\sqrt{5}}}{\sqrt{20+10\sqrt{5}}}\right)+\sqrt{5}\,\arctan\left(-\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+2*x)/(x^2+1)/(x^2+x-1)^(1/2),x)
```

```
[Out] (10*(-5^(1/2)-2+x)^2/(-5^(1/2)+2-x)^2-5*5^(1/2)*(-5^(1/2)-2+x)^2/(-5^(1/2)+
2-x)^2+10+5*5^(1/2))^(1/2)*5^(1/2)*(arctan(1/5*5^(1/2)*((5^(1/2)-2)*(-(-5^(
1/2)-2+x)^2/(-5^(1/2)+2-x)^2+4*5^(1/2)+9))^(1/2)*(20+10*5^(1/2))^(1/2)*(5^(
1/2)*(-5^(1/2)-2+x)^2/(-5^(1/2)+2-x)^2+2*(-5^(1/2)-2+x)^2/(-5^(1/2)+2-x)^2-
5^(1/2)+2)*(-5^(1/2)-2+x)/(-5^(1/2)+2-x)*(5^(1/2)-2)/((-5^(1/2)-2+x)^4/(-5^
(1/2)+2-x)^4-18*(-5^(1/2)-2+x)^2/(-5^(1/2)+2-x)^2+1))*5^(1/2)+arctanh((10*(
-5^(1/2)-2+x)^2/(-5^(1/2)+2-x)^2-5*5^(1/2)*(-5^(1/2)-2+x)^2/(-5^(1/2)+2-x)^
2+10+5*5^(1/2))^(1/2)/(20+10*5^(1/2))^(1/2))+2*arctan(1/5*5^(1/2)*((5^(1/2)
-2)*(-(-5^(1/2)-2+x)^2/(-5^(1/2)+2-x)^2+4*5^(1/2)+9))^(1/2)*(20+10*5^(1/2))
^(1/2)*(5^(1/2)*(-5^(1/2)-2+x)^2/(-5^(1/2)+2-x)^2+2*(-5^(1/2)-2+x)^2/(-5^(1
/2)+2-x)^2-5^(1/2)+2)*(-5^(1/2)-2+x)/(-5^(1/2)+2-x)*(5^(1/2)-2)/((-5^(1/2)-
2+x)^4/(-5^(1/2)+2-x)^4-18*(-5^(1/2)-2+x)^2/(-5^(1/2)+2-x)^2+1)))/(-5*(5^(1
/2)*(-5^(1/2)-2+x)^2/(-5^(1/2)+2-x)^2-2*(-5^(1/2)-2+x)^2/(-5^(1/2)+2-x)^2-5
^(1/2)-2)/((-5^(1/2)-2+x)/(-5^(1/2)+2-x)+1)^2)^(1/2)/((-5^(1/2)-2+x)/(-5^(1
/2)+2-x)+1)/(20+10*5^(1/2))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2x+1}{\sqrt{x^2+x-1}\left(x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)/(x^2+1)/(x^2+x-1)^(1/2),x, algorithm="maxima")
[Out] integrate((2*x + 1)/(sqrt(x^2 + x - 1)*(x^2 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{2\,x+1}{\left(x^2+1\right)\,\sqrt{x^2+x-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x + 1)/((x^2 + 1)*(x + x^2 - 1)^(1/2)),x)
[Out] int((2*x + 1)/((x^2 + 1)*(x + x^2 - 1)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2x+1}{\left(x^2+1\right)\sqrt{x^2+x-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)/(x**2+1)/(x**2+x-1)**(1/2),x)
[Out] Integral((2*x + 1)/((x**2 + 1)*sqrt(x**2 + x - 1)), x)
```

3.12 $\int \frac{a-c+bx}{\left(1+x^2\right)\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=484

$$-\frac{\sqrt{-a\left(2c-\sqrt{a^2-2ac+b^2+c^2}\right)+c\left(c-\sqrt{a^2-2ac+b^2+c^2}\right)+a^2+b^2}\,\tan^{-1}\left(\frac{b\sqrt{a^2-2ac+b^2+c^2}\cdots}{\sqrt{2}\sqrt[4]{a^2-2ac+b^2+c^2}\sqrt{-a\left(2c-\sqrt{a^2-\cdots}\right.}}\right.}{\sqrt{2}\sqrt[4]{a^2-2ac+b^2+c^2}}$$

```
[Out] -1/2*arctan(1/2*(b*(a^2-2*a*c+b^2+c^2)^(1/2)-x*(b^2+(a-c)*(a-c+(a^2-2*a*c+b
^2+c^2)^(1/2))))/(a^2-2*a*c+b^2+c^2)^(1/4)*2^(1/2)/(c*x^2+b*x+a)^(1/2)/(a^2
+b^2+c*(c-(a^2-2*a*c+b^2+c^2)^(1/2))-a*(2*c-(a^2-2*a*c+b^2+c^2)^(1/2)))^(1/
2))*(a^2+b^2+c*(c-(a^2-2*a*c+b^2+c^2)^(1/2))-a*(2*c-(a^2-2*a*c+b^2+c^2)^(1/
2)))^(1/2)/(a^2-2*a*c+b^2+c^2)^(1/4)*2^(1/2)-1/2*arctanh(1/2*(x*(b^2+(a-c)*
(a-c-(a^2-2*a*c+b^2+c^2)^(1/2)))+b*(a^2-2*a*c+b^2+c^2)^(1/2))/(a^2-2*a*c+b^
2+c^2)^(1/4)*2^(1/2)/(c*x^2+b*x+a)^(1/2)/(a^2+b^2+c*(c+(a^2-2*a*c+b^2+c^2)^
(1/2))-a*(2*c+(a^2-2*a*c+b^2+c^2)^(1/2)))^(1/2))*(a^2+b^2+c*(c+(a^2-2*a*c+b
^2+c^2)^(1/2))-a*(2*c+(a^2-2*a*c+b^2+c^2)^(1/2)))^(1/2)/(a^2-2*a*c+b^2+c^2)
^(1/4)*2^(1/2)
```

Rubi [A] time = 23.58, antiderivative size = 484, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {1036, 1030, 208, 205}

$$-\frac{\sqrt{-a\left(2c-\sqrt{a^2-2ac+b^2+c^2}\right)+c\left(c-\sqrt{a^2-2ac+b^2+c^2}\right)+a^2+b^2}\,\tan^{-1}\left(\frac{b\sqrt{a^2-2ac+b^2+c^2}\cdots}{\sqrt{2}\sqrt[4]{a^2-2ac+b^2+c^2}\sqrt{-a\left(2c-\sqrt{a^2-\cdots}\right.}}\right.}{\sqrt{2}\sqrt[4]{a^2-2ac+b^2+c^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - c + b*x)/((1 + x^2)*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] -((Sqrt[a^2 + b^2 + c*(c - Sqrt[a^2 + b^2 - 2*a*c + c^2]) - a*(2*c - Sqrt[a
^2 + b^2 - 2*a*c + c^2])]*ArcTan[(b*Sqrt[a^2 + b^2 - 2*a*c + c^2] - (b^2 +
(a - c)*(a - c + Sqrt[a^2 + b^2 - 2*a*c + c^2]))*x)/(Sqrt[2]*(a^2 + b^2 - 2
*a*c + c^2)^(1/4)*Sqrt[a^2 + b^2 + c*(c - Sqrt[a^2 + b^2 - 2*a*c + c^2]) -
a*(2*c - Sqrt[a^2 + b^2 - 2*a*c + c^2])]*Sqrt[a + b*x + c*x^2]))])/(Sqrt[2]*
(a^2 + b^2 - 2*a*c + c^2)^(1/4))) - (Sqrt[a^2 + b^2 + c*(c + Sqrt[a^2 + b^2
 - 2*a*c + c^2]) - a*(2*c + Sqrt[a^2 + b^2 - 2*a*c + c^2])]*ArcTanh[(b*Sqrt
[a^2 + b^2 - 2*a*c + c^2] + (b^2 + (a - c)*(a - c - Sqrt[a^2 + b^2 - 2*a*c
+ c^2]))*x)/(Sqrt[2]*(a^2 + b^2 - 2*a*c + c^2)^(1/4)*Sqrt[a^2 + b^2 + c*(c
+ Sqrt[a^2 + b^2 - 2*a*c + c^2]) - a*(2*c + Sqrt[a^2 + b^2 - 2*a*c + c^2])]
*Sqrt[a + b*x + c*x^2]])])/(Sqrt[2]*(a^2 + b^2 - 2*a*c + c^2)^(1/4))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1030

```
Int[((g_) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f
_.)*(x_)^2]), x_Symbol] :> Dist[-2*a*g*h, Subst[Int[1/Simp[2*a^2*g*h*c + a*
```

```
e*x^2, x], x], x, Simp[a*h - g*c*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[
{a, c, d, e, f, g, h}, x] && EqQ[a*h^2*e + 2*g*h*(c*d - a*f) - g^2*c*e, 0]
```

Rule 1036

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 + a*c*e^2, 2]}, Dist
[1/(2*q), Int[Simp[-(a*h*e) - g*(c*d - a*f - q) + (h*(c*d - a*f + q) - g*c*
e)*x, x]/((a + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Si
mp[-(a*h*e) - g*(c*d - a*f + q) + (h*(c*d - a*f - q) - g*c*e)*x, x]/((a + c
*x^2)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f, g, h}, x] &&
 NeQ[e^2 - 4*d*f, 0] && NegQ[-(a*c)]
```

Rubi steps

$$\int \frac{a-c+bx}{\left(1+x^2\right)\sqrt{a+bx+cx^2}}\,dx = -\frac{\int \frac{-b^2-(a-c)\left(a-c+\sqrt{a^2+b^2-2ac+c^2}\right)-b\sqrt{a^2+b^2-2ac+c^2}\,x}{\left(1+x^2\right)\sqrt{a+bx+cx^2}}\,dx}{2\sqrt{a^2+b^2-2ac+c^2}} + \frac{\int \frac{-b^2-(a-c)\left(a-c-\sqrt{a^2+b^2-2ac+c^2}\right.}{\left(1+x^2\right)\sqrt{a+bx+}}}{2\sqrt{a^2+b^2-2a}}$$

$$= \left(b\left(b^2+(a-c)\left(a-c-\sqrt{a^2+b^2-2ac+c^2}\right)\right)\right)\text{Subst}\left(\int \frac{}{-2b\sqrt{a^2+b^2-2ac+}}\right.$$

$$= -\frac{\sqrt{a^2+b^2+c\left(c-\sqrt{a^2+b^2-2ac+c^2}\right)-a\left(2c-\sqrt{a^2+b^2-2ac+c^2}\right)}\tanh^{-1}}{\sqrt{2}\sqrt[4]{a^2+b^2-}}$$

Mathematica [C] time = 0.08, size = 136, normalized size = 0.28

$$\frac{1}{2}i\left(\sqrt{a+ib-c}\tanh^{-1}\left(\frac{2a+b(x+i)+2icx}{2\sqrt{a+ib-c}\sqrt{a+x(b+cx)}}\right)-\sqrt{a-ib-c}\tanh^{-1}\left(\frac{2a+b(x-i)-2icx}{2\sqrt{a-ib-c}\sqrt{a+x(b+cx)}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - c + b*x)/((1 + x^2)*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (I/2)*(-(Sqrt[a - I*b - c]*ArcTanh[(2*a - (2*I)*c*x + b*(-I + x))/(2*Sqrt[a
 - I*b - c]*Sqrt[a + x*(b + c*x)])]) + Sqrt[a + I*b - c]*ArcTanh[(2*a + (2*
I)*c*x + b*(I + x))/(2*Sqrt[a + I*b - c]*Sqrt[a + x*(b + c*x)])])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a-c)/(x^2+1)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a-c)/(x^2+1)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming [a,b,c]=[42,62,91]Warning, choosing root of [1,0,%%%{-2,[2,0,0]%%%
}+%%%{4,[1,0,1]%%%}+%%%{4,[1,0,0]%%%}+%%%{-2,[0,2,0]%%%}+%%%{-2,[0,0,2]%%%}
+%%%{-4,[0,0,1]%%%},%%%{-8,[2,0,0]%%%}+%%%{16,[1,0,1]%%%}+%%%{-8,[0,2,0]%%%
}+%%%{-8,[0,0,2]%%%},%%%{1,[4,0,0]%%%}+%%%{-4,[3,0,1]%%%}+%%%{4,[3,0,0]%%%}
+%%%{2,[2,2,0]%%%}+%%%{6,[2,0,2]%%%}+%%%{-12,[2,0,1]%%%}+%%%{-4,[1,2,1]%%%}
+%%%{4,[1,2,0]%%%}+%%%{-4,[1,0,3]%%%}+%%%{12,[1,0,2]%%%}+%%%{1,[0,4,0]%%%}+
%%%{2,[0,2,2]%%%}+%%%{-4,[0,2,1]%%%}+%%%{-4,[0,2,0]%%%}+%%%{1,[0,0,4]%%%}+%
%%{-4,[0,0,3]%%%}] at parameters values [-27,26,-89]Warning, need to choose
 a branch for the root of a polynomial with parameters. This might be wrong
.The choice was done assuming [a,b]=[-66,-52]Evaluation time: 7.83Unable to
 convert to real %%%{1.0,[2]%%%}+%%%{132.0,[1]%%%}+%%%{7060.0,[0]%%%} Error
: Bad Argument Value
```

maple [B] time = 0.52, size = 6871419, normalized size = 14197.15

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a-c)/(x^2+1)/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{bx + a - c}{\sqrt{cx^2 + bx + a}\left(x^2 + 1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a-c)/(x^2+1)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a - c)/(sqrt(c*x^2 + b*x + a)*(x^2 + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a - c + b\,x}{\left(x^2 + 1\right)\,\sqrt{c\,x^2 + b\,x + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - c + b*x)/((x^2 + 1)*(a + b*x + c*x^2)^(1/2)),x)
```

```
[Out] int((a - c + b*x)/((x^2 + 1)*(a + b*x + c*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + bx - c}{\left(x^2 + 1\right)\sqrt{a + bx + cx^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a-c)/(x**2+1)/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((a + b*x - c)/((x**2 + 1)*sqrt(a + b*x + c*x**2)), x)
```

3.13 $\int \frac{(A+Bx)(a+bx+cx^2)}{d+ex+fx^2}\,dx$

Optimal. Leaf size=184

$$-\frac{\log\left(d+ex+fx^2\right)\left(Af(ce-bf)-B\left(af^2-bef-cdf+ce^2\right)\right)}{2f^3}-\frac{\tanh^{-1}\left(\frac{e+2fx}{\sqrt{e^2-4df}}\right)\left(Af\left(2af^2-bef-2cdf+ce^2\right)\right.}{f^3\sqrt{e^2}}$$

```
[Out] -(-A*c*f-B*b*f+B*c*e)*x/f^2+1/2*B*c*x^2/f-1/2*(A*f*(-b*f+c*e)-B*(a*f^2-b*e*
f-c*d*f+c*e^2))*ln(f*x^2+e*x+d)/f^3-(A*f*(2*a*f^2-b*e*f-2*c*d*f+c*e^2)+B*(f
*(-a*e*f-2*b*d*f+b*e^2)-c*(-3*d*e*f+e^3)))*arctanh((2*f*x+e)/(-4*d*f+e^2)^(
1/2))/f^3/(-4*d*f+e^2)^(1/2)
```

Rubi [A] time = 0.35, antiderivative size = 182, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {1628, 634, 618, 206, 628}

$$-\frac{\log\left(d+ex+fx^2\right)\left(Bf(be-af)+Af(ce-bf)-Bc\left(e^2-df\right)\right)}{2f^3}-\frac{\tanh^{-1}\left(\frac{e+2fx}{\sqrt{e^2-4df}}\right)\left(Af\left(2af^2-bef-2cdf+c\right.\right.}{f^3\sqrt{e^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x)*(a + b*x + c*x^2))/(d + e*x + f*x^2),x]
```

```
[Out] -(((B*c*e - b*B*f - A*c*f)*x)/f^2) + (B*c*x^2)/(2*f) - ((B*f*(b*e^2 - 2*b*d
*f - a*e*f) - B*c*(e^3 - 3*d*e*f) + A*f*(c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2)
)*ArcTanh[(e + 2*f*x)/Sqrt[e^2 - 4*d*f]])/(f^3*Sqrt[e^2 - 4*d*f]) - ((B*f*(
b*e - a*f) + A*f*(c*e - b*f) - B*c*(e^2 - d*f))*Log[d + e*x + f*x^2])/(2*f^
3)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
```

```
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)}{d+ex+fx^2}\,dx = \int\left(-\frac{Bce-bBf-Acf}{f^2}+\frac{Bcx}{f}+\frac{-Af(cd-af)+Bd(ce-bf)-\left(Bf(be-af\right.}{f^2\left(d+ex+f\right.}\right.$$

$$= -\frac{(Bce-bBf-Acf)x}{f^2}+\frac{Bcx^2}{2f}+\frac{\int\frac{-Af(cd-af)+Bd(ce-bf)-\left(Bf(be-af)+Af(ce-bf)-Bc\right.}{d+ex+fx^2}}{f^2}$$

$$= -\frac{(Bce-bBf-Acf)x}{f^2}+\frac{Bcx^2}{2f}+\frac{\left(-Bf(be-af)-Af(ce-bf)+Bc\left(e^2-df\right.\right.}{2f^3}$$

$$= -\frac{(Bce-bBf-Acf)x}{f^2}+\frac{Bcx^2}{2f}-\frac{\left(Bf(be-af)+Af(ce-bf)-Bc\left(e^2-df\right)\right)}{2f^3}$$

$$= -\frac{(Bce-bBf-Acf)x}{f^2}+\frac{Bcx^2}{2f}-\frac{\left(Bf\left(be^2-2bdf-aef\right)-Bc\left(e^3-3def\right)+\right.}{f^3\sqrt{}}$$

Mathematica [A] time = 0.21, size = 175, normalized size = 0.95

$$\frac{\log(d+x(e+fx))\left(Bf(af-be)+Af(bf-ce)+Bc\left(e^2-df\right)\right)-\frac{2\tan^{-1}\left(\frac{e+2fx}{\sqrt{4df-e^2}}\right)\left(Af\left(-2af^2+bef+2cdf-ce^2\right)+Bf\left(aef+2\right.\right.}{\sqrt{4df-e^2}}}{2f^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x)*(a + b*x + c*x^2))/(d + e*x + f*x^2),x]
```

```
[Out] (2*f*(-(B*c*e) + b*B*f + A*c*f)*x + B*c*f^2*x^2 - (2*(B*f*(-(b*e^2) + 2*b*d
*f + a*e*f) + B*c*(e^3 - 3*d*e*f) + A*f*(-(c*e^2) + 2*c*d*f + b*e*f - 2*a*f
^2))*ArcTan[(e + 2*f*x)/Sqrt[-e^2 + 4*d*f]])/Sqrt[-e^2 + 4*d*f] + (B*f*(-(b
*e) + a*f) + A*f*(-(c*e) + b*f) + B*c*(e^2 - d*f))*Log[d + x*(e + f*x)])/(2
*f^3)
```

fricas [A] time = 1.39, size = 583, normalized size = 3.17

$$\left[\frac{\left(Bce^2f^2-4\,Bcdf^3\right)x^2-\left(Bce^3-2\,Aaf^3+(2\,(Bb+Ac)d+(Ba+Ab)e)f^2-\left(3\,Bcde+(Bb+Ac)e^2\right)f\right)\sqrt{e^2-4}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(c*x^2+b*x+a)/(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] [1/2*((B*c*e^2*f^2 - 4*B*c*d*f^3)*x^2 - (B*c*e^3 - 2*A*a*f^3 + (2*(B*b + A*
c)*d + (B*a + A*b)*e)*f^2 - (3*B*c*d*e + (B*b + A*c)*e^2)*f)*sqrt(e^2 - 4*d
*f)*log((2*f^2*x^2 + 2*e*f*x + e^2 - 2*d*f - sqrt(e^2 - 4*d*f)*(2*f*x + e))
/(f*x^2 + e*x + d)) - 2*(B*c*e^3*f + 4*(B*b + A*c)*d*f^3 - (4*B*c*d*e + (B*
b + A*c)*e^2)*f^2)*x + (B*c*e^4 - 4*(B*a + A*b)*d*f^3 + (4*B*c*d^2 + 4*(B*b
 + A*c)*d*e + (B*a + A*b)*e^2)*f^2 - (5*B*c*d*e^2 + (B*b + A*c)*e^3)*f)*log
(f*x^2 + e*x + d))/(e^2*f^3 - 4*d*f^4), 1/2*((B*c*e^2*f^2 - 4*B*c*d*f^3)*x^
2 + 2*(B*c*e^3 - 2*A*a*f^3 + (2*(B*b + A*c)*d + (B*a + A*b)*e)*f^2 - (3*B*c
*d*e + (B*b + A*c)*e^2)*f)*sqrt(-e^2 + 4*d*f)*arctan(-sqrt(-e^2 + 4*d*f)*(2
```

```
*f*x + e)/(e^2 - 4*d*f)) - 2*(B*c*e^3*f + 4*(B*b + A*c)*d*f^3 - (4*B*c*d*e
+ (B*b + A*c)*e^2)*f^2)*x + (B*c*e^4 - 4*(B*a + A*b)*d*f^3 + (4*B*c*d^2 + 4
*(B*b + A*c)*d*e + (B*a + A*b)*e^2)*f^2 - (5*B*c*d*e^2 + (B*b + A*c)*e^3)*f
)*log(f*x^2 + e*x + d))/(e^2*f^3 - 4*d*f^4)]
```

giac [A] time = 0.17, size = 191, normalized size = 1.04

$$\frac{Bcfx^2 + 2\,Bbfx + 2\,Acfx - 2\,Bcxe}{2\,f^2} - \frac{\left(Bcdf - Baf^2 - Abf^2 + Bbfe + Acfe - Bce^2\right)\log\left(fx^2 + xe + d\right)}{2\,f^3} - \frac{\left(2\,Bbdf\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(c*x^2+b*x+a)/(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] 1/2*(B*c*f*x^2 + 2*B*b*f*x + 2*A*c*f*x - 2*B*c*x*e)/f^2 - 1/2*(B*c*d*f - B*
a*f^2 - A*b*f^2 + B*b*f*e + A*c*f*e - B*c*e^2)*log(f*x^2 + x*e + d)/f^3 - (
2*B*b*d*f^2 + 2*A*c*d*f^2 - 2*A*a*f^3 - 3*B*c*d*f*e + B*a*f^2*e + A*b*f^2*e
 - B*b*f*e^2 - A*c*f*e^2 + B*c*e^3)*arctan((2*f*x + e)/sqrt(4*d*f - e^2))/(
sqrt(4*d*f - e^2)*f^3)
```

maple [B] time = 0.01, size = 510, normalized size = 2.77

$$\frac{2Aa\arctan\left(\frac{2fx+e}{\sqrt{4df-e^2}}\right)}{\sqrt{4df-e^2}} - \frac{Abe\arctan\left(\frac{2fx+e}{\sqrt{4df-e^2}}\right)}{\sqrt{4df-e^2}\,f} - \frac{2Acd\arctan\left(\frac{2fx+e}{\sqrt{4df-e^2}}\right)}{\sqrt{4df-e^2}\,f} + \frac{Ac\,e^2\arctan\left(\frac{2fx+e}{\sqrt{4df-e^2}}\right)}{\sqrt{4df-e^2}\,f^2} - \frac{Bae\arctan\left(\frac{2}{\sqrt{4}}\right.}{\sqrt{4df-e^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x+A)*(c*x^2+b*x+a)/(f*x^2+e*x+d),x)
```

```
[Out] 1/2*B*c/f*x^2+A*c/f*x+B*b/f*x-1/f^2*B*c*e*x+1/2/f*ln(f*x^2+e*x+d)*A*b-1/2/f
^2*ln(f*x^2+e*x+d)*A*c*e+1/2/f*ln(f*x^2+e*x+d)*B*a-1/2/f^2*ln(f*x^2+e*x+d)*
B*b*e-1/2/f^2*ln(f*x^2+e*x+d)*B*c*d+1/2/f^3*ln(f*x^2+e*x+d)*B*c*e^2+2/(4*d*
f-e^2)^(1/2)*arctan((2*f*x+e)/(4*d*f-e^2)^(1/2))*A*a-2/f/(4*d*f-e^2)^(1/2)*
arctan((2*f*x+e)/(4*d*f-e^2)^(1/2))*A*c*d-2/f/(4*d*f-e^2)^(1/2)*arctan((2*f
*x+e)/(4*d*f-e^2)^(1/2))*B*b*d+3/f^2/(4*d*f-e^2)^(1/2)*arctan((2*f*x+e)/(4*
d*f-e^2)^(1/2))*B*c*d*e-1/f/(4*d*f-e^2)^(1/2)*arctan((2*f*x+e)/(4*d*f-e^2)^
(1/2))*e*A*b+1/f^2/(4*d*f-e^2)^(1/2)*arctan((2*f*x+e)/(4*d*f-e^2)^(1/2))*e^
2*A*c-1/f/(4*d*f-e^2)^(1/2)*arctan((2*f*x+e)/(4*d*f-e^2)^(1/2))*e*B*a+1/f^2
/(4*d*f-e^2)^(1/2)*arctan((2*f*x+e)/(4*d*f-e^2)^(1/2))*e^2*B*b-1/f^3/(4*d*f
-e^2)^(1/2)*arctan((2*f*x+e)/(4*d*f-e^2)^(1/2))*e^3*B*c
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(c*x^2+b*x+a)/(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for mo
re details)Is 4*d*f-e^2 positive or negative?
```

mupad [B] time = 3.85, size = 273, normalized size = 1.48

$$x\left(\frac{A\,c+B\,b}{f} - \frac{B\,c\,e}{f^2}\right) - \frac{\ln\left(f\,x^2+e\,x+d\right)\left(B\,c\,e^4 - 4\,A\,b\,d\,f^3 - 4\,B\,a\,d\,f^3 - A\,c\,e^3\,f - B\,b\,e^3\,f + A\,b\,e^2\,f^2 + B\,a\,e^2\right.}{2\left(4\,d\,f^4 - e^2\,f^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x)*(a + b*x + c*x^2))/(d + e*x + f*x^2),x)
```

```
[Out] x*((A*c + B*b)/f - (B*c*e)/f^2) - (log(d + e*x + f*x^2)*(B*c*e^4 - 4*A*b*d*
f^3 - 4*B*a*d*f^3 - A*c*e^3*f - B*b*e^3*f + A*b*e^2*f^2 + B*a*e^2*f^2 + 4*B
*c*d^2*f^2 + 4*A*c*d*e*f^2 + 4*B*b*d*e*f^2 - 5*B*c*d*e^2*f))/(2*(4*d*f^4 -
e^2*f^3)) - (atan(e/(4*d*f - e^2)^(1/2) + (2*f*x)/(4*d*f - e^2)^(1/2))*(B*c
*e^3 - 2*A*a*f^3 + A*b*e*f^2 + 2*A*c*d*f^2 + B*a*e*f^2 + 2*B*b*d*f^2 - A*c*
e^2*f - B*b*e^2*f - 3*B*c*d*e*f))/(f^3*(4*d*f - e^2)^(1/2)) + (B*c*x^2)/(2*
f)
```

sympy [B] time = 16.65, size = 1260, normalized size = 6.85

$$\frac{Bcx^2}{2f}+x\left(\frac{Ac}{f}+\frac{Bb}{f}-\frac{Bce}{f^2}\right)+\left(-\frac{\sqrt{-4df+e^2}\left(-2Aaf^3+Abef^2+2Acdf^2-Ace^2f+Baef^2+2Bbdf^2-Bbe^2f\right.}{2f^3\left(4df-e^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(c*x**2+b*x+a)/(f*x**2+e*x+d),x)
```

```
[Out] B*c*x**2/(2*f) + x*(A*c/f + B*b/f - B*c*e/f**2) + (-sqrt(-4*d*f + e**2)*(-2
*A*a*f**3 + A*b*e*f**2 + 2*A*c*d*f**2 - A*c*e**2*f + B*a*e*f**2 + 2*B*b*d*f
**2 - B*b*e**2*f - 3*B*c*d*e*f + B*c*e**3)/(2*f**3*(4*d*f - e**2)) + (A*b*f
**2 - A*c*e*f + B*a*f**2 - B*b*e*f - B*c*d*f + B*c*e**2)/(2*f**3))*log(x +
(-A*a*e*f**2 + 2*A*b*d*f**2 - A*c*d*e*f + 2*B*a*d*f**2 - B*b*d*e*f - 2*B*c*
d**2*f + B*c*d*e**2 - 4*d*f**3*(-sqrt(-4*d*f + e**2)*(-2*A*a*f**3 + A*b*e*f
**2 + 2*A*c*d*f**2 - A*c*e**2*f + B*a*e*f**2 + 2*B*b*d*f**2 - B*b*e**2*f -
3*B*c*d*e*f + B*c*e**3)/(2*f**3*(4*d*f - e**2)) + (A*b*f**2 - A*c*e*f + B*a
*f**2 - B*b*e*f - B*c*d*f + B*c*e**2)/(2*f**3)) + e**2*f**2*(-sqrt(-4*d*f +
 e**2)*(-2*A*a*f**3 + A*b*e*f**2 + 2*A*c*d*f**2 - A*c*e**2*f + B*a*e*f**2 +
 2*B*b*d*f**2 - B*b*e**2*f - 3*B*c*d*e*f + B*c*e**3)/(2*f**3*(4*d*f - e**2)
) + (A*b*f**2 - A*c*e*f + B*a*f**2 - B*b*e*f - B*c*d*f + B*c*e**2)/(2*f**3)
))/(-2*A*a*f**3 + A*b*e*f**2 + 2*A*c*d*f**2 - A*c*e**2*f + B*a*e*f**2 + 2*B
*b*d*f**2 - B*b*e**2*f - 3*B*c*d*e*f + B*c*e**3)) + (sqrt(-4*d*f + e**2)*(-
2*A*a*f**3 + A*b*e*f**2 + 2*A*c*d*f**2 - A*c*e**2*f + B*a*e*f**2 + 2*B*b*d*
f**2 - B*b*e**2*f - 3*B*c*d*e*f + B*c*e**3)/(2*f**3*(4*d*f - e**2)) + (A*b*
f**2 - A*c*e*f + B*a*f**2 - B*b*e*f - B*c*d*f + B*c*e**2)/(2*f**3))*log(x +
 (-A*a*e*f**2 + 2*A*b*d*f**2 - A*c*d*e*f + 2*B*a*d*f**2 - B*b*d*e*f - 2*B*c
*d**2*f + B*c*d*e**2 - 4*d*f**3*(sqrt(-4*d*f + e**2)*(-2*A*a*f**3 + A*b*e*f
**2 + 2*A*c*d*f**2 - A*c*e**2*f + B*a*e*f**2 + 2*B*b*d*f**2 - B*b*e**2*f -
3*B*c*d*e*f + B*c*e**3)/(2*f**3*(4*d*f - e**2)) + (A*b*f**2 - A*c*e*f + B*a
*f**2 - B*b*e*f - B*c*d*f + B*c*e**2)/(2*f**3)) + e**2*f**2*(sqrt(-4*d*f +
e**2)*(-2*A*a*f**3 + A*b*e*f**2 + 2*A*c*d*f**2 - A*c*e**2*f + B*a*e*f**2 +
2*B*b*d*f**2 - B*b*e**2*f - 3*B*c*d*e*f + B*c*e**3)/(2*f**3*(4*d*f - e**2))
 + (A*b*f**2 - A*c*e*f + B*a*f**2 - B*b*e*f - B*c*d*f + B*c*e**2)/(2*f**3))
)/(-2*A*a*f**3 + A*b*e*f**2 + 2*A*c*d*f**2 - A*c*e**2*f + B*a*e*f**2 + 2*B*
b*d*f**2 - B*b*e**2*f - 3*B*c*d*e*f + B*c*e**3))
```

3.14 $\int \frac{(A+Bx)\left(a+bx+cx^2\right)^2}{d+ex+fx^2}\,dx$

Optimal. Leaf size=542

$$\frac{\log\left(d+ex+fx^2\right)\left(B\left(-f^2\left(-a^2f^2+2abef-\left(b^2\left(e^2-df\right)\right)\right)+2cf\left(af\left(e^2-df\right)-b\left(e^3-2def\right)\right)+c^2\left(d^2f^2-3d\right.\right.\right.}{2f^5}$$

```
[Out] (B*(-b*f+c*e)*(f*(-2*a*f+b*e)-c*(-2*d*f+e^2))+A*f*(b^2*f^2-2*c*f*(-a*f+b*e)
+c^2*(-d*f+e^2)))*x/f^4-1/2*(A*c*f*(-2*b*f+c*e)-B*(b^2*f^2-2*c*f*(-a*f+b*e)
+c^2*(-d*f+e^2)))*x^2/f^3-1/3*c*(-A*c*f-2*B*b*f+B*c*e)*x^3/f^2+1/4*B*c^2*x^
4/f+1/2*(A*f*(-b*f+c*e)*(f*(-2*a*f+b*e)-c*(-2*d*f+e^2))+B*(c^2*(d^2*f^2-3*d
*e^2*f+e^4)-f^2*(2*a*b*e*f-a^2*f^2-b^2*(-d*f+e^2))+2*c*f*(a*f*(-d*f+e^2)-b*
(-2*d*e*f+e^3))))*ln(f*x^2+e*x+d)/f^5-(A*f*(c^2*(2*d^2*f^2-4*d*e^2*f+e^4)-f
^2*(2*a*b*e*f-2*a^2*f^2-b^2*(-2*d*f+e^2))+2*c*f*(a*f*(-2*d*f+e^2)-b*(-3*d*e
*f+e^3)))-B*(c^2*(5*d^2*e*f^2-5*d*e^3*f+e^5)+f^2*(a^2*e*f^2-2*a*b*f*(-2*d*f
+e^2)+b^2*(-3*d*e*f+e^3))+2*c*f*(a*e*f*(-3*d*f+e^2)-b*(2*d^2*f^2-4*d*e^2*f+
e^4))))*arctanh((2*f*x+e)/(-4*d*f+e^2)^(1/2))/f^5/(-4*d*f+e^2)^(1/2)
```

Rubi [A] time = 1.10, antiderivative size = 542, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1011, 634, 618, 206, 628}

$$\frac{\log\left(d+ex+fx^2\right)\left(B\left(-f^2\left(-a^2f^2+2abef+b^2\left(-\left(e^2-df\right)\right)\right)+2cf\left(af\left(e^2-df\right)-b\left(e^3-2def\right)\right)+c^2\left(d^2f^2-\right.\right.\right.}{2f^5}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x)*(a + b*x + c*x^2)^2)/(d + e*x + f*x^2),x]
```

```
[Out] ((B*(c*e - b*f)*(f*(b*e - 2*a*f) - c*(e^2 - 2*d*f)) + A*f*(b^2*f^2 - 2*c*f*
(b*e - a*f) + c^2*(e^2 - d*f)))*x)/f^4 - ((A*c*f*(c*e - 2*b*f) - B*(b^2*f^2
 - 2*c*f*(b*e - a*f) + c^2*(e^2 - d*f)))*x^2)/(2*f^3) - (c*(B*c*e - 2*b*B*f
 - A*c*f)*x^3)/(3*f^2) + (B*c^2*x^4)/(4*f) - ((A*f*(c^2*(e^4 - 4*d*e^2*f +
2*d^2*f^2) - f^2*(2*a*b*e*f - 2*a^2*f^2 - b^2*(e^2 - 2*d*f)) + 2*c*f*(a*f*(
e^2 - 2*d*f) - b*(e^3 - 3*d*e*f))) - B*(c^2*(e^5 - 5*d*e^3*f + 5*d^2*e*f^2)
 + f^2*(a^2*e*f^2 - 2*a*b*f*(e^2 - 2*d*f) + b^2*(e^3 - 3*d*e*f)) + 2*c*f*(a
*e*f*(e^2 - 3*d*f) - b*(e^4 - 4*d*e^2*f + 2*d^2*f^2))))*ArcTanh[(e + 2*f*x)
/Sqrt[e^2 - 4*d*f]])/(f^5*Sqrt[e^2 - 4*d*f]) + ((A*f*(c*e - b*f)*(f*(b*e -
2*a*f) - c*(e^2 - 2*d*f)) + B*(c^2*(e^4 - 3*d*e^2*f + d^2*f^2) - f^2*(2*a*b
*e*f - a^2*f^2 - b^2*(e^2 - d*f)) + 2*c*f*(a*f*(e^2 - d*f) - b*(e^3 - 2*d*e
*f))))*Log[d + e*x + f*x^2])/(2*f^5)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1011

```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e
_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x +
c*x^2)^p*(d + e*x + f*x^2)^q*(g + h*x), x], x] /; FreeQ[{a, b, c, d, e, f,
g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && IGtQ[p, 0] && In
tegerQ[q]
```

Rubi steps

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^2}{d+ex+fx^2}\,dx = \int\left(\frac{B(ce-bf)\left(f(be-2af)-c\left(e^2-2df\right)\right)+Af\left(b^2f^2-2cf(be-af)+c^2 \cdots}{f^4}\right.$$

$$= \frac{\left(B(ce-bf)\left(f(be-2af)-c\left(e^2-2df\right)\right)+Af\left(b^2f^2-2cf(be-af)+c^2\left(e^2 \cdots\right.\right.}{f^4}$$

$$= \frac{\left(B(ce-bf)\left(f(be-2af)-c\left(e^2-2df\right)\right)+Af\left(b^2f^2-2cf(be-af)+c^2\left(e^2 \cdots\right.\right.}{f^4}$$

$$= \frac{\left(B(ce-bf)\left(f(be-2af)-c\left(e^2-2df\right)\right)+Af\left(b^2f^2-2cf(be-af)+c^2\left(e^2 \cdots\right.\right.}{f^4}$$

$$= \frac{\left(B(ce-bf)\left(f(be-2af)-c\left(e^2-2df\right)\right)+Af\left(b^2f^2-2cf(be-af)+c^2\left(e^2 \cdots\right.\right.}{f^4}$$

Mathematica [A] time = 0.60, size = 535, normalized size = 0.99

$$\frac{6\log(d+x(e+fx))\left(B\left(f^2\left(a^2f^2-2abef+b^2\left(e^2-df\right)\right)-2cf\left(af\left(df-e^2\right)+b\left(e^3-2def\right)\right)+c^2\left(d^2f^2-3a \cdots\right.\right.\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x)*(a + b*x + c*x^2)^2)/(d + e*x + f*x^2),x]
```

```
[Out] (12*f*(-(B*(c*e - b*f)*(f*(-(b*e) + 2*a*f) + c*(e^2 - 2*d*f))) + A*f*(b^2*f
^2 + 2*c*f*(-(b*e) + a*f) + c^2*(e^2 - d*f)))*x + 6*f^2*(A*c*f*(-(c*e) + 2*
b*f) + B*(b^2*f^2 + 2*c*f*(-(b*e) + a*f) + c^2*(e^2 - d*f)))*x^2 + 4*c*f^3*
(-(B*c*e) + 2*b*B*f + A*c*f)*x^3 + 3*B*c^2*f^4*x^4 - (12*(-(A*f*(c^2*(e^4 -
 4*d*e^2*f + 2*d^2*f^2) + f^2*(-2*a*b*e*f + 2*a^2*f^2 + b^2*(e^2 - 2*d*f))
+ 2*c*f*(a*f*(e^2 - 2*d*f) - b*(e^3 - 3*d*e*f)))) + B*(c^2*(e^5 - 5*d*e^3*f
 + 5*d^2*e*f^2) + f^2*(a^2*e*f^2 + 2*a*b*f*(-e^2 + 2*d*f) + b^2*(e^3 - 3*d*
e*f)) - 2*c*f*(-(a*e*f*(e^2 - 3*d*f)) + b*(e^4 - 4*d*e^2*f + 2*d^2*f^2))))*
ArcTan[(e + 2*f*x)/Sqrt[-e^2 + 4*d*f]])/Sqrt[-e^2 + 4*d*f] + 6*(A*f*(-(c*e)
 + b*f)*(f*(-(b*e) + 2*a*f) + c*(e^2 - 2*d*f)) + B*(c^2*(e^4 - 3*d*e^2*f +
```

```
d^2*f^2) + f^2*(-2*a*b*e*f + a^2*f^2 + b^2*(e^2 - d*f)) - 2*c*f*(a*f*(-e^2
+ d*f) + b*(e^3 - 2*d*e*f))))*Log[d + x*(e + f*x)])/(12*f^5)
```

fricas [A] time = 1.96, size = 1837, normalized size = 3.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(c*x^2+b*x+a)^2/(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] [1/12*(3*(B*c^2*e^2*f^4 - 4*B*c^2*d*f^5)*x^4 - 4*(B*c^2*e^3*f^3 + 4*(2*B*b*
c + A*c^2)*d*f^5 - (4*B*c^2*d*e + (2*B*b*c + A*c^2)*e^2)*f^4)*x^3 + 6*(B*c^
2*e^4*f^2 - 4*(B*b^2 + 2*(B*a + A*b)*c)*d*f^5 + (4*B*c^2*d^2 + 4*(2*B*b*c +
 A*c^2)*d*e + (B*b^2 + 2*(B*a + A*b)*c)*e^2)*f^4 - (5*B*c^2*d*e^2 + (2*B*b*
c + A*c^2)*e^3)*f^3)*x^2 - 6*(B*c^2*e^5 - 2*A*a^2*f^5 + (2*(2*B*a*b + A*b^2
 + 2*A*a*c)*d + (B*a^2 + 2*A*a*b)*e)*f^4 - (2*(2*B*b*c + A*c^2)*d^2 + 3*(B*
b^2 + 2*(B*a + A*b)*c)*d*e + (2*B*a*b + A*b^2 + 2*A*a*c)*e^2)*f^3 + (5*B*c^
2*d^2*e + 4*(2*B*b*c + A*c^2)*d*e^2 + (B*b^2 + 2*(B*a + A*b)*c)*e^3)*f^2 -
(5*B*c^2*d*e^3 + (2*B*b*c + A*c^2)*e^4)*f)*sqrt(e^2 - 4*d*f)*log((2*f^2*x^2
 + 2*e*f*x + e^2 - 2*d*f - sqrt(e^2 - 4*d*f)*(2*f*x + e))/(f*x^2 + e*x + d)
) - 12*(B*c^2*e^5*f + 4*(2*B*a*b + A*b^2 + 2*A*a*c)*d*f^5 - (4*(2*B*b*c + A
*c^2)*d^2 + 4*(B*b^2 + 2*(B*a + A*b)*c)*d*e + (2*B*a*b + A*b^2 + 2*A*a*c)*e
^2)*f^4 + (8*B*c^2*d^2*e + 5*(2*B*b*c + A*c^2)*d*e^2 + (B*b^2 + 2*(B*a + A*
b)*c)*e^3)*f^3 - (6*B*c^2*d*e^3 + (2*B*b*c + A*c^2)*e^4)*f^2)*x + 6*(B*c^2*
e^6 - 4*(B*a^2 + 2*A*a*b)*d*f^5 + (4*(B*b^2 + 2*(B*a + A*b)*c)*d^2 + 4*(2*B
*a*b + A*b^2 + 2*A*a*c)*d*e + (B*a^2 + 2*A*a*b)*e^2)*f^4 - (4*B*c^2*d^3 + 8
*(2*B*b*c + A*c^2)*d^2*e + 5*(B*b^2 + 2*(B*a + A*b)*c)*d*e^2 + (2*B*a*b + A
*b^2 + 2*A*a*c)*e^3)*f^3 + (13*B*c^2*d^2*e^2 + 6*(2*B*b*c + A*c^2)*d*e^3 +
(B*b^2 + 2*(B*a + A*b)*c)*e^4)*f^2 - (7*B*c^2*d*e^4 + (2*B*b*c + A*c^2)*e^5
)*f)*log(f*x^2 + e*x + d))/(e^2*f^5 - 4*d*f^6), 1/12*(3*(B*c^2*e^2*f^4 - 4*
B*c^2*d*f^5)*x^4 - 4*(B*c^2*e^3*f^3 + 4*(2*B*b*c + A*c^2)*d*f^5 - (4*B*c^2*
d*e + (2*B*b*c + A*c^2)*e^2)*f^4)*x^3 + 6*(B*c^2*e^4*f^2 - 4*(B*b^2 + 2*(B*
a + A*b)*c)*d*f^5 + (4*B*c^2*d^2 + 4*(2*B*b*c + A*c^2)*d*e + (B*b^2 + 2*(B*
a + A*b)*c)*e^2)*f^4 - (5*B*c^2*d*e^2 + (2*B*b*c + A*c^2)*e^3)*f^3)*x^2 + 1
2*(B*c^2*e^5 - 2*A*a^2*f^5 + (2*(2*B*a*b + A*b^2 + 2*A*a*c)*d + (B*a^2 + 2*
A*a*b)*e)*f^4 - (2*(2*B*b*c + A*c^2)*d^2 + 3*(B*b^2 + 2*(B*a + A*b)*c)*d*e
+ (2*B*a*b + A*b^2 + 2*A*a*c)*e^2)*f^3 + (5*B*c^2*d^2*e + 4*(2*B*b*c + A*c^
2)*d*e^2 + (B*b^2 + 2*(B*a + A*b)*c)*e^3)*f^2 - (5*B*c^2*d*e^3 + (2*B*b*c +
 A*c^2)*e^4)*f)*sqrt(-e^2 + 4*d*f)*arctan(-sqrt(-e^2 + 4*d*f)*(2*f*x + e)/(
e^2 - 4*d*f)) - 12*(B*c^2*e^5*f + 4*(2*B*a*b + A*b^2 + 2*A*a*c)*d*f^5 - (4*
(2*B*b*c + A*c^2)*d^2 + 4*(B*b^2 + 2*(B*a + A*b)*c)*d*e + (2*B*a*b + A*b^2
+ 2*A*a*c)*e^2)*f^4 + (8*B*c^2*d^2*e + 5*(2*B*b*c + A*c^2)*d*e^2 + (B*b^2 +
 2*(B*a + A*b)*c)*e^3)*f^3 - (6*B*c^2*d*e^3 + (2*B*b*c + A*c^2)*e^4)*f^2)*x
 + 6*(B*c^2*e^6 - 4*(B*a^2 + 2*A*a*b)*d*f^5 + (4*(B*b^2 + 2*(B*a + A*b)*c)*
d^2 + 4*(2*B*a*b + A*b^2 + 2*A*a*c)*d*e + (B*a^2 + 2*A*a*b)*e^2)*f^4 - (4*B
*c^2*d^3 + 8*(2*B*b*c + A*c^2)*d^2*e + 5*(B*b^2 + 2*(B*a + A*b)*c)*d*e^2 +
(2*B*a*b + A*b^2 + 2*A*a*c)*e^3)*f^3 + (13*B*c^2*d^2*e^2 + 6*(2*B*b*c + A*c
^2)*d*e^3 + (B*b^2 + 2*(B*a + A*b)*c)*e^4)*f^2 - (7*B*c^2*d*e^4 + (2*B*b*c
+ A*c^2)*e^5)*f)*log(f*x^2 + e*x + d))/(e^2*f^5 - 4*d*f^6)]
```

giac [A] time = 0.25, size = 738, normalized size = 1.36

$$\frac{3\,Bc^2f^3x^4 + 8\,Bbcf^3x^3 + 4\,Ac^2f^3x^3 - 4\,Bc^2f^2x^3e - 6\,Bc^2df^2x^2 + 6\,Bb^2f^3x^2 + 12\,Bacf^3x^2 + 12\,Abcf^3x^2 - 12\,Bb}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(c*x^2+b*x+a)^2/(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] 1/12*(3*B*c^2*f^3*x^4 + 8*B*b*c*f^3*x^3 + 4*A*c^2*f^3*x^3 - 4*B*c^2*f^2*x^3
*e - 6*B*c^2*d*f^2*x^2 + 6*B*b^2*f^3*x^2 + 12*B*a*c*f^3*x^2 + 12*A*b*c*f^3*
x^2 - 12*B*b*c*f^2*x^2*e - 6*A*c^2*f^2*x^2*e - 24*B*b*c*d*f^2*x - 12*A*c^2*
d*f^2*x + 24*B*a*b*f^3*x + 12*A*b^2*f^3*x + 24*A*a*c*f^3*x + 6*B*c^2*f*x^2*
e^2 + 24*B*c^2*d*f*x*e - 12*B*b^2*f^2*x*e - 24*B*a*c*f^2*x*e - 24*A*b*c*f^2
*x*e + 24*B*b*c*f*x*e^2 + 12*A*c^2*f*x*e^2 - 12*B*c^2*x*e^3)/f^4 + 1/2*(B*c
^2*d^2*f^2 - B*b^2*d*f^3 - 2*B*a*c*d*f^3 - 2*A*b*c*d*f^3 + B*a^2*f^4 + 2*A*
a*b*f^4 + 4*B*b*c*d*f^2*e + 2*A*c^2*d*f^2*e - 2*B*a*b*f^3*e - A*b^2*f^3*e -
 2*A*a*c*f^3*e - 3*B*c^2*d*f*e^2 + B*b^2*f^2*e^2 + 2*B*a*c*f^2*e^2 + 2*A*b*
c*f^2*e^2 - 2*B*b*c*f*e^3 - A*c^2*f*e^3 + B*c^2*e^4)*log(f*x^2 + x*e + d)/f
^5 + (4*B*b*c*d^2*f^3 + 2*A*c^2*d^2*f^3 - 4*B*a*b*d*f^4 - 2*A*b^2*d*f^4 - 4
*A*a*c*d*f^4 + 2*A*a^2*f^5 - 5*B*c^2*d^2*f^2*e + 3*B*b^2*d*f^3*e + 6*B*a*c*
d*f^3*e + 6*A*b*c*d*f^3*e - B*a^2*f^4*e - 2*A*a*b*f^4*e - 8*B*b*c*d*f^2*e^2
 - 4*A*c^2*d*f^2*e^2 + 2*B*a*b*f^3*e^2 + A*b^2*f^3*e^2 + 2*A*a*c*f^3*e^2 +
5*B*c^2*d*f*e^3 - B*b^2*f^2*e^3 - 2*B*a*c*f^2*e^3 - 2*A*b*c*f^2*e^3 + 2*B*b
*c*f*e^4 + A*c^2*f*e^4 - B*c^2*e^5)*arctan((2*f*x + e)/sqrt(4*d*f - e^2))/(
sqrt(4*d*f - e^2)*f^5)
```

maple [B] time = 0.01, size = 1672, normalized size = 3.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x+A)*(c*x^2+b*x+a)^2/(f*x^2+e*x+d),x)
```

```
[Out] 1/4*B*c^2/f*x^4-2*B*b*c*d/f^2*x+1/2/f*ln(f*x^2+e*x+d)*B*a^2+2/(4*d*f-e^2)^(
1/2)*arctan((2*f*x+e)/(4*d*f-e^2)^(1/2))*A*a^2+1/3*A*c^2/f*x^3+1/2*B*b^2/f*
x^2+A*b^2/f*x-A*c^2*d/f^2*x+2*B*a*b/f*x+B*a*c/f*x^2-1/2*B*c^2*d/f^2*x^2+2*A
*a*c/f*x+2/3*B*b*c/f*x^3+A*b*c/f*x^2-8/f^3/(4*d*f-e^2)^(1/2)*arctan((2*f*x+
e)/(4*d*f-e^2)^(1/2))*B*b*c*d*e^2+6/f^2/(4*d*f-e^2)^(1/2)*arctan((2*f*x+e)/
(4*d*f-e^2)^(1/2))*A*b*c*d*e+6/f^2/(4*d*f-e^2)^(1/2)*arctan((2*f*x+e)/(4*d*
f-e^2)^(1/2))*B*a*c*d*e+1/f^3*A*c^2*e^2*x-1/f^2*B*b^2*e*x-1/f^4*B*c^2*e^3*x
-1/2/f^2*A*x^2*c^2*e+1/2/f^3*B*x^2*c^2*e^2-1/3/f^2*B*x^3*c^2*e-1/2/f^2*ln(f
*x^2+e*x+d)*A*b^2*e-1/2/f^4*ln(f*x^2+e*x+d)*A*c^2*e^3-1/2/f^2*ln(f*x^2+e*x+
d)*B*b^2*d+1/2/f^3*ln(f*x^2+e*x+d)*B*b^2*e^2+1/2/f^3*ln(f*x^2+e*x+d)*B*c^2*
d^2+1/2/f^5*ln(f*x^2+e*x+d)*B*c^2*e^4+1/f*ln(f*x^2+e*x+d)*A*a*b-1/f^2*B*x^2
*b*c*e-2/f^2*A*b*c*e*x-2/f^2*B*a*c*e*x+2/f^3*B*b*c*e^2*x+2/f^3*B*c^2*d*e*x+
1/f^2/(4*d*f-e^2)^(1/2)*arctan((2*f*x+e)/(4*d*f-e^2)^(1/2))*e^2*A*b^2-1/f/(
4*d*f-e^2)^(1/2)*arctan((2*f*x+e)/(4*d*f-e^2)^(1/2))*e*B*a^2-1/f^3/(4*d*f-e
^2)^(1/2)*arctan((2*f*x+e)/(4*d*f-e^2)^(1/2))*e^3*B*b^2-1/f^5/(4*d*f-e^2)^(
1/2)*arctan((2*f*x+e)/(4*d*f-e^2)^(1/2))*e^5*B*c^2-2/f/(4*d*f-e^2)^(1/2)*ar
ctan((2*f*x+e)/(4*d*f-e^2)^(1/2))*A*b^2*d+2/f^2/(4*d*f-e^2)^(1/2)*arctan((2
*f*x+e)/(4*d*f-e^2)^(1/2))*A*c^2*d^2-1/f^2*ln(f*x^2+e*x+d)*A*a*c*e-1/f^2*ln
(f*x^2+e*x+d)*A*b*c*d+1/f^3*ln(f*x^2+e*x+d)*A*b*c*e^2+1/f^3*ln(f*x^2+e*x+d)
*A*c^2*d*e-1/f^2*ln(f*x^2+e*x+d)*B*a*b*e-1/f^2*ln(f*x^2+e*x+d)*B*a*c*d+1/f^
3*ln(f*x^2+e*x+d)*B*a*c*e^2-1/f^4*ln(f*x^2+e*x+d)*B*b*c*e^3-3/2/f^4*ln(f*x^
2+e*x+d)*B*c^2*d*e^2+1/f^4/(4*d*f-e^2)^(1/2)*arctan((2*f*x+e)/(4*d*f-e^2)^(
1/2))*e^4*A*c^2-2/f^3/(4*d*f-e^2)^(1/2)*arctan((2*f*x+e)/(4*d*f-e^2)^(1/2))
*e^3*A*b*c+2/f^2/(4*d*f-e^2)^(1/2)*arctan((2*f*x+e)/(4*d*f-e^2)^(1/2))*e^2*
B*a*b-2/f^3/(4*d*f-e^2)^(1/2)*arctan((2*f*x+e)/(4*d*f-e^2)^(1/2))*e^3*B*a*c
+2/f^3*ln(f*x^2+e*x+d)*B*b*c*d*e-4/f/(4*d*f-e^2)^(1/2)*arctan((2*f*x+e)/(4*
d*f-e^2)^(1/2))*A*a*c*d-4/f^3/(4*d*f-e^2)^(1/2)*arctan((2*f*x+e)/(4*d*f-e^2
)^(1/2))*A*c^2*d*e^2-4/f/(4*d*f-e^2)^(1/2)*arctan((2*f*x+e)/(4*d*f-e^2)^(1/
2))*B*a*b*d+3/f^2/(4*d*f-e^2)^(1/2)*arctan((2*f*x+e)/(4*d*f-e^2)^(1/2))*B*b
^2*d*e+4/f^2/(4*d*f-e^2)^(1/2)*arctan((2*f*x+e)/(4*d*f-e^2)^(1/2))*B*b*c*d^
2-5/f^3/(4*d*f-e^2)^(1/2)*arctan((2*f*x+e)/(4*d*f-e^2)^(1/2))*B*c^2*d^2*e-2
/f/(4*d*f-e^2)^(1/2)*arctan((2*f*x+e)/(4*d*f-e^2)^(1/2))*e*A*a*b+5/f^4/(4*d
*f-e^2)^(1/2)*arctan((2*f*x+e)/(4*d*f-e^2)^(1/2))*B*c^2*d*e^3+2/f^4/(4*d*f-
e^2)^(1/2)*arctan((2*f*x+e)/(4*d*f-e^2)^(1/2))*e^4*B*b*c+2/f^2/(4*d*f-e^2)^
(1/2)*arctan((2*f*x+e)/(4*d*f-e^2)^(1/2))*e^2*A*a*c
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(c*x^2+b*x+a)^2/(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for mo
re details)Is 4*d*f-e^2 positive or negative?
```

mupad [B] time = 4.85, size = 893, normalized size = 1.65

$$x^3\left(\frac{A\,c^2+2\,B\,b\,c}{3\,f}-\frac{B\,c^2\,e}{3\,f^2}\right)+x\left(\frac{A\,b^2+2\,B\,a\,b+2\,A\,a\,c}{f}-\frac{d\left(\frac{A\,c^2+2\,B\,b\,c}{f}-\frac{B\,c^2\,e}{f^2}\right)}{f}+\frac{e\left(\frac{e\left(\frac{A\,c^2+2\,B\,b\,c}{f}-\frac{B\,c^2\,e}{f^2}\right)}{f}-\frac{B\,b^2+2\,A\,c}{f}\right.}{f}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x)*(a + b*x + c*x^2)^2)/(d + e*x + f*x^2),x)
```

```
[Out] x^3*((A*c^2 + 2*B*b*c)/(3*f) - (B*c^2*e)/(3*f^2)) + x*((A*b^2 + 2*A*a*c + 2
*B*a*b)/f - (d*((A*c^2 + 2*B*b*c)/f - (B*c^2*e)/f^2))/f + (e*((e*((A*c^2 +
2*B*b*c)/f - (B*c^2*e)/f^2))/f - (B*b^2 + 2*A*b*c + 2*B*a*c)/f + (B*c^2*d)/
f^2))/f) - x^2*((e*((A*c^2 + 2*B*b*c)/f - (B*c^2*e)/f^2))/(2*f) - (B*b^2 +
2*A*b*c + 2*B*a*c)/(2*f) + (B*c^2*d)/(2*f^2)) - (log(d + e*x + f*x^2)*(B*c^
2*e^6 - 4*B*a^2*d*f^5 - A*c^2*e^5*f - A*b^2*e^3*f^3 + B*a^2*e^2*f^4 + 4*B*b
^2*d^2*f^4 + B*b^2*e^4*f^2 - 4*B*c^2*d^3*f^3 + 6*A*c^2*d*e^3*f^2 - 8*A*c^2*
d^2*e*f^3 - 5*B*b^2*d*e^2*f^3 - 8*A*a*b*d*f^5 - 2*B*b*c*e^5*f + 13*B*c^2*d^
2*e^2*f^2 + 2*A*a*b*e^2*f^4 - 2*A*a*c*e^3*f^3 + 8*A*b*c*d^2*f^4 - 2*B*a*b*e
^3*f^3 + 8*B*a*c*d^2*f^4 + 2*A*b*c*e^4*f^2 + 2*B*a*c*e^4*f^2 + 4*A*b^2*d*e*
f^4 - 7*B*c^2*d*e^4*f - 10*A*b*c*d*e^2*f^3 - 10*B*a*c*d*e^2*f^3 + 12*B*b*c*
d*e^3*f^2 - 16*B*b*c*d^2*e*f^3 + 8*A*a*c*d*e*f^4 + 8*B*a*b*d*e*f^4))/(2*(4*
d*f^6 - e^2*f^5)) + (B*c^2*x^4)/(4*f) + (atan(e/(4*d*f - e^2)^(1/2) + (2*f*
x)/(4*d*f - e^2)^(1/2))*(2*A*a^2*f^5 - B*c^2*e^5 - 2*A*b^2*d*f^4 - B*a^2*e*
f^4 + A*c^2*e^4*f + A*b^2*e^2*f^3 + 2*A*c^2*d^2*f^3 - B*b^2*e^3*f^2 - 4*A*c
^2*d*e^2*f^2 - 5*B*c^2*d^2*e*f^2 - 2*A*a*b*e*f^4 - 4*A*a*c*d*f^4 - 4*B*a*b*
d*f^4 + 2*B*b*c*e^4*f + 2*A*a*c*e^2*f^3 + 2*B*a*b*e^2*f^3 - 2*A*b*c*e^3*f^2
 - 2*B*a*c*e^3*f^2 + 4*B*b*c*d^2*f^3 + 3*B*b^2*d*e*f^3 + 5*B*c^2*d*e^3*f -
8*B*b*c*d*e^2*f^2 + 6*A*b*c*d*e*f^3 + 6*B*a*c*d*e*f^3))/(f^5*(4*d*f - e^2)^
(1/2))
```

sympy [B] time = 145.64, size = 4663, normalized size = 8.60

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(c*x**2+b*x+a)**2/(f*x**2+e*x+d),x)
```

```
[Out] B*c**2*x**4/(4*f) + x**3*(A*c**2/(3*f) + 2*B*b*c/(3*f) - B*c**2*e/(3*f**2))
 + x**2*(A*b*c/f - A*c**2*e/(2*f**2) + B*a*c/f + B*b**2/(2*f) - B*b*c*e/f**
2 - B*c**2*d/(2*f**2) + B*c**2*e**2/(2*f**3)) + x*(2*A*a*c/f + A*b**2/f - 2
*A*b*c*e/f**2 - A*c**2*d/f**2 + A*c**2*e**2/f**3 + 2*B*a*b/f - 2*B*a*c*e/f*
*2 - B*b**2*e/f**2 - 2*B*b*c*d/f**2 + 2*B*b*c*e**2/f**3 + 2*B*c**2*d*e/f**3
 - B*c**2*e**3/f**4) + (-sqrt(-4*d*f + e**2)*(-2*A*a**2*f**5 + 2*A*a*b*e*f*
```

```
*4 + 4*A*a*c*d*f**4 - 2*A*a*c*e**2*f**3 + 2*A*b**2*d*f**4 - A*b**2*e**2*f**
3 - 6*A*b*c*d*e*f**3 + 2*A*b*c*e**3*f**2 - 2*A*c**2*d**2*f**3 + 4*A*c**2*d*
e**2*f**2 - A*c**2*e**4*f + B*a**2*e*f**4 + 4*B*a*b*d*f**4 - 2*B*a*b*e**2*f
**3 - 6*B*a*c*d*e*f**3 + 2*B*a*c*e**3*f**2 - 3*B*b**2*d*e*f**3 + B*b**2*e**
3*f**2 - 4*B*b*c*d**2*f**3 + 8*B*b*c*d*e**2*f**2 - 2*B*b*c*e**4*f + 5*B*c**
2*d**2*e*f**2 - 5*B*c**2*d*e**3*f + B*c**2*e**5)/(2*f**5*(4*d*f - e**2)) +
(2*A*a*b*f**4 - 2*A*a*c*e*f**3 - A*b**2*e*f**3 - 2*A*b*c*d*f**3 + 2*A*b*c*e
**2*f**2 + 2*A*c**2*d*e*f**2 - A*c**2*e**3*f + B*a**2*f**4 - 2*B*a*b*e*f**3
 - 2*B*a*c*d*f**3 + 2*B*a*c*e**2*f**2 - B*b**2*d*f**3 + B*b**2*e**2*f**2 +
4*B*b*c*d*e*f**2 - 2*B*b*c*e**3*f + B*c**2*d**2*f**2 - 3*B*c**2*d*e**2*f +
B*c**2*e**4)/(2*f**5))*log(x + (-A*a**2*e*f**4 + 4*A*a*b*d*f**4 - 2*A*a*c*d
*e*f**3 - A*b**2*d*e*f**3 - 4*A*b*c*d**2*f**3 + 2*A*b*c*d*e**2*f**2 + 3*A*c
**2*d**2*e*f**2 - A*c**2*d*e**3*f + 2*B*a**2*d*f**4 - 2*B*a*b*d*e*f**3 - 4*
B*a*c*d**2*f**3 + 2*B*a*c*d*e**2*f**2 - 2*B*b**2*d**2*f**3 + B*b**2*d*e**2*
f**2 + 6*B*b*c*d**2*e*f**2 - 2*B*b*c*d*e**3*f + 2*B*c**2*d**3*f**2 - 4*B*c*
*2*d**2*e**2*f + B*c**2*d*e**4 - 4*d*f**5*(-sqrt(-4*d*f + e**2)*(-2*A*a**2*
f**5 + 2*A*a*b*e*f**4 + 4*A*a*c*d*f**4 - 2*A*a*c*e**2*f**3 + 2*A*b**2*d*f**
4 - A*b**2*e**2*f**3 - 6*A*b*c*d*e*f**3 + 2*A*b*c*e**3*f**2 - 2*A*c**2*d**2
*f**3 + 4*A*c**2*d*e**2*f**2 - A*c**2*e**4*f + B*a**2*e*f**4 + 4*B*a*b*d*f*
*4 - 2*B*a*b*e**2*f**3 - 6*B*a*c*d*e*f**3 + 2*B*a*c*e**3*f**2 - 3*B*b**2*d*
e*f**3 + B*b**2*e**3*f**2 - 4*B*b*c*d**2*f**3 + 8*B*b*c*d*e**2*f**2 - 2*B*b
*c*e**4*f + 5*B*c**2*d**2*e*f**2 - 5*B*c**2*d*e**3*f + B*c**2*e**5)/(2*f**5
*(4*d*f - e**2)) + (2*A*a*b*f**4 - 2*A*a*c*e*f**3 - A*b**2*e*f**3 - 2*A*b*c
*d*f**3 + 2*A*b*c*e**2*f**2 + 2*A*c**2*d*e*f**2 - A*c**2*e**3*f + B*a**2*f*
*4 - 2*B*a*b*e*f**3 - 2*B*a*c*d*f**3 + 2*B*a*c*e**2*f**2 - B*b**2*d*f**3 +
B*b**2*e**2*f**2 + 4*B*b*c*d*e*f**2 - 2*B*b*c*e**3*f + B*c**2*d**2*f**2 - 3
*B*c**2*d*e**2*f + B*c**2*e**4)/(2*f**5)) + e**2*f**4*(-sqrt(-4*d*f + e**2)
*(-2*A*a**2*f**5 + 2*A*a*b*e*f**4 + 4*A*a*c*d*f**4 - 2*A*a*c*e**2*f**3 + 2*
A*b**2*d*f**4 - A*b**2*e**2*f**3 - 6*A*b*c*d*e*f**3 + 2*A*b*c*e**3*f**2 - 2
*A*c**2*d**2*f**3 + 4*A*c**2*d*e**2*f**2 - A*c**2*e**4*f + B*a**2*e*f**4 +
4*B*a*b*d*f**4 - 2*B*a*b*e**2*f**3 - 6*B*a*c*d*e*f**3 + 2*B*a*c*e**3*f**2 -
 3*B*b**2*d*e*f**3 + B*b**2*e**3*f**2 - 4*B*b*c*d**2*f**3 + 8*B*b*c*d*e**2*
f**2 - 2*B*b*c*e**4*f + 5*B*c**2*d**2*e*f**2 - 5*B*c**2*d*e**3*f + B*c**2*e
**5)/(2*f**5*(4*d*f - e**2)) + (2*A*a*b*f**4 - 2*A*a*c*e*f**3 - A*b**2*e*f*
*3 - 2*A*b*c*d*f**3 + 2*A*b*c*e**2*f**2 + 2*A*c**2*d*e*f**2 - A*c**2*e**3*f
 + B*a**2*f**4 - 2*B*a*b*e*f**3 - 2*B*a*c*d*f**3 + 2*B*a*c*e**2*f**2 - B*b*
*2*d*f**3 + B*b**2*e**2*f**2 + 4*B*b*c*d*e*f**2 - 2*B*b*c*e**3*f + B*c**2*d
**2*f**2 - 3*B*c**2*d*e**2*f + B*c**2*e**4)/(2*f**5)))/(-2*A*a**2*f**5 + 2*
A*a*b*e*f**4 + 4*A*a*c*d*f**4 - 2*A*a*c*e**2*f**3 + 2*A*b**2*d*f**4 - A*b**
2*e**2*f**3 - 6*A*b*c*d*e*f**3 + 2*A*b*c*e**3*f**2 - 2*A*c**2*d**2*f**3 + 4
*A*c**2*d*e**2*f**2 - A*c**2*e**4*f + B*a**2*e*f**4 + 4*B*a*b*d*f**4 - 2*B*
a*b*e**2*f**3 - 6*B*a*c*d*e*f**3 + 2*B*a*c*e**3*f**2 - 3*B*b**2*d*e*f**3 +
B*b**2*e**3*f**2 - 4*B*b*c*d**2*f**3 + 8*B*b*c*d*e**2*f**2 - 2*B*b*c*e**4*f
 + 5*B*c**2*d**2*e*f**2 - 5*B*c**2*d*e**3*f + B*c**2*e**5)) + (sqrt(-4*d*f
+ e**2)*(-2*A*a**2*f**5 + 2*A*a*b*e*f**4 + 4*A*a*c*d*f**4 - 2*A*a*c*e**2*f*
*3 + 2*A*b**2*d*f**4 - A*b**2*e**2*f**3 - 6*A*b*c*d*e*f**3 + 2*A*b*c*e**3*f
**2 - 2*A*c**2*d**2*f**3 + 4*A*c**2*d*e**2*f**2 - A*c**2*e**4*f + B*a**2*e*
f**4 + 4*B*a*b*d*f**4 - 2*B*a*b*e**2*f**3 - 6*B*a*c*d*e*f**3 + 2*B*a*c*e**3
*f**2 - 3*B*b**2*d*e*f**3 + B*b**2*e**3*f**2 - 4*B*b*c*d**2*f**3 + 8*B*b*c*
d*e**2*f**2 - 2*B*b*c*e**4*f + 5*B*c**2*d**2*e*f**2 - 5*B*c**2*d*e**3*f + B
*c**2*e**5)/(2*f**5*(4*d*f - e**2)) + (2*A*a*b*f**4 - 2*A*a*c*e*f**3 - A*b*
*2*e*f**3 - 2*A*b*c*d*f**3 + 2*A*b*c*e**2*f**2 + 2*A*c**2*d*e*f**2 - A*c**2
*e**3*f + B*a**2*f**4 - 2*B*a*b*e*f**3 - 2*B*a*c*d*f**3 + 2*B*a*c*e**2*f**2
 - B*b**2*d*f**3 + B*b**2*e**2*f**2 + 4*B*b*c*d*e*f**2 - 2*B*b*c*e**3*f + B
*c**2*d**2*f**2 - 3*B*c**2*d*e**2*f + B*c**2*e**4)/(2*f**5))*log(x + (-A*a*
*2*e*f**4 + 4*A*a*b*d*f**4 - 2*A*a*c*d*e*f**3 - A*b**2*d*e*f**3 - 4*A*b*c*d
**2*f**3 + 2*A*b*c*d*e**2*f**2 + 3*A*c**2*d**2*e*f**2 - A*c**2*d*e**3*f + 2
*B*a**2*d*f**4 - 2*B*a*b*d*e*f**3 - 4*B*a*c*d**2*f**3 + 2*B*a*c*d*e**2*f**2
 - 2*B*b**2*d**2*f**3 + B*b**2*d*e**2*f**2 + 6*B*b*c*d**2*e*f**2 - 2*B*b*c*
```

```
d*e**3*f + 2*B*c**2*d**3*f**2 - 4*B*c**2*d**2*e**2*f + B*c**2*d*e**4 - 4*d*
f**5*(sqrt(-4*d*f + e**2)*(-2*A*a**2*f**5 + 2*A*a*b*e*f**4 + 4*A*a*c*d*f**4
 - 2*A*a*c*e**2*f**3 + 2*A*b**2*d*f**4 - A*b**2*e**2*f**3 - 6*A*b*c*d*e*f**
3 + 2*A*b*c*e**3*f**2 - 2*A*c**2*d**2*f**3 + 4*A*c**2*d*e**2*f**2 - A*c**2*
e**4*f + B*a**2*e*f**4 + 4*B*a*b*d*f**4 - 2*B*a*b*e**2*f**3 - 6*B*a*c*d*e*f
**3 + 2*B*a*c*e**3*f**2 - 3*B*b**2*d*e*f**3 + B*b**2*e**3*f**2 - 4*B*b*c*d*
*2*f**3 + 8*B*b*c*d*e**2*f**2 - 2*B*b*c*e**4*f + 5*B*c**2*d**2*e*f**2 - 5*B
*c**2*d*e**3*f + B*c**2*e**5)/(2*f**5*(4*d*f - e**2)) + (2*A*a*b*f**4 - 2*A
*a*c*e*f**3 - A*b**2*e*f**3 - 2*A*b*c*d*f**3 + 2*A*b*c*e**2*f**2 + 2*A*c**2
*d*e*f**2 - A*c**2*e**3*f + B*a**2*f**4 - 2*B*a*b*e*f**3 - 2*B*a*c*d*f**3 +
 2*B*a*c*e**2*f**2 - B*b**2*d*f**3 + B*b**2*e**2*f**2 + 4*B*b*c*d*e*f**2 -
2*B*b*c*e**3*f + B*c**2*d**2*f**2 - 3*B*c**2*d*e**2*f + B*c**2*e**4)/(2*f**
5)) + e**2*f**4*(sqrt(-4*d*f + e**2)*(-2*A*a**2*f**5 + 2*A*a*b*e*f**4 + 4*A
*a*c*d*f**4 - 2*A*a*c*e**2*f**3 + 2*A*b**2*d*f**4 - A*b**2*e**2*f**3 - 6*A*
b*c*d*e*f**3 + 2*A*b*c*e**3*f**2 - 2*A*c**2*d**2*f**3 + 4*A*c**2*d*e**2*f**
2 - A*c**2*e**4*f + B*a**2*e*f**4 + 4*B*a*b*d*f**4 - 2*B*a*b*e**2*f**3 - 6*
B*a*c*d*e*f**3 + 2*B*a*c*e**3*f**2 - 3*B*b**2*d*e*f**3 + B*b**2*e**3*f**2 -
 4*B*b*c*d**2*f**3 + 8*B*b*c*d*e**2*f**2 - 2*B*b*c*e**4*f + 5*B*c**2*d**2*e
*f**2 - 5*B*c**2*d*e**3*f + B*c**2*e**5)/(2*f**5*(4*d*f - e**2)) + (2*A*a*b
*f**4 - 2*A*a*c*e*f**3 - A*b**2*e*f**3 - 2*A*b*c*d*f**3 + 2*A*b*c*e**2*f**2
 + 2*A*c**2*d*e*f**2 - A*c**2*e**3*f + B*a**2*f**4 - 2*B*a*b*e*f**3 - 2*B*a
*c*d*f**3 + 2*B*a*c*e**2*f**2 - B*b**2*d*f**3 + B*b**2*e**2*f**2 + 4*B*b*c*
d*e*f**2 - 2*B*b*c*e**3*f + B*c**2*d**2*f**2 - 3*B*c**2*d*e**2*f + B*c**2*e
**4)/(2*f**5)))/(-2*A*a**2*f**5 + 2*A*a*b*e*f**4 + 4*A*a*c*d*f**4 - 2*A*a*c
*e**2*f**3 + 2*A*b**2*d*f**4 - A*b**2*e**2*f**3 - 6*A*b*c*d*e*f**3 + 2*A*b*
c*e**3*f**2 - 2*A*c**2*d**2*f**3 + 4*A*c**2*d*e**2*f**2 - A*c**2*e**4*f + B
*a**2*e*f**4 + 4*B*a*b*d*f**4 - 2*B*a*b*e**2*f**3 - 6*B*a*c*d*e*f**3 + 2*B*
a*c*e**3*f**2 - 3*B*b**2*d*e*f**3 + B*b**2*e**3*f**2 - 4*B*b*c*d**2*f**3 +
8*B*b*c*d*e**2*f**2 - 2*B*b*c*e**4*f + 5*B*c**2*d**2*e*f**2 - 5*B*c**2*d*e*
*3*f + B*c**2*e**5))
```

3.15 $\int \frac{A+Bx}{\left(a+bx+cx^2\right)\left(d+ex+fx^2\right)}\,dx$

Optimal. Leaf size=406

$$\frac{\tanh^{-1}\left(\frac{e+2fx}{\sqrt{e^2-4df}}\right)\left(B(aef-2bdf+cde)-A\left(2af^2-bef-2cdf+ce^2\right)\right)}{\sqrt{e^2-4df}\left(f\left(a^2f-abe+b^2d\right)-c\left(bde-a\left(e^2-2df\right)\right)+c^2d^2\right)}+\frac{\log\left(a+bx+cx^2\right)(-aBf+Abf}{2\left(f\left(a^2f-abe+b^2d\right)-c\left(bde-a\left(\right.\right.\right.}$$

```
[Out] 1/2*(A*b*f-A*c*e-B*a*f+B*c*d)*ln(c*x^2+b*x+a)/(c^2*d^2+f*(a^2*f-a*b*e+b^2*d
)-c*(b*d*e-a*(-2*d*f+e^2)))-1/2*(A*b*f-A*c*e-B*a*f+B*c*d)*ln(f*x^2+e*x+d)/(
c^2*d^2+f*(a^2*f-a*b*e+b^2*d)-c*(b*d*e-a*(-2*d*f+e^2)))-(A*b^2*f+2*c*(-A*a*
f+A*c*d+B*a*e)-b*(A*c*e+B*a*f+B*c*d))*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))
/(c^2*d^2+f*(a^2*f-a*b*e+b^2*d)-c*(b*d*e-a*(-2*d*f+e^2)))/(-4*a*c+b^2)^(1/2
)+(B*(a*e*f-2*b*d*f+c*d*e)-A*(2*a*f^2-b*e*f-2*c*d*f+c*e^2))*arctanh((2*f*x+
e)/(-4*d*f+e^2)^(1/2))/(c^2*d^2+f*(a^2*f-a*b*e+b^2*d)-c*(b*d*e-a*(-2*d*f+e^
2)))/(-4*d*f+e^2)^(1/2)
```

Rubi [A] time = 0.48, antiderivative size = 398, normalized size of antiderivative = 0.98, number of steps used = 9, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1022, 634, 618, 206, 628}

$$\frac{\tanh^{-1}\left(\frac{e+2fx}{\sqrt{e^2-4df}}\right)\left(B(aef-2bdf+cde)-A\left(2af^2-bef-2cdf+ce^2\right)\right)}{\sqrt{e^2-4df}\left(f\left(a^2f-abe+b^2d\right)+ac\left(e^2-2df\right)-bcde+c^2d^2\right)}+\frac{\log\left(a+bx+cx^2\right)(-aBf+Abf}{2\left(f\left(a^2f-abe+b^2d\right)+ac\left(e^2-2d\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x)/((a + b*x + c*x^2)*(d + e*x + f*x^2)),x]
```

```
[Out] -(((A*b^2*f + 2*c*(A*c*d + a*B*e - a*A*f) - b*(B*c*d + A*c*e + a*B*f))*ArcT
anh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(Sqrt[b^2 - 4*a*c]*(c^2*d^2 - b*c*d*e +
 f*(b^2*d - a*b*e + a^2*f) + a*c*(e^2 - 2*d*f)))) + ((B*(c*d*e - 2*b*d*f +
a*e*f) - A*(c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2))*ArcTanh[(e + 2*f*x)/Sqrt[e^
2 - 4*d*f]])/(Sqrt[e^2 - 4*d*f]*(c^2*d^2 - b*c*d*e + f*(b^2*d - a*b*e + a^2
*f) + a*c*(e^2 - 2*d*f))) + ((B*c*d - A*c*e + A*b*f - a*B*f)*Log[a + b*x +
c*x^2])/(2*(c^2*d^2 - b*c*d*e + f*(b^2*d - a*b*e + a^2*f) + a*c*(e^2 - 2*d*
f))) - ((B*c*d - A*c*e + A*b*f - a*B*f)*Log[d + e*x + f*x^2])/(2*(c^2*d^2 -
 b*c*d*e + f*(b^2*d - a*b*e + a^2*f) + a*c*(e^2 - 2*d*f)))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1022

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*((d_) + (e_.)*
(x_) + (f_.)*(x_)^2)), x_Symbol] :> With[{q = Simplify[c^2*d^2 - b*c*d*e +
a*c*e^2 + b^2*d*f - 2*a*c*d*f - a*b*e*f + a^2*f^2]}, Dist[1/q, Int[Simp[g*c
^2*d - g*b*c*e + a*h*c*e + g*b^2*f - a*b*h*f - a*g*c*f + c*(h*c*d - g*c*e +
 g*b*f - a*h*f)*x, x]/(a + b*x + c*x^2), x], x] + Dist[1/q, Int[Simp[-(h*c*
d*e) + g*c*e^2 + b*h*d*f - g*c*d*f - g*b*e*f + a*g*f^2 - f*(h*c*d - g*c*e +
 g*b*f - a*h*f)*x, x]/(d + e*x + f*x^2), x], x] /; NeQ[q, 0]] /; FreeQ[{a,
b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]
```

Rubi steps

$$\int \frac{A+Bx}{\left(a+bx+cx^2\right)\left(d+ex+fx^2\right)}\,dx = \frac{\int \frac{aB(ce-bf)+A\left(c^2d+b^2f-c(be+af)\right)+c(Bcd-Ace+Abf-aBf)x}{a+bx+cx^2}\,dx}{c^2d^2-bcde+f\left(b^2d-abe+a^2f\right)+ac\left(e^2-2df\right)} + \frac{\int \frac{-Af(be-af)+Ac\left(e^2-d\ldots\right.}{\ldots}}{c^2d^2-bcde+f\left(b\ldots\right.}$$

$$= \frac{(Bcd-Ace+Abf-aBf)\int \frac{b+2cx}{a+bx+cx^2}\,dx}{2\left(c^2d^2-bcde+f\left(b^2d-abe+a^2f\right)+ac\left(e^2-2df\right)\right)} - \frac{(Bcd-Ace\ldots}{2\left(c^2d^2-bcde+\ldots\right.}$$

$$= \frac{(Bcd-Ace+Abf-aBf)\log\left(a+bx+cx^2\right)}{2\left(c^2d^2-bcde+f\left(b^2d-abe+a^2f\right)+ac\left(e^2-2df\right)\right)} - \frac{(Bcd-Ace+\ldots}{2\left(c^2d^2-bcde+\ldots\right.}$$

$$= -\frac{\left(Ab^2f+2c(Acd+aBe-aAf)-b(Bcd+Ace+aBf)\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\ldots}{\sqrt{b^2-4ac}\left(c^2d^2-bcde+f\left(b^2d-abe+a^2f\right)+ac\left(e^2-2df\right)\right)}$$

Mathematica [A] time = 0.46, size = 267, normalized size = 0.66

$$\frac{\frac{2\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)\left(-b(aBf+Ace+Bcd)+2c(-aAf+aBe+Acd)+Ab^2f\right)}{\sqrt{4ac-b^2}} - \frac{2\tan^{-1}\left(\frac{e+2fx}{\sqrt{4df-e^2}}\right)\left(A\left(-2af^2+bef+2cdf-ce^2\right)+B(aef-2bdf+cde)\right)}{\sqrt{4df-e^2}} + \log(a\ldots}{2\left(f\left(a^2f-abe+b^2d\right)+ac\left(e^2-2df\right)\ldots\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x)/((a + b*x + c*x^2)*(d + e*x + f*x^2)),x]
```

```
[Out] ((2*(A*b^2*f + 2*c*(A*c*d + a*B*e - a*A*f) - b*(B*c*d + A*c*e + a*B*f))*Arc
Tan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] - (2*(B*(c*d*e - 2*
b*d*f + a*e*f) + A*(-(c*e^2) + 2*c*d*f + b*e*f - 2*a*f^2))*ArcTan[(e + 2*f*
x)/Sqrt[-e^2 + 4*d*f]])/Sqrt[-e^2 + 4*d*f] + (B*c*d - A*c*e + A*b*f - a*B*f
)*Log[a + x*(b + c*x)] + (-(B*c*d) + A*c*e - A*b*f + a*B*f)*Log[d + x*(e +
f*x)])/(2*(c^2*d^2 - b*c*d*e + f*(b^2*d - a*b*e + a^2*f) + a*c*(e^2 - 2*d*f
)))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(c*x^2+b*x+a)/(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.17, size = 416, normalized size = 1.02

$$\frac{\left(Bcd - Baf + Abf - Ace\right)\log\left(cx^2 + bx + a\right)}{2\left(c^2d^2 + b^2df - 2\,acdf + a^2f^2 - bcde - abfe + ace^2\right)} - \frac{\left(Bcd - Baf + Abf - Ace\right)\log\left(fx^2 + xe + d\right)}{2\left(c^2d^2 + b^2df - 2\,acdf + a^2f^2 - bcde - abfe + ace^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(c*x^2+b*x+a)/(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] 1/2*(B*c*d - B*a*f + A*b*f - A*c*e)*log(c*x^2 + b*x + a)/(c^2*d^2 + b^2*d*f
 - 2*a*c*d*f + a^2*f^2 - b*c*d*e - a*b*f*e + a*c*e^2) - 1/2*(B*c*d - B*a*f
+ A*b*f - A*c*e)*log(f*x^2 + x*e + d)/(c^2*d^2 + b^2*d*f - 2*a*c*d*f + a^2*
f^2 - b*c*d*e - a*b*f*e + a*c*e^2) - (B*b*c*d - 2*A*c^2*d + B*a*b*f - A*b^2
*f + 2*A*a*c*f - 2*B*a*c*e + A*b*c*e)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c)
)/((c^2*d^2 + b^2*d*f - 2*a*c*d*f + a^2*f^2 - b*c*d*e - a*b*f*e + a*c*e^2)*
sqrt(-b^2 + 4*a*c)) + (2*B*b*d*f - 2*A*c*d*f + 2*A*a*f^2 - B*c*d*e - B*a*f*
e - A*b*f*e + A*c*e^2)*arctan((2*f*x + e)/sqrt(4*d*f - e^2))/((c^2*d^2 + b^
2*d*f - 2*a*c*d*f + a^2*f^2 - b*c*d*e - a*b*f*e + a*c*e^2)*sqrt(4*d*f - e^2
))
```

maple [B] time = 0.01, size = 1698, normalized size = 4.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x+A)/(c*x^2+b*x+a)/(f*x^2+e*x+d),x)
```

```
[Out] 1/2/(a^2*f^2-a*b*e*f-2*a*c*d*f+a*c*e^2+b^2*d*f-b*c*d*e+c^2*d^2)*ln(c*x^2+b*
x+a)*A*b*f-1/2/(a^2*f^2-a*b*e*f-2*a*c*d*f+a*c*e^2+b^2*d*f-b*c*d*e+c^2*d^2)*
c*ln(c*x^2+b*x+a)*A*e-1/2/(a^2*f^2-a*b*e*f-2*a*c*d*f+a*c*e^2+b^2*d*f-b*c*d*
e+c^2*d^2)*ln(c*x^2+b*x+a)*B*a*f+1/2/(a^2*f^2-a*b*e*f-2*a*c*d*f+a*c*e^2+b^2
*d*f-b*c*d*e+c^2*d^2)*c*ln(c*x^2+b*x+a)*B*d-2/(a^2*f^2-a*b*e*f-2*a*c*d*f+a*
c*e^2+b^2*d*f-b*c*d*e+c^2*d^2)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^
2)^(1/2))*A*a*c*f+1/(a^2*f^2-a*b*e*f-2*a*c*d*f+a*c*e^2+b^2*d*f-b*c*d*e+c^2*
d^2)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*A*b^2*f-1/(a^2*f
^2-a*b*e*f-2*a*c*d*f+a*c*e^2+b^2*d*f-b*c*d*e+c^2*d^2)/(4*a*c-b^2)^(1/2)*arc
tan((2*c*x+b)/(4*a*c-b^2)^(1/2))*A*b*c*e+2/(a^2*f^2-a*b*e*f-2*a*c*d*f+a*c*e
^2+b^2*d*f-b*c*d*e+c^2*d^2)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^
(1/2))*A*c^2*d-1/(a^2*f^2-a*b*e*f-2*a*c*d*f+a*c*e^2+b^2*d*f-b*c*d*e+c^2*d^2
)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*B*a*b*f+2/(a^2*f^2-
a*b*e*f-2*a*c*d*f+a*c*e^2+b^2*d*f-b*c*d*e+c^2*d^2)/(4*a*c-b^2)^(1/2)*arctan
((2*c*x+b)/(4*a*c-b^2)^(1/2))*B*a*c*e-1/(a^2*f^2-a*b*e*f-2*a*c*d*f+a*c*e^2+
b^2*d*f-b*c*d*e+c^2*d^2)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/
2))*B*b*c*d-1/2/(a^2*f^2-a*b*e*f-2*a*c*d*f+a*c*e^2+b^2*d*f-b*c*d*e+c^2*d^2)
*f*ln(f*x^2+e*x+d)*A*b+1/2/(a^2*f^2-a*b*e*f-2*a*c*d*f+a*c*e^2+b^2*d*f-b*c*d
*e+c^2*d^2)*ln(f*x^2+e*x+d)*A*c*e+1/2/(a^2*f^2-a*b*e*f-2*a*c*d*f+a*c*e^2+b^
2*d*f-b*c*d*e+c^2*d^2)*f*ln(f*x^2+e*x+d)*B*a-1/2/(a^2*f^2-a*b*e*f-2*a*c*d*f
+a*c*e^2+b^2*d*f-b*c*d*e+c^2*d^2)*ln(f*x^2+e*x+d)*B*c*d+2/(a^2*f^2-a*b*e*f-
2*a*c*d*f+a*c*e^2+b^2*d*f-b*c*d*e+c^2*d^2)/(4*d*f-e^2)^(1/2)*arctan((2*f*x+
e)/(4*d*f-e^2)^(1/2))*A*a*f^2-1/(a^2*f^2-a*b*e*f-2*a*c*d*f+a*c*e^2+b^2*d*f-
b*c*d*e+c^2*d^2)/(4*d*f-e^2)^(1/2)*arctan((2*f*x+e)/(4*d*f-e^2)^(1/2))*A*b*
e*f-2/(a^2*f^2-a*b*e*f-2*a*c*d*f+a*c*e^2+b^2*d*f-b*c*d*e+c^2*d^2)/(4*d*f-e^
2)^(1/2)*arctan((2*f*x+e)/(4*d*f-e^2)^(1/2))*A*c*d*f+1/(a^2*f^2-a*b*e*f-2*a
*c*d*f+a*c*e^2+b^2*d*f-b*c*d*e+c^2*d^2)/(4*d*f-e^2)^(1/2)*arctan((2*f*x+e)/
(4*d*f-e^2)^(1/2))*A*c*e^2-1/(a^2*f^2-a*b*e*f-2*a*c*d*f+a*c*e^2+b^2*d*f-b*c
```

```
*d*e+c^2*d^2)/(4*d*f-e^2)^(1/2)*arctan((2*f*x+e)/(4*d*f-e^2)^(1/2))*B*a*e*f
+2/(a^2*f^2-a*b*e*f-2*a*c*d*f+a*c*e^2+b^2*d*f-b*c*d*e+c^2*d^2)/(4*d*f-e^2)^
(1/2)*arctan((2*f*x+e)/(4*d*f-e^2)^(1/2))*B*b*d*f-1/(a^2*f^2-a*b*e*f-2*a*c*
d*f+a*c*e^2+b^2*d*f-b*c*d*e+c^2*d^2)/(4*d*f-e^2)^(1/2)*arctan((2*f*x+e)/(4*
d*f-e^2)^(1/2))*B*c*d*e
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(c*x^2+b*x+a)/(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for mo
re details)Is 4*d*f-e^2 positive or negative?
```

mupad [**F(-1)**] time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x)/((a + b*x + c*x^2)*(d + e*x + f*x^2)),x)
```

```
[Out] \text{Hanged}
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(c*x**2+b*x+a)/(f*x**2+e*x+d),x)
```

```
[Out] Timed out
```

3.16 $\int \frac{A+Bx}{\left(a+bx+cx^2\right)^2\left(d+ex+fx^2\right)}\,dx$

Optimal. Leaf size=1075

$$-\frac{Ac(2ace-b(cd+af))+(Ab-aB)\left(fb^2+2c^2d-c(be+2af)\right)+c\left(Afb^2-(Bcd+Ace+aBf)b+2c(Acd+\right.}{\left(b^2-4ac\right)\left((cd-af)^2-(bd-ae)(ce-bf)\right)\left(cx^2+bx+a\right)}$$

```
[Out] (-A*c*(2*a*c*e-b*(a*f+c*d))-(A*b-B*a)*(2*c^2*d+b^2*f-c*(2*a*f+b*e))-c*(A*b^
2*f+2*c*(-A*a*f+A*c*d+B*a*e)-b*(A*c*e+B*a*f+B*c*d))*x)/(-4*a*c+b^2)/((-a*f+
c*d)^2-(-a*e+b*d)*(-b*f+c*e))/(c*x^2+b*x+a)-(b^5*(-A*e+B*d)*f^2-2*b^4*f*(B*
c*d*e-A*(a*f^2-c*d*f+c*e^2))-4*c^2*(A*(c^3*d^3-3*a^3*f^3-a^2*c*f*(-7*d*f+e^
2)+a*c^2*d*(-5*d*f+3*e^2))-a*B*e*(c^2*d^2-3*a^2*f^2-a*c*(-2*d*f+e^2)))-4*b^
2*c*(B*c^2*d^2*e+A*f*(2*c^2*d^2+3*a^2*f^2+3*a*c*(-d*f+e^2)))+2*b*c*(B*(c^3*
d^3+3*a^3*f^3+a*c^2*d*(-7*d*f+e^2)+3*a^2*c*f*(d*f+e^2))+A*c*e*(3*c^2*d^2+3*
a^2*f^2+a*c*(2*d*f+3*e^2)))-b^3*(A*c*e*(-4*a*f^2-2*c*d*f+c*e^2)+B*(4*a*c*d*
f^2+a^2*f^3-c^2*d*(5*d*f+e^2))))*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/(-4*
a*c+b^2)^(3/2)/(c^2*d^2+f*(a^2*f-a*b*e+b^2*d)-c*(b*d*e-a*(-2*d*f+e^2)))^2+1
/2*(A*(-b*f+c*e)*(f*(-2*a*f+b*e)-c*(-2*d*f+e^2))-B*(2*c*d*f*(-a*f+b*e)-f^2*
(-a^2*f+b^2*d)-c^2*d*(-d*f+e^2)))*ln(c*x^2+b*x+a)/(c^2*d^2+f*(a^2*f-a*b*e+b
^2*d)-c*(b*d*e-a*(-2*d*f+e^2)))^2-1/2*(A*(-b*f+c*e)*(f*(-2*a*f+b*e)-c*(-2*d
*f+e^2))-B*(2*c*d*f*(-a*f+b*e)-f^2*(-a^2*f+b^2*d)-c^2*d*(-d*f+e^2)))*ln(f*x
^2+e*x+d)/(c^2*d^2+f*(a^2*f-a*b*e+b^2*d)-c*(b*d*e-a*(-2*d*f+e^2)))^2+(B*(c^
2*d*e*(-3*d*f+e^2)-2*c*d*f*(-a*e*f-2*b*d*f+b*e^2)+f^2*(a^2*e*f-4*a*b*d*f+b^
2*d*e))-A*(c^2*(2*d^2*f^2-4*d*e^2*f+e^4)-f^2*(2*a*b*e*f-2*a^2*f^2-b^2*(-2*d
*f+e^2))+2*c*f*(a*f*(-2*d*f+e^2)-b*(-3*d*e*f+e^3))))*arctanh((2*f*x+e)/(-4*
d*f+e^2)^(1/2))/(c^2*d^2+f*(a^2*f-a*b*e+b^2*d)-c*(b*d*e-a*(-2*d*f+e^2)))^2/
(-4*d*f+e^2)^(1/2)
```

Rubi [A] time = 4.18, antiderivative size = 1067, normalized size of antiderivative = 0.99, number of steps used = 10, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1016, 1072, 634, 618, 206, 628}

$$-\frac{Ac(2ace-b(cd+af))+(Ab-aB)\left(fb^2+2c^2d-c(be+2af)\right)+c\left(Afb^2-(Bcd+Ace+aBf)b+2c(Acd+\right.}{\left(b^2-4ac\right)\left((cd-af)^2-(bd-ae)(ce-bf)\right)\left(cx^2+bx+a\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x)/((a + b*x + c*x^2)^2*(d + e*x + f*x^2)),x]
```

```
[Out] -((A*c*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e +
2*a*f)) + c*(A*b^2*f + 2*c*(A*c*d + a*B*e - a*A*f) - b*(B*c*d + A*c*e + a*B
*f))*x)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(a + b*x +
 c*x^2))) - ((b^5*(B*d - A*e)*f^2 - 2*b^4*f*(B*c*d*e - a*A*f^2 - A*c*(e^2 -
 d*f)) - 4*c^2*(A*(c^3*d^3 - 3*a^3*f^3 - a^2*c*f*(e^2 - 7*d*f) + a*c^2*d*(3
*e^2 - 5*d*f)) - a*B*e*(c^2*d^2 - 3*a^2*f^2 - a*c*(e^2 - 2*d*f))) - 4*b^2*(
B*c^3*d^2*e + A*c*f*(2*c^2*d^2 + 3*a^2*f^2 + 3*a*c*(e^2 - d*f))) + 2*b*c*(B
*(c^3*d^3 + 3*a^3*f^3 + a*c^2*d*(e^2 - 7*d*f) + 3*a^2*c*f*(e^2 + d*f)) + A*
c*e*(3*c^2*d^2 + 3*a^2*f^2 + a*c*(3*e^2 + 2*d*f))) - b^3*(A*c*e*(c*e^2 - 2*
c*d*f - 4*a*f^2) + B*(4*a*c*d*f^2 + a^2*f^3 - c^2*d*(e^2 + 5*d*f))))*ArcTan
h[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/((b^2 - 4*a*c)^(3/2)*(c^2*d^2 - b*c*d*e +
 f*(b^2*d - a*b*e + a^2*f) + a*c*(e^2 - 2*d*f))^2) + ((B*(c^2*d*e*(e^2 - 3*
d*f) - 2*c*d*f*(b*e^2 - 2*b*d*f - a*e*f) + f^2*(b^2*d*e - 4*a*b*d*f + a^2*e
*f)) - A*(c^2*(e^4 - 4*d*e^2*f + 2*d^2*f^2) - f^2*(2*a*b*e*f - 2*a^2*f^2 -
b^2*(e^2 - 2*d*f)) + 2*c*f*(a*f*(e^2 - 2*d*f) - b*(e^3 - 3*d*e*f))))*ArcTan
h[(e + 2*f*x)/Sqrt[e^2 - 4*d*f]])/(Sqrt[e^2 - 4*d*f]*(c^2*d^2 - b*c*d*e + f
*(b^2*d - a*b*e + a^2*f) + a*c*(e^2 - 2*d*f))^2) + ((A*(c*e - b*f)*(f*(b*e
```

```
- 2*a*f) - c*(e^2 - 2*d*f)) - B*(2*c*d*f*(b*e - a*f) - f^2*(b^2*d - a^2*f)
- c^2*d*(e^2 - d*f)))*Log[a + b*x + c*x^2])/(2*(c^2*d^2 - b*c*d*e + f*(b^2*
d - a*b*e + a^2*f) + a*c*(e^2 - 2*d*f))^2) - ((A*(c*e - b*f)*(f*(b*e - 2*a*
f) - c*(e^2 - 2*d*f)) - B*(2*c*d*f*(b*e - a*f) - f^2*(b^2*d - a^2*f) - c^2*
d*(e^2 - d*f)))*Log[d + e*x + f*x^2])/(2*(c^2*d^2 - b*c*d*e + f*(b^2*d - a*
b*e + a^2*f) + a*c*(e^2 - 2*d*f))^2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1016

```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e
_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((a + b*x + c*x^2)^(p + 1)
*(d + e*x + f*x^2)^(q + 1)*(g*c*(2*a*c*e - b*(c*d + a*f)) + (g*b - a*h)*(2*
c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(g*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))
 - h*(b*c*d - 2*a*c*e + a*b*f))*x))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d -
a*e)*(c*e - b*f))*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*
d - a*e)*(c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*
x^2)^q*Simp[(b*h - 2*g*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)
 + (b^2*(g*f) - b*(h*c*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) - a*(-(h*c*e
))))*(a*f*(p + 1) - c*d*(p + 2)) - e*((g*c)*(2*a*c*e - b*(c*d + a*f)) + (g*
b - a*h)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (2*f*((g*c)*(2*
a*c*e - b*(c*d + a*f)) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(
p + q + 2) - (b^2*g*f - b*(h*c*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) - a*
(-(h*c*e))))*(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(g*f) - b*(h*c
*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) + a*h*c*e))*(2*p + 2*q + 5)*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, q}, x] && NeQ[b^2 - 4*a*c, 0] &
& NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (b*d - a*e)*(c*e
 - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1])
```

Rule 1072

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)), x_Symbol] :> With[{q = c^2*d^2 - b*c*
d*e + a*c*e^2 + b^2*d*f - 2*a*c*d*f - a*b*e*f + a^2*f^2}, Dist[1/q, Int[(A*
c^2*d - a*c*C*d - A*b*c*e + a*B*c*e + A*b^2*f - a*b*B*f - a*A*c*f + a^2*C*f
 + c*(B*c*d - b*C*d - A*c*e + a*C*e + A*b*f - a*B*f)*x)/(a + b*x + c*x^2),
```

```
x], x] + Dist[1/q, Int[(c*C*d^2 - B*c*d*e + A*c*e^2 + b*B*d*f - A*c*d*f - a
*C*d*f - A*b*e*f + a*A*f^2 - f*(B*c*d - b*C*d - A*c*e + a*C*e + A*b*f - a*B
*f)*x)/(d + e*x + f*x^2), x], x] /; NeQ[q, 0]] /; FreeQ[{a, b, c, d, e, f,
A, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]
```

Rubi steps

$$\int \frac{A+Bx}{\left(a+bx+cx^2\right)^2\left(d+ex+fx^2\right)}\,dx = -\frac{Ac(2ace-b(cd+af))+(Ab-aB)\left(2c^2d+b^2f-c(be+2af)\right)+c\left(\right.}{\left(b^2-4ac\right)\left((cd-af)^2-(bd-ae)(ce\right.}$$

$$= -\frac{Ac(2ace-b(cd+af))+(Ab-aB)\left(2c^2d+b^2f-c(be+2af)\right)+c\left(\right.}{\left(b^2-4ac\right)\left((cd-af)^2-(bd-ae)(ce\right.}$$

$$= -\frac{Ac(2ace-b(cd+af))+(Ab-aB)\left(2c^2d+b^2f-c(be+2af)\right)+c\left(\right.}{\left(b^2-4ac\right)\left((cd-af)^2-(bd-ae)(ce\right.}$$

$$= -\frac{Ac(2ace-b(cd+af))+(Ab-aB)\left(2c^2d+b^2f-c(be+2af)\right)+c\left(\right.}{\left(b^2-4ac\right)\left((cd-af)^2-(bd-ae)(ce\right.}$$

$$= -\frac{Ac(2ace-b(cd+af))+(Ab-aB)\left(2c^2d+b^2f-c(be+2af)\right)+c\left(\right.}{\left(b^2-4ac\right)\left((cd-af)^2-(bd-ae)(ce\right.}$$

Mathematica [A] time = 6.70, size = 952, normalized size = 0.89

$$-\frac{2\left(c^2d^2-bced+f\left(fa^2-bea+b^2d\right)+ac\left(e^2-2df\right)\right)\left(A\left(fb^3+c(fx-e)b^2+c(c(d-ex)-3af)b+2c^2(cdx+a(e-fx))\right)+B\left(2cfa^2-\left(fb^2+c(fx-e)b+2c^2(d-ex)\right)a-\right.\right.}{\left(b^2-4ac\right)(a+x(b+cx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x)/((a + b*x + c*x^2)^2*(d + e*x + f*x^2)),x]
```

```
[Out] ((-2*(c^2*d^2 - b*c*d*e + f*(b^2*d - a*b*e + a^2*f) + a*c*(e^2 - 2*d*f))*(A
*(b^3*f + b^2*c*(-e + f*x) + b*c*(-3*a*f + c*(d - e*x)) + 2*c^2*(c*d*x + a*
(e - f*x))) + B*(2*a^2*c*f - b*c^2*d*x - a*(b^2*f + 2*c^2*(d - e*x) + b*c*(
-e + f*x)))))/((b^2 - 4*a*c)*(a + x*(b + c*x))) - (2*(b^5*(B*d - A*e)*f^2 +
 2*b^4*f*(-(B*c*d*e) + a*A*f^2 + A*c*(e^2 - d*f)) - 4*b^2*(B*c^3*d^2*e + A*
c*f*(2*c^2*d^2 + 3*a^2*f^2 + 3*a*c*(e^2 - d*f))) + 2*b*c*(B*(c^3*d^3 + 3*a^
3*f^3 + a*c^2*d*(e^2 - 7*d*f) + 3*a^2*c*f*(e^2 + d*f)) + A*c*e*(3*c^2*d^2 +
 3*a^2*f^2 + a*c*(3*e^2 + 2*d*f))) + 4*c^2*(a*B*e*(c^2*d^2 - 3*a^2*f^2 - a*
c*(e^2 - 2*d*f)) + A*(-(c^3*d^3) + 3*a^3*f^3 + a^2*c*f*(e^2 - 7*d*f) + a*c^
2*d*(-3*e^2 + 5*d*f))) + b^3*(A*c*e*(-(c*e^2) + 2*c*d*f + 4*a*f^2) + B*(-4*
a*c*d*f^2 - a^2*f^3 + c^2*d*(e^2 + 5*d*f))))*ArcTan[(b + 2*c*x)/Sqrt[-b^2 +
 4*a*c]])/(-b^2 + 4*a*c)^(3/2) + (2*(B*(c^2*d*e*(-e^2 + 3*d*f) - 2*c*d*f*(-
(b*e^2) + 2*b*d*f + a*e*f) + f^2*(-(b^2*d*e) + 4*a*b*d*f - a^2*e*f)) + A*(c
^2*(e^4 - 4*d*e^2*f + 2*d^2*f^2) + f^2*(-2*a*b*e*f + 2*a^2*f^2 + b^2*(e^2 -
 2*d*f)) + 2*c*f*(a*f*(e^2 - 2*d*f) - b*(e^3 - 3*d*e*f))))*ArcTan[(e + 2*f*
x)/Sqrt[-e^2 + 4*d*f]])/Sqrt[-e^2 + 4*d*f] - (A*(c*e - b*f)*(f*(-(b*e) + 2*
a*f) + c*(e^2 - 2*d*f)) + B*(2*c*d*f*(b*e - a*f) + f^2*(-(b^2*d) + a^2*f) +
 c^2*d*(-e^2 + d*f)))*Log[a + x*(b + c*x)] + (A*(c*e - b*f)*(f*(-(b*e) + 2*
a*f) + c*(e^2 - 2*d*f)) + B*(2*c*d*f*(b*e - a*f) + f^2*(-(b^2*d) + a^2*f) +
```

```
 c^2*d*(-e^2 + d*f)))*Log[d + x*(e + f*x)])/(2*(c^2*d^2 - b*c*d*e + f*(b^2*
d - a*b*e + a^2*f) + a*c*(e^2 - 2*d*f))^2)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(c*x^2+b*x+a)^2/(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 0.45, size = 3226, normalized size = 3.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(c*x^2+b*x+a)^2/(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] -1/2*(B*c^2*d^2*f - B*b^2*d*f^2 - 2*B*a*c*d*f^2 + 2*A*b*c*d*f^2 + B*a^2*f^3
 - 2*A*a*b*f^3 + 2*B*b*c*d*f*e - 2*A*c^2*d*f*e + A*b^2*f^2*e + 2*A*a*c*f^2*
e - B*c^2*d*e^2 - 2*A*b*c*f*e^2 + A*c^2*e^3)*log(c*x^2 + b*x + a)/(c^4*d^4
+ 2*b^2*c^2*d^3*f - 4*a*c^3*d^3*f + b^4*d^2*f^2 - 4*a*b^2*c*d^2*f^2 + 6*a^2
*c^2*d^2*f^2 + 2*a^2*b^2*d*f^3 - 4*a^3*c*d*f^3 + a^4*f^4 - 2*b*c^3*d^3*e -
2*b^3*c*d^2*f*e + 2*a*b*c^2*d^2*f*e - 2*a*b^3*d*f^2*e + 2*a^2*b*c*d*f^2*e -
 2*a^3*b*f^3*e + b^2*c^2*d^2*e^2 + 2*a*c^3*d^2*e^2 + 4*a*b^2*c*d*f*e^2 - 4*
a^2*c^2*d*f*e^2 + a^2*b^2*f^2*e^2 + 2*a^3*c*f^2*e^2 - 2*a*b*c^2*d*e^3 - 2*a
^2*b*c*f*e^3 + a^2*c^2*e^4) + 1/2*(B*c^2*d^2*f - B*b^2*d*f^2 - 2*B*a*c*d*f^
2 + 2*A*b*c*d*f^2 + B*a^2*f^3 - 2*A*a*b*f^3 + 2*B*b*c*d*f*e - 2*A*c^2*d*f*e
 + A*b^2*f^2*e + 2*A*a*c*f^2*e - B*c^2*d*e^2 - 2*A*b*c*f*e^2 + A*c^2*e^3)*l
og(f*x^2 + x*e + d)/(c^4*d^4 + 2*b^2*c^2*d^3*f - 4*a*c^3*d^3*f + b^4*d^2*f^
2 - 4*a*b^2*c*d^2*f^2 + 6*a^2*c^2*d^2*f^2 + 2*a^2*b^2*d*f^3 - 4*a^3*c*d*f^3
 + a^4*f^4 - 2*b*c^3*d^3*e - 2*b^3*c*d^2*f*e + 2*a*b*c^2*d^2*f*e - 2*a*b^3*
d*f^2*e + 2*a^2*b*c*d*f^2*e - 2*a^3*b*f^3*e + b^2*c^2*d^2*e^2 + 2*a*c^3*d^2
*e^2 + 4*a*b^2*c*d*f*e^2 - 4*a^2*c^2*d*f*e^2 + a^2*b^2*f^2*e^2 + 2*a^3*c*f^
2*e^2 - 2*a*b*c^2*d*e^3 - 2*a^2*b*c*f*e^3 + a^2*c^2*e^4) + (2*B*b*c^4*d^3 -
 4*A*c^5*d^3 + 5*B*b^3*c^2*d^2*f - 14*B*a*b*c^3*d^2*f - 8*A*b^2*c^3*d^2*f +
 20*A*a*c^4*d^2*f + B*b^5*d*f^2 - 4*B*a*b^3*c*d*f^2 - 2*A*b^4*c*d*f^2 + 6*B
*a^2*b*c^2*d*f^2 + 12*A*a*b^2*c^2*d*f^2 - 28*A*a^2*c^3*d*f^2 - B*a^2*b^3*f^
3 + 2*A*a*b^4*f^3 + 6*B*a^3*b*c*f^3 - 12*A*a^2*b^2*c*f^3 + 12*A*a^3*c^2*f^3
 - 4*B*b^2*c^3*d^2*e + 4*B*a*c^4*d^2*e + 6*A*b*c^4*d^2*e - 2*B*b^4*c*d*f*e
+ 2*A*b^3*c^2*d*f*e + 8*B*a^2*c^3*d*f*e + 4*A*a*b*c^3*d*f*e - A*b^5*f^2*e +
 4*A*a*b^3*c*f^2*e - 12*B*a^3*c^2*f^2*e + 6*A*a^2*b*c^2*f^2*e + B*b^3*c^2*d
*e^2 + 2*B*a*b*c^3*d*e^2 - 12*A*a*c^4*d*e^2 + 2*A*b^4*c*f*e^2 + 6*B*a^2*b*c
^2*f*e^2 - 12*A*a*b^2*c^2*f*e^2 + 4*A*a^2*c^3*f*e^2 - A*b^3*c^2*e^3 - 4*B*a
^2*c^3*e^3 + 6*A*a*b*c^3*e^3)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^2*
c^4*d^4 - 4*a*c^5*d^4 + 2*b^4*c^2*d^3*f - 12*a*b^2*c^3*d^3*f + 16*a^2*c^4*d
^3*f + b^6*d^2*f^2 - 8*a*b^4*c*d^2*f^2 + 22*a^2*b^2*c^2*d^2*f^2 - 24*a^3*c^
3*d^2*f^2 + 2*a^2*b^4*d*f^3 - 12*a^3*b^2*c*d*f^3 + 16*a^4*c^2*d*f^3 + a^4*b
^2*f^4 - 4*a^5*c*f^4 - 2*b^3*c^3*d^3*e + 8*a*b*c^4*d^3*e - 2*b^5*c*d^2*f*e
+ 10*a*b^3*c^2*d^2*f*e - 8*a^2*b*c^3*d^2*f*e - 2*a*b^5*d*f^2*e + 10*a^2*b^3
*c*d*f^2*e - 8*a^3*b*c^2*d*f^2*e - 2*a^3*b^3*f^3*e + 8*a^4*b*c*f^3*e + b^4*
c^2*d^2*e^2 - 2*a*b^2*c^3*d^2*e^2 - 8*a^2*c^4*d^2*e^2 + 4*a*b^4*c*d*f*e^2 -
 20*a^2*b^2*c^2*d*f*e^2 + 16*a^3*c^3*d*f*e^2 + a^2*b^4*f^2*e^2 - 2*a^3*b^2*
c*f^2*e^2 - 8*a^4*c^2*f^2*e^2 - 2*a*b^3*c^2*d*e^3 + 8*a^2*b*c^3*d*e^3 - 2*a
^2*b^3*c*f*e^3 + 8*a^3*b*c^2*f*e^3 + a^2*b^2*c^2*e^4 - 4*a^3*c^3*e^4)*sqrt(
-b^2 + 4*a*c)) - (4*B*b*c*d^2*f^2 - 2*A*c^2*d^2*f^2 - 4*B*a*b*d*f^3 + 2*A*b
^2*d*f^3 + 4*A*a*c*d*f^3 - 2*A*a^2*f^4 - 3*B*c^2*d^2*f*e + B*b^2*d*f^2*e +
2*B*a*c*d*f^2*e - 6*A*b*c*d*f^2*e + B*a^2*f^3*e + 2*A*a*b*f^3*e - 2*B*b*c*d
*f*e^2 + 4*A*c^2*d*f*e^2 - A*b^2*f^2*e^2 - 2*A*a*c*f^2*e^2 + B*c^2*d*e^3 +
```

```
2*A*b*c*f*e^3 - A*c^2*e^4)*arctan((2*f*x + e)/sqrt(4*d*f - e^2))/((c^4*d^4
+ 2*b^2*c^2*d^3*f - 4*a*c^3*d^3*f + b^4*d^2*f^2 - 4*a*b^2*c*d^2*f^2 + 6*a^2
*c^2*d^2*f^2 + 2*a^2*b^2*d*f^3 - 4*a^3*c*d*f^3 + a^4*f^4 - 2*b*c^3*d^3*e -
2*b^3*c*d^2*f*e + 2*a*b*c^2*d^2*f*e - 2*a*b^3*d*f^2*e + 2*a^2*b*c*d*f^2*e -
 2*a^3*b*f^3*e + b^2*c^2*d^2*e^2 + 2*a*c^3*d^2*e^2 + 4*a*b^2*c*d*f*e^2 - 4*
a^2*c^2*d*f*e^2 + a^2*b^2*f^2*e^2 + 2*a^3*c*f^2*e^2 - 2*a*b*c^2*d*e^3 - 2*a
^2*b*c*f*e^3 + a^2*c^2*e^4)*sqrt(4*d*f - e^2)) + (2*B*a*c^4*d^3 - A*b*c^4*d
^3 + 3*B*a*b^2*c^2*d^2*f - 2*A*b^3*c^2*d^2*f - 6*B*a^2*c^3*d^2*f + 5*A*a*b*
c^3*d^2*f + B*a*b^4*d*f^2 - A*b^5*d*f^2 - 4*B*a^2*b^2*c*d*f^2 + 5*A*a*b^3*c
*d*f^2 + 6*B*a^3*c^2*d*f^2 - 7*A*a^2*b*c^2*d*f^2 + B*a^3*b^2*f^3 - A*a^2*b^
3*f^3 - 2*B*a^4*c*f^3 + 3*A*a^3*b*c*f^3 - 3*B*a*b*c^3*d^2*e + 2*A*b^2*c^3*d
^2*e - 2*A*a*c^4*d^2*e - 2*B*a*b^3*c*d*f*e + 2*A*b^4*c*d*f*e + 2*B*a^2*b*c^
2*d*f*e - 6*A*a*b^2*c^2*d*f*e + 4*A*a^2*c^3*d*f*e - B*a^2*b^3*f^2*e + A*a*b
^4*f^2*e + B*a^3*b*c*f^2*e - 2*A*a^2*b^2*c*f^2*e - 2*A*a^3*c^2*f^2*e + B*a*
b^2*c^2*d*e^2 - A*b^3*c^2*d*e^2 + 2*B*a^2*c^3*d*e^2 + A*a*b*c^3*d*e^2 + 2*B
*a^2*b^2*c*f*e^2 - 2*A*a*b^3*c*f*e^2 - 2*B*a^3*c^2*f*e^2 + 5*A*a^2*b*c^2*f*
e^2 - B*a^2*b*c^2*e^3 + A*a*b^2*c^2*e^3 - 2*A*a^2*c^3*e^3 + (B*b*c^4*d^3 -
2*A*c^5*d^3 + B*b^3*c^2*d^2*f - B*a*b*c^3*d^2*f - 3*A*b^2*c^3*d^2*f + 6*A*a
*c^4*d^2*f + B*a*b^3*c*d*f^2 - A*b^4*c*d*f^2 - B*a^2*b*c^2*d*f^2 + 4*A*a*b^
2*c^2*d*f^2 - 6*A*a^2*c^3*d*f^2 + B*a^3*b*c*f^3 - A*a^2*b^2*c*f^3 + 2*A*a^3
*c^2*f^3 - B*b^2*c^3*d^2*e - 2*B*a*c^4*d^2*e + 3*A*b*c^4*d^2*e - 4*B*a*b^2*
c^2*d*f*e + 2*A*b^3*c^2*d*f*e + 4*B*a^2*c^3*d*f*e - 2*A*a*b*c^3*d*f*e - B*a
^2*b^2*c*f^2*e + A*a*b^3*c*f^2*e - 2*B*a^3*c^2*f^2*e - A*a^2*b*c^2*f^2*e +
3*B*a*b*c^3*d*e^2 - A*b^2*c^3*d*e^2 - 2*A*a*c^4*d*e^2 + 3*B*a^2*b*c^2*f*e^2
 - 2*A*a*b^2*c^2*f*e^2 + 2*A*a^2*c^3*f*e^2 - 2*B*a^2*c^3*e^3 + A*a*b*c^3*e^
3)*x)/((c^2*d^2 + b^2*d*f - 2*a*c*d*f + a^2*f^2 - b*c*d*e - a*b*f*e + a*c*e
^2)^2*(c*x^2 + b*x + a)*(b^2 - 4*a*c))
```

maple [B] time = 0.05, size = 51470, normalized size = 47.88

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x+A)/(c*x^2+b*x+a)^2/(f*x^2+e*x+d),x)
```

```
[Out] result too large to display
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(c*x^2+b*x+a)^2/(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for mo
re details)Is 4*d*f-e^2 positive or negative?
```

mupad [B] time = 30.31, size = 118429, normalized size = 110.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x)/((a + b*x + c*x^2)^2*(d + e*x + f*x^2)),x)
```

```
[Out] symsum(log((x*(4*A^3*b^3*c^4*f^6 + 16*B^3*a^3*c^4*f^6 - 3*B^3*a^2*b^2*c^3*f
^6 + 4*B^3*a^2*c^5*e^2*f^4 + B^3*b^2*c^5*d^2*f^4 - 16*A^3*a*b*c^5*f^6 + 16*
A^3*a*c^6*e*f^5 + 20*A^2*B*a^2*c^5*f^6 - 3*A^2*B*b^4*c^3*f^6 + 4*A^2*B*c^7*
```

d^2*f^4 - 16*B^3*a^2*c^5*d*f^5 - 4*A^3*b^2*c^5*e*f^5 + 6*B^3*a*b^2*c^4*d*f^5 - 4*B^3*a^2*b*c^4*e*f^5 + A^2*B*b^2*c^5*e^2*f^4 - 24*A^2*B*a*c^6*d*f^5 + 6*A*B^2*a*b^3*c^3*f^6 - 28*A*B^2*a^2*b*c^4*f^6 + 8*A^2*B*a*b^2*c^4*f^6 - 4*A*B^2*b*c^6*d^2*f^4 + 8*A*B^2*a^2*c^5*e*f^5 - 6*A*B^2*b^3*c^4*d*f^5 + 8*A^2*B*b^2*c^5*d*f^5 + 2*A^2*B*b^3*c^4*e*f^5 - 4*B^3*a*b*c^5*d*e*f^4 - 4*A*B^2*a*b*c^5*e^2*f^4 + 2*A*B^2*a*b^2*c^4*e*f^5 + 2*A*B^2*b^2*c^5*d*e*f^4 + 16*A*B^2*a*b*c^5*d*f^5 - 12*A^2*B*a*b*c^5*e*f^5 + 8*A*B^2*a*c^6*d*e*f^4 - 4*A^2*B*b*c^6*d*e*f^4))/(16*a^2*c^6*d^4 + a^4*b^4*f^4 + 16*a^4*c^4*e^4 + b^4*c^4*d^4 + 16*a^6*c^2*f^4 + b^8*d^2*f^2 - 8*a*b^2*c^5*d^4 - 8*a^5*b^2*c*f^4 + 2*a^2*b^6*d*f^3 - 2*a^3*b^5*e*f^3 - 64*a^3*c^5*d^3*f - 64*a^5*c^3*d*f^3 - 2*b^5*c^3*d^3*e + 2*b^6*c^2*d^3*f + a^2*b^4*c^2*e^4 - 8*a^3*b^2*c^3*e^4 + 32*a^3*c^5*d^2*e^2 + a^2*b^6*e^2*f^2 + 96*a^4*c^4*d^2*f^2 + b^6*c^2*d^2*e^2 + 32*a^5*c^3*e^2*f^2 - 2*a*b^7*d*e*f^2 - 2*b^7*c*d^2*e*f + 54*a^2*b^4*c^2*d^2*f^2 - 112*a^3*b^2*c^3*d^2*f^2 + 16*a*b^3*c^4*d^3*e - 2*a*b^5*c^2*d*e^3 - 32*a^2*b*c^5*d^3*e - 32*a^3*b*c^4*d*e^3 - 20*a*b^4*c^3*d^3*f - 12*a*b^6*c*d^2*f^2 - 20*a^3*b^4*c*d*f^3 - 2*a^2*b^5*c*e^3*f - 32*a^4*b*c^3*e^3*f + 16*a^4*b^3*c*e*f^3 - 32*a^5*b*c^2*e*f^3 - 64*a^4*c^4*d*e^2*f - 6*a*b^4*c^3*d^2*e^2 + 16*a^2*b^3*c^3*d*e^3 + 64*a^2*b^2*c^4*d^3*f + 64*a^4*b^2*c^2*d*f^3 + 16*a^3*b^3*c^2*e^3*f - 6*a^3*b^4*c*e^2*f^2 - 48*a^2*b^3*c^3*d^2*e*f - 36*a^2*b^4*c^2*d*e^2*f + 96*a^3*b^2*c^3*d*e^2*f - 48*a^3*b^3*c^2*d*e*f^2 + 4*a*b^6*c*d*e^2*f + 18*a*b^5*c^2*d^2*e*f + 18*a^2*b^5*c*d*e*f^2 + 32*a^3*b*c^4*d^2*e*f + 32*a^4*b*c^3*d*e*f^2) - root(48416*a^6*b^2*c^6*d^4*e^2*f^4*z^4 - 41544*a^5*b^4*c^5*d^4*e^2*f^4*z^4 - 31872*a^7*b^2*c^5*d^3*e^2*f^5*z^4 - 31872*a^5*b^2*c^7*d^5*e^2*f^3*z^4 - 29184*a^6*b^2*c^6*d^3*e^4*f^3*z^4 + 28800*a^5*b^4*c^5*d^3*e^4*f^3*z^4 + 21510*a^4*b^6*c^4*d^4*e^2*f^4*z^4 + 21408*a^6*b^4*c^4*d^3*e^2*f^5*z^4 + 21408*a^4*b^4*c^6*d^5*e^2*f^3*z^4 - 18112*a^7*b^3*c^4*d^2*e^3*f^5*z^4 - 18112*a^4*b^3*c^7*d^5*e^3*f^2*z^4 - 15600*a^5*b^5*c^4*d^3*e^3*f^4*z^4 - 15600*a^4*b^5*c^5*d^4*e^3*f^3*z^4 + 15296*a^6*b^3*c^5*d^3*e^3*f^4*z^4 + 15296*a^5*b^3*c^6*d^4*e^3*f^3*z^4 + 14016*a^7*b^2*c^5*d^2*e^4*f^4*z^4 + 14016*a^5*b^2*c^7*d^4*e^4*f^2*z^4 - 13920*a^4*b^6*c^4*d^3*e^4*f^3*z^4 - 11648*a^6*b^3*c^5*d^2*e^5*f^3*z^4 - 11648*a^5*b^3*c^6*d^3*e^5*f^2*z^4 + 10432*a^6*b^2*c^6*d^2*e^6*f^2*z^4 + 9008*a^6*b^5*c^3*d^2*e^3*f^5*z^4 + 9008*a^3*b^5*c^6*d^5*e^3*f^2*z^4 + 8544*a^5*b^5*c^4*d^2*e^5*f^3*z^4 + 8544*a^4*b^5*c^5*d^3*e^5*f^2*z^4 - 8496*a^5*b^4*c^5*d^2*e^6*f^2*z^4 + 7488*a^8*b^2*c^4*d^2*e^2*f^6*z^4 + 7488*a^4*b^2*c^8*d^6*e^2*f^2*z^4 + 7380*a^4*b^7*c^3*d^3*e^3*f^4*z^4 + 7380*a^3*b^7*c^4*d^4*e^3*f^3*z^4 - 6720*a^3*b^8*c^3*d^4*e^2*f^4*z^4 - 5784*a^5*b^6*c^3*d^3*e^2*f^5*z^4 - 5784*a^3*b^6*c^5*d^5*e^2*f^3*z^4 - 3440*a^6*b^4*c^4*d^2*e^4*f^4*z^4 - 3440*a^4*b^4*c^6*d^4*e^4*f^2*z^4 + 3360*a^3*b^8*c^3*d^3*e^4*f^3*z^4 + 3140*a^4*b^6*c^4*d^2*e^6*f^2*z^4 - 2760*a^4*b^7*c^3*d^2*e^5*f^3*z^4 - 2760*a^3*b^7*c^4*d^3*e^5*f^2*z^4 - 1764*a^5*b^7*c^2*d^2*e^3*f^5*z^4 - 1764*a^2*b^7*c^5*d^5*e^3*f^2*z^4 - 1640*a^3*b^9*c^2*d^3*e^3*f^4*z^4 - 1640*a^2*b^9*c^3*d^4*e^3*f^3*z^4 - 1604*a^6*b^6*c^2*d^2*e^2*f^6*z^4 - 1604*a^2*b^6*c^6*d^6*e^2*f^2*z^4 - 1500*a^5*b^6*c^3*d^2*e^4*f^4*z^4 - 1500*a^3*b^6*c^5*d^4*e^4*f^2*z^4 + 1140*a^2*b^10*c^2*d^4*e^2*f^4*z^4 + 810*a^4*b^8*c^2*d^2*e^4*f^4*z^4 + 810*a^2*b^8*c^4*d^4*e^4*f^2*z^4 - 544*a^3*b^8*c^3*d^2*e^6*f^2*z^4 + 416*a^3*b^9*c^2*d^2*e^5*f^3*z^4 + 416*a^2*b^9*c^3*d^3*e^5*f^2*z^4 - 384*a^2*b^10*c^2*d^3*e^4*f^3*z^4 + 180*a^4*b^8*c^2*d^3*e^2*f^5*z^4 + 180*a^2*b^8*c^4*d^5*e^2*f^3*z^4 + 48*a^7*b^4*c^3*d^2*e^2*f^6*z^4 + 48*a^3*b^4*c^7*d^6*e^2*f^2*z^4 + 36*a^2*b^10*c^2*d^2*e^6*f^2*z^4 - 1024*a^10*b*c^3*d*e*f^8*z^4 - 1024*a^3*b*c^10*d^8*e*f*z^4 - 192*a^8*b^5*c*d*e*f^8*z^4 - 192*a*b^5*c^8*d^8*e*f*z^4 + 16128*a^7*b^3*c^4*d^3*e*f^6*z^4 + 16128*a^4*b^3*c^7*d^6*e*f^3*z^4 - 11712*a^6*b^5*c^3*d^3*e*f^6*z^4 - 11712*a^3*b^5*c^6*d^6*e*f^3*z^4 + 11520*a^8*b*c^5*d^2*e^3*f^5*z^4 + 11520*a^5*b*c^8*d^5*e^3*f^2*z^4 - 9984*a^6*b^3*c^5*d^4*e*f^5*z^4 - 9984*a^5*b^3*c^6*d^5*e*f^4*z^4 + 8640*a^5*b^5*c^4*d^4*e*f^5*z^4 + 8640*a^4*b^5*c^5*d^5*e*f^4*z^4 - 7424*a^7*b*c^6*d^3*e^3*f^4*z^4 - 7424*a^6*b*c^7*d^4*e^3*f^3*z^4 - 6912*a^8*b^3*c^3*d^2*e*f^7*z^4 - 6912*a^3*b^3*c^8*d^7*e*f^2*z^4 + 4800*a^7*b^3*c^4*d*e^5*f^4*z^4 + 4800*a^4*b^3*c^7*d^4*e^5*f*z^4 + 4608*a^7*b*c^6*d^2*e^5*f^3*z^4 + 4608*a^6*b*c^7*d^3*e^5*f^2*z^4 - 4560*a^4*b^7*c^3*d^4*e*

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f^5*z^4 - 4560*a^3*b^7*c^4*d^5*e*f^4*z^4 + 4176*a^5*b^7*c^2*d^3*e*f^6*z^4 +
 4176*a^2*b^7*c^5*d^6*e*f^3*z^4 + 3264*a^7*b^5*c^2*d^2*e*f^7*z^4 + 3264*a^2
*b^5*c^7*d^7*e*f^2*z^4 + 3008*a^8*b^3*c^3*d*e^3*f^6*z^4 + 3008*a^3*b^3*c^8*
d^6*e^3*f*z^4 + 2880*a^6*b^3*c^5*d*e^7*f^2*z^4 + 2880*a^5*b^3*c^6*d^2*e^7*f
*z^4 - 2240*a^7*b^4*c^3*d*e^4*f^5*z^4 - 2240*a^3*b^4*c^7*d^5*e^4*f*z^4 - 14
88*a^5*b^5*c^4*d*e^7*f^2*z^4 - 1488*a^4*b^5*c^5*d^2*e^7*f*z^4 + 1440*a^3*b^
9*c^2*d^4*e*f^5*z^4 + 1440*a^2*b^9*c^3*d^5*e*f^4*z^4 - 1328*a^6*b^5*c^3*d*e
^5*f^4*z^4 - 1328*a^3*b^5*c^6*d^4*e^5*f*z^4 - 1152*a^7*b^2*c^5*d*e^6*f^3*z^
4 - 1152*a^5*b^2*c^7*d^3*e^6*f*z^4 - 1120*a^6*b^4*c^4*d*e^6*f^3*z^4 - 1120*
a^4*b^4*c^6*d^3*e^6*f*z^4 + 912*a^6*b^6*c^2*d*e^4*f^5*z^4 + 912*a^2*b^6*c^6
*d^5*e^4*f*z^4 + 872*a^5*b^6*c^3*d*e^6*f^3*z^4 + 872*a^3*b^6*c^5*d^3*e^6*f*
z^4 + 768*a^8*b^2*c^4*d*e^4*f^5*z^4 + 768*a^4*b^2*c^8*d^5*e^4*f*z^4 - 672*a
^8*b^4*c^2*d*e^2*f^7*z^4 - 672*a^2*b^4*c^8*d^7*e^2*f*z^4 - 624*a^7*b^5*c^2*
d*e^3*f^6*z^4 - 624*a^2*b^5*c^7*d^6*e^3*f*z^4 + 480*a^5*b^8*c*d^2*e^2*f^6*z
^4 + 480*a*b^8*c^5*d^6*e^2*f^2*z^4 + 316*a^4*b^7*c^3*d*e^7*f^2*z^4 + 316*a^
3*b^7*c^4*d^2*e^7*f*z^4 - 204*a^4*b^8*c^2*d*e^6*f^3*z^4 - 204*a^2*b^8*c^4*d
^3*e^6*f*z^4 + 168*a^3*b^10*c*d^3*e^2*f^5*z^4 + 168*a*b^10*c^3*d^5*e^2*f^3*
z^4 + 156*a^2*b^11*c*d^3*e^3*f^4*z^4 + 156*a*b^11*c^2*d^4*e^3*f^3*z^4 + 128
*a^9*b^2*c^3*d*e^2*f^7*z^4 + 128*a^3*b^2*c^9*d^7*e^2*f*z^4 - 124*a^3*b^10*c
*d^2*e^4*f^4*z^4 - 124*a*b^10*c^3*d^4*e^4*f^2*z^4 + 100*a^4*b^9*c*d^2*e^3*f
^5*z^4 + 100*a*b^9*c^4*d^5*e^3*f^2*z^4 + 36*a^5*b^7*c^2*d*e^5*f^4*z^4 + 36*
a^2*b^7*c^5*d^4*e^5*f*z^4 - 24*a^3*b^9*c^2*d*e^7*f^2*z^4 - 24*a^2*b^11*c*d^
2*e^5*f^3*z^4 - 24*a^2*b^9*c^3*d^2*e^7*f*z^4 - 24*a*b^11*c^2*d^3*e^5*f^2*z^
4 - 9216*a^8*b*c^5*d^3*e*f^6*z^4 - 9216*a^5*b*c^8*d^6*e*f^3*z^4 - 5376*a^8*
b*c^5*d*e^5*f^4*z^4 - 5376*a^5*b*c^8*d^4*e^5*f*z^4 + 5120*a^9*b*c^4*d^2*e*f
^7*z^4 + 5120*a^7*b*c^6*d^4*e*f^5*z^4 + 5120*a^6*b*c^7*d^5*e*f^4*z^4 + 5120
*a^4*b*c^9*d^7*e*f^2*z^4 - 4352*a^9*b*c^4*d*e^3*f^6*z^4 - 4352*a^4*b*c^9*d^
6*e^3*f*z^4 - 1792*a^7*b*c^6*d*e^7*f^2*z^4 - 1792*a^6*b*c^7*d^2*e^7*f*z^4 -
 1600*a^6*b^2*c^6*d*e^8*f*z^4 + 912*a^5*b^4*c^5*d*e^8*f*z^4 + 768*a^9*b^3*c
^2*d*e*f^8*z^4 + 768*a^2*b^3*c^9*d^8*e*f*z^4 - 720*a^4*b^9*c*d^3*e*f^6*z^4
- 720*a*b^9*c^4*d^6*e*f^3*z^4 - 656*a^6*b^7*c*d^2*e*f^7*z^4 - 656*a*b^7*c^6
*d^7*e*f^2*z^4 - 240*a^2*b^11*c*d^4*e*f^5*z^4 - 240*a*b^11*c^2*d^5*e*f^4*z^
4 + 216*a^7*b^6*c*d*e^2*f^7*z^4 + 216*a*b^6*c^7*d^7*e^2*f*z^4 - 204*a^4*b^6
*c^4*d*e^8*f*z^4 - 144*a^5*b^8*c*d*e^4*f^5*z^4 - 144*a*b^8*c^5*d^5*e^4*f*z^
4 - 84*a*b^12*c*d^4*e^2*f^4*z^4 + 36*a^4*b^9*c*d*e^5*f^4*z^4 + 36*a*b^9*c^4
*d^4*e^5*f*z^4 + 20*a^6*b^7*c*d*e^3*f^6*z^4 + 20*a*b^7*c^6*d^6*e^3*f*z^4 +
16*a^3*b^10*c*d*e^6*f^3*z^4 + 16*a^3*b^8*c^3*d*e^8*f*z^4 + 16*a*b^12*c*d^3*
e^4*f^3*z^4 + 16*a*b^10*c^3*d^3*e^6*f*z^4 + 48*b^11*c^3*d^6*e*f^3*z^4 + 48*
b^9*c^5*d^7*e*f^2*z^4 - 20*b^8*c^6*d^7*e^2*f*z^4 + 8*b^10*c^4*d^5*e^4*f*z^4
 - 4*b^13*c*d^4*e^3*f^3*z^4 - 4*b^11*c^3*d^4*e^5*f*z^4 + 4*b^9*c^5*d^6*e^3*
f*z^4 + 3072*a^9*c^5*d*e^4*f^5*z^4 + 3072*a^5*c^9*d^5*e^4*f*z^4 + 2560*a^8*
c^6*d*e^6*f^3*z^4 + 2560*a^6*c^8*d^3*e^6*f*z^4 + 1536*a^10*c^4*d*e^2*f^7*z^
4 + 1536*a^4*c^10*d^7*e^2*f*z^4 + 48*a^5*b^9*d^2*e*f^7*z^4 + 48*a^3*b^11*d^
3*e*f^6*z^4 - 20*a^6*b^8*d*e^2*f^7*z^4 + 8*a^4*b^10*d*e^4*f^5*z^4 + 4*a^5*b
^9*d*e^3*f^6*z^4 - 4*a^3*b^11*d*e^5*f^4*z^4 - 4*a*b^13*d^3*e^3*f^4*z^4 + 76
8*a^9*b*c^4*e^5*f^5*z^4 + 768*a^8*b*c^5*e^7*f^3*z^4 + 256*a^10*b*c^3*e^3*f^
7*z^4 - 192*a^6*b^3*c^5*e^9*f*z^4 - 68*a^7*b^6*c*e^4*f^6*z^4 + 48*a^8*b^5*c
*e^3*f^7*z^4 + 48*a^5*b^5*c^4*e^9*f*z^4 + 36*a^6*b^7*c*e^5*f^5*z^4 - 12*a^9
*b^4*c*e^2*f^8*z^4 - 4*a^4*b^9*c*e^7*f^3*z^4 - 4*a^4*b^7*c^3*e^9*f*z^4 + 38
4*a^5*b^8*c*d^3*f^7*z^4 + 384*a*b^8*c^5*d^7*f^3*z^4 + 288*a^3*b^10*c*d^4*f^
6*z^4 + 288*a*b^10*c^3*d^6*f^4*z^4 + 224*a^7*b^6*c*d^2*f^8*z^4 + 224*a*b^6*
c^7*d^8*f^2*z^4 - 192*a^10*b^2*c^2*d*f^9*z^4 - 192*a^2*b^2*c^10*d^9*f*z^4 +
 768*a^5*b*c^8*d^3*e^7*z^4 + 768*a^4*b*c^9*d^5*e^5*z^4 + 256*a^3*b*c^10*d^7
*e^3*z^4 - 192*a^5*b^3*c^6*d*e^9*z^4 - 68*a*b^6*c^7*d^6*e^4*z^4 + 48*a^4*b^
5*c^5*d*e^9*z^4 + 48*a*b^5*c^8*d^7*e^3*z^4 + 36*a*b^7*c^6*d^5*e^5*z^4 - 12*
a*b^4*c^9*d^8*e^2*z^4 - 4*a^3*b^7*c^4*d*e^9*z^4 - 4*a*b^9*c^4*d^3*e^7*z^4 +
 16*b^13*c*d^5*e*f^4*z^4 + 16*b^7*c^7*d^8*e*f*z^4 + 768*a^7*c^7*d*e^8*f*z^4
 + 16*a^7*b^7*d*e*f^8*z^4 + 16*a*b^13*d^4*e*f^5*z^4 + 256*a^7*b*c^6*e^9*f*z
^4 + 80*a*b^12*c*d^5*f^5*z^4 + 48*a^9*b^4*c*d*f^9*z^4 + 48*a*b^4*c^9*d^9*f*
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z^4 + 256*a^6*b*c^7*d*e^9*z^4 - 42*b^10*c^4*d^6*e^2*f^2*z^4 - 20*b^12*c^2*d
^5*e^2*f^3*z^4 + 6*b^12*c^2*d^4*e^4*f^2*z^4 + 4*b^11*c^3*d^5*e^3*f^2*z^4 -
24960*a^7*c^7*d^4*e^2*f^4*z^4 + 18944*a^8*c^6*d^3*e^2*f^5*z^4 + 18944*a^6*c
^8*d^5*e^2*f^3*z^4 + 14336*a^7*c^7*d^3*e^4*f^3*z^4 - 9984*a^8*c^6*d^2*e^4*f
^4*z^4 - 9984*a^6*c^8*d^4*e^4*f^2*z^4 - 7936*a^9*c^5*d^2*e^2*f^6*z^4 - 7936
*a^5*c^9*d^6*e^2*f^2*z^4 - 4352*a^7*c^7*d^2*e^6*f^2*z^4 - 42*a^4*b^10*d^2*e
^2*f^6*z^4 - 20*a^2*b^12*d^3*e^2*f^5*z^4 + 6*a^2*b^12*d^2*e^4*f^4*z^4 + 4*a
^3*b^11*d^2*e^3*f^5*z^4 - 480*a^8*b^2*c^4*e^6*f^4*z^4 + 440*a^7*b^4*c^3*e^6
*f^4*z^4 - 320*a^8*b^3*c^3*e^5*f^5*z^4 - 320*a^7*b^3*c^4*e^7*f^3*z^4 + 240*
a^8*b^4*c^2*e^4*f^6*z^4 + 240*a^6*b^4*c^4*e^8*f^2*z^4 - 192*a^9*b^3*c^2*e^3
*f^7*z^4 - 192*a^9*b^2*c^3*e^4*f^6*z^4 - 192*a^7*b^2*c^5*e^8*f^2*z^4 - 90*a
^6*b^6*c^2*e^6*f^4*z^4 - 68*a^5*b^6*c^3*e^8*f^2*z^4 + 48*a^10*b^2*c^2*e^2*f
^8*z^4 - 48*a^7*b^5*c^2*e^5*f^5*z^4 - 48*a^6*b^5*c^3*e^7*f^3*z^4 + 36*a^5*b
^7*c^2*e^7*f^3*z^4 + 6*a^4*b^8*c^2*e^8*f^2*z^4 - 33920*a^6*b^2*c^6*d^5*f^5*
z^4 + 27936*a^5*b^4*c^5*d^5*f^5*z^4 + 26112*a^7*b^2*c^5*d^4*f^6*z^4 + 26112
*a^5*b^2*c^7*d^6*f^4*z^4 - 20352*a^6*b^4*c^4*d^4*f^6*z^4 - 20352*a^4*b^4*c^
6*d^6*f^4*z^4 - 13080*a^4*b^6*c^4*d^5*f^5*z^4 - 11520*a^8*b^2*c^4*d^3*f^7*z
^4 - 11520*a^4*b^2*c^8*d^7*f^3*z^4 + 8736*a^5*b^6*c^3*d^4*f^6*z^4 + 8736*a^
3*b^6*c^5*d^6*f^4*z^4 + 7488*a^7*b^4*c^3*d^3*f^7*z^4 + 7488*a^3*b^4*c^7*d^7
*f^3*z^4 + 3840*a^3*b^8*c^3*d^5*f^5*z^4 + 2560*a^9*b^2*c^3*d^2*f^8*z^4 + 25
60*a^3*b^2*c^9*d^8*f^2*z^4 - 2416*a^6*b^6*c^2*d^3*f^7*z^4 - 2416*a^2*b^6*c^
6*d^7*f^3*z^4 - 2160*a^4*b^8*c^2*d^4*f^6*z^4 - 2160*a^2*b^8*c^4*d^6*f^4*z^4
 - 1152*a^8*b^4*c^2*d^2*f^8*z^4 - 1152*a^2*b^4*c^8*d^8*f^2*z^4 - 720*a^2*b^
10*c^2*d^5*f^5*z^4 - 480*a^4*b^2*c^8*d^4*e^6*z^4 + 440*a^3*b^4*c^7*d^4*e^6*
z^4 - 320*a^4*b^3*c^7*d^3*e^7*z^4 - 320*a^3*b^3*c^8*d^5*e^5*z^4 + 240*a^4*b
^4*c^6*d^2*e^8*z^4 + 240*a^2*b^4*c^8*d^6*e^4*z^4 - 192*a^5*b^2*c^7*d^2*e^8*
z^4 - 192*a^3*b^2*c^9*d^6*e^4*z^4 - 192*a^2*b^3*c^9*d^7*e^3*z^4 - 90*a^2*b^
6*c^6*d^4*e^6*z^4 - 68*a^3*b^6*c^5*d^2*e^8*z^4 - 48*a^3*b^5*c^6*d^3*e^7*z^4
 - 48*a^2*b^5*c^7*d^5*e^5*z^4 + 48*a^2*b^2*c^10*d^8*e^2*z^4 + 36*a^2*b^7*c^
5*d^3*e^7*z^4 + 6*a^2*b^8*c^4*d^2*e^8*z^4 - 4*b^6*c^8*d^9*f*z^4 + 256*a^11*
c^3*d*f^9*z^4 + 256*a^3*c^11*d^9*f*z^4 - 4*a^8*b^6*d*f^9*z^4 - 384*a^9*c^5*
e^6*f^4*z^4 - 256*a^10*c^4*e^4*f^6*z^4 - 256*a^8*c^6*e^8*f^2*z^4 - 64*a^11*
c^3*e^2*f^8*z^4 - 24*b^10*c^4*d^7*f^3*z^4 - 16*b^12*c^2*d^6*f^4*z^4 - 16*b^
8*c^6*d^8*f^2*z^4 + 17920*a^7*c^7*d^5*f^5*z^4 - 14336*a^8*c^6*d^4*f^6*z^4 -
 14336*a^6*c^8*d^6*f^4*z^4 + 7168*a^9*c^5*d^3*f^7*z^4 + 7168*a^5*c^9*d^7*f^
3*z^4 - 2048*a^10*c^4*d^2*f^8*z^4 - 2048*a^4*c^10*d^8*f^2*z^4 + 6*b^8*c^6*d
^6*e^4*z^4 + 6*a^6*b^8*e^4*f^6*z^4 - 4*b^9*c^5*d^5*e^5*z^4 - 4*b^7*c^7*d^7*
e^3*z^4 - 4*a^7*b^7*e^3*f^7*z^4 - 4*a^5*b^9*e^5*f^5*z^4 - 384*a^5*c^9*d^4*e
^6*z^4 - 256*a^6*c^8*d^2*e^8*z^4 - 256*a^4*c^10*d^6*e^4*z^4 - 64*a^3*c^11*d
^8*e^2*z^4 - 24*a^4*b^10*d^3*f^7*z^4 - 16*a^6*b^8*d^2*f^8*z^4 - 16*a^2*b^12
*d^4*f^6*z^4 + 48*a^6*b^2*c^6*e^10*z^4 - 12*a^5*b^4*c^5*e^10*z^4 - 4*b^14*d
^5*f^5*z^4 - 64*a^7*c^7*e^10*z^4 + b^14*d^4*e^2*f^4*z^4 + b^10*c^4*d^4*e^6*
z^4 + b^6*c^8*d^8*e^2*z^4 + a^8*b^6*e^2*f^8*z^4 + a^4*b^10*e^6*f^4*z^4 + a^
4*b^6*c^4*e^10*z^4 - 4820*A*B*a^4*b*c^5*d^2*e^2*f^4*z^2 + 2976*A*B*a^3*b*c^
6*d^3*e^2*f^3*z^2 - 2328*A*B*a^3*b*c^6*d^2*e^4*f^2*z^2 + 1848*A*B*a^2*b^4*c
^4*d^3*e*f^4*z^2 - 1768*A*B*a^3*b^4*c^3*d^2*e*f^5*z^2 + 1528*A*B*a^4*b^2*c^
4*d^2*e*f^5*z^2 - 1136*A*B*a^3*b^2*c^5*d^3*e*f^4*z^2 - 974*A*B*a^4*b^3*c^3*
d*e^2*f^5*z^2 + 692*A*B*a^2*b*c^7*d^4*e^2*f^2*z^2 + 588*A*B*a*b^6*c^3*d^2*e
^3*f^3*z^2 - 580*A*B*a^3*b^3*c^4*d*e^4*f^3*z^2 + 488*A*B*a^3*b^4*c^3*d*e^3*
f^4*z^2 - 444*A*B*a^2*b^2*c^6*d^2*e^5*f*z^2 - 412*A*B*a*b^5*c^4*d^2*e^4*f^2
*z^2 + 366*A*B*a^2*b^6*c^2*d^2*e*f^5*z^2 - 352*A*B*a^2*b^2*c^6*d^4*e*f^3*z^
2 + 326*A*B*a^2*b^4*c^4*d*e^5*f^2*z^2 + 324*A*B*a*b^5*c^4*d^3*e^2*f^3*z^2 -
 302*A*B*a*b^3*c^6*d^4*e^2*f^2*z^2 - 296*A*B*a*b^7*c^2*d^2*e^2*f^4*z^2 + 12
2*A*B*a^4*b^2*c^4*d*e^3*f^4*z^2 - 122*A*B*a^2*b^6*c^2*d*e^3*f^4*z^2 - 84*A*
B*a^3*b^2*c^5*d*e^5*f^2*z^2 + 72*A*B*a*b^4*c^5*d^3*e^3*f^2*z^2 - 64*A*B*a^2
*b^5*c^3*d*e^4*f^3*z^2 + 60*A*B*a^3*b^5*c^2*d*e^2*f^5*z^2 + 1312*A*B*a^5*b*
c^4*d*e^2*f^5*z^2 + 1040*A*B*a^4*b*c^5*d*e^4*f^3*z^2 - 500*A*B*a*b^6*c^3*d^
3*e*f^4*z^2 - 376*A*B*a*b^2*c^7*d^5*e*f^2*z^2 + 276*A*B*a^4*b^4*c^2*d*e*f^6
*z^2 - 262*A*B*a^2*b^3*c^5*d*e^6*f*z^2 + 238*A*B*a*b^2*c^7*d^4*e^3*f*z^2 +
```

232*A*B*a^5*b^2*c^3*d*e*f^6*z^2 - 176*A*B*a^2*b*c^7*d^3*e^4*f*z^2 - 120*A*B
*a*b^6*c^3*d*e^5*f^2*z^2 - 108*A*B*a*b^4*c^5*d^4*e*f^3*z^2 + 68*A*B*a*b^7*c
^2*d*e^4*f^3*z^2 + 68*A*B*a*b^4*c^5*d^2*e^5*f*z^2 + 46*A*B*a^2*b^7*c*d*e^2*
f^5*z^2 - 36*A*B*a*b^3*c^6*d^3*e^4*f*z^2 - 1932*A*B*a^2*b^3*c^5*d^3*e^2*f^3
*z^2 - 1818*A*B*a^2*b^4*c^4*d^2*e^3*f^3*z^2 + 1620*A*B*a^3*b^3*c^4*d^2*e^2*
f^4*z^2 + 1560*A*B*a^2*b^3*c^5*d^2*e^4*f^2*z^2 + 1244*A*B*a^3*b^2*c^5*d^2*e
^3*f^3*z^2 + 820*A*B*a^2*b^2*c^6*d^3*e^3*f^2*z^2 + 480*A*B*a^2*b^5*c^3*d^2*
e^2*f^4*z^2 + 352*A*B*a^3*b*c^6*d*e^6*f*z^2 - 108*A*B*a^3*b^6*c*d*e*f^6*z^2
 + 82*A*B*a*b^5*c^4*d*e^6*f*z^2 - 64*A*B*a*b*c^8*d^5*e^2*f*z^2 + 16*A*B*a*b
^8*c*d^2*e*f^5*z^2 - 4*A*B*a*b^8*c*d*e^3*f^4*z^2 + 16*B^2*a*b*c^8*d^6*e*f*z
^2 + 56*A*B*b^2*c^8*d^6*e*f*z^2 - 8*A*B*b^9*c*d*e^4*f^3*z^2 - 8*A*B*b^7*c^3
*d*e^6*f*z^2 - 800*A*B*a^6*c^4*d*e*f^6*z^2 + 10*A*B*a^2*b^8*d*e*f^6*z^2 - 6
*A*B*a*b^9*d*e^2*f^5*z^2 - 12*A*B*a^5*b^4*c*e*f^7*z^2 + 912*A*B*a^6*b*c^3*d
*f^7*z^2 + 192*A*B*a^4*b^5*c*d*f^7*z^2 + 192*A*B*a*b*c^8*d^6*f^2*z^2 - 20*A
*B*a*b^4*c^5*d*e^7*z^2 + 4*A*B*a*b*c^8*d^4*e^4*z^2 + 2144*B^2*a^4*b*c^5*d^3
*e*f^4*z^2 - 1120*B^2*a^3*b*c^6*d^4*e*f^3*z^2 - 688*B^2*a^5*b*c^4*d^2*e*f^5
*z^2 - 256*B^2*a^3*b*c^6*d^2*e^5*f*z^2 + 152*B^2*a*b^3*c^6*d^5*e*f^2*z^2 +
120*B^2*a^5*b^3*c^2*d*e*f^6*z^2 - 116*B^2*a^5*b*c^4*d*e^3*f^4*z^2 + 110*B^2
*a*b^7*c^2*d^3*e*f^4*z^2 - 80*B^2*a^2*b*c^7*d^5*e*f^2*z^2 - 72*B^2*a*b^5*c^
4*d^4*e*f^3*z^2 - 48*B^2*a^4*b*c^5*d*e^5*f^2*z^2 - 46*B^2*a*b^3*c^6*d^4*e^3
*f*z^2 - 44*B^2*a*b^4*c^5*d^3*e^4*f*z^2 - 34*B^2*a*b^5*c^4*d^2*e^5*f*z^2 +
20*B^2*a^2*b*c^7*d^4*e^3*f*z^2 - 10*B^2*a^3*b^6*c*d*e^2*f^5*z^2 - 10*B^2*a^
2*b^7*c*d^2*e*f^5*z^2 - 10*B^2*a*b^2*c^7*d^5*e^2*f*z^2 - 7*B^2*a^2*b^4*c^4*
d*e^6*f*z^2 - 6*B^2*a^3*b^2*c^5*d*e^6*f*z^2 + 4*B^2*a*b^8*c*d^2*e^2*f^4*z^2
 - 2*B^2*a^2*b^7*c*d*e^3*f^4*z^2 + 3196*A^2*a^4*b*c^5*d*e^3*f^4*z^2 - 3184*
A^2*a^4*b*c^5*d^2*e*f^5*z^2 + 1568*A^2*a^3*b*c^6*d^3*e*f^4*z^2 + 1504*A^2*a
^3*b*c^6*d*e^5*f^2*z^2 - 656*A^2*a^4*b^3*c^3*d*e*f^6*z^2 - 400*A^2*a*b^6*c^
3*d*e^4*f^3*z^2 + 314*A^2*a*b^5*c^4*d*e^5*f^2*z^2 - 264*A^2*a^3*b^5*c^2*d*e
*f^6*z^2 + 240*A^2*a^2*b^2*c^6*d*e^6*f*z^2 - 224*A^2*a^2*b*c^7*d^4*e*f^3*z^
2 + 216*A^2*a*b^5*c^4*d^3*e*f^4*z^2 - 192*A^2*a^2*b*c^7*d^2*e^5*f*z^2 + 178
*A^2*a*b^7*c^2*d*e^3*f^4*z^2 - 154*A^2*a*b^7*c^2*d^2*e*f^5*z^2 + 128*A^2*a*
b^3*c^6*d^4*e*f^3*z^2 + 106*A^2*a*b^3*c^6*d^2*e^5*f*z^2 - 12*A^2*a*b^2*c^7*
d^3*e^4*f*z^2 - 58*A*B*b^8*c^2*d^2*e^3*f^3*z^2 + 40*A*B*b^7*c^3*d^2*e^4*f^2
*z^2 - 28*A*B*b^7*c^3*d^3*e^2*f^3*z^2 - 24*A*B*b^5*c^5*d^4*e^2*f^2*z^2 - 20
*A*B*b^6*c^4*d^3*e^3*f^2*z^2 + 2768*A*B*a^4*c^6*d^2*e^3*f^3*z^2 - 1712*A*B*
a^3*c^7*d^3*e^3*f^2*z^2 - 156*A*B*a^4*b^2*c^4*e^5*f^3*z^2 + 146*A*B*a^4*b^3
*c^3*e^4*f^4*z^2 - 106*A*B*a^5*b^2*c^3*e^3*f^5*z^2 + 90*A*B*a^5*b^3*c^2*e^2
*f^6*z^2 + 38*A*B*a^3*b^3*c^4*e^6*f^2*z^2 - 36*A*B*a^3*b^5*c^2*e^4*f^4*z^2
+ 16*A*B*a^3*b^4*c^3*e^5*f^3*z^2 - 9*A*B*a^4*b^4*c^2*e^3*f^5*z^2 - 8*A*B*a^
2*b^5*c^3*e^6*f^2*z^2 + 2*A*B*a^2*b^6*c^2*e^5*f^3*z^2 + 920*A*B*a^4*b^3*c^3
*d^2*f^6*z^2 - 480*A*B*a^2*b^5*c^3*d^3*f^5*z^2 - 336*A*B*a^2*b^3*c^5*d^4*f^
4*z^2 - 272*A*B*a^3*b^3*c^4*d^3*f^5*z^2 + 240*A*B*a^3*b^5*c^2*d^2*f^6*z^2 -
 32*A*B*a*c^9*d^6*e*f*z^2 - 792*B^2*a^2*b^3*c^5*d^3*e^3*f^2*z^2 + 714*B^2*a
^2*b^4*c^4*d^3*e^2*f^3*z^2 - 572*B^2*a^3*b^2*c^5*d^3*e^2*f^3*z^2 - 475*B^2*
a^2*b^2*c^6*d^4*e^2*f^2*z^2 + 265*B^2*a^4*b^2*c^4*d^2*e^2*f^4*z^2 + 260*B^2
*a^3*b^3*c^4*d^2*e^3*f^3*z^2 - 212*B^2*a^3*b^4*c^3*d^2*e^2*f^4*z^2 + 180*B^
2*a^3*b^2*c^5*d^2*e^4*f^2*z^2 - 158*B^2*a^2*b^4*c^4*d^2*e^4*f^2*z^2 + 47*B^
2*a^2*b^6*c^2*d^2*e^2*f^4*z^2 + 16*B^2*a^2*b^5*c^3*d^2*e^3*f^3*z^2 + 2752*A
^2*a^3*b^2*c^5*d^2*e^2*f^4*z^2 - 2148*A^2*a^2*b^4*c^4*d^2*e^2*f^4*z^2 + 206
4*A^2*a^2*b^3*c^5*d^2*e^3*f^3*z^2 - 424*A^2*a^2*b^2*c^6*d^3*e^2*f^3*z^2 - 1
98*A^2*a^2*b^2*c^6*d^2*e^4*f^2*z^2 - 272*B^2*a^6*b*c^3*d*e*f^6*z^2 - 24*B^2
*a^4*b^5*c*d*e*f^6*z^2 + 1808*A^2*a^5*b*c^4*d*e*f^6*z^2 - 244*A^2*a*b*c^8*d
^4*e^3*f*z^2 + 208*A^2*a*b*c^8*d^5*e*f^2*z^2 + 134*A^2*a^2*b^7*c*d*e*f^6*z^
2 - 76*A^2*a*b^4*c^5*d*e^6*f*z^2 + 4*A^2*a*b^8*c*d*e^2*f^5*z^2 + 148*A*B*b^
4*c^6*d^5*e*f^2*z^2 + 65*A*B*b^6*c^4*d^4*e*f^3*z^2 + 46*A*B*b^8*c^2*d^3*e*f
^4*z^2 - 38*A*B*b^3*c^7*d^5*e^2*f*z^2 + 34*A*B*b^9*c*d^2*e^2*f^4*z^2 - 29*A
*B*b^4*c^6*d^4*e^3*f*z^2 + 20*A*B*b^5*c^5*d^3*e^4*f*z^2 + 12*A*B*b^8*c^2*d*
e^5*f^2*z^2 - 7*A*B*b^6*c^4*d^2*e^5*f*z^2 - 2880*A*B*a^4*c^6*d^3*e*f^4*z^2
+ 2784*A*B*a^5*c^5*d^2*e*f^5*z^2 - 1112*A*B*a^5*c^5*d*e^3*f^4*z^2 + 896*A*B

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*a^3*c^7*d^4*e*f^3*z^2 + 848*A*B*a^3*c^7*d^2*e^5*f*z^2 - 560*A*B*a^4*c^6*d*
e^5*f^2*z^2 + 96*A*B*a^2*c^8*d^5*e*f^2*z^2 - 88*A*B*a^2*c^8*d^4*e^3*f*z^2 -
 100*A*B*a^6*b*c^3*e^2*f^6*z^2 - 76*A*B*a^5*b*c^4*e^4*f^4*z^2 + 48*A*B*a^6*
b^2*c^2*e*f^7*z^2 - 42*A*B*a^3*b^2*c^5*e^7*f*z^2 + 36*A*B*a^4*b*c^5*e^6*f^2
*z^2 - 24*A*B*a^4*b^5*c*e^2*f^6*z^2 + 10*A*B*a^3*b^6*c*e^3*f^5*z^2 + 7*A*B*
a^2*b^4*c^4*e^7*f*z^2 + 2*A*B*a^2*b^7*c*e^4*f^4*z^2 - 2496*A*B*a^5*b*c^4*d^
2*f^6*z^2 + 1872*A*B*a^4*b*c^5*d^3*f^5*z^2 - 744*A*B*a^5*b^3*c^2*d*f^7*z^2
- 720*A*B*a^2*b*c^7*d^5*f^3*z^2 + 504*A*B*a*b^3*c^6*d^5*f^3*z^2 + 256*A*B*a
^3*b*c^6*d^4*f^4*z^2 + 168*A*B*a*b^7*c^2*d^3*f^5*z^2 - 144*A*B*a^2*b^7*c*d^
2*f^6*z^2 + 144*A*B*a*b^5*c^4*d^4*f^4*z^2 + 66*A*B*a^2*b^2*c^6*d*e^7*z^2 -
36*A*B*a*b^2*c^7*d^3*e^5*z^2 + 20*A*B*a*b^3*c^6*d^2*e^6*z^2 + 12*A*B*a^2*b*
c^7*d^2*e^6*z^2 + 1208*B^2*a^3*b*c^6*d^3*e^3*f^2*z^2 - 848*B^2*a^3*b^3*c^4*
d^3*e*f^4*z^2 + 672*B^2*a^2*b^3*c^5*d^4*e*f^3*z^2 - 632*B^2*a^4*b*c^5*d^2*e
^3*f^3*z^2 + 432*B^2*a^4*b^3*c^3*d^2*e*f^5*z^2 + 276*B^2*a^2*b^2*c^6*d^3*e^
4*f*z^2 - 196*B^2*a*b^6*c^3*d^3*e^2*f^3*z^2 - 168*B^2*a^2*b^5*c^3*d^3*e*f^4
*z^2 + 154*B^2*a^2*b^3*c^5*d^2*e^5*f*z^2 + 148*B^2*a*b^5*c^4*d^3*e^3*f^2*z^
2 + 96*B^2*a*b^4*c^5*d^4*e^2*f^2*z^2 - 72*B^2*a^3*b^5*c^2*d^2*e*f^5*z^2 + 7
0*B^2*a^5*b^2*c^3*d*e^2*f^5*z^2 - 60*B^2*a^4*b^3*c^3*d*e^3*f^4*z^2 + 52*B^2
*a*b^6*c^3*d^2*e^4*f^2*z^2 + 36*B^2*a^4*b^2*c^4*d*e^4*f^3*z^2 - 32*B^2*a*b^
7*c^2*d^2*e^3*f^3*z^2 + 24*B^2*a^3*b^5*c^2*d*e^3*f^4*z^2 + 15*B^2*a^4*b^4*c
^2*d*e^2*f^5*z^2 - 8*B^2*a^3*b^4*c^3*d*e^4*f^3*z^2 + 8*B^2*a^2*b^5*c^3*d*e^
5*f^2*z^2 - 2*B^2*a^3*b^3*c^4*d*e^5*f^2*z^2 - 2*B^2*a^2*b^6*c^2*d*e^4*f^3*z
^2 - 3176*A^2*a^3*b*c^6*d^2*e^3*f^3*z^2 - 2252*A^2*a^4*b^2*c^4*d*e^2*f^5*z^
2 + 1952*A^2*a^3*b^4*c^3*d*e^2*f^5*z^2 - 1496*A^2*a^3*b^3*c^4*d*e^3*f^4*z^2
 + 1378*A^2*a^2*b^4*c^4*d*e^4*f^3*z^2 + 1184*A^2*a^3*b^3*c^4*d^2*e*f^5*z^2
- 1166*A^2*a^2*b^3*c^5*d*e^5*f^2*z^2 - 1164*A^2*a^3*b^2*c^5*d*e^4*f^3*z^2 -
 1152*A^2*a^2*b^3*c^5*d^3*e*f^4*z^2 + 578*A^2*a*b^6*c^3*d^2*e^2*f^4*z^2 - 5
48*A^2*a*b^5*c^4*d^2*e^3*f^3*z^2 + 440*A^2*a*b^2*c^7*d^4*e^2*f^2*z^2 - 412*
A^2*a^2*b^6*c^2*d*e^2*f^5*z^2 - 360*A^2*a*b^3*c^6*d^3*e^3*f^2*z^2 + 312*A^2
*a*b^4*c^5*d^3*e^2*f^3*z^2 + 248*A^2*a^2*b*c^7*d^3*e^3*f^2*z^2 - 224*A^2*a^
2*b^5*c^3*d*e^3*f^4*z^2 + 216*A^2*a^2*b^5*c^3*d^2*e*f^5*z^2 + 52*A^2*a*b^4*
c^5*d^2*e^4*f^2*z^2 - 16*B^2*b^3*c^7*d^6*e*f*z^2 - 14*B^2*b^9*c*d^3*e*f^4*z
^2 + 32*B^2*a^4*c^6*d*e^6*f*z^2 - 20*A^2*b^9*c*d*e^3*f^4*z^2 + 18*A^2*b^9*c
*d^2*e*f^5*z^2 + 8*A^2*b^6*c^4*d*e^6*f*z^2 - 360*A^2*a^3*c^7*d*e^6*f*z^2 +
136*A^2*a*c^9*d^5*e^2*f*z^2 + 2*B^2*a^3*b^7*d*e*f^6*z^2 + 2*B^2*a*b^9*d^2*e
*f^5*z^2 + 12*B^2*a^4*b*c^5*e^7*f*z^2 - 204*A^2*a^3*b*c^6*e^7*f*z^2 - 128*A
^2*a^6*b*c^3*e*f^7*z^2 - 48*A^2*a*b^5*c^4*e^7*f*z^2 - 36*B^2*a^5*b^4*c*d*f^
7*z^2 - 24*A^2*a^4*b^5*c*e*f^7*z^2 - 16*B^2*a*b^8*c*d^3*f^5*z^2 - 164*A^2*a
^3*b^6*c*d*f^7*z^2 - 16*A^2*a*b^8*c*d^2*f^6*z^2 + 4*B^2*a^3*b*c^6*d*e^7*z^2
 - 4*B^2*a*b*c^8*d^5*e^3*z^2 + 48*A^2*a*b*c^8*d^3*e^5*z^2 + 36*A^2*a^2*b*c^
7*d*e^7*z^2 - 6*A^2*a*b^3*c^6*d*e^7*z^2 + 136*A*B*a^6*c^4*e^3*f^5*z^2 - 96*
A*B*b^5*c^5*d^5*f^3*z^2 + 80*A*B*a^5*c^5*e^5*f^3*z^2 - 72*A*B*b^3*c^7*d^6*f
^2*z^2 - 24*A*B*b^7*c^3*d^4*f^4*z^2 + 14*A*B*b^3*c^7*d^4*e^4*z^2 - 14*A*B*b
^2*c^8*d^5*e^3*z^2 - 2*A*B*b^5*c^5*d^2*e^6*z^2 - 2*A*B*b^4*c^6*d^3*e^5*z^2
+ 2*A*B*a^3*b^7*e^2*f^6*z^2 - A*B*a^2*b^8*e^3*f^5*z^2 + 16*A*B*a^2*c^8*d^3*
e^5*z^2 - 2*A*B*a^2*b^3*c^5*e^8*z^2 + 22*B^2*b^8*c^2*d^3*e^2*f^3*z^2 - 12*B
^2*b^7*c^3*d^3*e^3*f^2*z^2 + 12*B^2*b^6*c^4*d^4*e^2*f^2*z^2 - 6*B^2*b^8*c^2
*d^2*e^4*f^2*z^2 - 864*B^2*a^4*c^6*d^3*e^2*f^3*z^2 + 496*B^2*a^3*c^7*d^4*e^
2*f^2*z^2 + 224*B^2*a^5*c^5*d^2*e^2*f^4*z^2 + 136*B^2*a^4*c^6*d^2*e^4*f^2*z
^2 - 53*A^2*b^8*c^2*d^2*e^2*f^4*z^2 + 52*A^2*b^7*c^3*d^2*e^3*f^3*z^2 + 52*A
^2*b^5*c^5*d^3*e^3*f^2*z^2 - 36*A^2*b^6*c^4*d^3*e^2*f^3*z^2 - 12*A^2*b^4*c^
6*d^4*e^2*f^2*z^2 - 9*A^2*b^6*c^4*d^2*e^4*f^2*z^2 + 836*A^2*a^4*c^6*d^2*e^2
*f^4*z^2 - 668*A^2*a^2*c^8*d^4*e^2*f^2*z^2 + 656*A^2*a^3*c^7*d^2*e^4*f^2*z^
2 + 368*A^2*a^3*c^7*d^3*e^2*f^3*z^2 - 45*B^2*a^6*b^2*c^2*e^2*f^6*z^2 - 18*B
^2*a^5*b^2*c^3*e^4*f^4*z^2 - 9*B^2*a^4*b^2*c^4*e^6*f^2*z^2 - 6*B^2*a^5*b^3*
c^2*e^3*f^5*z^2 + 3*B^2*a^4*b^4*c^2*e^4*f^4*z^2 - 2*B^2*a^4*b^3*c^3*e^5*f^3
*z^2 - 580*B^2*a^4*b^2*c^4*d^3*f^5*z^2 + 536*B^2*a^3*b^4*c^3*d^3*f^5*z^2 +
471*A^2*a^4*b^2*c^4*e^4*f^4*z^2 - 436*A^2*a^3*b^4*c^3*e^4*f^4*z^2 - 348*B^2
*a^4*b^4*c^2*d^2*f^6*z^2 + 316*B^2*a^2*b^2*c^6*d^5*f^3*z^2 + 310*A^2*a^3*b^
```

3*c^4*e^5*f^3*z^2 + 232*A^2*a^5*b^2*c^3*e^2*f^6*z^2 - 229*A^2*a^2*b^4*c^4*e
^6*f^2*z^2 - 216*A^2*a^4*b^4*c^2*e^2*f^6*z^2 + 204*A^2*a^4*b^3*c^3*e^3*f^5*
z^2 + 200*B^2*a^5*b^2*c^3*d^2*f^6*z^2 + 150*A^2*a^3*b^2*c^5*e^6*f^2*z^2 - 1
20*B^2*a^2*b^4*c^4*d^4*f^4*z^2 + 91*A^2*a^2*b^6*c^2*e^4*f^4*z^2 + 72*A^2*a^
3*b^5*c^2*e^3*f^5*z^2 - 66*B^2*a^2*b^6*c^2*d^3*f^5*z^2 + 44*A^2*a^2*b^5*c^3
*e^5*f^3*z^2 - 16*B^2*a^3*b^2*c^5*d^4*f^4*z^2 + 1952*A^2*a^4*b^2*c^4*d^2*f^
6*z^2 - 1792*A^2*a^3*b^2*c^5*d^3*f^5*z^2 - 1272*A^2*a^3*b^4*c^3*d^2*f^6*z^2
 + 976*A^2*a^2*b^2*c^6*d^4*f^4*z^2 + 960*A^2*a^2*b^4*c^4*d^3*f^5*z^2 + 282*
A^2*a^2*b^6*c^2*d^2*f^6*z^2 - 45*B^2*a^2*b^2*c^6*d^2*e^6*z^2 - 48*A^2*b*c^9
*d^6*e*f*z^2 - 14*A^2*a*b^9*d*e*f^6*z^2 - 7*A*B*b^10*d^2*e*f^5*z^2 + 2*A*B*
b^10*d*e^3*f^4*z^2 - 64*A*B*a^7*c^3*e*f^7*z^2 - 16*A*B*b^9*c*d^3*f^5*z^2 +
8*A*B*a^4*c^6*e^7*f*z^2 + 4*A*B*b*c^9*d^6*e^2*z^2 + 2*A*B*b^6*c^4*d*e^7*z^2
 - 120*A*B*a^3*c^7*d*e^7*z^2 - 16*A*B*a^3*b^7*d*f^7*z^2 + 16*A*B*a*b^9*d^2*
f^6*z^2 + 8*A*B*a*c^9*d^5*e^3*z^2 + 12*A*B*a^3*b*c^6*e^8*z^2 - 48*B^2*b^5*c
^5*d^5*e*f^2*z^2 + 15*B^2*b^4*c^6*d^5*e^2*f*z^2 - 14*B^2*b^7*c^3*d^4*e*f^3*
z^2 + 4*B^2*b^9*c*d^2*e^3*f^3*z^2 + 4*B^2*b^7*c^3*d^2*e^5*f*z^2 + 4*B^2*b^5
*c^5*d^4*e^3*f*z^2 - B^2*b^6*c^4*d^3*e^4*f*z^2 - 336*B^2*a^3*c^7*d^3*e^4*f*
z^2 + 112*B^2*a^5*c^5*d*e^4*f^3*z^2 - 112*A^2*b^3*c^7*d^5*e*f^2*z^2 + 80*B^
2*a^6*c^4*d*e^2*f^5*z^2 - 48*A^2*b^5*c^5*d^4*e*f^3*z^2 + 36*A^2*b^8*c^2*d*e
^4*f^3*z^2 + 36*A^2*b^3*c^7*d^4*e^3*f*z^2 - 28*A^2*b^7*c^3*d*e^5*f^2*z^2 +
20*A^2*b^2*c^8*d^5*e^2*f*z^2 + 16*B^2*a^2*c^8*d^5*e^2*f*z^2 - 14*A^2*b^7*c^
3*d^3*e*f^4*z^2 - 14*A^2*b^4*c^6*d^3*e^4*f*z^2 - 10*A^2*b^5*c^5*d^2*e^5*f*z
^2 - 1008*A^2*a^4*c^6*d*e^4*f^3*z^2 - 760*A^2*a^5*c^5*d*e^2*f^5*z^2 + 272*A
^2*a^2*c^8*d^3*e^4*f*z^2 + 48*B^2*a^5*b*c^4*e^5*f^3*z^2 + 36*B^2*a^6*b*c^3*
e^3*f^5*z^2 + 12*B^2*a^5*b^4*c*e^2*f^6*z^2 - 624*A^2*a^4*b*c^5*e^5*f^3*z^2
- 548*A^2*a^5*b*c^4*e^3*f^5*z^2 + 182*A^2*a^2*b^3*c^5*e^7*f*z^2 - 180*B^2*a
*b^4*c^5*d^5*f^3*z^2 + 132*B^2*a^6*b^2*c^2*d*f^7*z^2 + 108*B^2*a^3*b^6*c*d^
2*f^6*z^2 + 96*A^2*a^5*b^3*c^2*e*f^7*z^2 + 68*A^2*a*b^6*c^3*e^6*f^2*z^2 + 5
8*A^2*a^3*b^6*c*e^2*f^6*z^2 - 56*B^2*a*b^2*c^7*d^6*f^2*z^2 - 38*A^2*a^2*b^7
*c*e^3*f^5*z^2 - 36*A^2*a*b^7*c^2*e^5*f^3*z^2 + 20*B^2*a*b^6*c^3*d^4*f^4*z^
2 - 736*A^2*a^5*b^2*c^3*d*f^7*z^2 + 624*A^2*a^4*b^4*c^2*d*f^7*z^2 - 416*A^2
*a*b^2*c^7*d^5*f^3*z^2 - 276*A^2*a*b^4*c^5*d^4*f^4*z^2 - 196*A^2*a*b^6*c^3*
d^3*f^5*z^2 + 8*B^2*a*b^4*c^5*d^2*e^6*z^2 + 6*B^2*a*b^2*c^7*d^4*e^4*z^2 + 2
*B^2*a^2*b^3*c^5*d*e^7*z^2 + 2*B^2*a*b^3*c^6*d^3*e^5*z^2 - 18*A^2*a*b^2*c^7
*d^2*e^6*z^2 - 16*A*B*b*c^9*d^7*f*z^2 - B^2*b^10*d^2*e^2*f^4*z^2 + 48*B^2*a
^7*c^3*e^2*f^6*z^2 - 36*B^2*a^6*c^4*e^4*f^4*z^2 + 31*B^2*b^6*c^4*d^5*f^3*z^
2 - 24*B^2*a^5*c^5*e^6*f^2*z^2 + 20*B^2*b^4*c^6*d^6*f^2*z^2 - 6*A^2*b^8*c^2
*e^6*f^2*z^2 + 2*B^2*b^8*c^2*d^4*f^4*z^2 - 768*B^2*a^5*c^5*d^3*f^5*z^2 + 51
2*B^2*a^6*c^4*d^2*f^6*z^2 + 512*B^2*a^4*c^6*d^4*f^4*z^2 + 232*A^2*a^5*c^5*e
^4*f^4*z^2 + 188*A^2*a^4*c^6*e^6*f^2*z^2 - 128*B^2*a^3*c^7*d^5*f^3*z^2 + 92
*A^2*a^6*c^4*e^2*f^6*z^2 + 80*A^2*b^4*c^6*d^5*f^3*z^2 + 64*A^2*b^2*c^8*d^6*
f^2*z^2 + 31*A^2*b^6*c^4*d^4*f^4*z^2 + 14*A^2*b^8*c^2*d^3*f^5*z^2 - 5*B^2*b
^4*c^6*d^4*e^4*z^2 + 4*B^2*b^3*c^7*d^5*e^3*z^2 + 2*B^2*b^5*c^5*d^3*e^5*z^2
- B^2*b^6*c^4*d^2*e^6*z^2 - B^2*b^2*c^8*d^6*e^2*z^2 - B^2*a^4*b^6*e^2*f^6*z
^2 - 1152*A^2*a^3*c^7*d^4*f^4*z^2 + 1008*A^2*a^4*c^6*d^3*f^5*z^2 + 624*A^2*
a^2*c^8*d^5*f^3*z^2 - 288*A^2*a^5*c^5*d^2*f^6*z^2 + 56*B^2*a^3*c^7*d^2*e^6*
z^2 - 10*B^2*a^2*b^8*d^2*f^6*z^2 - 9*A^2*b^2*c^8*d^4*e^4*z^2 - 5*A^2*a^2*b^
8*e^2*f^6*z^2 - 4*B^2*a^2*c^8*d^4*e^4*z^2 + 3*A^2*b^4*c^6*d^2*e^6*z^2 - 2*A
^2*b^3*c^7*d^3*e^5*z^2 - 36*A^2*a^2*c^8*d^2*e^6*z^2 - 48*A^2*a^6*b^2*c^2*f^
8*z^2 - 45*A^2*a^2*b^2*c^6*e^8*z^2 + 4*A^2*b^10*d*e^2*f^5*z^2 + 4*B^2*b^2*c
^8*d^7*f*z^2 + 4*A^2*b^9*c*e^5*f^3*z^2 + 4*A^2*b^7*c^3*e^7*f*z^2 - 128*B^2*
a^7*c^3*d*f^7*z^2 - 160*A^2*a*c^9*d^6*f^2*z^2 - 112*A^2*a^6*c^4*d*f^7*z^2 +
 12*A^2*b*c^9*d^5*e^3*z^2 + 4*A^2*a*b^9*e^3*f^5*z^2 + 3*B^2*a^4*b^6*d*f^7*z
^2 + 2*A^2*a^3*b^7*e*f^7*z^2 - 24*A^2*a*c^9*d^4*e^4*z^2 + 14*A^2*a^2*b^8*d*
f^7*z^2 + 12*A^2*a^5*b^4*c*f^8*z^2 + 12*A^2*a*b^4*c^5*e^8*z^2 + A*B*a^4*b^6
*e*f^7*z^2 + B^2*a^2*b^8*d*e^2*f^5*z^2 + 16*A^2*c^10*d^7*f*z^2 + 3*B^2*b^10
*d^3*f^5*z^2 - A^2*b^10*e^4*f^4*z^2 - 4*A^2*c^10*d^6*e^2*z^2 - A^2*b^10*d^2
*f^6*z^2 + 64*A^2*a^7*c^3*f^8*z^2 - 4*B^2*a^4*c^6*e^8*z^2 - A^2*b^6*c^4*e^8
*z^2 + 48*A^2*a^3*c^7*e^8*z^2 - A^2*a^4*b^6*f^8*z^2 + 720*A^2*B*a*b^2*c^5*d

```
^2*e^2*f^3*z - 600*A^2*B*a^2*b^2*c^4*d*e^2*f^4*z + 576*A*B^2*a^2*b^2*c^4*d^
2*e*f^4*z + 348*A*B^2*a*b^2*c^5*d^2*e^3*f^2*z - 336*A*B^2*a^2*b*c^5*d^2*e^2
*f^3*z - 260*A*B^2*a*b^3*c^4*d^2*e^2*f^3*z - 240*A*B^2*a^2*b^2*c^4*d*e^3*f^
3*z + 196*A*B^2*a^2*b^3*c^3*d*e^2*f^4*z + 172*A^2*B*a*b*c^6*d*e^5*f*z + 20*
A*B^2*a*b^6*c*d*e*f^5*z - 912*A^2*B*a^2*b*c^5*d^2*e*f^4*z - 644*A^2*B*a*b*c
^6*d^2*e^3*f^2*z - 432*A*B^2*a*b^2*c^5*d^3*e*f^3*z + 372*A^2*B*a^2*b*c^5*d*
e^3*f^3*z - 330*A^2*B*a*b^2*c^5*d*e^4*f^2*z + 312*A*B^2*a*b*c^6*d^3*e^2*f^2
*z - 208*A*B^2*a^3*b^2*c^3*d*e*f^5*z + 192*A^2*B*a^2*b^3*c^3*d*e*f^5*z + 17
2*A^2*B*a*b^3*c^4*d*e^3*f^3*z + 108*A*B^2*a^2*b*c^5*d*e^4*f^2*z + 104*A*B^2
*a^3*b*c^4*d*e^2*f^4*z - 80*A^2*B*a*b^3*c^4*d^2*e*f^4*z + 68*A^2*B*a*b^4*c^
3*d*e^2*f^4*z - 60*A*B^2*a*b^5*c^2*d*e^2*f^4*z + 58*A*B^2*a*b^3*c^4*d*e^4*f
^2*z - 36*A*B^2*a*b^4*c^3*d^2*e*f^4*z - 24*A*B^2*a^2*b^4*c^2*d*e*f^5*z + 24
*A*B^2*a*b^4*c^3*d*e^3*f^3*z + 592*A^2*B*a*b*c^6*d^3*e*f^3*z + 240*A^2*B*a^
3*b*c^4*d*e*f^5*z - 132*A*B^2*a*b*c^6*d^2*e^4*f*z - 60*A*B^2*a*b^2*c^5*d*e^
5*f*z - 48*A^2*B*a*b^5*c^2*d*e*f^5*z + 20*B^3*a*b*c^6*d^3*e^3*f*z + 16*B^3*
a^4*b*c^3*d*e*f^5*z - 16*B^3*a*b*c^6*d^4*e*f^2*z + 12*B^3*a^2*b*c^5*d*e^5*f
*z + 320*A^3*a*b*c^6*d*e^4*f^2*z + 40*A^3*a*b^4*c^3*d*e*f^5*z - 48*A^2*B*b*
c^7*d^4*e*f^2*z - 44*A^2*B*b^3*c^5*d*e^5*f*z - 20*A*B^2*b*c^7*d^4*e^2*f*z +
 14*A*B^2*b^4*c^4*d*e^5*f*z + 12*A^2*B*b*c^7*d^3*e^3*f*z + 4*A*B^2*b^7*c*d*
e^2*f^4*z + 160*A*B^2*a^4*c^4*d*e*f^5*z + 152*A^2*B*a*c^7*d^2*e^4*f*z - 40*
A*B^2*a*c^7*d^3*e^3*f*z + 32*A*B^2*a*c^7*d^4*e*f^2*z - 16*A*B^2*a^2*c^6*d*e
^5*f*z + 128*A^2*B*a^4*b*c^3*e*f^6*z + 42*A^2*B*a*b^2*c^5*e^6*f*z + 24*A^2*
B*a^2*b^5*c*e*f^6*z - 12*A*B^2*a^3*b^4*c*e*f^6*z - 12*A*B^2*a^2*b*c^5*e^6*f
*z - 10*A^2*B*a*b^6*c*e^2*f^5*z - 160*A*B^2*a*b*c^6*d^4*f^3*z + 112*A*B^2*a
^4*b*c^3*d*f^6*z - 24*A*B^2*a^2*b^5*c*d*f^6*z - 84*B^3*a*b^2*c^5*d^3*e^2*f^
2*z - 80*B^3*a^2*b^3*c^3*d^2*e*f^4*z - 60*B^3*a^2*b*c^5*d^2*e^3*f^2*z - 20*
B^3*a^3*b^2*c^3*d*e^2*f^4*z - 20*B^3*a*b^3*c^4*d^2*e^3*f^2*z - 9*B^3*a^2*b^
2*c^4*d*e^4*f^2*z - 8*B^3*a*b^4*c^3*d^2*e^2*f^3*z + 6*B^3*a^2*b^4*c^2*d*e^2
*f^4*z - 4*B^3*a^2*b^3*c^3*d*e^3*f^3*z - 216*A^2*B*b^4*c^4*d^2*e^2*f^3*z +
196*A^2*B*b^3*c^5*d^2*e^3*f^2*z - 108*A*B^2*b^3*c^5*d^3*e^2*f^2*z - 94*A*B^
2*b^4*c^4*d^2*e^3*f^2*z + 88*A^2*B*b^2*c^6*d^3*e^2*f^2*z + 80*A*B^2*b^5*c^3
*d^2*e^2*f^3*z + 360*A^2*B*a^2*c^6*d^2*e^2*f^3*z + 8*A*B^2*a^2*c^6*d^2*e^3*
f^2*z + 153*A^2*B*a^2*b^2*c^4*e^4*f^3*z - 144*A^2*B*a^2*b^3*c^3*e^3*f^4*z +
 80*A^2*B*a^3*b^2*c^3*e^2*f^5*z + 36*A*B^2*a^3*b^2*c^3*e^3*f^4*z + 12*A^2*B
*a^2*b^4*c^2*e^2*f^5*z + 12*A*B^2*a^3*b^3*c^2*e^2*f^5*z + 9*A*B^2*a^2*b^2*c
^4*e^5*f^2*z - 6*A*B^2*a^2*b^4*c^2*e^3*f^4*z + 4*A*B^2*a^2*b^3*c^3*e^4*f^3*
z + 480*A^2*B*a^2*b^2*c^4*d^2*f^5*z - 176*A*B^2*a^2*b^3*c^3*d^2*f^5*z - 10*
A^2*B*a*b^6*c*d*f^6*z + 16*A*B^2*a*b*c^6*d*e^6*z + 80*B^3*a*b^3*c^4*d^3*e*f
^3*z - 48*B^3*a^3*b*c^4*d^2*e*f^4*z + 48*B^3*a^2*b*c^5*d^3*e*f^3*z + 44*B^3
*a^3*b*c^4*d*e^3*f^3*z + 24*B^3*a*b^5*c^2*d^2*e*f^4*z + 18*B^3*a*b^2*c^5*d^
2*e^4*f*z + 696*A^3*a^2*b*c^5*d*e^2*f^4*z - 504*A^3*a*b*c^6*d^2*e^2*f^3*z -
 192*A^3*a*b^2*c^5*d*e^3*f^3*z - 144*A^3*a^2*b^2*c^4*d*e*f^5*z + 96*A^3*a*b
^2*c^5*d^2*e*f^4*z - 72*A^3*a*b^3*c^4*d*e^2*f^4*z - 208*A^2*B*b^3*c^5*d^3*e
*f^3*z + 152*A*B^2*b^4*c^4*d^3*e*f^3*z + 80*A^2*B*b^5*c^3*d^2*e*f^4*z + 75*
A^2*B*b^4*c^4*d*e^4*f^2*z - 59*A^2*B*b^2*c^6*d^2*e^4*f*z - 52*A^2*B*b^5*c^3
*d*e^3*f^3*z + 42*A*B^2*b^3*c^5*d^2*e^4*f*z - 21*A*B^2*b^6*c^2*d^2*e*f^4*z
- 16*A*B^2*b^5*c^3*d*e^4*f^2*z + 16*A*B^2*b^2*c^6*d^4*e*f^2*z + 16*A*B^2*b^
2*c^6*d^3*e^3*f*z + 11*A^2*B*b^6*c^2*d*e^2*f^4*z + 4*A*B^2*b^6*c^2*d*e^3*f^
3*z - 256*A^2*B*a*c^7*d^3*e^2*f^2*z - 96*A*B^2*a^3*c^5*d^2*e*f^4*z - 36*A^2
*B*a^2*c^6*d*e^4*f^2*z - 32*A^2*B*a^3*c^5*d*e^2*f^4*z - 32*A*B^2*a^2*c^6*d^
3*e*f^3*z + 8*A*B^2*a^3*c^5*d*e^3*f^3*z - 96*A^2*B*a^3*b^3*c^2*e*f^6*z + 68
*A^2*B*a^3*b*c^4*e^3*f^4*z - 60*A*B^2*a^4*b*c^3*e^2*f^5*z - 60*A*B^2*a^3*b*
c^4*e^4*f^3*z + 48*A*B^2*a^4*b^2*c^2*e*f^6*z - 38*A^2*B*a*b^3*c^4*e^5*f^2*z
 - 36*A^2*B*a^2*b*c^5*e^5*f^2*z + 36*A^2*B*a*b^5*c^2*e^3*f^4*z - 16*A^2*B*a
*b^4*c^3*e^4*f^3*z + 384*A*B^2*a^2*b*c^5*d^3*f^4*z - 352*A*B^2*a^3*b*c^4*d^
2*f^5*z - 288*A^2*B*a*b^2*c^5*d^3*f^4*z - 160*A^2*B*a^3*b^2*c^3*d*f^6*z - 1
48*A^2*B*a*b^4*c^3*d^2*f^5*z + 112*A*B^2*a*b^3*c^4*d^3*f^4*z + 72*A^2*B*a^2
*b^4*c^2*d*f^6*z + 72*A*B^2*a*b^5*c^2*d^2*f^5*z + 48*A*B^2*a^3*b^3*c^2*d*f^
6*z + 102*B^3*a^2*b^2*c^4*d^2*e^2*f^3*z - 32*B^3*b^5*c^3*d^3*e*f^3*z - 8*B^
```

3*b^3*c^5*d^3*e^3*f*z - 7*B^3*b^4*c^4*d^2*e^4*f*z + 5*B^3*b^2*c^6*d^4*e^2*f
*z + 80*A^3*b^2*c^6*d^3*e*f^3*z - 74*A^3*b^3*c^5*d*e^4*f^2*z - 64*A^3*b^4*c
^4*d^2*e*f^4*z + 60*A^3*b^4*c^4*d*e^3*f^3*z - 48*B^3*a^4*c^4*d*e^2*f^4*z -
24*B^3*a^3*c^5*d*e^4*f^2*z + 20*B^3*a^2*c^6*d^2*e^4*f*z - 16*A^3*b^5*c^3*d*
e^2*f^4*z + 8*A^3*b*c^7*d^3*e^2*f^2*z + 480*A^3*a^2*c^6*d^2*e*f^4*z - 392*A
^3*a^2*c^6*d*e^3*f^3*z + 280*A^3*a*c^7*d^2*e^3*f^2*z - 4*B^3*a^4*b*c^3*e^3*
f^4*z - 200*A^3*a^3*b*c^4*e^2*f^5*z - 144*A^3*a^2*b*c^5*e^4*f^3*z + 48*B^3*
a*b^2*c^5*d^4*f^3*z + 42*A^3*a*b^2*c^5*e^5*f^2*z - 36*B^3*a^4*b^2*c^2*d*f^6
*z - 32*A^3*a^3*b^2*c^3*e*f^6*z - 24*A^3*a^2*b^4*c^2*e*f^6*z - 24*A^3*a*b^5
*c^2*e^2*f^5*z + 10*A^3*a*b^3*c^4*e^4*f^3*z - 4*B^3*a*b^4*c^3*d^3*f^4*z - 4
*A^3*a*b^4*c^3*e^3*f^4*z - 480*A^3*a^2*b*c^5*d^2*f^5*z - 160*A^3*a^2*b^3*c^
3*d*f^6*z + 128*A^3*a*b^3*c^4*d^2*f^5*z + 8*A^2*B*b^5*c^3*e^5*f^2*z - 2*A^2
*B*b^6*c^2*e^4*f^3*z + 112*A^2*B*b^4*c^4*d^3*f^4*z - 92*A^2*B*a^4*c^4*e^2*f
^5*z - 64*A^2*B*a^3*c^5*e^4*f^3*z - 64*A*B^2*b^5*c^3*d^3*f^4*z + 24*A*B^2*a
^4*c^4*e^3*f^4*z + 24*A*B^2*a^3*c^5*e^5*f^2*z + 16*A^2*B*b^2*c^6*d^4*f^3*z
+ 16*A*B^2*b^3*c^5*d^4*f^3*z - A^2*B*b^6*c^2*d^2*f^5*z + 448*A^2*B*a^3*c^5*
d^2*f^5*z - 352*A^2*B*a^2*c^6*d^3*f^4*z - 5*A*B^2*b^2*c^6*d^2*e^5*z - 48*A^
2*B*a^4*b^2*c^2*f^7*z - 2*B^3*b^7*c*d^2*e*f^4*z + 34*A^3*b^2*c^6*d*e^5*f*z
+ 16*A^3*b*c^7*d^2*e^4*f*z + 2*A^3*b^6*c^2*d*e*f^5*z - 416*A^3*a^3*c^5*d*e*
f^5*z - 224*A^3*a*c^7*d^3*e*f^3*z + 12*B^3*a^3*b^4*c*d*f^6*z - 10*B^3*a*b^6
*c*d^2*f^5*z + 416*A^3*a^3*b*c^4*d*f^6*z + 224*A^3*a*b*c^6*d^3*f^4*z + 24*A
^3*a*b^5*c^2*d*f^6*z - 4*B^3*a*b*c^6*d^2*e^5*z + 20*A^2*B*c^8*d^4*e^2*f*z -
7*A^2*B*b^4*c^4*e^6*f*z - 2*A^2*B*b^7*c*e^3*f^4*z - 64*A*B^2*a^5*c^3*e*f^6
*z + 16*A*B^2*b*c^7*d^5*f^2*z - 8*A^2*B*a^2*c^6*e^6*f*z - 2*A*B^2*b^7*c*d^2
*f^5*z - 272*A^2*B*a^4*c^4*d*f^6*z + 128*A^2*B*a*c^7*d^4*f^3*z + 9*A^2*B*b^
2*c^6*d*e^6*z - 4*A*B^2*b^3*c^5*d*e^6*z + 4*A*B^2*b*c^7*d^3*e^4*z + 8*A*B^2
*a*c^7*d^2*e^5*z + 12*A^2*B*a^3*b^4*c*f^7*z + 30*B^3*b^4*c^4*d^3*e^2*f^2*z
+ 8*B^3*b^5*c^3*d^2*e^3*f^2*z - 2*B^3*b^6*c^2*d^2*e^2*f^3*z + 152*A^3*b^3*c
^5*d^2*e^2*f^3*z - 108*A^3*b^2*c^6*d^2*e^3*f^2*z + 48*B^3*a^3*c^5*d^2*e^2*f
^3*z - 16*B^3*a^2*c^6*d^3*e^2*f^2*z - 3*B^3*a^4*b^2*c^2*e^2*f^5*z - 120*B^3
*a^2*b^2*c^4*d^3*f^4*z + 112*B^3*a^3*b^2*c^3*d^2*f^5*z + 112*A^3*a^2*b^3*c^
3*e^2*f^5*z + 12*A^3*a^2*b^2*c^4*e^3*f^4*z - 120*A^3*a*c^7*d*e^5*f*z - 52*A
^3*a*b*c^6*e^6*f*z + 10*A^3*a*b^6*c*e*f^6*z - 2*A*B^2*b^8*d*e*f^5*z - 2*A^2
*B*a*b^7*e*f^6*z - 24*A^2*B*a*c^7*d*e^6*z + 2*A*B^2*a*b^7*d*f^6*z - 12*A^2*
B*a*b*c^6*e^7*z - 2*A^3*b^7*c*d*f^6*z - 4*A^3*b*c^7*d*e^6*z + 16*B^3*a^5*c^
3*e^2*f^5*z + 11*B^3*b^6*c^2*d^3*f^4*z - 11*A^3*b^4*c^4*e^5*f^2*z - 8*B^3*b
^4*c^4*d^4*f^3*z - 4*B^3*b^2*c^6*d^5*f^2*z + 4*B^3*a^4*c^4*e^4*f^3*z + 4*A^
3*b^5*c^3*e^4*f^3*z - A^3*b^6*c^2*e^3*f^4*z + 136*A^3*a^3*c^5*e^3*f^4*z + 6
8*A^3*a^2*c^6*e^5*f^2*z - 64*A^3*b^3*c^5*d^3*f^4*z + 2*B^3*b^3*c^5*d^2*e^5*
z - B^3*b^2*c^6*d^3*e^4*z + 96*A^3*a^3*b^3*c^2*f^7*z + A*B^2*a^2*b^6*e*f^6*
z + 32*A^3*c^8*d^4*e*f^2*z - 24*A^3*c^8*d^3*e^3*f*z + 10*A^3*b^3*c^5*e^6*f*
z + 2*A^3*b^7*c*e^2*f^5*z + 128*A^3*a^4*c^4*e*f^6*z - 32*A^3*b*c^7*d^4*f^3*
z - 4*B^3*a^2*c^6*d*e^6*z - B^3*a^2*b^6*d*f^6*z - 128*A^3*a^4*b*c^3*f^7*z -
24*A^3*a^2*b^5*c*f^7*z - 16*A^2*B*c^8*d^5*f^2*z - 4*A^2*B*c^8*d^3*e^4*z +
64*A^2*B*a^5*c^3*f^7*z + 2*A^2*B*b^3*c^5*e^7*z + 4*A*B^2*a^2*c^6*e^7*z - A^
2*B*a^2*b^6*f^7*z + 4*A^3*c^8*d^2*e^5*z - 3*A^3*b^2*c^6*e^7*z + A^2*B*b^8*d
*f^6*z - A^3*b^8*e*f^6*z + 16*A^3*a*c^7*e^7*z + 2*A^3*a*b^7*f^7*z + A^2*B*b
^8*e^2*f^5*z + B^3*b^8*d^2*f^5*z - 48*A^2*B^2*a*b*c^4*d*e*f^4 + 28*A*B^3*a*
b^2*c^3*d*e*f^4 - 16*A*B^3*a*b*c^4*d*e^2*f^3 + 16*A^3*B*a*c^5*d*e*f^4 + 32*
A^3*B*a*b*c^4*d*f^5 + 12*A^2*B^2*b^3*c^3*d*e*f^4 + 5*A*B^3*b^2*c^4*d^2*e*f^
3 + 4*A*B^3*b^3*c^3*d*e^2*f^3 + 24*A^2*B^2*a*c^5*d*e^2*f^3 + 24*A^2*B^2*a^2
*b*c^3*e*f^5 + 12*A^2*B^2*a*b*c^4*e^3*f^3 - 6*A^2*B^2*a*b^3*c^2*e*f^5 + 4*A
*B^3*a^2*b*c^3*e^2*f^4 + 3*A*B^3*a^2*b^2*c^2*e*f^5 - 18*A^2*B^2*a*b^2*c^3*d
*f^5 - 4*B^4*a^2*b*c^3*d*e*f^4 + 4*B^4*a*b*c^4*d^2*e*f^3 - 6*A*B^3*b^4*c^2*
d*e*f^4 + 4*A^3*B*b*c^5*d*e^2*f^3 - 2*A^3*B*b^2*c^4*d*e*f^4 - 8*A*B^3*a^2*c
^4*d*e*f^4 - 8*A*B^3*a*c^5*d^2*e*f^3 + 26*A^3*B*a*b^2*c^3*e*f^5 + 8*A^3*B*a
*b*c^4*e^2*f^4 + 32*A*B^3*a*b*c^4*d^2*f^4 - 28*A*B^3*a^2*b*c^3*d*f^5 + 6*A*
B^3*a*b^3*c^2*d*f^5 - 9*A^2*B^2*b^2*c^4*d*e^2*f^3 - 18*A^2*B^2*a*b^2*c^3*e^
2*f^4 - 4*A^3*B*c^6*d^2*e*f^3 - 3*A^3*B*b^4*c^2*e*f^5 - 44*A^3*B*a^2*c^4*e*

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f^5 - 16*A^3*B*a*c^5*e^3*f^3 - 16*A*B^3*a^3*c^3*e*f^5 - 10*A^3*B*b^3*c^3*d*
f^5 - 4*A^3*B*b*c^5*d^2*f^4 - 4*A*B^3*b*c^5*d^3*f^3 - 28*A^3*B*a^2*b*c^3*f^
6 + 6*A^3*B*a*b^3*c^2*f^6 - 4*A^4*b*c^5*d*e*f^4 - 20*A^4*a*b*c^4*e*f^5 + 3*
A^2*B^2*b^4*c^2*e^2*f^4 - 2*A^2*B^2*b^3*c^3*e^3*f^3 + 12*A^2*B^2*a^2*c^4*e^
2*f^4 + 9*A^2*B^2*b^2*c^4*d^2*f^4 - 3*A^2*B^2*a^2*b^2*c^2*f^6 - 2*B^4*b^3*c
^3*d^2*e*f^3 + 4*B^4*a^2*c^4*d*e^2*f^3 - 10*B^4*a*b^2*c^3*d^2*f^4 - 3*B^4*a
^2*b^2*c^2*d*f^5 + 3*A^3*B*b^2*c^4*e^3*f^3 - 2*A^3*B*b^3*c^3*e^2*f^4 - 10*A
*B^3*b^3*c^3*d^2*f^4 - 4*A*B^3*a^2*c^4*e^3*f^3 + 3*A^2*B^2*b^4*c^2*d*f^5 +
36*A^2*B^2*a^2*c^4*d*f^5 - 24*A^2*B^2*a*c^5*d^2*f^4 + 4*A^2*B^2*c^6*d^3*f^3
 + 16*A^2*B^2*a^3*c^3*f^6 + 4*A^4*b^3*c^3*e*f^5 + 16*B^4*a^3*c^3*d*f^5 + 16
*A^4*a*c^5*e^2*f^4 + 8*A^4*b^2*c^4*d*f^5 - 8*A^4*a*b^2*c^3*f^6 - 24*A^4*a*c
^5*d*f^5 + 3*B^4*b^4*c^2*d^2*f^4 - 3*A^4*b^2*c^4*e^2*f^4 + 4*A^4*c^6*d^2*f^
4 + 36*A^4*a^2*c^4*f^6 + B^4*b^2*c^4*d^3*f^3, z, k)*(root(48416*a^6*b^2*c^6
*d^4*e^2*f^4*z^4 - 41544*a^5*b^4*c^5*d^4*e^2*f^4*z^4 - 31872*a^7*b^2*c^5*d^
3*e^2*f^5*z^4 - 31872*a^5*b^2*c^7*d^5*e^2*f^3*z^4 - 29184*a^6*b^2*c^6*d^3*e
^4*f^3*z^4 + 28800*a^5*b^4*c^5*d^3*e^4*f^3*z^4 + 21510*a^4*b^6*c^4*d^4*e^2*
f^4*z^4 + 21408*a^6*b^4*c^4*d^3*e^2*f^5*z^4 + 21408*a^4*b^4*c^6*d^5*e^2*f^3
*z^4 - 18112*a^7*b^3*c^4*d^2*e^3*f^5*z^4 - 18112*a^4*b^3*c^7*d^5*e^3*f^2*z^
4 - 15600*a^5*b^5*c^4*d^3*e^3*f^4*z^4 - 15600*a^4*b^5*c^5*d^4*e^3*f^3*z^4 +
 15296*a^6*b^3*c^5*d^3*e^3*f^4*z^4 + 15296*a^5*b^3*c^6*d^4*e^3*f^3*z^4 + 14
016*a^7*b^2*c^5*d^2*e^4*f^4*z^4 + 14016*a^5*b^2*c^7*d^4*e^4*f^2*z^4 - 13920
*a^4*b^6*c^4*d^3*e^4*f^3*z^4 - 11648*a^6*b^3*c^5*d^2*e^5*f^3*z^4 - 11648*a^
5*b^3*c^6*d^3*e^5*f^2*z^4 + 10432*a^6*b^2*c^6*d^2*e^6*f^2*z^4 + 9008*a^6*b^
5*c^3*d^2*e^3*f^5*z^4 + 9008*a^3*b^5*c^6*d^5*e^3*f^2*z^4 + 8544*a^5*b^5*c^4
*d^2*e^5*f^3*z^4 + 8544*a^4*b^5*c^5*d^3*e^5*f^2*z^4 - 8496*a^5*b^4*c^5*d^2*
e^6*f^2*z^4 + 7488*a^8*b^2*c^4*d^2*e^2*f^6*z^4 + 7488*a^4*b^2*c^8*d^6*e^2*f
^2*z^4 + 7380*a^4*b^7*c^3*d^3*e^3*f^4*z^4 + 7380*a^3*b^7*c^4*d^4*e^3*f^3*z^
4 - 6720*a^3*b^8*c^3*d^4*e^2*f^4*z^4 - 5784*a^5*b^6*c^3*d^3*e^2*f^5*z^4 - 5
784*a^3*b^6*c^5*d^5*e^2*f^3*z^4 - 3440*a^6*b^4*c^4*d^2*e^4*f^4*z^4 - 3440*a
^4*b^4*c^6*d^4*e^4*f^2*z^4 + 3360*a^3*b^8*c^3*d^3*e^4*f^3*z^4 + 3140*a^4*b^
6*c^4*d^2*e^6*f^2*z^4 - 2760*a^4*b^7*c^3*d^2*e^5*f^3*z^4 - 2760*a^3*b^7*c^4
*d^3*e^5*f^2*z^4 - 1764*a^5*b^7*c^2*d^2*e^3*f^5*z^4 - 1764*a^2*b^7*c^5*d^5*
e^3*f^2*z^4 - 1640*a^3*b^9*c^2*d^3*e^3*f^4*z^4 - 1640*a^2*b^9*c^3*d^4*e^3*f
^3*z^4 - 1604*a^6*b^6*c^2*d^2*e^2*f^6*z^4 - 1604*a^2*b^6*c^6*d^6*e^2*f^2*z^
4 - 1500*a^5*b^6*c^3*d^2*e^4*f^4*z^4 - 1500*a^3*b^6*c^5*d^4*e^4*f^2*z^4 + 1
140*a^2*b^10*c^2*d^4*e^2*f^4*z^4 + 810*a^4*b^8*c^2*d^2*e^4*f^4*z^4 + 810*a^
2*b^8*c^4*d^4*e^4*f^2*z^4 - 544*a^3*b^8*c^3*d^2*e^6*f^2*z^4 + 416*a^3*b^9*c
^2*d^2*e^5*f^3*z^4 + 416*a^2*b^9*c^3*d^3*e^5*f^2*z^4 - 384*a^2*b^10*c^2*d^3
*e^4*f^3*z^4 + 180*a^4*b^8*c^2*d^3*e^2*f^5*z^4 + 180*a^2*b^8*c^4*d^5*e^2*f^
3*z^4 + 48*a^7*b^4*c^3*d^2*e^2*f^6*z^4 + 48*a^3*b^4*c^7*d^6*e^2*f^2*z^4 + 3
6*a^2*b^10*c^2*d^2*e^6*f^2*z^4 - 1024*a^10*b*c^3*d*e*f^8*z^4 - 1024*a^3*b*c
^10*d^8*e*f*z^4 - 192*a^8*b^5*c*d*e*f^8*z^4 - 192*a*b^5*c^8*d^8*e*f*z^4 + 1
6128*a^7*b^3*c^4*d^3*e*f^6*z^4 + 16128*a^4*b^3*c^7*d^6*e*f^3*z^4 - 11712*a^
6*b^5*c^3*d^3*e*f^6*z^4 - 11712*a^3*b^5*c^6*d^6*e*f^3*z^4 + 11520*a^8*b*c^5
*d^2*e^3*f^5*z^4 + 11520*a^5*b*c^8*d^5*e^3*f^2*z^4 - 9984*a^6*b^3*c^5*d^4*e
*f^5*z^4 - 9984*a^5*b^3*c^6*d^5*e*f^4*z^4 + 8640*a^5*b^5*c^4*d^4*e*f^5*z^4
+ 8640*a^4*b^5*c^5*d^5*e*f^4*z^4 - 7424*a^7*b*c^6*d^3*e^3*f^4*z^4 - 7424*a^
6*b*c^7*d^4*e^3*f^3*z^4 - 6912*a^8*b^3*c^3*d^2*e*f^7*z^4 - 6912*a^3*b^3*c^8
*d^7*e*f^2*z^4 + 4800*a^7*b^3*c^4*d*e^5*f^4*z^4 + 4800*a^4*b^3*c^7*d^4*e^5*
f*z^4 + 4608*a^7*b*c^6*d^2*e^5*f^3*z^4 + 4608*a^6*b*c^7*d^3*e^5*f^2*z^4 - 4
560*a^4*b^7*c^3*d^4*e*f^5*z^4 - 4560*a^3*b^7*c^4*d^5*e*f^4*z^4 + 4176*a^5*b
^7*c^2*d^3*e*f^6*z^4 + 4176*a^2*b^7*c^5*d^6*e*f^3*z^4 + 3264*a^7*b^5*c^2*d^
2*e*f^7*z^4 + 3264*a^2*b^5*c^7*d^7*e*f^2*z^4 + 3008*a^8*b^3*c^3*d*e^3*f^6*z
^4 + 3008*a^3*b^3*c^8*d^6*e^3*f*z^4 + 2880*a^6*b^3*c^5*d*e^7*f^2*z^4 + 2880
*a^5*b^3*c^6*d^2*e^7*f*z^4 - 2240*a^7*b^4*c^3*d*e^4*f^5*z^4 - 2240*a^3*b^4*
c^7*d^5*e^4*f*z^4 - 1488*a^5*b^5*c^4*d*e^7*f^2*z^4 - 1488*a^4*b^5*c^5*d^2*e
^7*f*z^4 + 1440*a^3*b^9*c^2*d^4*e*f^5*z^4 + 1440*a^2*b^9*c^3*d^5*e*f^4*z^4
- 1328*a^6*b^5*c^3*d*e^5*f^4*z^4 - 1328*a^3*b^5*c^6*d^4*e^5*f*z^4 - 1152*a^
7*b^2*c^5*d*e^6*f^3*z^4 - 1152*a^5*b^2*c^7*d^3*e^6*f*z^4 - 1120*a^6*b^4*c^4
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*d*e^6*f^3*z^4 - 1120*a^4*b^4*c^6*d^3*e^6*f*z^4 + 912*a^6*b^6*c^2*d*e^4*f^5
*z^4 + 912*a^2*b^6*c^6*d^5*e^4*f*z^4 + 872*a^5*b^6*c^3*d*e^6*f^3*z^4 + 872*
a^3*b^6*c^5*d^3*e^6*f*z^4 + 768*a^8*b^2*c^4*d*e^4*f^5*z^4 + 768*a^4*b^2*c^8
*d^5*e^4*f*z^4 - 672*a^8*b^4*c^2*d*e^2*f^7*z^4 - 672*a^2*b^4*c^8*d^7*e^2*f*
z^4 - 624*a^7*b^5*c^2*d*e^3*f^6*z^4 - 624*a^2*b^5*c^7*d^6*e^3*f*z^4 + 480*a
^5*b^8*c*d^2*e^2*f^6*z^4 + 480*a*b^8*c^5*d^6*e^2*f^2*z^4 + 316*a^4*b^7*c^3*
d*e^7*f^2*z^4 + 316*a^3*b^7*c^4*d^2*e^7*f*z^4 - 204*a^4*b^8*c^2*d*e^6*f^3*z
^4 - 204*a^2*b^8*c^4*d^3*e^6*f*z^4 + 168*a^3*b^10*c*d^3*e^2*f^5*z^4 + 168*a
*b^10*c^3*d^5*e^2*f^3*z^4 + 156*a^2*b^11*c*d^3*e^3*f^4*z^4 + 156*a*b^11*c^2
*d^4*e^3*f^3*z^4 + 128*a^9*b^2*c^3*d*e^2*f^7*z^4 + 128*a^3*b^2*c^9*d^7*e^2*
f*z^4 - 124*a^3*b^10*c*d^2*e^4*f^4*z^4 - 124*a*b^10*c^3*d^4*e^4*f^2*z^4 + 1
00*a^4*b^9*c*d^2*e^3*f^5*z^4 + 100*a*b^9*c^4*d^5*e^3*f^2*z^4 + 36*a^5*b^7*c
^2*d*e^5*f^4*z^4 + 36*a^2*b^7*c^5*d^4*e^5*f*z^4 - 24*a^3*b^9*c^2*d*e^7*f^2*
z^4 - 24*a^2*b^11*c*d^2*e^5*f^3*z^4 - 24*a^2*b^9*c^3*d^2*e^7*f*z^4 - 24*a*b
^11*c^2*d^3*e^5*f^2*z^4 - 9216*a^8*b*c^5*d^3*e*f^6*z^4 - 9216*a^5*b*c^8*d^6
*e*f^3*z^4 - 5376*a^8*b*c^5*d*e^5*f^4*z^4 - 5376*a^5*b*c^8*d^4*e^5*f*z^4 +
5120*a^9*b*c^4*d^2*e*f^7*z^4 + 5120*a^7*b*c^6*d^4*e*f^5*z^4 + 5120*a^6*b*c^
7*d^5*e*f^4*z^4 + 5120*a^4*b*c^9*d^7*e*f^2*z^4 - 4352*a^9*b*c^4*d*e^3*f^6*z
^4 - 4352*a^4*b*c^9*d^6*e^3*f*z^4 - 1792*a^7*b*c^6*d*e^7*f^2*z^4 - 1792*a^6
*b*c^7*d^2*e^7*f*z^4 - 1600*a^6*b^2*c^6*d*e^8*f*z^4 + 912*a^5*b^4*c^5*d*e^8
*f*z^4 + 768*a^9*b^3*c^2*d*e*f^8*z^4 + 768*a^2*b^3*c^9*d^8*e*f*z^4 - 720*a^
4*b^9*c*d^3*e*f^6*z^4 - 720*a*b^9*c^4*d^6*e*f^3*z^4 - 656*a^6*b^7*c*d^2*e*f
^7*z^4 - 656*a*b^7*c^6*d^7*e*f^2*z^4 - 240*a^2*b^11*c*d^4*e*f^5*z^4 - 240*a
*b^11*c^2*d^5*e*f^4*z^4 + 216*a^7*b^6*c*d*e^2*f^7*z^4 + 216*a*b^6*c^7*d^7*e
^2*f*z^4 - 204*a^4*b^6*c^4*d*e^8*f*z^4 - 144*a^5*b^8*c*d*e^4*f^5*z^4 - 144*
a*b^8*c^5*d^5*e^4*f*z^4 - 84*a*b^12*c*d^4*e^2*f^4*z^4 + 36*a^4*b^9*c*d*e^5*
f^4*z^4 + 36*a*b^9*c^4*d^4*e^5*f*z^4 + 20*a^6*b^7*c*d*e^3*f^6*z^4 + 20*a*b^
7*c^6*d^6*e^3*f*z^4 + 16*a^3*b^10*c*d*e^6*f^3*z^4 + 16*a^3*b^8*c^3*d*e^8*f*
z^4 + 16*a*b^12*c*d^3*e^4*f^3*z^4 + 16*a*b^10*c^3*d^3*e^6*f*z^4 + 48*b^11*c
^3*d^6*e*f^3*z^4 + 48*b^9*c^5*d^7*e*f^2*z^4 - 20*b^8*c^6*d^7*e^2*f*z^4 + 8*
b^10*c^4*d^5*e^4*f*z^4 - 4*b^13*c*d^4*e^3*f^3*z^4 - 4*b^11*c^3*d^4*e^5*f*z^
4 + 4*b^9*c^5*d^6*e^3*f*z^4 + 3072*a^9*c^5*d*e^4*f^5*z^4 + 3072*a^5*c^9*d^5
*e^4*f*z^4 + 2560*a^8*c^6*d*e^6*f^3*z^4 + 2560*a^6*c^8*d^3*e^6*f*z^4 + 1536
*a^10*c^4*d*e^2*f^7*z^4 + 1536*a^4*c^10*d^7*e^2*f*z^4 + 48*a^5*b^9*d^2*e*f^
7*z^4 + 48*a^3*b^11*d^3*e*f^6*z^4 - 20*a^6*b^8*d*e^2*f^7*z^4 + 8*a^4*b^10*d
*e^4*f^5*z^4 + 4*a^5*b^9*d*e^3*f^6*z^4 - 4*a^3*b^11*d*e^5*f^4*z^4 - 4*a*b^1
3*d^3*e^3*f^4*z^4 + 768*a^9*b*c^4*e^5*f^5*z^4 + 768*a^8*b*c^5*e^7*f^3*z^4 +
 256*a^10*b*c^3*e^3*f^7*z^4 - 192*a^6*b^3*c^5*e^9*f*z^4 - 68*a^7*b^6*c*e^4*
f^6*z^4 + 48*a^8*b^5*c*e^3*f^7*z^4 + 48*a^5*b^5*c^4*e^9*f*z^4 + 36*a^6*b^7*
c*e^5*f^5*z^4 - 12*a^9*b^4*c*e^2*f^8*z^4 - 4*a^4*b^9*c*e^7*f^3*z^4 - 4*a^4*
b^7*c^3*e^9*f*z^4 + 384*a^5*b^8*c*d^3*f^7*z^4 + 384*a*b^8*c^5*d^7*f^3*z^4 +
 288*a^3*b^10*c*d^4*f^6*z^4 + 288*a*b^10*c^3*d^6*f^4*z^4 + 224*a^7*b^6*c*d^
2*f^8*z^4 + 224*a*b^6*c^7*d^8*f^2*z^4 - 192*a^10*b^2*c^2*d*f^9*z^4 - 192*a^
2*b^2*c^10*d^9*f*z^4 + 768*a^5*b*c^8*d^3*e^7*z^4 + 768*a^4*b*c^9*d^5*e^5*z^
4 + 256*a^3*b*c^10*d^7*e^3*z^4 - 192*a^5*b^3*c^6*d*e^9*z^4 - 68*a*b^6*c^7*d
^6*e^4*z^4 + 48*a^4*b^5*c^5*d*e^9*z^4 + 48*a*b^5*c^8*d^7*e^3*z^4 + 36*a*b^7
*c^6*d^5*e^5*z^4 - 12*a*b^4*c^9*d^8*e^2*z^4 - 4*a^3*b^7*c^4*d*e^9*z^4 - 4*a
*b^9*c^4*d^3*e^7*z^4 + 16*b^13*c*d^5*e*f^4*z^4 + 16*b^7*c^7*d^8*e*f*z^4 + 7
68*a^7*c^7*d*e^8*f*z^4 + 16*a^7*b^7*d*e*f^8*z^4 + 16*a*b^13*d^4*e*f^5*z^4 +
 256*a^7*b*c^6*e^9*f*z^4 + 80*a*b^12*c*d^5*f^5*z^4 + 48*a^9*b^4*c*d*f^9*z^4
 + 48*a*b^4*c^9*d^9*f*z^4 + 256*a^6*b*c^7*d*e^9*z^4 - 42*b^10*c^4*d^6*e^2*f
^2*z^4 - 20*b^12*c^2*d^5*e^2*f^3*z^4 + 6*b^12*c^2*d^4*e^4*f^2*z^4 + 4*b^11*
c^3*d^5*e^3*f^2*z^4 - 24960*a^7*c^7*d^4*e^2*f^4*z^4 + 18944*a^8*c^6*d^3*e^2
*f^5*z^4 + 18944*a^6*c^8*d^5*e^2*f^3*z^4 + 14336*a^7*c^7*d^3*e^4*f^3*z^4 -
9984*a^8*c^6*d^2*e^4*f^4*z^4 - 9984*a^6*c^8*d^4*e^4*f^2*z^4 - 7936*a^9*c^5*
d^2*e^2*f^6*z^4 - 7936*a^5*c^9*d^6*e^2*f^2*z^4 - 4352*a^7*c^7*d^2*e^6*f^2*z
^4 - 42*a^4*b^10*d^2*e^2*f^6*z^4 - 20*a^2*b^12*d^3*e^2*f^5*z^4 + 6*a^2*b^12
*d^2*e^4*f^4*z^4 + 4*a^3*b^11*d^2*e^3*f^5*z^4 - 480*a^8*b^2*c^4*e^6*f^4*z^4
 + 440*a^7*b^4*c^3*e^6*f^4*z^4 - 320*a^8*b^3*c^3*e^5*f^5*z^4 - 320*a^7*b^3*
```

c^4*e^7*f^3*z^4 + 240*a^8*b^4*c^2*e^4*f^6*z^4 + 240*a^6*b^4*c^4*e^8*f^2*z^4
- 192*a^9*b^3*c^2*e^3*f^7*z^4 - 192*a^9*b^2*c^3*e^4*f^6*z^4 - 192*a^7*b^2*
c^5*e^8*f^2*z^4 - 90*a^6*b^6*c^2*e^6*f^4*z^4 - 68*a^5*b^6*c^3*e^8*f^2*z^4 +
48*a^10*b^2*c^2*e^2*f^8*z^4 - 48*a^7*b^5*c^2*e^5*f^5*z^4 - 48*a^6*b^5*c^3*
e^7*f^3*z^4 + 36*a^5*b^7*c^2*e^7*f^3*z^4 + 6*a^4*b^8*c^2*e^8*f^2*z^4 - 3392
0*a^6*b^2*c^6*d^5*f^5*z^4 + 27936*a^5*b^4*c^5*d^5*f^5*z^4 + 26112*a^7*b^2*c
^5*d^4*f^6*z^4 + 26112*a^5*b^2*c^7*d^6*f^4*z^4 - 20352*a^6*b^4*c^4*d^4*f^6*
z^4 - 20352*a^4*b^4*c^6*d^6*f^4*z^4 - 13080*a^4*b^6*c^4*d^5*f^5*z^4 - 11520
*a^8*b^2*c^4*d^3*f^7*z^4 - 11520*a^4*b^2*c^8*d^7*f^3*z^4 + 8736*a^5*b^6*c^3
*d^4*f^6*z^4 + 8736*a^3*b^6*c^5*d^6*f^4*z^4 + 7488*a^7*b^4*c^3*d^3*f^7*z^4
+ 7488*a^3*b^4*c^7*d^7*f^3*z^4 + 3840*a^3*b^8*c^3*d^5*f^5*z^4 + 2560*a^9*b^
2*c^3*d^2*f^8*z^4 + 2560*a^3*b^2*c^9*d^8*f^2*z^4 - 2416*a^6*b^6*c^2*d^3*f^7
*z^4 - 2416*a^2*b^6*c^6*d^7*f^3*z^4 - 2160*a^4*b^8*c^2*d^4*f^6*z^4 - 2160*a
^2*b^8*c^4*d^6*f^4*z^4 - 1152*a^8*b^4*c^2*d^2*f^8*z^4 - 1152*a^2*b^4*c^8*d^
8*f^2*z^4 - 720*a^2*b^10*c^2*d^5*f^5*z^4 - 480*a^4*b^2*c^8*d^4*e^6*z^4 + 44
0*a^3*b^4*c^7*d^4*e^6*z^4 - 320*a^4*b^3*c^7*d^3*e^7*z^4 - 320*a^3*b^3*c^8*d
^5*e^5*z^4 + 240*a^4*b^4*c^6*d^2*e^8*z^4 + 240*a^2*b^4*c^8*d^6*e^4*z^4 - 19
2*a^5*b^2*c^7*d^2*e^8*z^4 - 192*a^3*b^2*c^9*d^6*e^4*z^4 - 192*a^2*b^3*c^9*d
^7*e^3*z^4 - 90*a^2*b^6*c^6*d^4*e^6*z^4 - 68*a^3*b^6*c^5*d^2*e^8*z^4 - 48*a
^3*b^5*c^6*d^3*e^7*z^4 - 48*a^2*b^5*c^7*d^5*e^5*z^4 + 48*a^2*b^2*c^10*d^8*e
^2*z^4 + 36*a^2*b^7*c^5*d^3*e^7*z^4 + 6*a^2*b^8*c^4*d^2*e^8*z^4 - 4*b^6*c^8
*d^9*f*z^4 + 256*a^11*c^3*d*f^9*z^4 + 256*a^3*c^11*d^9*f*z^4 - 4*a^8*b^6*d*
f^9*z^4 - 384*a^9*c^5*e^6*f^4*z^4 - 256*a^10*c^4*e^4*f^6*z^4 - 256*a^8*c^6*
e^8*f^2*z^4 - 64*a^11*c^3*e^2*f^8*z^4 - 24*b^10*c^4*d^7*f^3*z^4 - 16*b^12*c
^2*d^6*f^4*z^4 - 16*b^8*c^6*d^8*f^2*z^4 + 17920*a^7*c^7*d^5*f^5*z^4 - 14336
*a^8*c^6*d^4*f^6*z^4 - 14336*a^6*c^8*d^6*f^4*z^4 + 7168*a^9*c^5*d^3*f^7*z^4
+ 7168*a^5*c^9*d^7*f^3*z^4 - 2048*a^10*c^4*d^2*f^8*z^4 - 2048*a^4*c^10*d^8
*f^2*z^4 + 6*b^8*c^6*d^6*e^4*z^4 + 6*a^6*b^8*e^4*f^6*z^4 - 4*b^9*c^5*d^5*e^
5*z^4 - 4*b^7*c^7*d^7*e^3*z^4 - 4*a^7*b^7*e^3*f^7*z^4 - 4*a^5*b^9*e^5*f^5*z
^4 - 384*a^5*c^9*d^4*e^6*z^4 - 256*a^6*c^8*d^2*e^8*z^4 - 256*a^4*c^10*d^6*e
^4*z^4 - 64*a^3*c^11*d^8*e^2*z^4 - 24*a^4*b^10*d^3*f^7*z^4 - 16*a^6*b^8*d^2
*f^8*z^4 - 16*a^2*b^12*d^4*f^6*z^4 + 48*a^6*b^2*c^6*e^10*z^4 - 12*a^5*b^4*c
^5*e^10*z^4 - 4*b^14*d^5*f^5*z^4 - 64*a^7*c^7*e^10*z^4 + b^14*d^4*e^2*f^4*z
^4 + b^10*c^4*d^4*e^6*z^4 + b^6*c^8*d^8*e^2*z^4 + a^8*b^6*e^2*f^8*z^4 + a^4
*b^10*e^6*f^4*z^4 + a^4*b^6*c^4*e^10*z^4 - 4820*A*B*a^4*b*c^5*d^2*e^2*f^4*z
^2 + 2976*A*B*a^3*b*c^6*d^3*e^2*f^3*z^2 - 2328*A*B*a^3*b*c^6*d^2*e^4*f^2*z^
2 + 1848*A*B*a^2*b^4*c^4*d^3*e*f^4*z^2 - 1768*A*B*a^3*b^4*c^3*d^2*e*f^5*z^2
+ 1528*A*B*a^4*b^2*c^4*d^2*e*f^5*z^2 - 1136*A*B*a^3*b^2*c^5*d^3*e*f^4*z^2
- 974*A*B*a^4*b^3*c^3*d*e^2*f^5*z^2 + 692*A*B*a^2*b*c^7*d^4*e^2*f^2*z^2 + 5
88*A*B*a*b^6*c^3*d^2*e^3*f^3*z^2 - 580*A*B*a^3*b^3*c^4*d*e^4*f^3*z^2 + 488*
A*B*a^3*b^4*c^3*d*e^3*f^4*z^2 - 444*A*B*a^2*b^2*c^6*d^2*e^5*f*z^2 - 412*A*B
*a*b^5*c^4*d^2*e^4*f^2*z^2 + 366*A*B*a^2*b^6*c^2*d^2*e*f^5*z^2 - 352*A*B*a^
2*b^2*c^6*d^4*e*f^3*z^2 + 326*A*B*a^2*b^4*c^4*d*e^5*f^2*z^2 + 324*A*B*a*b^5
*c^4*d^3*e^2*f^3*z^2 - 302*A*B*a*b^3*c^6*d^4*e^2*f^2*z^2 - 296*A*B*a*b^7*c^
2*d^2*e^2*f^4*z^2 + 122*A*B*a^4*b^2*c^4*d*e^3*f^4*z^2 - 122*A*B*a^2*b^6*c^2
*d*e^3*f^4*z^2 - 84*A*B*a^3*b^2*c^5*d*e^5*f^2*z^2 + 72*A*B*a*b^4*c^5*d^3*e^
3*f^2*z^2 - 64*A*B*a^2*b^5*c^3*d*e^4*f^3*z^2 + 60*A*B*a^3*b^5*c^2*d*e^2*f^5
*z^2 + 1312*A*B*a^5*b*c^4*d*e^2*f^5*z^2 + 1040*A*B*a^4*b*c^5*d*e^4*f^3*z^2
- 500*A*B*a*b^6*c^3*d^3*e*f^4*z^2 - 376*A*B*a*b^2*c^7*d^5*e*f^2*z^2 + 276*A
*B*a^4*b^4*c^2*d*e*f^6*z^2 - 262*A*B*a^2*b^3*c^5*d*e^6*f*z^2 + 238*A*B*a*b^
2*c^7*d^4*e^3*f*z^2 + 232*A*B*a^5*b^2*c^3*d*e*f^6*z^2 - 176*A*B*a^2*b*c^7*d
^3*e^4*f*z^2 - 120*A*B*a*b^6*c^3*d*e^5*f^2*z^2 - 108*A*B*a*b^4*c^5*d^4*e*f^
3*z^2 + 68*A*B*a*b^7*c^2*d*e^4*f^3*z^2 + 68*A*B*a*b^4*c^5*d^2*e^5*f*z^2 + 4
6*A*B*a^2*b^7*c*d*e^2*f^5*z^2 - 36*A*B*a*b^3*c^6*d^3*e^4*f*z^2 - 1932*A*B*a
^2*b^3*c^5*d^3*e^2*f^3*z^2 - 1818*A*B*a^2*b^4*c^4*d^2*e^3*f^3*z^2 + 1620*A*
B*a^3*b^3*c^4*d^2*e^2*f^4*z^2 + 1560*A*B*a^2*b^3*c^5*d^2*e^4*f^2*z^2 + 1244
*A*B*a^3*b^2*c^5*d^2*e^3*f^3*z^2 + 820*A*B*a^2*b^2*c^6*d^3*e^3*f^2*z^2 + 48
0*A*B*a^2*b^5*c^3*d^2*e^2*f^4*z^2 + 352*A*B*a^3*b*c^6*d*e^6*f*z^2 - 108*A*B
*a^3*b^6*c*d*e*f^6*z^2 + 82*A*B*a*b^5*c^4*d*e^6*f*z^2 - 64*A*B*a*b*c^8*d^5*

e^2*f*z^2 + 16*A*B*a*b^8*c*d^2*e*f^5*z^2 - 4*A*B*a*b^8*c*d*e^3*f^4*z^2 + 16
*B^2*a*b*c^8*d^6*e*f*z^2 + 56*A*B*b^2*c^8*d^6*e*f*z^2 - 8*A*B*b^9*c*d*e^4*f
^3*z^2 - 8*A*B*b^7*c^3*d*e^6*f*z^2 - 800*A*B*a^6*c^4*d*e*f^6*z^2 + 10*A*B*a
^2*b^8*d*e*f^6*z^2 - 6*A*B*a*b^9*d*e^2*f^5*z^2 - 12*A*B*a^5*b^4*c*e*f^7*z^2
 + 912*A*B*a^6*b*c^3*d*f^7*z^2 + 192*A*B*a^4*b^5*c*d*f^7*z^2 + 192*A*B*a*b*
c^8*d^6*f^2*z^2 - 20*A*B*a*b^4*c^5*d*e^7*z^2 + 4*A*B*a*b*c^8*d^4*e^4*z^2 +
2144*B^2*a^4*b*c^5*d^3*e*f^4*z^2 - 1120*B^2*a^3*b*c^6*d^4*e*f^3*z^2 - 688*B
^2*a^5*b*c^4*d^2*e*f^5*z^2 - 256*B^2*a^3*b*c^6*d^2*e^5*f*z^2 + 152*B^2*a*b^
3*c^6*d^5*e*f^2*z^2 + 120*B^2*a^5*b^3*c^2*d*e*f^6*z^2 - 116*B^2*a^5*b*c^4*d
*e^3*f^4*z^2 + 110*B^2*a*b^7*c^2*d^3*e*f^4*z^2 - 80*B^2*a^2*b*c^7*d^5*e*f^2
*z^2 - 72*B^2*a*b^5*c^4*d^4*e*f^3*z^2 - 48*B^2*a^4*b*c^5*d*e^5*f^2*z^2 - 46
*B^2*a*b^3*c^6*d^4*e^3*f*z^2 - 44*B^2*a*b^4*c^5*d^3*e^4*f*z^2 - 34*B^2*a*b^
5*c^4*d^2*e^5*f*z^2 + 20*B^2*a^2*b*c^7*d^4*e^3*f*z^2 - 10*B^2*a^3*b^6*c*d*e
^2*f^5*z^2 - 10*B^2*a^2*b^7*c*d^2*e*f^5*z^2 - 10*B^2*a*b^2*c^7*d^5*e^2*f*z^
2 - 7*B^2*a^2*b^4*c^4*d*e^6*f*z^2 - 6*B^2*a^3*b^2*c^5*d*e^6*f*z^2 + 4*B^2*a
*b^8*c*d^2*e^2*f^4*z^2 - 2*B^2*a^2*b^7*c*d*e^3*f^4*z^2 + 3196*A^2*a^4*b*c^5
*d*e^3*f^4*z^2 - 3184*A^2*a^4*b*c^5*d^2*e*f^5*z^2 + 1568*A^2*a^3*b*c^6*d^3*
e*f^4*z^2 + 1504*A^2*a^3*b*c^6*d*e^5*f^2*z^2 - 656*A^2*a^4*b^3*c^3*d*e*f^6*
z^2 - 400*A^2*a*b^6*c^3*d*e^4*f^3*z^2 + 314*A^2*a*b^5*c^4*d*e^5*f^2*z^2 - 2
64*A^2*a^3*b^5*c^2*d*e*f^6*z^2 + 240*A^2*a^2*b^2*c^6*d*e^6*f*z^2 - 224*A^2*
a^2*b*c^7*d^4*e*f^3*z^2 + 216*A^2*a*b^5*c^4*d^3*e*f^4*z^2 - 192*A^2*a^2*b*c
^7*d^2*e^5*f*z^2 + 178*A^2*a*b^7*c^2*d*e^3*f^4*z^2 - 154*A^2*a*b^7*c^2*d^2*
e*f^5*z^2 + 128*A^2*a*b^3*c^6*d^4*e*f^3*z^2 + 106*A^2*a*b^3*c^6*d^2*e^5*f*z
^2 - 12*A^2*a*b^2*c^7*d^3*e^4*f*z^2 - 58*A*B*b^8*c^2*d^2*e^3*f^3*z^2 + 40*A
*B*b^7*c^3*d^2*e^4*f^2*z^2 - 28*A*B*b^7*c^3*d^3*e^2*f^3*z^2 - 24*A*B*b^5*c^
5*d^4*e^2*f^2*z^2 - 20*A*B*b^6*c^4*d^3*e^3*f^2*z^2 + 2768*A*B*a^4*c^6*d^2*e
^3*f^3*z^2 - 1712*A*B*a^3*c^7*d^3*e^3*f^2*z^2 - 156*A*B*a^4*b^2*c^4*e^5*f^3
*z^2 + 146*A*B*a^4*b^3*c^3*e^4*f^4*z^2 - 106*A*B*a^5*b^2*c^3*e^3*f^5*z^2 +
90*A*B*a^5*b^3*c^2*e^2*f^6*z^2 + 38*A*B*a^3*b^3*c^4*e^6*f^2*z^2 - 36*A*B*a^
3*b^5*c^2*e^4*f^4*z^2 + 16*A*B*a^3*b^4*c^3*e^5*f^3*z^2 - 9*A*B*a^4*b^4*c^2*
e^3*f^5*z^2 - 8*A*B*a^2*b^5*c^3*e^6*f^2*z^2 + 2*A*B*a^2*b^6*c^2*e^5*f^3*z^2
 + 920*A*B*a^4*b^3*c^3*d^2*f^6*z^2 - 480*A*B*a^2*b^5*c^3*d^3*f^5*z^2 - 336*
A*B*a^2*b^3*c^5*d^4*f^4*z^2 - 272*A*B*a^3*b^3*c^4*d^3*f^5*z^2 + 240*A*B*a^3
*b^5*c^2*d^2*f^6*z^2 - 32*A*B*a*c^9*d^6*e*f*z^2 - 792*B^2*a^2*b^3*c^5*d^3*e
^3*f^2*z^2 + 714*B^2*a^2*b^4*c^4*d^3*e^2*f^3*z^2 - 572*B^2*a^3*b^2*c^5*d^3*
e^2*f^3*z^2 - 475*B^2*a^2*b^2*c^6*d^4*e^2*f^2*z^2 + 265*B^2*a^4*b^2*c^4*d^2
*e^2*f^4*z^2 + 260*B^2*a^3*b^3*c^4*d^2*e^3*f^3*z^2 - 212*B^2*a^3*b^4*c^3*d^
2*e^2*f^4*z^2 + 180*B^2*a^3*b^2*c^5*d^2*e^4*f^2*z^2 - 158*B^2*a^2*b^4*c^4*d
^2*e^4*f^2*z^2 + 47*B^2*a^2*b^6*c^2*d^2*e^2*f^4*z^2 + 16*B^2*a^2*b^5*c^3*d^
2*e^3*f^3*z^2 + 2752*A^2*a^3*b^2*c^5*d^2*e^2*f^4*z^2 - 2148*A^2*a^2*b^4*c^4
*d^2*e^2*f^4*z^2 + 2064*A^2*a^2*b^3*c^5*d^2*e^3*f^3*z^2 - 424*A^2*a^2*b^2*c
^6*d^3*e^2*f^3*z^2 - 198*A^2*a^2*b^2*c^6*d^2*e^4*f^2*z^2 - 272*B^2*a^6*b*c^
3*d*e*f^6*z^2 - 24*B^2*a^4*b^5*c*d*e*f^6*z^2 + 1808*A^2*a^5*b*c^4*d*e*f^6*z
^2 - 244*A^2*a*b*c^8*d^4*e^3*f*z^2 + 208*A^2*a*b*c^8*d^5*e*f^2*z^2 + 134*A^
2*a^2*b^7*c*d*e*f^6*z^2 - 76*A^2*a*b^4*c^5*d*e^6*f*z^2 + 4*A^2*a*b^8*c*d*e^
2*f^5*z^2 + 148*A*B*b^4*c^6*d^5*e*f^2*z^2 + 65*A*B*b^6*c^4*d^4*e*f^3*z^2 +
46*A*B*b^8*c^2*d^3*e*f^4*z^2 - 38*A*B*b^3*c^7*d^5*e^2*f*z^2 + 34*A*B*b^9*c*
d^2*e^2*f^4*z^2 - 29*A*B*b^4*c^6*d^4*e^3*f*z^2 + 20*A*B*b^5*c^5*d^3*e^4*f*z
^2 + 12*A*B*b^8*c^2*d*e^5*f^2*z^2 - 7*A*B*b^6*c^4*d^2*e^5*f*z^2 - 2880*A*B*
a^4*c^6*d^3*e*f^4*z^2 + 2784*A*B*a^5*c^5*d^2*e*f^5*z^2 - 1112*A*B*a^5*c^5*d
*e^3*f^4*z^2 + 896*A*B*a^3*c^7*d^4*e*f^3*z^2 + 848*A*B*a^3*c^7*d^2*e^5*f*z^
2 - 560*A*B*a^4*c^6*d*e^5*f^2*z^2 + 96*A*B*a^2*c^8*d^5*e*f^2*z^2 - 88*A*B*a
^2*c^8*d^4*e^3*f*z^2 - 100*A*B*a^6*b*c^3*e^2*f^6*z^2 - 76*A*B*a^5*b*c^4*e^4
*f^4*z^2 + 48*A*B*a^6*b^2*c^2*e*f^7*z^2 - 42*A*B*a^3*b^2*c^5*e^7*f*z^2 + 36
*A*B*a^4*b*c^5*e^6*f^2*z^2 - 24*A*B*a^4*b^5*c*e^2*f^6*z^2 + 10*A*B*a^3*b^6*
c*e^3*f^5*z^2 + 7*A*B*a^2*b^4*c^4*e^7*f*z^2 + 2*A*B*a^2*b^7*c*e^4*f^4*z^2 -
 2496*A*B*a^5*b*c^4*d^2*f^6*z^2 + 1872*A*B*a^4*b*c^5*d^3*f^5*z^2 - 744*A*B*
a^5*b^3*c^2*d*f^7*z^2 - 720*A*B*a^2*b*c^7*d^5*f^3*z^2 + 504*A*B*a*b^3*c^6*d
^5*f^3*z^2 + 256*A*B*a^3*b*c^6*d^4*f^4*z^2 + 168*A*B*a*b^7*c^2*d^3*f^5*z^2

- 144*A*B*a^2*b^7*c*d^2*f^6*z^2 + 144*A*B*a*b^5*c^4*d^4*f^4*z^2 + 66*A*B*a^
2*b^2*c^6*d*e^7*z^2 - 36*A*B*a*b^2*c^7*d^3*e^5*z^2 + 20*A*B*a*b^3*c^6*d^2*e
^6*z^2 + 12*A*B*a^2*b*c^7*d^2*e^6*z^2 + 1208*B^2*a^3*b*c^6*d^3*e^3*f^2*z^2
- 848*B^2*a^3*b^3*c^4*d^3*e*f^4*z^2 + 672*B^2*a^2*b^3*c^5*d^4*e*f^3*z^2 - 6
32*B^2*a^4*b*c^5*d^2*e^3*f^3*z^2 + 432*B^2*a^4*b^3*c^3*d^2*e*f^5*z^2 + 276*
B^2*a^2*b^2*c^6*d^3*e^4*f*z^2 - 196*B^2*a*b^6*c^3*d^3*e^2*f^3*z^2 - 168*B^2
*a^2*b^5*c^3*d^3*e*f^4*z^2 + 154*B^2*a^2*b^3*c^5*d^2*e^5*f*z^2 + 148*B^2*a*
b^5*c^4*d^3*e^3*f^2*z^2 + 96*B^2*a*b^4*c^5*d^4*e^2*f^2*z^2 - 72*B^2*a^3*b^5
*c^2*d^2*e*f^5*z^2 + 70*B^2*a^5*b^2*c^3*d*e^2*f^5*z^2 - 60*B^2*a^4*b^3*c^3*
d*e^3*f^4*z^2 + 52*B^2*a*b^6*c^3*d^2*e^4*f^2*z^2 + 36*B^2*a^4*b^2*c^4*d*e^4
*f^3*z^2 - 32*B^2*a*b^7*c^2*d^2*e^3*f^3*z^2 + 24*B^2*a^3*b^5*c^2*d*e^3*f^4*
z^2 + 15*B^2*a^4*b^4*c^2*d*e^2*f^5*z^2 - 8*B^2*a^3*b^4*c^3*d*e^4*f^3*z^2 +
8*B^2*a^2*b^5*c^3*d*e^5*f^2*z^2 - 2*B^2*a^3*b^3*c^4*d*e^5*f^2*z^2 - 2*B^2*a
^2*b^6*c^2*d*e^4*f^3*z^2 - 3176*A^2*a^3*b*c^6*d^2*e^3*f^3*z^2 - 2252*A^2*a^
4*b^2*c^4*d*e^2*f^5*z^2 + 1952*A^2*a^3*b^4*c^3*d*e^2*f^5*z^2 - 1496*A^2*a^3
*b^3*c^4*d*e^3*f^4*z^2 + 1378*A^2*a^2*b^4*c^4*d*e^4*f^3*z^2 + 1184*A^2*a^3*
b^3*c^4*d^2*e*f^5*z^2 - 1166*A^2*a^2*b^3*c^5*d*e^5*f^2*z^2 - 1164*A^2*a^3*b
^2*c^5*d*e^4*f^3*z^2 - 1152*A^2*a^2*b^3*c^5*d^3*e*f^4*z^2 + 578*A^2*a*b^6*c
^3*d^2*e^2*f^4*z^2 - 548*A^2*a*b^5*c^4*d^2*e^3*f^3*z^2 + 440*A^2*a*b^2*c^7*
d^4*e^2*f^2*z^2 - 412*A^2*a^2*b^6*c^2*d*e^2*f^5*z^2 - 360*A^2*a*b^3*c^6*d^3
*e^3*f^2*z^2 + 312*A^2*a*b^4*c^5*d^3*e^2*f^3*z^2 + 248*A^2*a^2*b*c^7*d^3*e^
3*f^2*z^2 - 224*A^2*a^2*b^5*c^3*d*e^3*f^4*z^2 + 216*A^2*a^2*b^5*c^3*d^2*e*f
^5*z^2 + 52*A^2*a*b^4*c^5*d^2*e^4*f^2*z^2 - 16*B^2*b^3*c^7*d^6*e*f*z^2 - 14
*B^2*b^9*c*d^3*e*f^4*z^2 + 32*B^2*a^4*c^6*d*e^6*f*z^2 - 20*A^2*b^9*c*d*e^3*
f^4*z^2 + 18*A^2*b^9*c*d^2*e*f^5*z^2 + 8*A^2*b^6*c^4*d*e^6*f*z^2 - 360*A^2*
a^3*c^7*d*e^6*f*z^2 + 136*A^2*a*c^9*d^5*e^2*f*z^2 + 2*B^2*a^3*b^7*d*e*f^6*z
^2 + 2*B^2*a*b^9*d^2*e*f^5*z^2 + 12*B^2*a^4*b*c^5*e^7*f*z^2 - 204*A^2*a^3*b
*c^6*e^7*f*z^2 - 128*A^2*a^6*b*c^3*e*f^7*z^2 - 48*A^2*a*b^5*c^4*e^7*f*z^2 -
36*B^2*a^5*b^4*c*d*f^7*z^2 - 24*A^2*a^4*b^5*c*e*f^7*z^2 - 16*B^2*a*b^8*c*d
^3*f^5*z^2 - 164*A^2*a^3*b^6*c*d*f^7*z^2 - 16*A^2*a*b^8*c*d^2*f^6*z^2 + 4*B
^2*a^3*b*c^6*d*e^7*z^2 - 4*B^2*a*b*c^8*d^5*e^3*z^2 + 48*A^2*a*b*c^8*d^3*e^5
*z^2 + 36*A^2*a^2*b*c^7*d*e^7*z^2 - 6*A^2*a*b^3*c^6*d*e^7*z^2 + 136*A*B*a^6
*c^4*e^3*f^5*z^2 - 96*A*B*b^5*c^5*d^5*f^3*z^2 + 80*A*B*a^5*c^5*e^5*f^3*z^2
- 72*A*B*b^3*c^7*d^6*f^2*z^2 - 24*A*B*b^7*c^3*d^4*f^4*z^2 + 14*A*B*b^3*c^7*
d^4*e^4*z^2 - 14*A*B*b^2*c^8*d^5*e^3*z^2 - 2*A*B*b^5*c^5*d^2*e^6*z^2 - 2*A*
B*b^4*c^6*d^3*e^5*z^2 + 2*A*B*a^3*b^7*e^2*f^6*z^2 - A*B*a^2*b^8*e^3*f^5*z^2
+ 16*A*B*a^2*c^8*d^3*e^5*z^2 - 2*A*B*a^2*b^3*c^5*e^8*z^2 + 22*B^2*b^8*c^2*
d^3*e^2*f^3*z^2 - 12*B^2*b^7*c^3*d^3*e^3*f^2*z^2 + 12*B^2*b^6*c^4*d^4*e^2*f
^2*z^2 - 6*B^2*b^8*c^2*d^2*e^4*f^2*z^2 - 864*B^2*a^4*c^6*d^3*e^2*f^3*z^2 +
496*B^2*a^3*c^7*d^4*e^2*f^2*z^2 + 224*B^2*a^5*c^5*d^2*e^2*f^4*z^2 + 136*B^2
*a^4*c^6*d^2*e^4*f^2*z^2 - 53*A^2*b^8*c^2*d^2*e^2*f^4*z^2 + 52*A^2*b^7*c^3*
d^2*e^3*f^3*z^2 + 52*A^2*b^5*c^5*d^3*e^3*f^2*z^2 - 36*A^2*b^6*c^4*d^3*e^2*f
^3*z^2 - 12*A^2*b^4*c^6*d^4*e^2*f^2*z^2 - 9*A^2*b^6*c^4*d^2*e^4*f^2*z^2 + 8
36*A^2*a^4*c^6*d^2*e^2*f^4*z^2 - 668*A^2*a^2*c^8*d^4*e^2*f^2*z^2 + 656*A^2*
a^3*c^7*d^2*e^4*f^2*z^2 + 368*A^2*a^3*c^7*d^3*e^2*f^3*z^2 - 45*B^2*a^6*b^2*
c^2*e^2*f^6*z^2 - 18*B^2*a^5*b^2*c^3*e^4*f^4*z^2 - 9*B^2*a^4*b^2*c^4*e^6*f^
2*z^2 - 6*B^2*a^5*b^3*c^2*e^3*f^5*z^2 + 3*B^2*a^4*b^4*c^2*e^4*f^4*z^2 - 2*B
^2*a^4*b^3*c^3*e^5*f^3*z^2 - 580*B^2*a^4*b^2*c^4*d^3*f^5*z^2 + 536*B^2*a^3*
b^4*c^3*d^3*f^5*z^2 + 471*A^2*a^4*b^2*c^4*e^4*f^4*z^2 - 436*A^2*a^3*b^4*c^3
*e^4*f^4*z^2 - 348*B^2*a^4*b^4*c^2*d^2*f^6*z^2 + 316*B^2*a^2*b^2*c^6*d^5*f^
3*z^2 + 310*A^2*a^3*b^3*c^4*e^5*f^3*z^2 + 232*A^2*a^5*b^2*c^3*e^2*f^6*z^2 -
229*A^2*a^2*b^4*c^4*e^6*f^2*z^2 - 216*A^2*a^4*b^4*c^2*e^2*f^6*z^2 + 204*A^
2*a^4*b^3*c^3*e^3*f^5*z^2 + 200*B^2*a^5*b^2*c^3*d^2*f^6*z^2 + 150*A^2*a^3*b
^2*c^5*e^6*f^2*z^2 - 120*B^2*a^2*b^4*c^4*d^4*f^4*z^2 + 91*A^2*a^2*b^6*c^2*e
^4*f^4*z^2 + 72*A^2*a^3*b^5*c^2*e^3*f^5*z^2 - 66*B^2*a^2*b^6*c^2*d^3*f^5*z^
2 + 44*A^2*a^2*b^5*c^3*e^5*f^3*z^2 - 16*B^2*a^3*b^2*c^5*d^4*f^4*z^2 + 1952*
A^2*a^4*b^2*c^4*d^2*f^6*z^2 - 1792*A^2*a^3*b^2*c^5*d^3*f^5*z^2 - 1272*A^2*a
^3*b^4*c^3*d^2*f^6*z^2 + 976*A^2*a^2*b^2*c^6*d^4*f^4*z^2 + 960*A^2*a^2*b^4*
c^4*d^3*f^5*z^2 + 282*A^2*a^2*b^6*c^2*d^2*f^6*z^2 - 45*B^2*a^2*b^2*c^6*d^2*

e^6*z^2 - 48*A^2*b*c^9*d^6*e*f*z^2 - 14*A^2*a*b^9*d*e*f^6*z^2 - 7*A*B*b^10*
d^2*e*f^5*z^2 + 2*A*B*b^10*d*e^3*f^4*z^2 - 64*A*B*a^7*c^3*e*f^7*z^2 - 16*A*
B*b^9*c*d^3*f^5*z^2 + 8*A*B*a^4*c^6*e^7*f*z^2 + 4*A*B*b*c^9*d^6*e^2*z^2 + 2
*A*B*b^6*c^4*d*e^7*z^2 - 120*A*B*a^3*c^7*d*e^7*z^2 - 16*A*B*a^3*b^7*d*f^7*z
^2 + 16*A*B*a*b^9*d^2*f^6*z^2 + 8*A*B*a*c^9*d^5*e^3*z^2 + 12*A*B*a^3*b*c^6*
e^8*z^2 - 48*B^2*b^5*c^5*d^5*e*f^2*z^2 + 15*B^2*b^4*c^6*d^5*e^2*f*z^2 - 14*
B^2*b^7*c^3*d^4*e*f^3*z^2 + 4*B^2*b^9*c*d^2*e^3*f^3*z^2 + 4*B^2*b^7*c^3*d^2
*e^5*f*z^2 + 4*B^2*b^5*c^5*d^4*e^3*f*z^2 - B^2*b^6*c^4*d^3*e^4*f*z^2 - 336*
B^2*a^3*c^7*d^3*e^4*f*z^2 + 112*B^2*a^5*c^5*d*e^4*f^3*z^2 - 112*A^2*b^3*c^7
*d^5*e*f^2*z^2 + 80*B^2*a^6*c^4*d*e^2*f^5*z^2 - 48*A^2*b^5*c^5*d^4*e*f^3*z^
2 + 36*A^2*b^8*c^2*d*e^4*f^3*z^2 + 36*A^2*b^3*c^7*d^4*e^3*f*z^2 - 28*A^2*b^
7*c^3*d*e^5*f^2*z^2 + 20*A^2*b^2*c^8*d^5*e^2*f*z^2 + 16*B^2*a^2*c^8*d^5*e^2
*f*z^2 - 14*A^2*b^7*c^3*d^3*e*f^4*z^2 - 14*A^2*b^4*c^6*d^3*e^4*f*z^2 - 10*A
^2*b^5*c^5*d^2*e^5*f*z^2 - 1008*A^2*a^4*c^6*d*e^4*f^3*z^2 - 760*A^2*a^5*c^5
*d*e^2*f^5*z^2 + 272*A^2*a^2*c^8*d^3*e^4*f*z^2 + 48*B^2*a^5*b*c^4*e^5*f^3*z
^2 + 36*B^2*a^6*b*c^3*e^3*f^5*z^2 + 12*B^2*a^5*b^4*c*e^2*f^6*z^2 - 624*A^2*
a^4*b*c^5*e^5*f^3*z^2 - 548*A^2*a^5*b*c^4*e^3*f^5*z^2 + 182*A^2*a^2*b^3*c^5
*e^7*f*z^2 - 180*B^2*a*b^4*c^5*d^5*f^3*z^2 + 132*B^2*a^6*b^2*c^2*d*f^7*z^2
+ 108*B^2*a^3*b^6*c*d^2*f^6*z^2 + 96*A^2*a^5*b^3*c^2*e*f^7*z^2 + 68*A^2*a*b
^6*c^3*e^6*f^2*z^2 + 58*A^2*a^3*b^6*c*e^2*f^6*z^2 - 56*B^2*a*b^2*c^7*d^6*f^
2*z^2 - 38*A^2*a^2*b^7*c*e^3*f^5*z^2 - 36*A^2*a*b^7*c^2*e^5*f^3*z^2 + 20*B^
2*a*b^6*c^3*d^4*f^4*z^2 - 736*A^2*a^5*b^2*c^3*d*f^7*z^2 + 624*A^2*a^4*b^4*c
^2*d*f^7*z^2 - 416*A^2*a*b^2*c^7*d^5*f^3*z^2 - 276*A^2*a*b^4*c^5*d^4*f^4*z^
2 - 196*A^2*a*b^6*c^3*d^3*f^5*z^2 + 8*B^2*a*b^4*c^5*d^2*e^6*z^2 + 6*B^2*a*b
^2*c^7*d^4*e^4*z^2 + 2*B^2*a^2*b^3*c^5*d*e^7*z^2 + 2*B^2*a*b^3*c^6*d^3*e^5*
z^2 - 18*A^2*a*b^2*c^7*d^2*e^6*z^2 - 16*A*B*b*c^9*d^7*f*z^2 - B^2*b^10*d^2*
e^2*f^4*z^2 + 48*B^2*a^7*c^3*e^2*f^6*z^2 - 36*B^2*a^6*c^4*e^4*f^4*z^2 + 31*
B^2*b^6*c^4*d^5*f^3*z^2 - 24*B^2*a^5*c^5*e^6*f^2*z^2 + 20*B^2*b^4*c^6*d^6*f
^2*z^2 - 6*A^2*b^8*c^2*e^6*f^2*z^2 + 2*B^2*b^8*c^2*d^4*f^4*z^2 - 768*B^2*a^
5*c^5*d^3*f^5*z^2 + 512*B^2*a^6*c^4*d^2*f^6*z^2 + 512*B^2*a^4*c^6*d^4*f^4*z
^2 + 232*A^2*a^5*c^5*e^4*f^4*z^2 + 188*A^2*a^4*c^6*e^6*f^2*z^2 - 128*B^2*a^
3*c^7*d^5*f^3*z^2 + 92*A^2*a^6*c^4*e^2*f^6*z^2 + 80*A^2*b^4*c^6*d^5*f^3*z^2
 + 64*A^2*b^2*c^8*d^6*f^2*z^2 + 31*A^2*b^6*c^4*d^4*f^4*z^2 + 14*A^2*b^8*c^2
*d^3*f^5*z^2 - 5*B^2*b^4*c^6*d^4*e^4*z^2 + 4*B^2*b^3*c^7*d^5*e^3*z^2 + 2*B^
2*b^5*c^5*d^3*e^5*z^2 - B^2*b^6*c^4*d^2*e^6*z^2 - B^2*b^2*c^8*d^6*e^2*z^2 -
 B^2*a^4*b^6*e^2*f^6*z^2 - 1152*A^2*a^3*c^7*d^4*f^4*z^2 + 1008*A^2*a^4*c^6*
d^3*f^5*z^2 + 624*A^2*a^2*c^8*d^5*f^3*z^2 - 288*A^2*a^5*c^5*d^2*f^6*z^2 + 5
6*B^2*a^3*c^7*d^2*e^6*z^2 - 10*B^2*a^2*b^8*d^2*f^6*z^2 - 9*A^2*b^2*c^8*d^4*
e^4*z^2 - 5*A^2*a^2*b^8*e^2*f^6*z^2 - 4*B^2*a^2*c^8*d^4*e^4*z^2 + 3*A^2*b^4
*c^6*d^2*e^6*z^2 - 2*A^2*b^3*c^7*d^3*e^5*z^2 - 36*A^2*a^2*c^8*d^2*e^6*z^2 -
 48*A^2*a^6*b^2*c^2*f^8*z^2 - 45*A^2*a^2*b^2*c^6*e^8*z^2 + 4*A^2*b^10*d*e^2
*f^5*z^2 + 4*B^2*b^2*c^8*d^7*f*z^2 + 4*A^2*b^9*c*e^5*f^3*z^2 + 4*A^2*b^7*c^
3*e^7*f*z^2 - 128*B^2*a^7*c^3*d*f^7*z^2 - 160*A^2*a*c^9*d^6*f^2*z^2 - 112*A
^2*a^6*c^4*d*f^7*z^2 + 12*A^2*b*c^9*d^5*e^3*z^2 + 4*A^2*a*b^9*e^3*f^5*z^2 +
 3*B^2*a^4*b^6*d*f^7*z^2 + 2*A^2*a^3*b^7*e*f^7*z^2 - 24*A^2*a*c^9*d^4*e^4*z
^2 + 14*A^2*a^2*b^8*d*f^7*z^2 + 12*A^2*a^5*b^4*c*f^8*z^2 + 12*A^2*a*b^4*c^5
*e^8*z^2 + A*B*a^4*b^6*e*f^7*z^2 + B^2*a^2*b^8*d*e^2*f^5*z^2 + 16*A^2*c^10*
d^7*f*z^2 + 3*B^2*b^10*d^3*f^5*z^2 - A^2*b^10*e^4*f^4*z^2 - 4*A^2*c^10*d^6*
e^2*z^2 - A^2*b^10*d^2*f^6*z^2 + 64*A^2*a^7*c^3*f^8*z^2 - 4*B^2*a^4*c^6*e^8
*z^2 - A^2*b^6*c^4*e^8*z^2 + 48*A^2*a^3*c^7*e^8*z^2 - A^2*a^4*b^6*f^8*z^2 +
 720*A^2*B*a*b^2*c^5*d^2*e^2*f^3*z - 600*A^2*B*a^2*b^2*c^4*d*e^2*f^4*z + 57
6*A*B^2*a^2*b^2*c^4*d^2*e*f^4*z + 348*A*B^2*a*b^2*c^5*d^2*e^3*f^2*z - 336*A
*B^2*a^2*b*c^5*d^2*e^2*f^3*z - 260*A*B^2*a*b^3*c^4*d^2*e^2*f^3*z - 240*A*B^
2*a^2*b^2*c^4*d*e^3*f^3*z + 196*A*B^2*a^2*b^3*c^3*d*e^2*f^4*z + 172*A^2*B*a
*b*c^6*d*e^5*f*z + 20*A*B^2*a*b^6*c*d*e*f^5*z - 912*A^2*B*a^2*b*c^5*d^2*e*f
^4*z - 644*A^2*B*a*b*c^6*d^2*e^3*f^2*z - 432*A*B^2*a*b^2*c^5*d^3*e*f^3*z +
372*A^2*B*a^2*b*c^5*d*e^3*f^3*z - 330*A^2*B*a*b^2*c^5*d*e^4*f^2*z + 312*A*B
^2*a*b*c^6*d^3*e^2*f^2*z - 208*A*B^2*a^3*b^2*c^3*d*e*f^5*z + 192*A^2*B*a^2*
b^3*c^3*d*e*f^5*z + 172*A^2*B*a*b^3*c^4*d*e^3*f^3*z + 108*A*B^2*a^2*b*c^5*d

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*e^4*f^2*z + 104*A*B^2*a^3*b*c^4*d*e^2*f^4*z - 80*A^2*B*a*b^3*c^4*d^2*e*f^4
*z + 68*A^2*B*a*b^4*c^3*d*e^2*f^4*z - 60*A*B^2*a*b^5*c^2*d*e^2*f^4*z + 58*A
*B^2*a*b^3*c^4*d*e^4*f^2*z - 36*A*B^2*a*b^4*c^3*d^2*e*f^4*z - 24*A*B^2*a^2*
b^4*c^2*d*e*f^5*z + 24*A*B^2*a*b^4*c^3*d*e^3*f^3*z + 592*A^2*B*a*b*c^6*d^3*
e*f^3*z + 240*A^2*B*a^3*b*c^4*d*e*f^5*z - 132*A*B^2*a*b*c^6*d^2*e^4*f*z - 6
0*A*B^2*a*b^2*c^5*d*e^5*f*z - 48*A^2*B*a*b^5*c^2*d*e*f^5*z + 20*B^3*a*b*c^6
*d^3*e^3*f*z + 16*B^3*a^4*b*c^3*d*e*f^5*z - 16*B^3*a*b*c^6*d^4*e*f^2*z + 12
*B^3*a^2*b*c^5*d*e^5*f*z + 320*A^3*a*b*c^6*d*e^4*f^2*z + 40*A^3*a*b^4*c^3*d
*e*f^5*z - 48*A^2*B*b*c^7*d^4*e*f^2*z - 44*A^2*B*b^3*c^5*d*e^5*f*z - 20*A*B
^2*b*c^7*d^4*e^2*f*z + 14*A*B^2*b^4*c^4*d*e^5*f*z + 12*A^2*B*b*c^7*d^3*e^3*
f*z + 4*A*B^2*b^7*c*d*e^2*f^4*z + 160*A*B^2*a^4*c^4*d*e*f^5*z + 152*A^2*B*a
*c^7*d^2*e^4*f*z - 40*A*B^2*a*c^7*d^3*e^3*f*z + 32*A*B^2*a*c^7*d^4*e*f^2*z
- 16*A*B^2*a^2*c^6*d*e^5*f*z + 128*A^2*B*a^4*b*c^3*e*f^6*z + 42*A^2*B*a*b^2
*c^5*e^6*f*z + 24*A^2*B*a^2*b^5*c*e*f^6*z - 12*A*B^2*a^3*b^4*c*e*f^6*z - 12
*A*B^2*a^2*b*c^5*e^6*f*z - 10*A^2*B*a*b^6*c*e^2*f^5*z - 160*A*B^2*a*b*c^6*d
^4*f^3*z + 112*A*B^2*a^4*b*c^3*d*f^6*z - 24*A*B^2*a^2*b^5*c*d*f^6*z - 84*B^
3*a*b^2*c^5*d^3*e^2*f^2*z - 80*B^3*a^2*b^3*c^3*d^2*e*f^4*z - 60*B^3*a^2*b*c
^5*d^2*e^3*f^2*z - 20*B^3*a^3*b^2*c^3*d*e^2*f^4*z - 20*B^3*a*b^3*c^4*d^2*e^
3*f^2*z - 9*B^3*a^2*b^2*c^4*d*e^4*f^2*z - 8*B^3*a*b^4*c^3*d^2*e^2*f^3*z + 6
*B^3*a^2*b^4*c^2*d*e^2*f^4*z - 4*B^3*a^2*b^3*c^3*d*e^3*f^3*z - 216*A^2*B*b^
4*c^4*d^2*e^2*f^3*z + 196*A^2*B*b^3*c^5*d^2*e^3*f^2*z - 108*A*B^2*b^3*c^5*d
^3*e^2*f^2*z - 94*A*B^2*b^4*c^4*d^2*e^3*f^2*z + 88*A^2*B*b^2*c^6*d^3*e^2*f^
2*z + 80*A*B^2*b^5*c^3*d^2*e^2*f^3*z + 360*A^2*B*a^2*c^6*d^2*e^2*f^3*z + 8*
A*B^2*a^2*c^6*d^2*e^3*f^2*z + 153*A^2*B*a^2*b^2*c^4*e^4*f^3*z - 144*A^2*B*a
^2*b^3*c^3*e^3*f^4*z + 80*A^2*B*a^3*b^2*c^3*e^2*f^5*z + 36*A*B^2*a^3*b^2*c^
3*e^3*f^4*z + 12*A^2*B*a^2*b^4*c^2*e^2*f^5*z + 12*A*B^2*a^3*b^3*c^2*e^2*f^5
*z + 9*A*B^2*a^2*b^2*c^4*e^5*f^2*z - 6*A*B^2*a^2*b^4*c^2*e^3*f^4*z + 4*A*B^
2*a^2*b^3*c^3*e^4*f^3*z + 480*A^2*B*a^2*b^2*c^4*d^2*f^5*z - 176*A*B^2*a^2*b
^3*c^3*d^2*f^5*z - 10*A^2*B*a*b^6*c*d*f^6*z + 16*A*B^2*a*b*c^6*d*e^6*z + 80
*B^3*a*b^3*c^4*d^3*e*f^3*z - 48*B^3*a^3*b*c^4*d^2*e*f^4*z + 48*B^3*a^2*b*c^
5*d^3*e*f^3*z + 44*B^3*a^3*b*c^4*d*e^3*f^3*z + 24*B^3*a*b^5*c^2*d^2*e*f^4*z
 + 18*B^3*a*b^2*c^5*d^2*e^4*f*z + 696*A^3*a^2*b*c^5*d*e^2*f^4*z - 504*A^3*a
*b*c^6*d^2*e^2*f^3*z - 192*A^3*a*b^2*c^5*d*e^3*f^3*z - 144*A^3*a^2*b^2*c^4*
d*e*f^5*z + 96*A^3*a*b^2*c^5*d^2*e*f^4*z - 72*A^3*a*b^3*c^4*d*e^2*f^4*z - 2
08*A^2*B*b^3*c^5*d^3*e*f^3*z + 152*A*B^2*b^4*c^4*d^3*e*f^3*z + 80*A^2*B*b^5
*c^3*d^2*e*f^4*z + 75*A^2*B*b^4*c^4*d*e^4*f^2*z - 59*A^2*B*b^2*c^6*d^2*e^4*
f*z - 52*A^2*B*b^5*c^3*d*e^3*f^3*z + 42*A*B^2*b^3*c^5*d^2*e^4*f*z - 21*A*B^
2*b^6*c^2*d^2*e*f^4*z - 16*A*B^2*b^5*c^3*d*e^4*f^2*z + 16*A*B^2*b^2*c^6*d^4
*e*f^2*z + 16*A*B^2*b^2*c^6*d^3*e^3*f*z + 11*A^2*B*b^6*c^2*d*e^2*f^4*z + 4*
A*B^2*b^6*c^2*d*e^3*f^3*z - 256*A^2*B*a*c^7*d^3*e^2*f^2*z - 96*A*B^2*a^3*c^
5*d^2*e*f^4*z - 36*A^2*B*a^2*c^6*d*e^4*f^2*z - 32*A^2*B*a^3*c^5*d*e^2*f^4*z
 - 32*A*B^2*a^2*c^6*d^3*e*f^3*z + 8*A*B^2*a^3*c^5*d*e^3*f^3*z - 96*A^2*B*a^
3*b^3*c^2*e*f^6*z + 68*A^2*B*a^3*b*c^4*e^3*f^4*z - 60*A*B^2*a^4*b*c^3*e^2*f
^5*z - 60*A*B^2*a^3*b*c^4*e^4*f^3*z + 48*A*B^2*a^4*b^2*c^2*e*f^6*z - 38*A^2
*B*a*b^3*c^4*e^5*f^2*z - 36*A^2*B*a^2*b*c^5*e^5*f^2*z + 36*A^2*B*a*b^5*c^2*
e^3*f^4*z - 16*A^2*B*a*b^4*c^3*e^4*f^3*z + 384*A*B^2*a^2*b*c^5*d^3*f^4*z -
352*A*B^2*a^3*b*c^4*d^2*f^5*z - 288*A^2*B*a*b^2*c^5*d^3*f^4*z - 160*A^2*B*a
^3*b^2*c^3*d*f^6*z - 148*A^2*B*a*b^4*c^3*d^2*f^5*z + 112*A*B^2*a*b^3*c^4*d^
3*f^4*z + 72*A^2*B*a^2*b^4*c^2*d*f^6*z + 72*A*B^2*a*b^5*c^2*d^2*f^5*z + 48*
A*B^2*a^3*b^3*c^2*d*f^6*z + 102*B^3*a^2*b^2*c^4*d^2*e^2*f^3*z - 32*B^3*b^5*
c^3*d^3*e*f^3*z - 8*B^3*b^3*c^5*d^3*e^3*f*z - 7*B^3*b^4*c^4*d^2*e^4*f*z + 5
*B^3*b^2*c^6*d^4*e^2*f*z + 80*A^3*b^2*c^6*d^3*e*f^3*z - 74*A^3*b^3*c^5*d*e^
4*f^2*z - 64*A^3*b^4*c^4*d^2*e*f^4*z + 60*A^3*b^4*c^4*d*e^3*f^3*z - 48*B^3*
a^4*c^4*d*e^2*f^4*z - 24*B^3*a^3*c^5*d*e^4*f^2*z + 20*B^3*a^2*c^6*d^2*e^4*f
*z - 16*A^3*b^5*c^3*d*e^2*f^4*z + 8*A^3*b*c^7*d^3*e^2*f^2*z + 480*A^3*a^2*c
^6*d^2*e*f^4*z - 392*A^3*a^2*c^6*d*e^3*f^3*z + 280*A^3*a*c^7*d^2*e^3*f^2*z
- 4*B^3*a^4*b*c^3*e^3*f^4*z - 200*A^3*a^3*b*c^4*e^2*f^5*z - 144*A^3*a^2*b*c
^5*e^4*f^3*z + 48*B^3*a*b^2*c^5*d^4*f^3*z + 42*A^3*a*b^2*c^5*e^5*f^2*z - 36
*B^3*a^4*b^2*c^2*d*f^6*z - 32*A^3*a^3*b^2*c^3*e*f^6*z - 24*A^3*a^2*b^4*c^2*
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e*f^6*z - 24*A^3*a*b^5*c^2*e^2*f^5*z + 10*A^3*a*b^3*c^4*e^4*f^3*z - 4*B^3*a
*b^4*c^3*d^3*f^4*z - 4*A^3*a*b^4*c^3*e^3*f^4*z - 480*A^3*a^2*b*c^5*d^2*f^5*
z - 160*A^3*a^2*b^3*c^3*d*f^6*z + 128*A^3*a*b^3*c^4*d^2*f^5*z + 8*A^2*B*b^5
*c^3*e^5*f^2*z - 2*A^2*B*b^6*c^2*e^4*f^3*z + 112*A^2*B*b^4*c^4*d^3*f^4*z -
92*A^2*B*a^4*c^4*e^2*f^5*z - 64*A^2*B*a^3*c^5*e^4*f^3*z - 64*A*B^2*b^5*c^3*
d^3*f^4*z + 24*A*B^2*a^4*c^4*e^3*f^4*z + 24*A*B^2*a^3*c^5*e^5*f^2*z + 16*A^
2*B*b^2*c^6*d^4*f^3*z + 16*A*B^2*b^3*c^5*d^4*f^3*z - A^2*B*b^6*c^2*d^2*f^5*
z + 448*A^2*B*a^3*c^5*d^2*f^5*z - 352*A^2*B*a^2*c^6*d^3*f^4*z - 5*A*B^2*b^2
*c^6*d^2*e^5*z - 48*A^2*B*a^4*b^2*c^2*f^7*z - 2*B^3*b^7*c*d^2*e*f^4*z + 34*
A^3*b^2*c^6*d*e^5*f*z + 16*A^3*b*c^7*d^2*e^4*f*z + 2*A^3*b^6*c^2*d*e*f^5*z
- 416*A^3*a^3*c^5*d*e*f^5*z - 224*A^3*a*c^7*d^3*e*f^3*z + 12*B^3*a^3*b^4*c*
d*f^6*z - 10*B^3*a*b^6*c*d^2*f^5*z + 416*A^3*a^3*b*c^4*d*f^6*z + 224*A^3*a*
b*c^6*d^3*f^4*z + 24*A^3*a*b^5*c^2*d*f^6*z - 4*B^3*a*b*c^6*d^2*e^5*z + 20*A
^2*B*c^8*d^4*e^2*f*z - 7*A^2*B*b^4*c^4*e^6*f*z - 2*A^2*B*b^7*c*e^3*f^4*z -
64*A*B^2*a^5*c^3*e*f^6*z + 16*A*B^2*b*c^7*d^5*f^2*z - 8*A^2*B*a^2*c^6*e^6*f
*z - 2*A*B^2*b^7*c*d^2*f^5*z - 272*A^2*B*a^4*c^4*d*f^6*z + 128*A^2*B*a*c^7*
d^4*f^3*z + 9*A^2*B*b^2*c^6*d*e^6*z - 4*A*B^2*b^3*c^5*d*e^6*z + 4*A*B^2*b*c
^7*d^3*e^4*z + 8*A*B^2*a*c^7*d^2*e^5*z + 12*A^2*B*a^3*b^4*c*f^7*z + 30*B^3*
b^4*c^4*d^3*e^2*f^2*z + 8*B^3*b^5*c^3*d^2*e^3*f^2*z - 2*B^3*b^6*c^2*d^2*e^2
*f^3*z + 152*A^3*b^3*c^5*d^2*e^2*f^3*z - 108*A^3*b^2*c^6*d^2*e^3*f^2*z + 48
*B^3*a^3*c^5*d^2*e^2*f^3*z - 16*B^3*a^2*c^6*d^3*e^2*f^2*z - 3*B^3*a^4*b^2*c
^2*e^2*f^5*z - 120*B^3*a^2*b^2*c^4*d^3*f^4*z + 112*B^3*a^3*b^2*c^3*d^2*f^5*
z + 112*A^3*a^2*b^3*c^3*e^2*f^5*z + 12*A^3*a^2*b^2*c^4*e^3*f^4*z - 120*A^3*
a*c^7*d*e^5*f*z - 52*A^3*a*b*c^6*e^6*f*z + 10*A^3*a*b^6*c*e*f^6*z - 2*A*B^2
*b^8*d*e*f^5*z - 2*A^2*B*a*b^7*e*f^6*z - 24*A^2*B*a*c^7*d*e^6*z + 2*A*B^2*a
*b^7*d*f^6*z - 12*A^2*B*a*b*c^6*e^7*z - 2*A^3*b^7*c*d*f^6*z - 4*A^3*b*c^7*d
*e^6*z + 16*B^3*a^5*c^3*e^2*f^5*z + 11*B^3*b^6*c^2*d^3*f^4*z - 11*A^3*b^4*c
^4*e^5*f^2*z - 8*B^3*b^4*c^4*d^4*f^3*z - 4*B^3*b^2*c^6*d^5*f^2*z + 4*B^3*a^
4*c^4*e^4*f^3*z + 4*A^3*b^5*c^3*e^4*f^3*z - A^3*b^6*c^2*e^3*f^4*z + 136*A^3
*a^3*c^5*e^3*f^4*z + 68*A^3*a^2*c^6*e^5*f^2*z - 64*A^3*b^3*c^5*d^3*f^4*z +
2*B^3*b^3*c^5*d^2*e^5*z - B^3*b^2*c^6*d^3*e^4*z + 96*A^3*a^3*b^3*c^2*f^7*z
+ A*B^2*a^2*b^6*e*f^6*z + 32*A^3*c^8*d^4*e*f^2*z - 24*A^3*c^8*d^3*e^3*f*z +
 10*A^3*b^3*c^5*e^6*f*z + 2*A^3*b^7*c*e^2*f^5*z + 128*A^3*a^4*c^4*e*f^6*z -
 32*A^3*b*c^7*d^4*f^3*z - 4*B^3*a^2*c^6*d*e^6*z - B^3*a^2*b^6*d*f^6*z - 128
*A^3*a^4*b*c^3*f^7*z - 24*A^3*a^2*b^5*c*f^7*z - 16*A^2*B*c^8*d^5*f^2*z - 4*
A^2*B*c^8*d^3*e^4*z + 64*A^2*B*a^5*c^3*f^7*z + 2*A^2*B*b^3*c^5*e^7*z + 4*A*
B^2*a^2*c^6*e^7*z - A^2*B*a^2*b^6*f^7*z + 4*A^3*c^8*d^2*e^5*z - 3*A^3*b^2*c
^6*e^7*z + A^2*B*b^8*d*f^6*z - A^3*b^8*e*f^6*z + 16*A^3*a*c^7*e^7*z + 2*A^3
*a*b^7*f^7*z + A^2*B*b^8*e^2*f^5*z + B^3*b^8*d^2*f^5*z - 48*A^2*B^2*a*b*c^4
*d*e*f^4 + 28*A*B^3*a*b^2*c^3*d*e*f^4 - 16*A*B^3*a*b*c^4*d*e^2*f^3 + 16*A^3
*B*a*c^5*d*e*f^4 + 32*A^3*B*a*b*c^4*d*f^5 + 12*A^2*B^2*b^3*c^3*d*e*f^4 + 5*
A*B^3*b^2*c^4*d^2*e*f^3 + 4*A*B^3*b^3*c^3*d*e^2*f^3 + 24*A^2*B^2*a*c^5*d*e^
2*f^3 + 24*A^2*B^2*a^2*b*c^3*e*f^5 + 12*A^2*B^2*a*b*c^4*e^3*f^3 - 6*A^2*B^2
*a*b^3*c^2*e*f^5 + 4*A*B^3*a^2*b*c^3*e^2*f^4 + 3*A*B^3*a^2*b^2*c^2*e*f^5 -
18*A^2*B^2*a*b^2*c^3*d*f^5 - 4*B^4*a^2*b*c^3*d*e*f^4 + 4*B^4*a*b*c^4*d^2*e*
f^3 - 6*A*B^3*b^4*c^2*d*e*f^4 + 4*A^3*B*b*c^5*d*e^2*f^3 - 2*A^3*B*b^2*c^4*d
*e*f^4 - 8*A*B^3*a^2*c^4*d*e*f^4 - 8*A*B^3*a*c^5*d^2*e*f^3 + 26*A^3*B*a*b^2
*c^3*e*f^5 + 8*A^3*B*a*b*c^4*e^2*f^4 + 32*A*B^3*a*b*c^4*d^2*f^4 - 28*A*B^3*
a^2*b*c^3*d*f^5 + 6*A*B^3*a*b^3*c^2*d*f^5 - 9*A^2*B^2*b^2*c^4*d*e^2*f^3 - 1
8*A^2*B^2*a*b^2*c^3*e^2*f^4 - 4*A^3*B*c^6*d^2*e*f^3 - 3*A^3*B*b^4*c^2*e*f^5
 - 44*A^3*B*a^2*c^4*e*f^5 - 16*A^3*B*a*c^5*e^3*f^3 - 16*A*B^3*a^3*c^3*e*f^5
 - 10*A^3*B*b^3*c^3*d*f^5 - 4*A^3*B*b*c^5*d^2*f^4 - 4*A*B^3*b*c^5*d^3*f^3 -
 28*A^3*B*a^2*b*c^3*f^6 + 6*A^3*B*a*b^3*c^2*f^6 - 4*A^4*b*c^5*d*e*f^4 - 20*
A^4*a*b*c^4*e*f^5 + 3*A^2*B^2*b^4*c^2*e^2*f^4 - 2*A^2*B^2*b^3*c^3*e^3*f^3 +
 12*A^2*B^2*a^2*c^4*e^2*f^4 + 9*A^2*B^2*b^2*c^4*d^2*f^4 - 3*A^2*B^2*a^2*b^2
*c^2*f^6 - 2*B^4*b^3*c^3*d^2*e*f^3 + 4*B^4*a^2*c^4*d*e^2*f^3 - 10*B^4*a*b^2
*c^3*d^2*f^4 - 3*B^4*a^2*b^2*c^2*d*f^5 + 3*A^3*B*b^2*c^4*e^3*f^3 - 2*A^3*B*
b^3*c^3*e^2*f^4 - 10*A*B^3*b^3*c^3*d^2*f^4 - 4*A*B^3*a^2*c^4*e^3*f^3 + 3*A^
2*B^2*b^4*c^2*d*f^5 + 36*A^2*B^2*a^2*c^4*d*f^5 - 24*A^2*B^2*a*c^5*d^2*f^4 +
```

4*A^2*B^2*c^6*d^3*f^3 + 16*A^2*B^2*a^3*c^3*f^6 + 4*A^4*b^3*c^3*e*f^5 + 16*
B^4*a^3*c^3*d*f^5 + 16*A^4*a*c^5*e^2*f^4 + 8*A^4*b^2*c^4*d*f^5 - 8*A^4*a*b^
2*c^3*f^6 - 24*A^4*a*c^5*d*f^5 + 3*B^4*b^4*c^2*d^2*f^4 - 3*A^4*b^2*c^4*e^2*
f^4 + 4*A^4*c^6*d^2*f^4 + 36*A^4*a^2*c^4*f^6 + B^4*b^2*c^4*d^3*f^3, z, k)*(
root(48416*a^6*b^2*c^6*d^4*e^2*f^4*z^4 - 41544*a^5*b^4*c^5*d^4*e^2*f^4*z^4
- 31872*a^7*b^2*c^5*d^3*e^2*f^5*z^4 - 31872*a^5*b^2*c^7*d^5*e^2*f^3*z^4 - 2
9184*a^6*b^2*c^6*d^3*e^4*f^3*z^4 + 28800*a^5*b^4*c^5*d^3*e^4*f^3*z^4 + 2151
0*a^4*b^6*c^4*d^4*e^2*f^4*z^4 + 21408*a^6*b^4*c^4*d^3*e^2*f^5*z^4 + 21408*a
^4*b^4*c^6*d^5*e^2*f^3*z^4 - 18112*a^7*b^3*c^4*d^2*e^3*f^5*z^4 - 18112*a^4*
b^3*c^7*d^5*e^3*f^2*z^4 - 15600*a^5*b^5*c^4*d^3*e^3*f^4*z^4 - 15600*a^4*b^5
*c^5*d^4*e^3*f^3*z^4 + 15296*a^6*b^3*c^5*d^3*e^3*f^4*z^4 + 15296*a^5*b^3*c^
6*d^4*e^3*f^3*z^4 + 14016*a^7*b^2*c^5*d^2*e^4*f^4*z^4 + 14016*a^5*b^2*c^7*d
^4*e^4*f^2*z^4 - 13920*a^4*b^6*c^4*d^3*e^4*f^3*z^4 - 11648*a^6*b^3*c^5*d^2*
e^5*f^3*z^4 - 11648*a^5*b^3*c^6*d^3*e^5*f^2*z^4 + 10432*a^6*b^2*c^6*d^2*e^6
*f^2*z^4 + 9008*a^6*b^5*c^3*d^2*e^3*f^5*z^4 + 9008*a^3*b^5*c^6*d^5*e^3*f^2*
z^4 + 8544*a^5*b^5*c^4*d^2*e^5*f^3*z^4 + 8544*a^4*b^5*c^5*d^3*e^5*f^2*z^4 -
8496*a^5*b^4*c^5*d^2*e^6*f^2*z^4 + 7488*a^8*b^2*c^4*d^2*e^2*f^6*z^4 + 7488
*a^4*b^2*c^8*d^6*e^2*f^2*z^4 + 7380*a^4*b^7*c^3*d^3*e^3*f^4*z^4 + 7380*a^3*
b^7*c^4*d^4*e^3*f^3*z^4 - 6720*a^3*b^8*c^3*d^4*e^2*f^4*z^4 - 5784*a^5*b^6*c
^3*d^3*e^2*f^5*z^4 - 5784*a^3*b^6*c^5*d^5*e^2*f^3*z^4 - 3440*a^6*b^4*c^4*d^
2*e^4*f^4*z^4 - 3440*a^4*b^4*c^6*d^4*e^4*f^2*z^4 + 3360*a^3*b^8*c^3*d^3*e^4
*f^3*z^4 + 3140*a^4*b^6*c^4*d^2*e^6*f^2*z^4 - 2760*a^4*b^7*c^3*d^2*e^5*f^3*
z^4 - 2760*a^3*b^7*c^4*d^3*e^5*f^2*z^4 - 1764*a^5*b^7*c^2*d^2*e^3*f^5*z^4 -
1764*a^2*b^7*c^5*d^5*e^3*f^2*z^4 - 1640*a^3*b^9*c^2*d^3*e^3*f^4*z^4 - 1640
*a^2*b^9*c^3*d^4*e^3*f^3*z^4 - 1604*a^6*b^6*c^2*d^2*e^2*f^6*z^4 - 1604*a^2*
b^6*c^6*d^6*e^2*f^2*z^4 - 1500*a^5*b^6*c^3*d^2*e^4*f^4*z^4 - 1500*a^3*b^6*c
^5*d^4*e^4*f^2*z^4 + 1140*a^2*b^10*c^2*d^4*e^2*f^4*z^4 + 810*a^4*b^8*c^2*d^
2*e^4*f^4*z^4 + 810*a^2*b^8*c^4*d^4*e^4*f^2*z^4 - 544*a^3*b^8*c^3*d^2*e^6*f
^2*z^4 + 416*a^3*b^9*c^2*d^2*e^5*f^3*z^4 + 416*a^2*b^9*c^3*d^3*e^5*f^2*z^4
- 384*a^2*b^10*c^2*d^3*e^4*f^3*z^4 + 180*a^4*b^8*c^2*d^3*e^2*f^5*z^4 + 180*
a^2*b^8*c^4*d^5*e^2*f^3*z^4 + 48*a^7*b^4*c^3*d^2*e^2*f^6*z^4 + 48*a^3*b^4*c
^7*d^6*e^2*f^2*z^4 + 36*a^2*b^10*c^2*d^2*e^6*f^2*z^4 - 1024*a^10*b*c^3*d*e*
f^8*z^4 - 1024*a^3*b*c^10*d^8*e*f*z^4 - 192*a^8*b^5*c*d*e*f^8*z^4 - 192*a*b
^5*c^8*d^8*e*f*z^4 + 16128*a^7*b^3*c^4*d^3*e*f^6*z^4 + 16128*a^4*b^3*c^7*d^
6*e*f^3*z^4 - 11712*a^6*b^5*c^3*d^3*e*f^6*z^4 - 11712*a^3*b^5*c^6*d^6*e*f^3
*z^4 + 11520*a^8*b*c^5*d^2*e^3*f^5*z^4 + 11520*a^5*b*c^8*d^5*e^3*f^2*z^4 -
9984*a^6*b^3*c^5*d^4*e*f^5*z^4 - 9984*a^5*b^3*c^6*d^5*e*f^4*z^4 + 8640*a^5*
b^5*c^4*d^4*e*f^5*z^4 + 8640*a^4*b^5*c^5*d^5*e*f^4*z^4 - 7424*a^7*b*c^6*d^3
*e^3*f^4*z^4 - 7424*a^6*b*c^7*d^4*e^3*f^3*z^4 - 6912*a^8*b^3*c^3*d^2*e*f^7*
z^4 - 6912*a^3*b^3*c^8*d^7*e*f^2*z^4 + 4800*a^7*b^3*c^4*d*e^5*f^4*z^4 + 480
0*a^4*b^3*c^7*d^4*e^5*f*z^4 + 4608*a^7*b*c^6*d^2*e^5*f^3*z^4 + 4608*a^6*b*c
^7*d^3*e^5*f^2*z^4 - 4560*a^4*b^7*c^3*d^4*e*f^5*z^4 - 4560*a^3*b^7*c^4*d^5*
e*f^4*z^4 + 4176*a^5*b^7*c^2*d^3*e*f^6*z^4 + 4176*a^2*b^7*c^5*d^6*e*f^3*z^4
+ 3264*a^7*b^5*c^2*d^2*e*f^7*z^4 + 3264*a^2*b^5*c^7*d^7*e*f^2*z^4 + 3008*a
^8*b^3*c^3*d*e^3*f^6*z^4 + 3008*a^3*b^3*c^8*d^6*e^3*f*z^4 + 2880*a^6*b^3*c^
5*d*e^7*f^2*z^4 + 2880*a^5*b^3*c^6*d^2*e^7*f*z^4 - 2240*a^7*b^4*c^3*d*e^4*f
^5*z^4 - 2240*a^3*b^4*c^7*d^5*e^4*f*z^4 - 1488*a^5*b^5*c^4*d*e^7*f^2*z^4 -
1488*a^4*b^5*c^5*d^2*e^7*f*z^4 + 1440*a^3*b^9*c^2*d^4*e*f^5*z^4 + 1440*a^2*
b^9*c^3*d^5*e*f^4*z^4 - 1328*a^6*b^5*c^3*d*e^5*f^4*z^4 - 1328*a^3*b^5*c^6*d
^4*e^5*f*z^4 - 1152*a^7*b^2*c^5*d*e^6*f^3*z^4 - 1152*a^5*b^2*c^7*d^3*e^6*f*
z^4 - 1120*a^6*b^4*c^4*d*e^6*f^3*z^4 - 1120*a^4*b^4*c^6*d^3*e^6*f*z^4 + 912
*a^6*b^6*c^2*d*e^4*f^5*z^4 + 912*a^2*b^6*c^6*d^5*e^4*f*z^4 + 872*a^5*b^6*c^
3*d*e^6*f^3*z^4 + 872*a^3*b^6*c^5*d^3*e^6*f*z^4 + 768*a^8*b^2*c^4*d*e^4*f^5
*z^4 + 768*a^4*b^2*c^8*d^5*e^4*f*z^4 - 672*a^8*b^4*c^2*d*e^2*f^7*z^4 - 672*
a^2*b^4*c^8*d^7*e^2*f*z^4 - 624*a^7*b^5*c^2*d*e^3*f^6*z^4 - 624*a^2*b^5*c^7
*d^6*e^3*f*z^4 + 480*a^5*b^8*c*d^2*e^2*f^6*z^4 + 480*a*b^8*c^5*d^6*e^2*f^2*
z^4 + 316*a^4*b^7*c^3*d*e^7*f^2*z^4 + 316*a^3*b^7*c^4*d^2*e^7*f*z^4 - 204*a
^4*b^8*c^2*d*e^6*f^3*z^4 - 204*a^2*b^8*c^4*d^3*e^6*f*z^4 + 168*a^3*b^10*c*d
^3*e^2*f^5*z^4 + 168*a*b^10*c^3*d^5*e^2*f^3*z^4 + 156*a^2*b^11*c*d^3*e^3*f^

4*z^4 + 156*a*b^11*c^2*d^4*e^3*f^3*z^4 + 128*a^9*b^2*c^3*d*e^2*f^7*z^4 + 12
8*a^3*b^2*c^9*d^7*e^2*f*z^4 - 124*a^3*b^10*c*d^2*e^4*f^4*z^4 - 124*a*b^10*c
^3*d^4*e^4*f^2*z^4 + 100*a^4*b^9*c*d^2*e^3*f^5*z^4 + 100*a*b^9*c^4*d^5*e^3*
f^2*z^4 + 36*a^5*b^7*c^2*d*e^5*f^4*z^4 + 36*a^2*b^7*c^5*d^4*e^5*f*z^4 - 24*
a^3*b^9*c^2*d*e^7*f^2*z^4 - 24*a^2*b^11*c*d^2*e^5*f^3*z^4 - 24*a^2*b^9*c^3*
d^2*e^7*f*z^4 - 24*a*b^11*c^2*d^3*e^5*f^2*z^4 - 9216*a^8*b*c^5*d^3*e*f^6*z^
4 - 9216*a^5*b*c^8*d^6*e*f^3*z^4 - 5376*a^8*b*c^5*d*e^5*f^4*z^4 - 5376*a^5*
b*c^8*d^4*e^5*f*z^4 + 5120*a^9*b*c^4*d^2*e*f^7*z^4 + 5120*a^7*b*c^6*d^4*e*f
^5*z^4 + 5120*a^6*b*c^7*d^5*e*f^4*z^4 + 5120*a^4*b*c^9*d^7*e*f^2*z^4 - 4352
*a^9*b*c^4*d*e^3*f^6*z^4 - 4352*a^4*b*c^9*d^6*e^3*f*z^4 - 1792*a^7*b*c^6*d*
e^7*f^2*z^4 - 1792*a^6*b*c^7*d^2*e^7*f*z^4 - 1600*a^6*b^2*c^6*d*e^8*f*z^4 +
 912*a^5*b^4*c^5*d*e^8*f*z^4 + 768*a^9*b^3*c^2*d*e*f^8*z^4 + 768*a^2*b^3*c^
9*d^8*e*f*z^4 - 720*a^4*b^9*c*d^3*e*f^6*z^4 - 720*a*b^9*c^4*d^6*e*f^3*z^4 -
 656*a^6*b^7*c*d^2*e*f^7*z^4 - 656*a*b^7*c^6*d^7*e*f^2*z^4 - 240*a^2*b^11*c
*d^4*e*f^5*z^4 - 240*a*b^11*c^2*d^5*e*f^4*z^4 + 216*a^7*b^6*c*d*e^2*f^7*z^4
 + 216*a*b^6*c^7*d^7*e^2*f*z^4 - 204*a^4*b^6*c^4*d*e^8*f*z^4 - 144*a^5*b^8*
c*d*e^4*f^5*z^4 - 144*a*b^8*c^5*d^5*e^4*f*z^4 - 84*a*b^12*c*d^4*e^2*f^4*z^4
 + 36*a^4*b^9*c*d*e^5*f^4*z^4 + 36*a*b^9*c^4*d^4*e^5*f*z^4 + 20*a^6*b^7*c*d
*e^3*f^6*z^4 + 20*a*b^7*c^6*d^6*e^3*f*z^4 + 16*a^3*b^10*c*d*e^6*f^3*z^4 + 1
6*a^3*b^8*c^3*d*e^8*f*z^4 + 16*a*b^12*c*d^3*e^4*f^3*z^4 + 16*a*b^10*c^3*d^3
*e^6*f*z^4 + 48*b^11*c^3*d^6*e*f^3*z^4 + 48*b^9*c^5*d^7*e*f^2*z^4 - 20*b^8*
c^6*d^7*e^2*f*z^4 + 8*b^10*c^4*d^5*e^4*f*z^4 - 4*b^13*c*d^4*e^3*f^3*z^4 - 4
*b^11*c^3*d^4*e^5*f*z^4 + 4*b^9*c^5*d^6*e^3*f*z^4 + 3072*a^9*c^5*d*e^4*f^5*
z^4 + 3072*a^5*c^9*d^5*e^4*f*z^4 + 2560*a^8*c^6*d*e^6*f^3*z^4 + 2560*a^6*c^
8*d^3*e^6*f*z^4 + 1536*a^10*c^4*d*e^2*f^7*z^4 + 1536*a^4*c^10*d^7*e^2*f*z^4
 + 48*a^5*b^9*d^2*e*f^7*z^4 + 48*a^3*b^11*d^3*e*f^6*z^4 - 20*a^6*b^8*d*e^2*
f^7*z^4 + 8*a^4*b^10*d*e^4*f^5*z^4 + 4*a^5*b^9*d*e^3*f^6*z^4 - 4*a^3*b^11*d
*e^5*f^4*z^4 - 4*a*b^13*d^3*e^3*f^4*z^4 + 768*a^9*b*c^4*e^5*f^5*z^4 + 768*a
^8*b*c^5*e^7*f^3*z^4 + 256*a^10*b*c^3*e^3*f^7*z^4 - 192*a^6*b^3*c^5*e^9*f*z
^4 - 68*a^7*b^6*c*e^4*f^6*z^4 + 48*a^8*b^5*c*e^3*f^7*z^4 + 48*a^5*b^5*c^4*e
^9*f*z^4 + 36*a^6*b^7*c*e^5*f^5*z^4 - 12*a^9*b^4*c*e^2*f^8*z^4 - 4*a^4*b^9*
c*e^7*f^3*z^4 - 4*a^4*b^7*c^3*e^9*f*z^4 + 384*a^5*b^8*c*d^3*f^7*z^4 + 384*a
*b^8*c^5*d^7*f^3*z^4 + 288*a^3*b^10*c*d^4*f^6*z^4 + 288*a*b^10*c^3*d^6*f^4*
z^4 + 224*a^7*b^6*c*d^2*f^8*z^4 + 224*a*b^6*c^7*d^8*f^2*z^4 - 192*a^10*b^2*
c^2*d*f^9*z^4 - 192*a^2*b^2*c^10*d^9*f*z^4 + 768*a^5*b*c^8*d^3*e^7*z^4 + 76
8*a^4*b*c^9*d^5*e^5*z^4 + 256*a^3*b*c^10*d^7*e^3*z^4 - 192*a^5*b^3*c^6*d*e^
9*z^4 - 68*a*b^6*c^7*d^6*e^4*z^4 + 48*a^4*b^5*c^5*d*e^9*z^4 + 48*a*b^5*c^8*
d^7*e^3*z^4 + 36*a*b^7*c^6*d^5*e^5*z^4 - 12*a*b^4*c^9*d^8*e^2*z^4 - 4*a^3*b
^7*c^4*d*e^9*z^4 - 4*a*b^9*c^4*d^3*e^7*z^4 + 16*b^13*c*d^5*e*f^4*z^4 + 16*b
^7*c^7*d^8*e*f*z^4 + 768*a^7*c^7*d*e^8*f*z^4 + 16*a^7*b^7*d*e*f^8*z^4 + 16*
a*b^13*d^4*e*f^5*z^4 + 256*a^7*b*c^6*e^9*f*z^4 + 80*a*b^12*c*d^5*f^5*z^4 +
48*a^9*b^4*c*d*f^9*z^4 + 48*a*b^4*c^9*d^9*f*z^4 + 256*a^6*b*c^7*d*e^9*z^4 -
 42*b^10*c^4*d^6*e^2*f^2*z^4 - 20*b^12*c^2*d^5*e^2*f^3*z^4 + 6*b^12*c^2*d^4
*e^4*f^2*z^4 + 4*b^11*c^3*d^5*e^3*f^2*z^4 - 24960*a^7*c^7*d^4*e^2*f^4*z^4 +
 18944*a^8*c^6*d^3*e^2*f^5*z^4 + 18944*a^6*c^8*d^5*e^2*f^3*z^4 + 14336*a^7*
c^7*d^3*e^4*f^3*z^4 - 9984*a^8*c^6*d^2*e^4*f^4*z^4 - 9984*a^6*c^8*d^4*e^4*f
^2*z^4 - 7936*a^9*c^5*d^2*e^2*f^6*z^4 - 7936*a^5*c^9*d^6*e^2*f^2*z^4 - 4352
*a^7*c^7*d^2*e^6*f^2*z^4 - 42*a^4*b^10*d^2*e^2*f^6*z^4 - 20*a^2*b^12*d^3*e^
2*f^5*z^4 + 6*a^2*b^12*d^2*e^4*f^4*z^4 + 4*a^3*b^11*d^2*e^3*f^5*z^4 - 480*a
^8*b^2*c^4*e^6*f^4*z^4 + 440*a^7*b^4*c^3*e^6*f^4*z^4 - 320*a^8*b^3*c^3*e^5*
f^5*z^4 - 320*a^7*b^3*c^4*e^7*f^3*z^4 + 240*a^8*b^4*c^2*e^4*f^6*z^4 + 240*a
^6*b^4*c^4*e^8*f^2*z^4 - 192*a^9*b^3*c^2*e^3*f^7*z^4 - 192*a^9*b^2*c^3*e^4*
f^6*z^4 - 192*a^7*b^2*c^5*e^8*f^2*z^4 - 90*a^6*b^6*c^2*e^6*f^4*z^4 - 68*a^5
*b^6*c^3*e^8*f^2*z^4 + 48*a^10*b^2*c^2*e^2*f^8*z^4 - 48*a^7*b^5*c^2*e^5*f^5
*z^4 - 48*a^6*b^5*c^3*e^7*f^3*z^4 + 36*a^5*b^7*c^2*e^7*f^3*z^4 + 6*a^4*b^8*
c^2*e^8*f^2*z^4 - 33920*a^6*b^2*c^6*d^5*f^5*z^4 + 27936*a^5*b^4*c^5*d^5*f^5
*z^4 + 26112*a^7*b^2*c^5*d^4*f^6*z^4 + 26112*a^5*b^2*c^7*d^6*f^4*z^4 - 2035
2*a^6*b^4*c^4*d^4*f^6*z^4 - 20352*a^4*b^4*c^6*d^6*f^4*z^4 - 13080*a^4*b^6*c
^4*d^5*f^5*z^4 - 11520*a^8*b^2*c^4*d^3*f^7*z^4 - 11520*a^4*b^2*c^8*d^7*f^3*

z^4 + 8736*a^5*b^6*c^3*d^4*f^6*z^4 + 8736*a^3*b^6*c^5*d^6*f^4*z^4 + 7488*a^
7*b^4*c^3*d^3*f^7*z^4 + 7488*a^3*b^4*c^7*d^7*f^3*z^4 + 3840*a^3*b^8*c^3*d^5
*f^5*z^4 + 2560*a^9*b^2*c^3*d^2*f^8*z^4 + 2560*a^3*b^2*c^9*d^8*f^2*z^4 - 24
16*a^6*b^6*c^2*d^3*f^7*z^4 - 2416*a^2*b^6*c^6*d^7*f^3*z^4 - 2160*a^4*b^8*c^
2*d^4*f^6*z^4 - 2160*a^2*b^8*c^4*d^6*f^4*z^4 - 1152*a^8*b^4*c^2*d^2*f^8*z^4
 - 1152*a^2*b^4*c^8*d^8*f^2*z^4 - 720*a^2*b^10*c^2*d^5*f^5*z^4 - 480*a^4*b^
2*c^8*d^4*e^6*z^4 + 440*a^3*b^4*c^7*d^4*e^6*z^4 - 320*a^4*b^3*c^7*d^3*e^7*z
^4 - 320*a^3*b^3*c^8*d^5*e^5*z^4 + 240*a^4*b^4*c^6*d^2*e^8*z^4 + 240*a^2*b^
4*c^8*d^6*e^4*z^4 - 192*a^5*b^2*c^7*d^2*e^8*z^4 - 192*a^3*b^2*c^9*d^6*e^4*z
^4 - 192*a^2*b^3*c^9*d^7*e^3*z^4 - 90*a^2*b^6*c^6*d^4*e^6*z^4 - 68*a^3*b^6*
c^5*d^2*e^8*z^4 - 48*a^3*b^5*c^6*d^3*e^7*z^4 - 48*a^2*b^5*c^7*d^5*e^5*z^4 +
 48*a^2*b^2*c^10*d^8*e^2*z^4 + 36*a^2*b^7*c^5*d^3*e^7*z^4 + 6*a^2*b^8*c^4*d
^2*e^8*z^4 - 4*b^6*c^8*d^9*f*z^4 + 256*a^11*c^3*d*f^9*z^4 + 256*a^3*c^11*d^
9*f*z^4 - 4*a^8*b^6*d*f^9*z^4 - 384*a^9*c^5*e^6*f^4*z^4 - 256*a^10*c^4*e^4*
f^6*z^4 - 256*a^8*c^6*e^8*f^2*z^4 - 64*a^11*c^3*e^2*f^8*z^4 - 24*b^10*c^4*d
^7*f^3*z^4 - 16*b^12*c^2*d^6*f^4*z^4 - 16*b^8*c^6*d^8*f^2*z^4 + 17920*a^7*c
^7*d^5*f^5*z^4 - 14336*a^8*c^6*d^4*f^6*z^4 - 14336*a^6*c^8*d^6*f^4*z^4 + 71
68*a^9*c^5*d^3*f^7*z^4 + 7168*a^5*c^9*d^7*f^3*z^4 - 2048*a^10*c^4*d^2*f^8*z
^4 - 2048*a^4*c^10*d^8*f^2*z^4 + 6*b^8*c^6*d^6*e^4*z^4 + 6*a^6*b^8*e^4*f^6*
z^4 - 4*b^9*c^5*d^5*e^5*z^4 - 4*b^7*c^7*d^7*e^3*z^4 - 4*a^7*b^7*e^3*f^7*z^4
 - 4*a^5*b^9*e^5*f^5*z^4 - 384*a^5*c^9*d^4*e^6*z^4 - 256*a^6*c^8*d^2*e^8*z^
4 - 256*a^4*c^10*d^6*e^4*z^4 - 64*a^3*c^11*d^8*e^2*z^4 - 24*a^4*b^10*d^3*f^
7*z^4 - 16*a^6*b^8*d^2*f^8*z^4 - 16*a^2*b^12*d^4*f^6*z^4 + 48*a^6*b^2*c^6*e
^10*z^4 - 12*a^5*b^4*c^5*e^10*z^4 - 4*b^14*d^5*f^5*z^4 - 64*a^7*c^7*e^10*z^
4 + b^14*d^4*e^2*f^4*z^4 + b^10*c^4*d^4*e^6*z^4 + b^6*c^8*d^8*e^2*z^4 + a^8
*b^6*e^2*f^8*z^4 + a^4*b^10*e^6*f^4*z^4 + a^4*b^6*c^4*e^10*z^4 - 4820*A*B*a
^4*b*c^5*d^2*e^2*f^4*z^2 + 2976*A*B*a^3*b*c^6*d^3*e^2*f^3*z^2 - 2328*A*B*a^
3*b*c^6*d^2*e^4*f^2*z^2 + 1848*A*B*a^2*b^4*c^4*d^3*e*f^4*z^2 - 1768*A*B*a^3
*b^4*c^3*d^2*e*f^5*z^2 + 1528*A*B*a^4*b^2*c^4*d^2*e*f^5*z^2 - 1136*A*B*a^3*
b^2*c^5*d^3*e*f^4*z^2 - 974*A*B*a^4*b^3*c^3*d*e^2*f^5*z^2 + 692*A*B*a^2*b*c
^7*d^4*e^2*f^2*z^2 + 588*A*B*a*b^6*c^3*d^2*e^3*f^3*z^2 - 580*A*B*a^3*b^3*c^
4*d*e^4*f^3*z^2 + 488*A*B*a^3*b^4*c^3*d*e^3*f^4*z^2 - 444*A*B*a^2*b^2*c^6*d
^2*e^5*f*z^2 - 412*A*B*a*b^5*c^4*d^2*e^4*f^2*z^2 + 366*A*B*a^2*b^6*c^2*d^2*
e*f^5*z^2 - 352*A*B*a^2*b^2*c^6*d^4*e*f^3*z^2 + 326*A*B*a^2*b^4*c^4*d*e^5*f
^2*z^2 + 324*A*B*a*b^5*c^4*d^3*e^2*f^3*z^2 - 302*A*B*a*b^3*c^6*d^4*e^2*f^2*
z^2 - 296*A*B*a*b^7*c^2*d^2*e^2*f^4*z^2 + 122*A*B*a^4*b^2*c^4*d*e^3*f^4*z^2
 - 122*A*B*a^2*b^6*c^2*d*e^3*f^4*z^2 - 84*A*B*a^3*b^2*c^5*d*e^5*f^2*z^2 + 7
2*A*B*a*b^4*c^5*d^3*e^3*f^2*z^2 - 64*A*B*a^2*b^5*c^3*d*e^4*f^3*z^2 + 60*A*B
*a^3*b^5*c^2*d*e^2*f^5*z^2 + 1312*A*B*a^5*b*c^4*d*e^2*f^5*z^2 + 1040*A*B*a^
4*b*c^5*d*e^4*f^3*z^2 - 500*A*B*a*b^6*c^3*d^3*e*f^4*z^2 - 376*A*B*a*b^2*c^7
*d^5*e*f^2*z^2 + 276*A*B*a^4*b^4*c^2*d*e*f^6*z^2 - 262*A*B*a^2*b^3*c^5*d*e^
6*f*z^2 + 238*A*B*a*b^2*c^7*d^4*e^3*f*z^2 + 232*A*B*a^5*b^2*c^3*d*e*f^6*z^2
 - 176*A*B*a^2*b*c^7*d^3*e^4*f*z^2 - 120*A*B*a*b^6*c^3*d*e^5*f^2*z^2 - 108*
A*B*a*b^4*c^5*d^4*e*f^3*z^2 + 68*A*B*a*b^7*c^2*d*e^4*f^3*z^2 + 68*A*B*a*b^4
*c^5*d^2*e^5*f*z^2 + 46*A*B*a^2*b^7*c*d*e^2*f^5*z^2 - 36*A*B*a*b^3*c^6*d^3*
e^4*f*z^2 - 1932*A*B*a^2*b^3*c^5*d^3*e^2*f^3*z^2 - 1818*A*B*a^2*b^4*c^4*d^2
*e^3*f^3*z^2 + 1620*A*B*a^3*b^3*c^4*d^2*e^2*f^4*z^2 + 1560*A*B*a^2*b^3*c^5*
d^2*e^4*f^2*z^2 + 1244*A*B*a^3*b^2*c^5*d^2*e^3*f^3*z^2 + 820*A*B*a^2*b^2*c^
6*d^3*e^3*f^2*z^2 + 480*A*B*a^2*b^5*c^3*d^2*e^2*f^4*z^2 + 352*A*B*a^3*b*c^6
*d*e^6*f*z^2 - 108*A*B*a^3*b^6*c*d*e*f^6*z^2 + 82*A*B*a*b^5*c^4*d*e^6*f*z^2
 - 64*A*B*a*b*c^8*d^5*e^2*f*z^2 + 16*A*B*a*b^8*c*d^2*e*f^5*z^2 - 4*A*B*a*b^
8*c*d*e^3*f^4*z^2 + 16*B^2*a*b*c^8*d^6*e*f*z^2 + 56*A*B*b^2*c^8*d^6*e*f*z^2
 - 8*A*B*b^9*c*d*e^4*f^3*z^2 - 8*A*B*b^7*c^3*d*e^6*f*z^2 - 800*A*B*a^6*c^4*
d*e*f^6*z^2 + 10*A*B*a^2*b^8*d*e*f^6*z^2 - 6*A*B*a*b^9*d*e^2*f^5*z^2 - 12*A
*B*a^5*b^4*c*e*f^7*z^2 + 912*A*B*a^6*b*c^3*d*f^7*z^2 + 192*A*B*a^4*b^5*c*d*
f^7*z^2 + 192*A*B*a*b*c^8*d^6*f^2*z^2 - 20*A*B*a*b^4*c^5*d*e^7*z^2 + 4*A*B*
a*b*c^8*d^4*e^4*z^2 + 2144*B^2*a^4*b*c^5*d^3*e*f^4*z^2 - 1120*B^2*a^3*b*c^6
*d^4*e*f^3*z^2 - 688*B^2*a^5*b*c^4*d^2*e*f^5*z^2 - 256*B^2*a^3*b*c^6*d^2*e^
5*f*z^2 + 152*B^2*a*b^3*c^6*d^5*e*f^2*z^2 + 120*B^2*a^5*b^3*c^2*d*e*f^6*z^2

- 116*B^2*a^5*b*c^4*d*e^3*f^4*z^2 + 110*B^2*a*b^7*c^2*d^3*e*f^4*z^2 - 80*B
^2*a^2*b*c^7*d^5*e*f^2*z^2 - 72*B^2*a*b^5*c^4*d^4*e*f^3*z^2 - 48*B^2*a^4*b*
c^5*d*e^5*f^2*z^2 - 46*B^2*a*b^3*c^6*d^4*e^3*f*z^2 - 44*B^2*a*b^4*c^5*d^3*e
^4*f*z^2 - 34*B^2*a*b^5*c^4*d^2*e^5*f*z^2 + 20*B^2*a^2*b*c^7*d^4*e^3*f*z^2
- 10*B^2*a^3*b^6*c*d*e^2*f^5*z^2 - 10*B^2*a^2*b^7*c*d^2*e*f^5*z^2 - 10*B^2*
a*b^2*c^7*d^5*e^2*f*z^2 - 7*B^2*a^2*b^4*c^4*d*e^6*f*z^2 - 6*B^2*a^3*b^2*c^5
*d*e^6*f*z^2 + 4*B^2*a*b^8*c*d^2*e^2*f^4*z^2 - 2*B^2*a^2*b^7*c*d*e^3*f^4*z^
2 + 3196*A^2*a^4*b*c^5*d*e^3*f^4*z^2 - 3184*A^2*a^4*b*c^5*d^2*e*f^5*z^2 + 1
568*A^2*a^3*b*c^6*d^3*e*f^4*z^2 + 1504*A^2*a^3*b*c^6*d*e^5*f^2*z^2 - 656*A^
2*a^4*b^3*c^3*d*e*f^6*z^2 - 400*A^2*a*b^6*c^3*d*e^4*f^3*z^2 + 314*A^2*a*b^5
*c^4*d*e^5*f^2*z^2 - 264*A^2*a^3*b^5*c^2*d*e*f^6*z^2 + 240*A^2*a^2*b^2*c^6*
d*e^6*f*z^2 - 224*A^2*a^2*b*c^7*d^4*e*f^3*z^2 + 216*A^2*a*b^5*c^4*d^3*e*f^4
*z^2 - 192*A^2*a^2*b*c^7*d^2*e^5*f*z^2 + 178*A^2*a*b^7*c^2*d*e^3*f^4*z^2 -
154*A^2*a*b^7*c^2*d^2*e*f^5*z^2 + 128*A^2*a*b^3*c^6*d^4*e*f^3*z^2 + 106*A^2
*a*b^3*c^6*d^2*e^5*f*z^2 - 12*A^2*a*b^2*c^7*d^3*e^4*f*z^2 - 58*A*B*b^8*c^2*
d^2*e^3*f^3*z^2 + 40*A*B*b^7*c^3*d^2*e^4*f^2*z^2 - 28*A*B*b^7*c^3*d^3*e^2*f
^3*z^2 - 24*A*B*b^5*c^5*d^4*e^2*f^2*z^2 - 20*A*B*b^6*c^4*d^3*e^3*f^2*z^2 +
2768*A*B*a^4*c^6*d^2*e^3*f^3*z^2 - 1712*A*B*a^3*c^7*d^3*e^3*f^2*z^2 - 156*A
*B*a^4*b^2*c^4*e^5*f^3*z^2 + 146*A*B*a^4*b^3*c^3*e^4*f^4*z^2 - 106*A*B*a^5*
b^2*c^3*e^3*f^5*z^2 + 90*A*B*a^5*b^3*c^2*e^2*f^6*z^2 + 38*A*B*a^3*b^3*c^4*e
^6*f^2*z^2 - 36*A*B*a^3*b^5*c^2*e^4*f^4*z^2 + 16*A*B*a^3*b^4*c^3*e^5*f^3*z^
2 - 9*A*B*a^4*b^4*c^2*e^3*f^5*z^2 - 8*A*B*a^2*b^5*c^3*e^6*f^2*z^2 + 2*A*B*a
^2*b^6*c^2*e^5*f^3*z^2 + 920*A*B*a^4*b^3*c^3*d^2*f^6*z^2 - 480*A*B*a^2*b^5*
c^3*d^3*f^5*z^2 - 336*A*B*a^2*b^3*c^5*d^4*f^4*z^2 - 272*A*B*a^3*b^3*c^4*d^3
*f^5*z^2 + 240*A*B*a^3*b^5*c^2*d^2*f^6*z^2 - 32*A*B*a*c^9*d^6*e*f*z^2 - 792
*B^2*a^2*b^3*c^5*d^3*e^3*f^2*z^2 + 714*B^2*a^2*b^4*c^4*d^3*e^2*f^3*z^2 - 57
2*B^2*a^3*b^2*c^5*d^3*e^2*f^3*z^2 - 475*B^2*a^2*b^2*c^6*d^4*e^2*f^2*z^2 + 2
65*B^2*a^4*b^2*c^4*d^2*e^2*f^4*z^2 + 260*B^2*a^3*b^3*c^4*d^2*e^3*f^3*z^2 -
212*B^2*a^3*b^4*c^3*d^2*e^2*f^4*z^2 + 180*B^2*a^3*b^2*c^5*d^2*e^4*f^2*z^2 -
158*B^2*a^2*b^4*c^4*d^2*e^4*f^2*z^2 + 47*B^2*a^2*b^6*c^2*d^2*e^2*f^4*z^2 +
16*B^2*a^2*b^5*c^3*d^2*e^3*f^3*z^2 + 2752*A^2*a^3*b^2*c^5*d^2*e^2*f^4*z^2
- 2148*A^2*a^2*b^4*c^4*d^2*e^2*f^4*z^2 + 2064*A^2*a^2*b^3*c^5*d^2*e^3*f^3*z
^2 - 424*A^2*a^2*b^2*c^6*d^3*e^2*f^3*z^2 - 198*A^2*a^2*b^2*c^6*d^2*e^4*f^2*
z^2 - 272*B^2*a^6*b*c^3*d*e*f^6*z^2 - 24*B^2*a^4*b^5*c*d*e*f^6*z^2 + 1808*A
^2*a^5*b*c^4*d*e*f^6*z^2 - 244*A^2*a*b*c^8*d^4*e^3*f*z^2 + 208*A^2*a*b*c^8*
d^5*e*f^2*z^2 + 134*A^2*a^2*b^7*c*d*e*f^6*z^2 - 76*A^2*a*b^4*c^5*d*e^6*f*z^
2 + 4*A^2*a*b^8*c*d*e^2*f^5*z^2 + 148*A*B*b^4*c^6*d^5*e*f^2*z^2 + 65*A*B*b^
6*c^4*d^4*e*f^3*z^2 + 46*A*B*b^8*c^2*d^3*e*f^4*z^2 - 38*A*B*b^3*c^7*d^5*e^2
*f*z^2 + 34*A*B*b^9*c*d^2*e^2*f^4*z^2 - 29*A*B*b^4*c^6*d^4*e^3*f*z^2 + 20*A
*B*b^5*c^5*d^3*e^4*f*z^2 + 12*A*B*b^8*c^2*d*e^5*f^2*z^2 - 7*A*B*b^6*c^4*d^2
*e^5*f*z^2 - 2880*A*B*a^4*c^6*d^3*e*f^4*z^2 + 2784*A*B*a^5*c^5*d^2*e*f^5*z^
2 - 1112*A*B*a^5*c^5*d*e^3*f^4*z^2 + 896*A*B*a^3*c^7*d^4*e*f^3*z^2 + 848*A*
B*a^3*c^7*d^2*e^5*f*z^2 - 560*A*B*a^4*c^6*d*e^5*f^2*z^2 + 96*A*B*a^2*c^8*d^
5*e*f^2*z^2 - 88*A*B*a^2*c^8*d^4*e^3*f*z^2 - 100*A*B*a^6*b*c^3*e^2*f^6*z^2
- 76*A*B*a^5*b*c^4*e^4*f^4*z^2 + 48*A*B*a^6*b^2*c^2*e*f^7*z^2 - 42*A*B*a^3*
b^2*c^5*e^7*f*z^2 + 36*A*B*a^4*b*c^5*e^6*f^2*z^2 - 24*A*B*a^4*b^5*c*e^2*f^6
*z^2 + 10*A*B*a^3*b^6*c*e^3*f^5*z^2 + 7*A*B*a^2*b^4*c^4*e^7*f*z^2 + 2*A*B*a
^2*b^7*c*e^4*f^4*z^2 - 2496*A*B*a^5*b*c^4*d^2*f^6*z^2 + 1872*A*B*a^4*b*c^5*
d^3*f^5*z^2 - 744*A*B*a^5*b^3*c^2*d*f^7*z^2 - 720*A*B*a^2*b*c^7*d^5*f^3*z^2
+ 504*A*B*a*b^3*c^6*d^5*f^3*z^2 + 256*A*B*a^3*b*c^6*d^4*f^4*z^2 + 168*A*B*
a*b^7*c^2*d^3*f^5*z^2 - 144*A*B*a^2*b^7*c*d^2*f^6*z^2 + 144*A*B*a*b^5*c^4*d
^4*f^4*z^2 + 66*A*B*a^2*b^2*c^6*d*e^7*z^2 - 36*A*B*a*b^2*c^7*d^3*e^5*z^2 +
20*A*B*a*b^3*c^6*d^2*e^6*z^2 + 12*A*B*a^2*b*c^7*d^2*e^6*z^2 + 1208*B^2*a^3*
b*c^6*d^3*e^3*f^2*z^2 - 848*B^2*a^3*b^3*c^4*d^3*e*f^4*z^2 + 672*B^2*a^2*b^3
*c^5*d^4*e*f^3*z^2 - 632*B^2*a^4*b*c^5*d^2*e^3*f^3*z^2 + 432*B^2*a^4*b^3*c^
3*d^2*e*f^5*z^2 + 276*B^2*a^2*b^2*c^6*d^3*e^4*f*z^2 - 196*B^2*a*b^6*c^3*d^3
*e^2*f^3*z^2 - 168*B^2*a^2*b^5*c^3*d^3*e*f^4*z^2 + 154*B^2*a^2*b^3*c^5*d^2*
e^5*f*z^2 + 148*B^2*a*b^5*c^4*d^3*e^3*f^2*z^2 + 96*B^2*a*b^4*c^5*d^4*e^2*f^
2*z^2 - 72*B^2*a^3*b^5*c^2*d^2*e*f^5*z^2 + 70*B^2*a^5*b^2*c^3*d*e^2*f^5*z^2

- 60*B^2*a^4*b^3*c^3*d*e^3*f^4*z^2 + 52*B^2*a*b^6*c^3*d^2*e^4*f^2*z^2 + 36
*B^2*a^4*b^2*c^4*d*e^4*f^3*z^2 - 32*B^2*a*b^7*c^2*d^2*e^3*f^3*z^2 + 24*B^2*
a^3*b^5*c^2*d*e^3*f^4*z^2 + 15*B^2*a^4*b^4*c^2*d*e^2*f^5*z^2 - 8*B^2*a^3*b^
4*c^3*d*e^4*f^3*z^2 + 8*B^2*a^2*b^5*c^3*d*e^5*f^2*z^2 - 2*B^2*a^3*b^3*c^4*d
*e^5*f^2*z^2 - 2*B^2*a^2*b^6*c^2*d*e^4*f^3*z^2 - 3176*A^2*a^3*b*c^6*d^2*e^3
*f^3*z^2 - 2252*A^2*a^4*b^2*c^4*d*e^2*f^5*z^2 + 1952*A^2*a^3*b^4*c^3*d*e^2*
f^5*z^2 - 1496*A^2*a^3*b^3*c^4*d*e^3*f^4*z^2 + 1378*A^2*a^2*b^4*c^4*d*e^4*f
^3*z^2 + 1184*A^2*a^3*b^3*c^4*d^2*e*f^5*z^2 - 1166*A^2*a^2*b^3*c^5*d*e^5*f^
2*z^2 - 1164*A^2*a^3*b^2*c^5*d*e^4*f^3*z^2 - 1152*A^2*a^2*b^3*c^5*d^3*e*f^4
*z^2 + 578*A^2*a*b^6*c^3*d^2*e^2*f^4*z^2 - 548*A^2*a*b^5*c^4*d^2*e^3*f^3*z^
2 + 440*A^2*a*b^2*c^7*d^4*e^2*f^2*z^2 - 412*A^2*a^2*b^6*c^2*d*e^2*f^5*z^2 -
360*A^2*a*b^3*c^6*d^3*e^3*f^2*z^2 + 312*A^2*a*b^4*c^5*d^3*e^2*f^3*z^2 + 24
8*A^2*a^2*b*c^7*d^3*e^3*f^2*z^2 - 224*A^2*a^2*b^5*c^3*d*e^3*f^4*z^2 + 216*A
^2*a^2*b^5*c^3*d^2*e*f^5*z^2 + 52*A^2*a*b^4*c^5*d^2*e^4*f^2*z^2 - 16*B^2*b^
3*c^7*d^6*e*f*z^2 - 14*B^2*b^9*c*d^3*e*f^4*z^2 + 32*B^2*a^4*c^6*d*e^6*f*z^2
- 20*A^2*b^9*c*d*e^3*f^4*z^2 + 18*A^2*b^9*c*d^2*e*f^5*z^2 + 8*A^2*b^6*c^4*
d*e^6*f*z^2 - 360*A^2*a^3*c^7*d*e^6*f*z^2 + 136*A^2*a*c^9*d^5*e^2*f*z^2 + 2
*B^2*a^3*b^7*d*e*f^6*z^2 + 2*B^2*a*b^9*d^2*e*f^5*z^2 + 12*B^2*a^4*b*c^5*e^7
*f*z^2 - 204*A^2*a^3*b*c^6*e^7*f*z^2 - 128*A^2*a^6*b*c^3*e*f^7*z^2 - 48*A^2
*a*b^5*c^4*e^7*f*z^2 - 36*B^2*a^5*b^4*c*d*f^7*z^2 - 24*A^2*a^4*b^5*c*e*f^7*
z^2 - 16*B^2*a*b^8*c*d^3*f^5*z^2 - 164*A^2*a^3*b^6*c*d*f^7*z^2 - 16*A^2*a*b
^8*c*d^2*f^6*z^2 + 4*B^2*a^3*b*c^6*d*e^7*z^2 - 4*B^2*a*b*c^8*d^5*e^3*z^2 +
48*A^2*a*b*c^8*d^3*e^5*z^2 + 36*A^2*a^2*b*c^7*d*e^7*z^2 - 6*A^2*a*b^3*c^6*d
*e^7*z^2 + 136*A*B*a^6*c^4*e^3*f^5*z^2 - 96*A*B*b^5*c^5*d^5*f^3*z^2 + 80*A*
B*a^5*c^5*e^5*f^3*z^2 - 72*A*B*b^3*c^7*d^6*f^2*z^2 - 24*A*B*b^7*c^3*d^4*f^4
*z^2 + 14*A*B*b^3*c^7*d^4*e^4*z^2 - 14*A*B*b^2*c^8*d^5*e^3*z^2 - 2*A*B*b^5*
c^5*d^2*e^6*z^2 - 2*A*B*b^4*c^6*d^3*e^5*z^2 + 2*A*B*a^3*b^7*e^2*f^6*z^2 - A
*B*a^2*b^8*e^3*f^5*z^2 + 16*A*B*a^2*c^8*d^3*e^5*z^2 - 2*A*B*a^2*b^3*c^5*e^8
*z^2 + 22*B^2*b^8*c^2*d^3*e^2*f^3*z^2 - 12*B^2*b^7*c^3*d^3*e^3*f^2*z^2 + 12
*B^2*b^6*c^4*d^4*e^2*f^2*z^2 - 6*B^2*b^8*c^2*d^2*e^4*f^2*z^2 - 864*B^2*a^4*
c^6*d^3*e^2*f^3*z^2 + 496*B^2*a^3*c^7*d^4*e^2*f^2*z^2 + 224*B^2*a^5*c^5*d^2
*e^2*f^4*z^2 + 136*B^2*a^4*c^6*d^2*e^4*f^2*z^2 - 53*A^2*b^8*c^2*d^2*e^2*f^4
*z^2 + 52*A^2*b^7*c^3*d^2*e^3*f^3*z^2 + 52*A^2*b^5*c^5*d^3*e^3*f^2*z^2 - 36
*A^2*b^6*c^4*d^3*e^2*f^3*z^2 - 12*A^2*b^4*c^6*d^4*e^2*f^2*z^2 - 9*A^2*b^6*c
^4*d^2*e^4*f^2*z^2 + 836*A^2*a^4*c^6*d^2*e^2*f^4*z^2 - 668*A^2*a^2*c^8*d^4*
e^2*f^2*z^2 + 656*A^2*a^3*c^7*d^2*e^4*f^2*z^2 + 368*A^2*a^3*c^7*d^3*e^2*f^3
*z^2 - 45*B^2*a^6*b^2*c^2*e^2*f^6*z^2 - 18*B^2*a^5*b^2*c^3*e^4*f^4*z^2 - 9*
B^2*a^4*b^2*c^4*e^6*f^2*z^2 - 6*B^2*a^5*b^3*c^2*e^3*f^5*z^2 + 3*B^2*a^4*b^4
*c^2*e^4*f^4*z^2 - 2*B^2*a^4*b^3*c^3*e^5*f^3*z^2 - 580*B^2*a^4*b^2*c^4*d^3*
f^5*z^2 + 536*B^2*a^3*b^4*c^3*d^3*f^5*z^2 + 471*A^2*a^4*b^2*c^4*e^4*f^4*z^2
- 436*A^2*a^3*b^4*c^3*e^4*f^4*z^2 - 348*B^2*a^4*b^4*c^2*d^2*f^6*z^2 + 316*
B^2*a^2*b^2*c^6*d^5*f^3*z^2 + 310*A^2*a^3*b^3*c^4*e^5*f^3*z^2 + 232*A^2*a^5
*b^2*c^3*e^2*f^6*z^2 - 229*A^2*a^2*b^4*c^4*e^6*f^2*z^2 - 216*A^2*a^4*b^4*c^
2*e^2*f^6*z^2 + 204*A^2*a^4*b^3*c^3*e^3*f^5*z^2 + 200*B^2*a^5*b^2*c^3*d^2*f
^6*z^2 + 150*A^2*a^3*b^2*c^5*e^6*f^2*z^2 - 120*B^2*a^2*b^4*c^4*d^4*f^4*z^2
+ 91*A^2*a^2*b^6*c^2*e^4*f^4*z^2 + 72*A^2*a^3*b^5*c^2*e^3*f^5*z^2 - 66*B^2*
a^2*b^6*c^2*d^3*f^5*z^2 + 44*A^2*a^2*b^5*c^3*e^5*f^3*z^2 - 16*B^2*a^3*b^2*c
^5*d^4*f^4*z^2 + 1952*A^2*a^4*b^2*c^4*d^2*f^6*z^2 - 1792*A^2*a^3*b^2*c^5*d^
3*f^5*z^2 - 1272*A^2*a^3*b^4*c^3*d^2*f^6*z^2 + 976*A^2*a^2*b^2*c^6*d^4*f^4*
z^2 + 960*A^2*a^2*b^4*c^4*d^3*f^5*z^2 + 282*A^2*a^2*b^6*c^2*d^2*f^6*z^2 - 4
5*B^2*a^2*b^2*c^6*d^2*e^6*z^2 - 48*A^2*b*c^9*d^6*e*f*z^2 - 14*A^2*a*b^9*d*e
*f^6*z^2 - 7*A*B*b^10*d^2*e*f^5*z^2 + 2*A*B*b^10*d*e^3*f^4*z^2 - 64*A*B*a^7
*c^3*e*f^7*z^2 - 16*A*B*b^9*c*d^3*f^5*z^2 + 8*A*B*a^4*c^6*e^7*f*z^2 + 4*A*B
*b*c^9*d^6*e^2*z^2 + 2*A*B*b^6*c^4*d*e^7*z^2 - 120*A*B*a^3*c^7*d*e^7*z^2 -
16*A*B*a^3*b^7*d*f^7*z^2 + 16*A*B*a*b^9*d^2*f^6*z^2 + 8*A*B*a*c^9*d^5*e^3*z
^2 + 12*A*B*a^3*b*c^6*e^8*z^2 - 48*B^2*b^5*c^5*d^5*e*f^2*z^2 + 15*B^2*b^4*c
^6*d^5*e^2*f*z^2 - 14*B^2*b^7*c^3*d^4*e*f^3*z^2 + 4*B^2*b^9*c*d^2*e^3*f^3*z
^2 + 4*B^2*b^7*c^3*d^2*e^5*f*z^2 + 4*B^2*b^5*c^5*d^4*e^3*f*z^2 - B^2*b^6*c^
4*d^3*e^4*f*z^2 - 336*B^2*a^3*c^7*d^3*e^4*f*z^2 + 112*B^2*a^5*c^5*d*e^4*f^3

*z^2 - 112*A^2*b^3*c^7*d^5*e*f^2*z^2 + 80*B^2*a^6*c^4*d*e^2*f^5*z^2 - 48*A^
2*b^5*c^5*d^4*e*f^3*z^2 + 36*A^2*b^8*c^2*d*e^4*f^3*z^2 + 36*A^2*b^3*c^7*d^4
*e^3*f*z^2 - 28*A^2*b^7*c^3*d*e^5*f^2*z^2 + 20*A^2*b^2*c^8*d^5*e^2*f*z^2 +
16*B^2*a^2*c^8*d^5*e^2*f*z^2 - 14*A^2*b^7*c^3*d^3*e*f^4*z^2 - 14*A^2*b^4*c^
6*d^3*e^4*f*z^2 - 10*A^2*b^5*c^5*d^2*e^5*f*z^2 - 1008*A^2*a^4*c^6*d*e^4*f^3
*z^2 - 760*A^2*a^5*c^5*d*e^2*f^5*z^2 + 272*A^2*a^2*c^8*d^3*e^4*f*z^2 + 48*B
^2*a^5*b*c^4*e^5*f^3*z^2 + 36*B^2*a^6*b*c^3*e^3*f^5*z^2 + 12*B^2*a^5*b^4*c*
e^2*f^6*z^2 - 624*A^2*a^4*b*c^5*e^5*f^3*z^2 - 548*A^2*a^5*b*c^4*e^3*f^5*z^2
 + 182*A^2*a^2*b^3*c^5*e^7*f*z^2 - 180*B^2*a*b^4*c^5*d^5*f^3*z^2 + 132*B^2*
a^6*b^2*c^2*d*f^7*z^2 + 108*B^2*a^3*b^6*c*d^2*f^6*z^2 + 96*A^2*a^5*b^3*c^2*
e*f^7*z^2 + 68*A^2*a*b^6*c^3*e^6*f^2*z^2 + 58*A^2*a^3*b^6*c*e^2*f^6*z^2 - 5
6*B^2*a*b^2*c^7*d^6*f^2*z^2 - 38*A^2*a^2*b^7*c*e^3*f^5*z^2 - 36*A^2*a*b^7*c
^2*e^5*f^3*z^2 + 20*B^2*a*b^6*c^3*d^4*f^4*z^2 - 736*A^2*a^5*b^2*c^3*d*f^7*z
^2 + 624*A^2*a^4*b^4*c^2*d*f^7*z^2 - 416*A^2*a*b^2*c^7*d^5*f^3*z^2 - 276*A^
2*a*b^4*c^5*d^4*f^4*z^2 - 196*A^2*a*b^6*c^3*d^3*f^5*z^2 + 8*B^2*a*b^4*c^5*d
^2*e^6*z^2 + 6*B^2*a*b^2*c^7*d^4*e^4*z^2 + 2*B^2*a^2*b^3*c^5*d*e^7*z^2 + 2*
B^2*a*b^3*c^6*d^3*e^5*z^2 - 18*A^2*a*b^2*c^7*d^2*e^6*z^2 - 16*A*B*b*c^9*d^7
*f*z^2 - B^2*b^10*d^2*e^2*f^4*z^2 + 48*B^2*a^7*c^3*e^2*f^6*z^2 - 36*B^2*a^6
*c^4*e^4*f^4*z^2 + 31*B^2*b^6*c^4*d^5*f^3*z^2 - 24*B^2*a^5*c^5*e^6*f^2*z^2
+ 20*B^2*b^4*c^6*d^6*f^2*z^2 - 6*A^2*b^8*c^2*e^6*f^2*z^2 + 2*B^2*b^8*c^2*d^
4*f^4*z^2 - 768*B^2*a^5*c^5*d^3*f^5*z^2 + 512*B^2*a^6*c^4*d^2*f^6*z^2 + 512
*B^2*a^4*c^6*d^4*f^4*z^2 + 232*A^2*a^5*c^5*e^4*f^4*z^2 + 188*A^2*a^4*c^6*e^
6*f^2*z^2 - 128*B^2*a^3*c^7*d^5*f^3*z^2 + 92*A^2*a^6*c^4*e^2*f^6*z^2 + 80*A
^2*b^4*c^6*d^5*f^3*z^2 + 64*A^2*b^2*c^8*d^6*f^2*z^2 + 31*A^2*b^6*c^4*d^4*f^
4*z^2 + 14*A^2*b^8*c^2*d^3*f^5*z^2 - 5*B^2*b^4*c^6*d^4*e^4*z^2 + 4*B^2*b^3*
c^7*d^5*e^3*z^2 + 2*B^2*b^5*c^5*d^3*e^5*z^2 - B^2*b^6*c^4*d^2*e^6*z^2 - B^2
*b^2*c^8*d^6*e^2*z^2 - B^2*a^4*b^6*e^2*f^6*z^2 - 1152*A^2*a^3*c^7*d^4*f^4*z
^2 + 1008*A^2*a^4*c^6*d^3*f^5*z^2 + 624*A^2*a^2*c^8*d^5*f^3*z^2 - 288*A^2*a
^5*c^5*d^2*f^6*z^2 + 56*B^2*a^3*c^7*d^2*e^6*z^2 - 10*B^2*a^2*b^8*d^2*f^6*z^
2 - 9*A^2*b^2*c^8*d^4*e^4*z^2 - 5*A^2*a^2*b^8*e^2*f^6*z^2 - 4*B^2*a^2*c^8*d
^4*e^4*z^2 + 3*A^2*b^4*c^6*d^2*e^6*z^2 - 2*A^2*b^3*c^7*d^3*e^5*z^2 - 36*A^2
*a^2*c^8*d^2*e^6*z^2 - 48*A^2*a^6*b^2*c^2*f^8*z^2 - 45*A^2*a^2*b^2*c^6*e^8*
z^2 + 4*A^2*b^10*d*e^2*f^5*z^2 + 4*B^2*b^2*c^8*d^7*f*z^2 + 4*A^2*b^9*c*e^5*
f^3*z^2 + 4*A^2*b^7*c^3*e^7*f*z^2 - 128*B^2*a^7*c^3*d*f^7*z^2 - 160*A^2*a*c
^9*d^6*f^2*z^2 - 112*A^2*a^6*c^4*d*f^7*z^2 + 12*A^2*b*c^9*d^5*e^3*z^2 + 4*A
^2*a*b^9*e^3*f^5*z^2 + 3*B^2*a^4*b^6*d*f^7*z^2 + 2*A^2*a^3*b^7*e*f^7*z^2 -
24*A^2*a*c^9*d^4*e^4*z^2 + 14*A^2*a^2*b^8*d*f^7*z^2 + 12*A^2*a^5*b^4*c*f^8*
z^2 + 12*A^2*a*b^4*c^5*e^8*z^2 + A*B*a^4*b^6*e*f^7*z^2 + B^2*a^2*b^8*d*e^2*
f^5*z^2 + 16*A^2*c^10*d^7*f*z^2 + 3*B^2*b^10*d^3*f^5*z^2 - A^2*b^10*e^4*f^4
*z^2 - 4*A^2*c^10*d^6*e^2*z^2 - A^2*b^10*d^2*f^6*z^2 + 64*A^2*a^7*c^3*f^8*z
^2 - 4*B^2*a^4*c^6*e^8*z^2 - A^2*b^6*c^4*e^8*z^2 + 48*A^2*a^3*c^7*e^8*z^2 -
 A^2*a^4*b^6*f^8*z^2 + 720*A^2*B*a*b^2*c^5*d^2*e^2*f^3*z - 600*A^2*B*a^2*b^
2*c^4*d*e^2*f^4*z + 576*A*B^2*a^2*b^2*c^4*d^2*e*f^4*z + 348*A*B^2*a*b^2*c^5
*d^2*e^3*f^2*z - 336*A*B^2*a^2*b*c^5*d^2*e^2*f^3*z - 260*A*B^2*a*b^3*c^4*d^
2*e^2*f^3*z - 240*A*B^2*a^2*b^2*c^4*d*e^3*f^3*z + 196*A*B^2*a^2*b^3*c^3*d*e
^2*f^4*z + 172*A^2*B*a*b*c^6*d*e^5*f*z + 20*A*B^2*a*b^6*c*d*e*f^5*z - 912*A
^2*B*a^2*b*c^5*d^2*e*f^4*z - 644*A^2*B*a*b*c^6*d^2*e^3*f^2*z - 432*A*B^2*a*
b^2*c^5*d^3*e*f^3*z + 372*A^2*B*a^2*b*c^5*d*e^3*f^3*z - 330*A^2*B*a*b^2*c^5
*d*e^4*f^2*z + 312*A*B^2*a*b*c^6*d^3*e^2*f^2*z - 208*A*B^2*a^3*b^2*c^3*d*e*
f^5*z + 192*A^2*B*a^2*b^3*c^3*d*e*f^5*z + 172*A^2*B*a*b^3*c^4*d*e^3*f^3*z +
 108*A*B^2*a^2*b*c^5*d*e^4*f^2*z + 104*A*B^2*a^3*b*c^4*d*e^2*f^4*z - 80*A^2
*B*a*b^3*c^4*d^2*e*f^4*z + 68*A^2*B*a*b^4*c^3*d*e^2*f^4*z - 60*A*B^2*a*b^5*
c^2*d*e^2*f^4*z + 58*A*B^2*a*b^3*c^4*d*e^4*f^2*z - 36*A*B^2*a*b^4*c^3*d^2*e
*f^4*z - 24*A*B^2*a^2*b^4*c^2*d*e*f^5*z + 24*A*B^2*a*b^4*c^3*d*e^3*f^3*z +
592*A^2*B*a*b*c^6*d^3*e*f^3*z + 240*A^2*B*a^3*b*c^4*d*e*f^5*z - 132*A*B^2*a
*b*c^6*d^2*e^4*f*z - 60*A*B^2*a*b^2*c^5*d*e^5*f*z - 48*A^2*B*a*b^5*c^2*d*e*
f^5*z + 20*B^3*a*b*c^6*d^3*e^3*f*z + 16*B^3*a^4*b*c^3*d*e*f^5*z - 16*B^3*a*
b*c^6*d^4*e*f^2*z + 12*B^3*a^2*b*c^5*d*e^5*f*z + 320*A^3*a*b*c^6*d*e^4*f^2*
z + 40*A^3*a*b^4*c^3*d*e*f^5*z - 48*A^2*B*b*c^7*d^4*e*f^2*z - 44*A^2*B*b^3*

c^5*d*e^5*f*z - 20*A*B^2*b*c^7*d^4*e^2*f*z + 14*A*B^2*b^4*c^4*d*e^5*f*z + 1
2*A^2*B*b*c^7*d^3*e^3*f*z + 4*A*B^2*b^7*c*d*e^2*f^4*z + 160*A*B^2*a^4*c^4*d
*e*f^5*z + 152*A^2*B*a*c^7*d^2*e^4*f*z - 40*A*B^2*a*c^7*d^3*e^3*f*z + 32*A*
B^2*a*c^7*d^4*e*f^2*z - 16*A*B^2*a^2*c^6*d*e^5*f*z + 128*A^2*B*a^4*b*c^3*e*
f^6*z + 42*A^2*B*a*b^2*c^5*e^6*f*z + 24*A^2*B*a^2*b^5*c*e*f^6*z - 12*A*B^2*
a^3*b^4*c*e*f^6*z - 12*A*B^2*a^2*b*c^5*e^6*f*z - 10*A^2*B*a*b^6*c*e^2*f^5*z
 - 160*A*B^2*a*b*c^6*d^4*f^3*z + 112*A*B^2*a^4*b*c^3*d*f^6*z - 24*A*B^2*a^2
*b^5*c*d*f^6*z - 84*B^3*a*b^2*c^5*d^3*e^2*f^2*z - 80*B^3*a^2*b^3*c^3*d^2*e*
f^4*z - 60*B^3*a^2*b*c^5*d^2*e^3*f^2*z - 20*B^3*a^3*b^2*c^3*d*e^2*f^4*z - 2
0*B^3*a*b^3*c^4*d^2*e^3*f^2*z - 9*B^3*a^2*b^2*c^4*d*e^4*f^2*z - 8*B^3*a*b^4
*c^3*d^2*e^2*f^3*z + 6*B^3*a^2*b^4*c^2*d*e^2*f^4*z - 4*B^3*a^2*b^3*c^3*d*e^
3*f^3*z - 216*A^2*B*b^4*c^4*d^2*e^2*f^3*z + 196*A^2*B*b^3*c^5*d^2*e^3*f^2*z
 - 108*A*B^2*b^3*c^5*d^3*e^2*f^2*z - 94*A*B^2*b^4*c^4*d^2*e^3*f^2*z + 88*A^
2*B*b^2*c^6*d^3*e^2*f^2*z + 80*A*B^2*b^5*c^3*d^2*e^2*f^3*z + 360*A^2*B*a^2*
c^6*d^2*e^2*f^3*z + 8*A*B^2*a^2*c^6*d^2*e^3*f^2*z + 153*A^2*B*a^2*b^2*c^4*e
^4*f^3*z - 144*A^2*B*a^2*b^3*c^3*e^3*f^4*z + 80*A^2*B*a^3*b^2*c^3*e^2*f^5*z
 + 36*A*B^2*a^3*b^2*c^3*e^3*f^4*z + 12*A^2*B*a^2*b^4*c^2*e^2*f^5*z + 12*A*B
^2*a^3*b^3*c^2*e^2*f^5*z + 9*A*B^2*a^2*b^2*c^4*e^5*f^2*z - 6*A*B^2*a^2*b^4*
c^2*e^3*f^4*z + 4*A*B^2*a^2*b^3*c^3*e^4*f^3*z + 480*A^2*B*a^2*b^2*c^4*d^2*f
^5*z - 176*A*B^2*a^2*b^3*c^3*d^2*f^5*z - 10*A^2*B*a*b^6*c*d*f^6*z + 16*A*B^
2*a*b*c^6*d*e^6*z + 80*B^3*a*b^3*c^4*d^3*e*f^3*z - 48*B^3*a^3*b*c^4*d^2*e*f
^4*z + 48*B^3*a^2*b*c^5*d^3*e*f^3*z + 44*B^3*a^3*b*c^4*d*e^3*f^3*z + 24*B^3
*a*b^5*c^2*d^2*e*f^4*z + 18*B^3*a*b^2*c^5*d^2*e^4*f*z + 696*A^3*a^2*b*c^5*d
*e^2*f^4*z - 504*A^3*a*b*c^6*d^2*e^2*f^3*z - 192*A^3*a*b^2*c^5*d*e^3*f^3*z
 - 144*A^3*a^2*b^2*c^4*d*e*f^5*z + 96*A^3*a*b^2*c^5*d^2*e*f^4*z - 72*A^3*a*b
^3*c^4*d*e^2*f^4*z - 208*A^2*B*b^3*c^5*d^3*e*f^3*z + 152*A*B^2*b^4*c^4*d^3*
e*f^3*z + 80*A^2*B*b^5*c^3*d^2*e*f^4*z + 75*A^2*B*b^4*c^4*d*e^4*f^2*z - 59*
A^2*B*b^2*c^6*d^2*e^4*f*z - 52*A^2*B*b^5*c^3*d*e^3*f^3*z + 42*A*B^2*b^3*c^5
*d^2*e^4*f*z - 21*A*B^2*b^6*c^2*d^2*e*f^4*z - 16*A*B^2*b^5*c^3*d*e^4*f^2*z
 + 16*A*B^2*b^2*c^6*d^4*e*f^2*z + 16*A*B^2*b^2*c^6*d^3*e^3*f*z + 11*A^2*B*b^
6*c^2*d*e^2*f^4*z + 4*A*B^2*b^6*c^2*d*e^3*f^3*z - 256*A^2*B*a*c^7*d^3*e^2*f
^2*z - 96*A*B^2*a^3*c^5*d^2*e*f^4*z - 36*A^2*B*a^2*c^6*d*e^4*f^2*z - 32*A^2
*B*a^3*c^5*d*e^2*f^4*z - 32*A*B^2*a^2*c^6*d^3*e*f^3*z + 8*A*B^2*a^3*c^5*d*e
^3*f^3*z - 96*A^2*B*a^3*b^3*c^2*e*f^6*z + 68*A^2*B*a^3*b*c^4*e^3*f^4*z - 60
*A*B^2*a^4*b*c^3*e^2*f^5*z - 60*A*B^2*a^3*b*c^4*e^4*f^3*z + 48*A*B^2*a^4*b^
2*c^2*e*f^6*z - 38*A^2*B*a*b^3*c^4*e^5*f^2*z - 36*A^2*B*a^2*b*c^5*e^5*f^2*z
 + 36*A^2*B*a*b^5*c^2*e^3*f^4*z - 16*A^2*B*a*b^4*c^3*e^4*f^3*z + 384*A*B^2*
a^2*b*c^5*d^3*f^4*z - 352*A*B^2*a^3*b*c^4*d^2*f^5*z - 288*A^2*B*a*b^2*c^5*d
^3*f^4*z - 160*A^2*B*a^3*b^2*c^3*d*f^6*z - 148*A^2*B*a*b^4*c^3*d^2*f^5*z +
112*A*B^2*a*b^3*c^4*d^3*f^4*z + 72*A^2*B*a^2*b^4*c^2*d*f^6*z + 72*A*B^2*a*b
^5*c^2*d^2*f^5*z + 48*A*B^2*a^3*b^3*c^2*d*f^6*z + 102*B^3*a^2*b^2*c^4*d^2*e
^2*f^3*z - 32*B^3*b^5*c^3*d^3*e*f^3*z - 8*B^3*b^3*c^5*d^3*e^3*f*z - 7*B^3*b
^4*c^4*d^2*e^4*f*z + 5*B^3*b^2*c^6*d^4*e^2*f*z + 80*A^3*b^2*c^6*d^3*e*f^3*z
 - 74*A^3*b^3*c^5*d*e^4*f^2*z - 64*A^3*b^4*c^4*d^2*e*f^4*z + 60*A^3*b^4*c^4
*d*e^3*f^3*z - 48*B^3*a^4*c^4*d*e^2*f^4*z - 24*B^3*a^3*c^5*d*e^4*f^2*z + 20
*B^3*a^2*c^6*d^2*e^4*f*z - 16*A^3*b^5*c^3*d*e^2*f^4*z + 8*A^3*b*c^7*d^3*e^2
*f^2*z + 480*A^3*a^2*c^6*d^2*e*f^4*z - 392*A^3*a^2*c^6*d*e^3*f^3*z + 280*A^
3*a*c^7*d^2*e^3*f^2*z - 4*B^3*a^4*b*c^3*e^3*f^4*z - 200*A^3*a^3*b*c^4*e^2*f
^5*z - 144*A^3*a^2*b*c^5*e^4*f^3*z + 48*B^3*a*b^2*c^5*d^4*f^3*z + 42*A^3*a*
b^2*c^5*e^5*f^2*z - 36*B^3*a^4*b^2*c^2*d*f^6*z - 32*A^3*a^3*b^2*c^3*e*f^6*z
 - 24*A^3*a^2*b^4*c^2*e*f^6*z - 24*A^3*a*b^5*c^2*e^2*f^5*z + 10*A^3*a*b^3*c
^4*e^4*f^3*z - 4*B^3*a*b^4*c^3*d^3*f^4*z - 4*A^3*a*b^4*c^3*e^3*f^4*z - 480*
A^3*a^2*b*c^5*d^2*f^5*z - 160*A^3*a^2*b^3*c^3*d*f^6*z + 128*A^3*a*b^3*c^4*d
^2*f^5*z + 8*A^2*B*b^5*c^3*e^5*f^2*z - 2*A^2*B*b^6*c^2*e^4*f^3*z + 112*A^2*
B*b^4*c^4*d^3*f^4*z - 92*A^2*B*a^4*c^4*e^2*f^5*z - 64*A^2*B*a^3*c^5*e^4*f^3
*z - 64*A*B^2*b^5*c^3*d^3*f^4*z + 24*A*B^2*a^4*c^4*e^3*f^4*z + 24*A*B^2*a^3
*c^5*e^5*f^2*z + 16*A^2*B*b^2*c^6*d^4*f^3*z + 16*A*B^2*b^3*c^5*d^4*f^3*z -
A^2*B*b^6*c^2*d^2*f^5*z + 448*A^2*B*a^3*c^5*d^2*f^5*z - 352*A^2*B*a^2*c^6*d
^3*f^4*z - 5*A*B^2*b^2*c^6*d^2*e^5*z - 48*A^2*B*a^4*b^2*c^2*f^7*z - 2*B^3*b

^7*c*d^2*e*f^4*z + 34*A^3*b^2*c^6*d*e^5*f*z + 16*A^3*b*c^7*d^2*e^4*f*z + 2*
A^3*b^6*c^2*d*e*f^5*z - 416*A^3*a^3*c^5*d*e*f^5*z - 224*A^3*a*c^7*d^3*e*f^3
*z + 12*B^3*a^3*b^4*c*d*f^6*z - 10*B^3*a*b^6*c*d^2*f^5*z + 416*A^3*a^3*b*c^
4*d*f^6*z + 224*A^3*a*b*c^6*d^3*f^4*z + 24*A^3*a*b^5*c^2*d*f^6*z - 4*B^3*a*
b*c^6*d^2*e^5*z + 20*A^2*B*c^8*d^4*e^2*f*z - 7*A^2*B*b^4*c^4*e^6*f*z - 2*A^
2*B*b^7*c*e^3*f^4*z - 64*A*B^2*a^5*c^3*e*f^6*z + 16*A*B^2*b*c^7*d^5*f^2*z -
 8*A^2*B*a^2*c^6*e^6*f*z - 2*A*B^2*b^7*c*d^2*f^5*z - 272*A^2*B*a^4*c^4*d*f^
6*z + 128*A^2*B*a*c^7*d^4*f^3*z + 9*A^2*B*b^2*c^6*d*e^6*z - 4*A*B^2*b^3*c^5
*d*e^6*z + 4*A*B^2*b*c^7*d^3*e^4*z + 8*A*B^2*a*c^7*d^2*e^5*z + 12*A^2*B*a^3
*b^4*c*f^7*z + 30*B^3*b^4*c^4*d^3*e^2*f^2*z + 8*B^3*b^5*c^3*d^2*e^3*f^2*z -
 2*B^3*b^6*c^2*d^2*e^2*f^3*z + 152*A^3*b^3*c^5*d^2*e^2*f^3*z - 108*A^3*b^2*
c^6*d^2*e^3*f^2*z + 48*B^3*a^3*c^5*d^2*e^2*f^3*z - 16*B^3*a^2*c^6*d^3*e^2*f
^2*z - 3*B^3*a^4*b^2*c^2*e^2*f^5*z - 120*B^3*a^2*b^2*c^4*d^3*f^4*z + 112*B^
3*a^3*b^2*c^3*d^2*f^5*z + 112*A^3*a^2*b^3*c^3*e^2*f^5*z + 12*A^3*a^2*b^2*c^
4*e^3*f^4*z - 120*A^3*a*c^7*d*e^5*f*z - 52*A^3*a*b*c^6*e^6*f*z + 10*A^3*a*b
^6*c*e*f^6*z - 2*A*B^2*b^8*d*e*f^5*z - 2*A^2*B*a*b^7*e*f^6*z - 24*A^2*B*a*c
^7*d*e^6*z + 2*A*B^2*a*b^7*d*f^6*z - 12*A^2*B*a*b*c^6*e^7*z - 2*A^3*b^7*c*d
*f^6*z - 4*A^3*b*c^7*d*e^6*z + 16*B^3*a^5*c^3*e^2*f^5*z + 11*B^3*b^6*c^2*d^
3*f^4*z - 11*A^3*b^4*c^4*e^5*f^2*z - 8*B^3*b^4*c^4*d^4*f^3*z - 4*B^3*b^2*c^
6*d^5*f^2*z + 4*B^3*a^4*c^4*e^4*f^3*z + 4*A^3*b^5*c^3*e^4*f^3*z - A^3*b^6*c
^2*e^3*f^4*z + 136*A^3*a^3*c^5*e^3*f^4*z + 68*A^3*a^2*c^6*e^5*f^2*z - 64*A^
3*b^3*c^5*d^3*f^4*z + 2*B^3*b^3*c^5*d^2*e^5*z - B^3*b^2*c^6*d^3*e^4*z + 96*
A^3*a^3*b^3*c^2*f^7*z + A*B^2*a^2*b^6*e*f^6*z + 32*A^3*c^8*d^4*e*f^2*z - 24
*A^3*c^8*d^3*e^3*f*z + 10*A^3*b^3*c^5*e^6*f*z + 2*A^3*b^7*c*e^2*f^5*z + 128
*A^3*a^4*c^4*e*f^6*z - 32*A^3*b*c^7*d^4*f^3*z - 4*B^3*a^2*c^6*d*e^6*z - B^3
*a^2*b^6*d*f^6*z - 128*A^3*a^4*b*c^3*f^7*z - 24*A^3*a^2*b^5*c*f^7*z - 16*A^
2*B*c^8*d^5*f^2*z - 4*A^2*B*c^8*d^3*e^4*z + 64*A^2*B*a^5*c^3*f^7*z + 2*A^2*
B*b^3*c^5*e^7*z + 4*A*B^2*a^2*c^6*e^7*z - A^2*B*a^2*b^6*f^7*z + 4*A^3*c^8*d
^2*e^5*z - 3*A^3*b^2*c^6*e^7*z + A^2*B*b^8*d*f^6*z - A^3*b^8*e*f^6*z + 16*A
^3*a*c^7*e^7*z + 2*A^3*a*b^7*f^7*z + A^2*B*b^8*e^2*f^5*z + B^3*b^8*d^2*f^5*
z - 48*A^2*B^2*a*b*c^4*d*e*f^4 + 28*A*B^3*a*b^2*c^3*d*e*f^4 - 16*A*B^3*a*b*
c^4*d*e^2*f^3 + 16*A^3*B*a*c^5*d*e*f^4 + 32*A^3*B*a*b*c^4*d*f^5 + 12*A^2*B^
2*b^3*c^3*d*e*f^4 + 5*A*B^3*b^2*c^4*d^2*e*f^3 + 4*A*B^3*b^3*c^3*d*e^2*f^3 +
 24*A^2*B^2*a*c^5*d*e^2*f^3 + 24*A^2*B^2*a^2*b*c^3*e*f^5 + 12*A^2*B^2*a*b*c
^4*e^3*f^3 - 6*A^2*B^2*a*b^3*c^2*e*f^5 + 4*A*B^3*a^2*b*c^3*e^2*f^4 + 3*A*B^
3*a^2*b^2*c^2*e*f^5 - 18*A^2*B^2*a*b^2*c^3*d*f^5 - 4*B^4*a^2*b*c^3*d*e*f^4
+ 4*B^4*a*b*c^4*d^2*e*f^3 - 6*A*B^3*b^4*c^2*d*e*f^4 + 4*A^3*B*b*c^5*d*e^2*f
^3 - 2*A^3*B*b^2*c^4*d*e*f^4 - 8*A*B^3*a^2*c^4*d*e*f^4 - 8*A*B^3*a*c^5*d^2*
e*f^3 + 26*A^3*B*a*b^2*c^3*e*f^5 + 8*A^3*B*a*b*c^4*e^2*f^4 + 32*A*B^3*a*b*c
^4*d^2*f^4 - 28*A*B^3*a^2*b*c^3*d*f^5 + 6*A*B^3*a*b^3*c^2*d*f^5 - 9*A^2*B^2
*b^2*c^4*d*e^2*f^3 - 18*A^2*B^2*a*b^2*c^3*e^2*f^4 - 4*A^3*B*c^6*d^2*e*f^3 -
 3*A^3*B*b^4*c^2*e*f^5 - 44*A^3*B*a^2*c^4*e*f^5 - 16*A^3*B*a*c^5*e^3*f^3 -
16*A*B^3*a^3*c^3*e*f^5 - 10*A^3*B*b^3*c^3*d*f^5 - 4*A^3*B*b*c^5*d^2*f^4 - 4
*A*B^3*b*c^5*d^3*f^3 - 28*A^3*B*a^2*b*c^3*f^6 + 6*A^3*B*a*b^3*c^2*f^6 - 4*A
^4*b*c^5*d*e*f^4 - 20*A^4*a*b*c^4*e*f^5 + 3*A^2*B^2*b^4*c^2*e^2*f^4 - 2*A^2
*B^2*b^3*c^3*e^3*f^3 + 12*A^2*B^2*a^2*c^4*e^2*f^4 + 9*A^2*B^2*b^2*c^4*d^2*f
^4 - 3*A^2*B^2*a^2*b^2*c^2*f^6 - 2*B^4*b^3*c^3*d^2*e*f^3 + 4*B^4*a^2*c^4*d*
e^2*f^3 - 10*B^4*a*b^2*c^3*d^2*f^4 - 3*B^4*a^2*b^2*c^2*d*f^5 + 3*A^3*B*b^2*
c^4*e^3*f^3 - 2*A^3*B*b^3*c^3*e^2*f^4 - 10*A*B^3*b^3*c^3*d^2*f^4 - 4*A*B^3*
a^2*c^4*e^3*f^3 + 3*A^2*B^2*b^4*c^2*d*f^5 + 36*A^2*B^2*a^2*c^4*d*f^5 - 24*A
^2*B^2*a*c^5*d^2*f^4 + 4*A^2*B^2*c^6*d^3*f^3 + 16*A^2*B^2*a^3*c^3*f^6 + 4*A
^4*b^3*c^3*e*f^5 + 16*B^4*a^3*c^3*d*f^5 + 16*A^4*a*c^5*e^2*f^4 + 8*A^4*b^2*
c^4*d*f^5 - 8*A^4*a*b^2*c^3*f^6 - 24*A^4*a*c^5*d*f^5 + 3*B^4*b^4*c^2*d^2*f^
4 - 3*A^4*b^2*c^4*e^2*f^4 + 4*A^4*c^6*d^2*f^4 + 36*A^4*a^2*c^4*f^6 + B^4*b^
2*c^4*d^3*f^3, z, k)*((64*a^9*c^4*e*f^8 - 64*a^6*c^7*e^7*f^2 - 64*a^7*c^6*e
^5*f^4 + 64*a^8*c^5*e^3*f^6 + 4*b^5*c^8*d^7*f^2 + 4*b^7*c^6*d^6*f^3 - 4*b^9
*c^4*d^5*f^4 - 4*b^11*c^2*d^4*f^5 - 612*a^2*b^5*c^6*d^5*f^4 - 712*a^2*b^7*c
^4*d^4*f^5 - 132*a^2*b^9*c^2*d^3*f^6 + 1696*a^3*b^3*c^7*d^5*f^4 + 2736*a^3*
b^5*c^5*d^4*f^5 + 896*a^3*b^7*c^3*d^3*f^6 - 5120*a^4*b^3*c^6*d^4*f^5 - 3140

*a^4*b^5*c^4*d^3*f^6 - 220*a^4*b^7*c^2*d^2*f^7 + 5664*a^5*b^3*c^5*d^3*f^6 +
1128*a^5*b^5*c^3*d^2*f^7 - 2560*a^6*b^3*c^4*d^2*f^7 + 4*a^3*b^6*c^4*e^7*f^
2 - 6*a^3*b^7*c^3*e^6*f^3 + 4*a^3*b^8*c^2*e^5*f^4 - 36*a^4*b^4*c^5*e^7*f^2
+ 57*a^4*b^5*c^4*e^6*f^3 - 37*a^4*b^6*c^3*e^5*f^4 + 7*a^4*b^7*c^2*e^4*f^5 +
96*a^5*b^2*c^6*e^7*f^2 - 168*a^5*b^3*c^5*e^6*f^3 + 100*a^5*b^4*c^4*e^5*f^4
- 3*a^5*b^5*c^3*e^4*f^5 - 10*a^5*b^6*c^2*e^3*f^6 - 48*a^6*b^2*c^5*e^5*f^4
- 56*a^6*b^3*c^4*e^4*f^5 + 36*a^6*b^4*c^3*e^3*f^6 - 13*a^6*b^5*c^2*e^2*f^7
- 64*a^7*b^2*c^4*e^3*f^6 + 56*a^7*b^3*c^3*e^2*f^7 - 1472*a^4*c^9*d^4*e^3*f^
2 - 1088*a^5*c^8*d^2*e^5*f^2 + 3584*a^5*c^8*d^3*e^3*f^3 - 3200*a^6*c^7*d^2*
e^3*f^4 - 2*b^7*c^6*d^5*e^2*f^2 - b^8*c^5*d^4*e^3*f^2 + 4*b^9*c^4*d^3*e^4*f
^2 - 5*b^9*c^4*d^4*e^2*f^3 - 6*b^10*c^3*d^3*e^3*f^3 + 4*b^11*c^2*d^3*e^2*f^
4 + 8*a*b^11*c*d^3*f^6 + 8*a^5*b^7*c*d*f^8 - 448*a^8*b*c^4*d*f^8 - 16*a^5*b
*c^7*e^8*f - a^6*b^6*c*e*f^8 + 128*a^5*c^8*d*e^7*f + 128*a^8*c^5*d*e*f^7 -
b^12*c*d^3*e*f^5 - 32*a*b^3*c^9*d^7*f^2 - 24*a*b^5*c^7*d^6*f^3 + 88*a*b^7*c
^5*d^5*f^4 + 88*a*b^9*c^3*d^4*f^5 + 64*a^2*b*c^10*d^7*f^2 + 128*a^3*b*c^9*d
^6*f^3 + 16*a^3*b^9*c*d^2*f^7 - 1600*a^4*b*c^8*d^5*f^4 + 3840*a^5*b*c^7*d^4
*f^5 - 4160*a^6*b*c^6*d^3*f^6 - 92*a^6*b^5*c^2*d*f^8 + 2176*a^7*b*c^5*d^2*f
^7 + 352*a^7*b^3*c^3*d*f^8 - a^3*b^5*c^5*e^8*f - a^3*b^9*c*e^4*f^5 + 8*a^4*
b^3*c^6*e^8*f + a^4*b^8*c*e^3*f^6 + a^5*b^7*c*e^2*f^7 + 144*a^6*b*c^6*e^6*f
^3 + 80*a^7*b*c^5*e^4*f^5 + 12*a^7*b^4*c^2*e*f^8 - 80*a^8*b*c^4*e^2*f^7 - 4
8*a^8*b^2*c^3*e*f^8 + 128*a^3*c^10*d^5*e^3*f - 448*a^3*c^10*d^6*e*f^2 + 256
*a^4*c^9*d^3*e^5*f + 2176*a^4*c^9*d^5*e*f^3 - 4160*a^5*c^8*d^4*e*f^4 + 896*
a^6*c^7*d*e^5*f^3 + 3840*a^6*c^7*d^3*e*f^5 + 896*a^7*c^6*d*e^3*f^5 - 1600*a
^7*c^6*d^2*e*f^6 - b^5*c^8*d^6*e^2*f + b^6*c^7*d^5*e^3*f - 5*b^6*c^7*d^6*e*
f^2 + b^7*c^6*d^4*e^4*f - b^8*c^5*d^3*e^5*f + 5*b^8*c^5*d^5*e*f^3 + 9*b^10*
c^3*d^4*e*f^4 + 8*a*b^3*c^9*d^6*e^2*f + 12*a*b^4*c^8*d^6*e*f^2 - 27*a*b^5*c
^7*d^4*e^4*f + 18*a*b^6*c^6*d^3*e^5*f - 154*a*b^6*c^6*d^5*e*f^3 + a*b^7*c^5
*d^2*e^6*f - 200*a*b^8*c^4*d^4*e*f^4 - 2*a*b^10*c^2*d^3*e*f^5 + a*b^11*c*d^
2*e^2*f^5 - 16*a^2*b*c^10*d^6*e^2*f + a^2*b^6*c^5*d*e^7*f + a^2*b^10*c*d*e^
3*f^5 - 19*a^2*b^10*c*d^2*e*f^6 - 304*a^3*b*c^9*d^4*e^4*f + 10*a^3*b^9*c*d*
e^2*f^6 - 304*a^4*b*c^8*d^2*e^6*f - 48*a^4*b^2*c^7*d*e^7*f - 160*a^5*b*c^7*
d*e^6*f^2 + 214*a^5*b^6*c^2*d*e*f^7 - 2048*a^6*b*c^6*d*e^4*f^4 - 792*a^6*b^
4*c^3*d*e*f^7 - 1824*a^7*b*c^5*d*e^2*f^6 + 928*a^7*b^2*c^4*d*e*f^7 + 78*a*b
^5*c^7*d^5*e^2*f^2 + 10*a*b^6*c^6*d^4*e^3*f^2 - 68*a*b^7*c^5*d^3*e^4*f^2 +
129*a*b^7*c^5*d^4*e^2*f^3 - 4*a*b^8*c^4*d^2*e^5*f^2 + 96*a*b^8*c^4*d^3*e^3*
f^3 + 6*a*b^9*c^3*d^2*e^4*f^3 - 52*a*b^9*c^3*d^3*e^2*f^4 - 4*a*b^10*c^2*d^2
*e^3*f^4 - 48*a^2*b^2*c^9*d^5*e^3*f + 144*a^2*b^2*c^9*d^6*e*f^2 + 168*a^2*b
^3*c^8*d^4*e^4*f - 80*a^2*b^4*c^7*d^3*e^5*f + 1128*a^2*b^4*c^7*d^5*e*f^3 -
27*a^2*b^5*c^6*d^2*e^6*f + 1481*a^2*b^6*c^5*d^4*e*f^4 - 4*a^2*b^7*c^4*d*e^6
*f^2 + 6*a^2*b^8*c^3*d*e^5*f^3 + 126*a^2*b^8*c^3*d^3*e*f^5 - 4*a^2*b^9*c^2*
d*e^4*f^4 + 992*a^3*b*c^9*d^5*e^2*f^2 + 32*a^3*b^2*c^8*d^3*e^5*f - 2912*a^3
*b^2*c^8*d^5*e*f^3 + 168*a^3*b^3*c^7*d^2*e^6*f - 4812*a^3*b^4*c^6*d^4*e*f^4
+ 22*a^3*b^5*c^5*d*e^6*f^2 - 68*a^3*b^6*c^4*d*e^5*f^3 - 860*a^3*b^6*c^4*d^
3*e*f^5 + 80*a^3*b^7*c^3*d*e^4*f^4 - 44*a^3*b^8*c^2*d*e^3*f^5 + 232*a^3*b^8
*c^2*d^2*e*f^6 + 1280*a^4*b*c^8*d^3*e^4*f^2 - 816*a^4*b*c^8*d^4*e^2*f^3 + 7
088*a^4*b^2*c^7*d^4*e*f^4 + 16*a^4*b^3*c^6*d*e^6*f^2 + 312*a^4*b^4*c^5*d*e^
5*f^3 + 2864*a^4*b^4*c^5*d^3*e*f^5 - 576*a^4*b^5*c^4*d*e^4*f^4 + 393*a^4*b^
6*c^3*d*e^3*f^5 - 995*a^4*b^6*c^3*d^2*e*f^6 - 78*a^4*b^7*c^2*d*e^2*f^6 + 99
2*a^5*b*c^7*d^2*e^4*f^3 - 3264*a^5*b*c^7*d^3*e^2*f^4 - 768*a^5*b^2*c^6*d*e^
5*f^3 - 5184*a^5*b^2*c^6*d^3*e*f^5 + 1792*a^5*b^3*c^5*d*e^4*f^4 - 1232*a^5*
b^4*c^4*d*e^3*f^5 + 1588*a^5*b^4*c^4*d^2*e*f^6 + 30*a^5*b^5*c^3*d*e^2*f^6 +
5008*a^6*b*c^6*d^2*e^2*f^5 + 976*a^6*b^2*c^5*d*e^3*f^5 - 16*a^6*b^2*c^5*d^
2*e*f^6 + 944*a^6*b^3*c^4*d*e^2*f^6 - 19*a^4*b^8*c*d*e*f^7 - 528*a^2*b^3*c^
8*d^5*e^2*f^2 - 124*a^2*b^4*c^7*d^4*e^3*f^2 + 432*a^2*b^5*c^6*d^3*e^4*f^2 -
843*a^2*b^5*c^6*d^4*e^2*f^3 + 83*a^2*b^6*c^5*d^2*e^5*f^2 - 722*a^2*b^6*c^5
*d^3*e^3*f^3 - 80*a^2*b^7*c^4*d^2*e^4*f^3 + 376*a^2*b^7*c^4*d^3*e^2*f^4 + 4
3*a^2*b^9*c^2*d^2*e^2*f^5 + 768*a^3*b^2*c^8*d^4*e^3*f^2 - 1216*a^3*b^3*c^7*
d^3*e^4*f^2 + 1832*a^3*b^3*c^7*d^4*e^2*f^3 - 540*a^3*b^4*c^6*d^2*e^5*f^2 +
2928*a^3*b^4*c^6*d^3*e^3*f^3 + 414*a^3*b^5*c^5*d^2*e^4*f^3 - 1740*a^3*b^5*c

^5*d^3*e^2*f^4 + 344*a^3*b^6*c^4*d^2*e^3*f^4 - 634*a^3*b^7*c^3*d^2*e^2*f^5
+ 1360*a^4*b^2*c^7*d^2*e^5*f^2 - 5664*a^4*b^2*c^7*d^3*e^3*f^3 - 1008*a^4*b^
3*c^6*d^2*e^4*f^3 + 4064*a^4*b^3*c^6*d^3*e^2*f^4 - 1928*a^4*b^4*c^5*d^2*e^3
*f^4 + 3065*a^4*b^5*c^4*d^2*e^2*f^5 + 4032*a^5*b^2*c^6*d^2*e^3*f^4 - 6376*a
^5*b^3*c^5*d^2*e^2*f^5)/(16*a^2*c^6*d^4 + a^4*b^4*f^4 + 16*a^4*c^4*e^4 + b^
4*c^4*d^4 + 16*a^6*c^2*f^4 + b^8*d^2*f^2 - 8*a*b^2*c^5*d^4 - 8*a^5*b^2*c*f^
4 + 2*a^2*b^6*d*f^3 - 2*a^3*b^5*e*f^3 - 64*a^3*c^5*d^3*f - 64*a^5*c^3*d*f^3
 - 2*b^5*c^3*d^3*e + 2*b^6*c^2*d^3*f + a^2*b^4*c^2*e^4 - 8*a^3*b^2*c^3*e^4
+ 32*a^3*c^5*d^2*e^2 + a^2*b^6*e^2*f^2 + 96*a^4*c^4*d^2*f^2 + b^6*c^2*d^2*e
^2 + 32*a^5*c^3*e^2*f^2 - 2*a*b^7*d*e*f^2 - 2*b^7*c*d^2*e*f + 54*a^2*b^4*c^
2*d^2*f^2 - 112*a^3*b^2*c^3*d^2*f^2 + 16*a*b^3*c^4*d^3*e - 2*a*b^5*c^2*d*e^
3 - 32*a^2*b*c^5*d^3*e - 32*a^3*b*c^4*d*e^3 - 20*a*b^4*c^3*d^3*f - 12*a*b^6
*c*d^2*f^2 - 20*a^3*b^4*c*d*f^3 - 2*a^2*b^5*c*e^3*f - 32*a^4*b*c^3*e^3*f +
16*a^4*b^3*c*e*f^3 - 32*a^5*b*c^2*e*f^3 - 64*a^4*c^4*d*e^2*f - 6*a*b^4*c^3*
d^2*e^2 + 16*a^2*b^3*c^3*d*e^3 + 64*a^2*b^2*c^4*d^3*f + 64*a^4*b^2*c^2*d*f^
3 + 16*a^3*b^3*c^2*e^3*f - 6*a^3*b^4*c*e^2*f^2 - 48*a^2*b^3*c^3*d^2*e*f - 3
6*a^2*b^4*c^2*d*e^2*f + 96*a^3*b^2*c^3*d*e^2*f - 48*a^3*b^3*c^2*d*e*f^2 + 4
*a*b^6*c*d*e^2*f + 18*a*b^5*c^2*d^2*e*f + 18*a^2*b^5*c*d*e*f^2 + 32*a^3*b*c
^4*d^2*e*f + 32*a^4*b*c^3*d*e*f^2) + (x*(128*a^9*c^4*f^9 - 2*a^6*b^6*c*f^9
- 640*a^8*c^5*d*f^8 + 96*a^5*c^8*e^8*f + 6*b^12*c*d^3*f^6 + 24*a^7*b^4*c^2*
f^9 - 96*a^8*b^2*c^3*f^9 + 128*a^2*c^11*d^7*f^2 - 640*a^3*c^10*d^6*f^3 + 11
52*a^4*c^9*d^5*f^4 - 640*a^5*c^8*d^4*f^5 - 640*a^6*c^7*d^3*f^6 + 1152*a^7*c
^6*d^2*f^7 + 288*a^6*c^7*e^6*f^3 + 416*a^7*c^6*e^4*f^5 + 352*a^8*c^5*e^2*f^
7 + 8*b^4*c^9*d^7*f^2 + 22*b^6*c^7*d^6*f^3 + 26*b^8*c^5*d^5*f^4 + 18*b^10*c
^3*d^4*f^5 + 672*a^2*b^2*c^9*d^6*f^3 + 1224*a^2*b^4*c^7*d^5*f^4 + 1202*a^2*
b^6*c^5*d^4*f^5 + 564*a^2*b^8*c^3*d^3*f^6 - 2048*a^3*b^2*c^8*d^5*f^4 - 2744
*a^3*b^4*c^6*d^4*f^5 - 1736*a^3*b^6*c^4*d^3*f^6 - 128*a^3*b^8*c^2*d^2*f^7 +
 2656*a^4*b^2*c^7*d^4*f^5 + 2648*a^4*b^4*c^5*d^3*f^6 + 570*a^4*b^6*c^3*d^2*
f^7 - 1344*a^5*b^2*c^6*d^3*f^6 - 904*a^5*b^4*c^4*d^2*f^7 - 160*a^6*b^2*c^5*
d^2*f^7 + 8*a^2*b^7*c^4*e^7*f^2 - 12*a^2*b^8*c^3*e^6*f^3 + 8*a^2*b^9*c^2*e^
5*f^4 - 90*a^3*b^5*c^5*e^7*f^2 + 132*a^3*b^6*c^4*e^6*f^3 - 76*a^3*b^7*c^3*e
^5*f^4 + 6*a^3*b^8*c^2*e^4*f^5 + 336*a^4*b^3*c^6*e^7*f^2 - 462*a^4*b^4*c^5*
e^6*f^3 + 164*a^4*b^5*c^4*e^5*f^4 + 106*a^4*b^6*c^3*e^4*f^5 - 56*a^4*b^7*c^
2*e^3*f^6 + 432*a^5*b^2*c^6*e^6*f^3 + 288*a^5*b^3*c^5*e^5*f^4 - 598*a^5*b^4
*c^4*e^4*f^5 + 102*a^5*b^5*c^3*e^3*f^6 + 90*a^5*b^6*c^2*e^2*f^7 + 720*a^6*b
^2*c^5*e^4*f^5 + 336*a^6*b^3*c^4*e^3*f^6 - 314*a^6*b^4*c^3*e^2*f^7 + 240*a^
7*b^2*c^4*e^2*f^7 + 64*a^3*c^10*d^5*e^2*f^2 - 768*a^4*c^9*d^3*e^4*f^2 + 416
*a^4*c^9*d^4*e^2*f^3 + 1856*a^5*c^8*d^2*e^4*f^3 - 1664*a^5*c^8*d^3*e^2*f^4
+ 2336*a^6*c^7*d^2*e^2*f^5 + 26*b^6*c^7*d^5*e^2*f^2 - 8*b^7*c^6*d^4*e^3*f^2
 - 10*b^8*c^5*d^3*e^4*f^2 + 58*b^8*c^5*d^4*e^2*f^3 + 8*b^9*c^4*d^2*e^5*f^2
- 12*b^9*c^4*d^3*e^3*f^3 - 12*b^10*c^3*d^2*e^4*f^3 + 36*b^10*c^3*d^3*e^2*f^
4 + 8*b^11*c^2*d^2*e^3*f^4 + 2*a^4*b^8*c*d*f^8 + 6*a^5*b^7*c*e*f^8 - 384*a^
8*b*c^4*e*f^8 - 64*a*b^2*c^10*d^7*f^2 - 216*a*b^4*c^8*d^6*f^3 - 300*a*b^6*c
^6*d^5*f^4 - 240*a*b^8*c^4*d^4*f^5 - 92*a*b^10*c^2*d^3*f^6 + 10*a^2*b^10*c*
d^2*f^7 - 12*a^5*b^6*c^2*d*f^8 - 40*a^6*b^4*c^3*d*f^8 + 384*a^7*b^2*c^4*d*f
^8 - 2*a^2*b^6*c^5*e^8*f - 2*a^2*b^10*c*e^4*f^5 + 22*a^3*b^4*c^6*e^8*f + 6*
a^3*b^9*c*e^3*f^6 - 80*a^4*b^2*c^7*e^8*f - 8*a^4*b^8*c*e^2*f^7 - 416*a^5*b*
c^7*e^7*f^2 - 960*a^6*b*c^6*e^5*f^4 - 72*a^6*b^5*c^2*e*f^8 - 928*a^7*b*c^5*
e^3*f^6 + 288*a^7*b^3*c^3*e*f^8 - 32*a^2*c^11*d^6*e^2*f + 32*a^3*c^10*d^4*e
^4*f + 160*a^4*c^9*d^2*e^6*f - 704*a^5*c^8*d*e^6*f^2 - 1536*a^6*c^7*d*e^4*f
^4 - 1472*a^7*c^6*d*e^2*f^6 - 2*b^4*c^9*d^6*e^2*f + 6*b^5*c^8*d^5*e^3*f - 2
4*b^5*c^8*d^6*e*f^2 - 8*b^6*c^7*d^4*e^4*f + 6*b^7*c^6*d^3*e^5*f - 58*b^7*c^
6*d^5*e*f^3 - 2*b^8*c^5*d^2*e^6*f - 60*b^9*c^4*d^4*e*f^4 - 26*b^11*c^2*d^3*
e*f^5 - 2*b^12*c*d^2*e^2*f^5 + 16*a*b^2*c^10*d^6*e^2*f - 48*a*b^3*c^9*d^5*e
^3*f + 192*a*b^3*c^9*d^6*e*f^2 + 66*a*b^4*c^8*d^4*e^4*f - 52*a*b^5*c^7*d^3*
e^5*f + 576*a*b^5*c^7*d^5*e*f^3 + 14*a*b^6*c^6*d^2*e^6*f + 718*a*b^7*c^5*d^
4*e*f^4 - 16*a*b^8*c^4*d*e^6*f^2 + 24*a*b^9*c^3*d*e^5*f^3 + 356*a*b^9*c^3*d
^3*e*f^5 - 16*a*b^10*c^2*d*e^4*f^4 + 96*a^2*b*c^10*d^5*e^3*f - 384*a^2*b*c^
10*d^6*e*f^2 - 42*a^2*b^5*c^6*d*e^7*f - 2*a^2*b^10*c*d*e^2*f^6 - 64*a^3*b*c

^9*d^3*e^5*f + 1792*a^3*b*c^9*d^5*e*f^3 + 144*a^3*b^3*c^7*d*e^7*f - 3712*a^4*b*c^8*d^4*e*f^4 + 14*a^4*b^7*c^2*d*e*f^7 + 2368*a^5*b*c^7*d*e^5*f^3 + 4608*a^5*b*c^7*d^3*e*f^5 + 192*a^5*b^5*c^3*d*e*f^7 + 3808*a^6*b*c^6*d*e^3*f^5 - 3712*a^6*b*c^6*d^2*e*f^6 - 1184*a^6*b^3*c^4*d*e*f^7 - 204*a*b^4*c^8*d^5*e^2*f^2 + 46*a*b^5*c^7*d^4*e^3*f^2 + 132*a*b^6*c^6*d^3*e^4*f^2 - 590*a*b^6*c^6*d^4*e^2*f^3 - 90*a*b^7*c^5*d^2*e^5*f^2 + 64*a*b^7*c^5*d^3*e^3*f^3 + 196*a*b^8*c^4*d^2*e^4*f^3 - 408*a*b^8*c^4*d^3*e^2*f^4 - 188*a*b^9*c^3*d^2*e^3*f^4 + 78*a*b^10*c^2*d^2*e^2*f^5 - 144*a^2*b^2*c^9*d^4*e^4*f + 128*a^2*b^3*c^8*d^3*e^5*f - 1824*a^2*b^3*c^8*d^5*e*f^3 - 6*a^2*b^4*c^7*d^2*e^6*f - 3096*a^2*b^5*c^6*d^4*e*f^4 + 190*a^2*b^6*c^5*d*e^6*f^2 - 316*a^2*b^7*c^4*d*e^5*f^3 - 1908*a^2*b^7*c^4*d^3*e*f^5 + 228*a^2*b^8*c^3*d*e^4*f^4 - 58*a^2*b^9*c^2*d*e^3*f^5 + 92*a^2*b^9*c^2*d^2*e*f^6 - 288*a^3*b*c^9*d^4*e^3*f^2 - 112*a^3*b^2*c^8*d^2*e^6*f + 5664*a^3*b^3*c^7*d^4*e*f^4 - 796*a^3*b^4*c^6*d*e^6*f^2 + 1524*a^3*b^5*c^5*d*e^5*f^3 + 5120*a^3*b^5*c^5*d^3*e*f^5 - 1176*a^3*b^6*c^4*d*e^4*f^4 + 240*a^3*b^7*c^3*d*e^3*f^5 - 116*a^3*b^7*c^3*d^2*e*f^6 + 68*a^3*b^8*c^2*d*e^2*f^6 + 192*a^4*b*c^8*d^2*e^5*f^2 + 2240*a^4*b*c^8*d^3*e^3*f^3 + 1344*a^4*b^2*c^7*d*e^6*f^2 - 3168*a^4*b^3*c^6*d*e^5*f^3 - 7232*a^4*b^3*c^6*d^3*e*f^5 + 2464*a^4*b^4*c^5*d*e^4*f^4 + 78*a^4*b^5*c^4*d*e^3*f^5 - 1160*a^4*b^5*c^4*d^2*e*f^6 - 574*a^4*b^6*c^3*d*e^2*f^6 - 4928*a^5*b*c^7*d^2*e^3*f^4 - 1152*a^5*b^2*c^6*d*e^4*f^4 - 2416*a^5*b^3*c^5*d*e^3*f^5 + 4096*a^5*b^3*c^5*d^2*e*f^6 + 1748*a^5*b^4*c^4*d*e^2*f^6 - 1280*a^6*b^2*c^5*d*e^2*f^6 + 4*a*b^7*c^5*d*e^7*f + 4*a*b^11*c*d*e^3*f^5 - 10*a*b^11*c*d^2*e*f^6 - 4*a^3*b^9*c*d*e*f^7 - 160*a^4*b*c^8*d*e^7*f + 1792*a^7*b*c^5*d*e*f^7 + 384*a^2*b^2*c^9*d^5*e^2*f^2 + 16*a^2*b^3*c^8*d^4*e^3*f^2 - 624*a^2*b^4*c^7*d^3*e^4*f^2 + 1962*a^2*b^4*c^7*d^4*e^2*f^3 + 348*a^2*b^5*c^6*d^2*e^5*f^2 + 204*a^2*b^5*c^6*d^3*e^3*f^3 - 1214*a^2*b^6*c^5*d^2*e^4*f^3 + 1636*a^2*b^6*c^5*d^3*e^2*f^4 + 1520*a^2*b^7*c^4*d^2*e^3*f^4 - 750*a^2*b^8*c^3*d^2*e^2*f^5 + 1216*a^3*b^2*c^8*d^3*e^4*f^2 - 2224*a^3*b^2*c^8*d^4*e^2*f^3 - 512*a^3*b^3*c^7*d^2*e^5*f^2 - 1632*a^3*b^3*c^7*d^3*e^3*f^3 + 3492*a^3*b^4*c^6*d^2*e^4*f^3 - 2824*a^3*b^4*c^6*d^3*e^2*f^4 - 5492*a^3*b^5*c^5*d^2*e^3*f^4 + 2868*a^3*b^6*c^4*d^2*e^2*f^5 - 4480*a^4*b^2*c^7*d^2*e^4*f^3 + 2432*a^4*b^2*c^7*d^3*e^2*f^4 + 8864*a^4*b^3*c^6*d^2*e^3*f^4 - 4206*a^4*b^4*c^5*d^2*e^2*f^5 + 432*a^5*b^2*c^6*d^2*e^2*f^5))/(16*a^2*c^6*d^4 + a^4*b^4*f^4 + 16*a^4*c^4*e^4 + b^4*c^4*d^4 + 16*a^6*c^2*f^4 + b^8*d^2*f^2 - 8*a*b^2*c^5*d^4 - 8*a^5*b^2*c*f^4 + 2*a^2*b^6*d*f^3 - 2*a^3*b^5*e*f^3 - 64*a^3*c^5*d^3*f - 64*a^5*c^3*d*f^3 - 2*b^5*c^3*d^3*e + 2*b^6*c^2*d^3*f + a^2*b^4*c^2*e^4 - 8*a^3*b^2*c^3*e^4 + 32*a^3*c^5*d^2*e^2 + a^2*b^6*e^2*f^2 + 96*a^4*c^4*d^2*f^2 + b^6*c^2*d^2*e^2 + 32*a^5*c^3*e^2*f^2 - 2*a*b^7*d*e*f^2 - 2*b^7*c*d^2*e*f + 54*a^2*b^4*c^2*d^2*f^2 - 112*a^3*b^2*c^3*d^2*f^2 + 16*a*b^3*c^4*d^3*e - 2*a*b^5*c^2*d*e^3 - 32*a^2*b*c^5*d^3*e - 32*a^3*b*c^4*d*e^3 - 20*a*b^4*c^3*d^3*f - 12*a*b^6*c*d^2*f^2 - 20*a^3*b^4*c*d*f^3 - 2*a^2*b^5*c*e^3*f - 32*a^4*b*c^3*e^3*f + 16*a^4*b^3*c*e*f^3 - 32*a^5*b*c^2*e*f^3 - 64*a^4*c^4*d*e^2*f - 6*a*b^4*c^3*d^2*e^2 + 16*a^2*b^3*c^3*d*e^3 + 64*a^2*b^2*c^4*d^3*f + 64*a^4*b^2*c^2*d*f^3 + 16*a^3*b^3*c^2*e^3*f - 6*a^3*b^4*c*e^2*f^2 - 48*a^2*b^3*c^3*d^2*e*f - 36*a^2*b^4*c^2*d*e^2*f + 96*a^3*b^2*c^3*d*e^2*f - 48*a^3*b^3*c^2*d*e*f^2 + 4*a*b^6*c*d*e^2*f + 18*a*b^5*c^2*d^2*e*f + 18*a^2*b^5*c*d*e*f^2 + 32*a^3*b*c^4*d^2*e*f + 32*a^4*b*c^3*d*e*f^2)) - (64*A*a^7*c^4*f^8 - A*a^4*b^6*c*f^8 + 32*A*a*c^10*d^6*f^2 - 352*A*a^6*c^5*d*f^7 - A*b^10*c*d^2*f^6 + 8*B*a^4*c^7*e^7*f - 64*B*a^7*c^4*e*f^7 + 12*A*a^5*b^4*c^2*f^8 - 48*A*a^6*b^2*c^3*f^8 - 224*A*a^2*c^9*d^5*f^3 + 640*A*a^3*c^8*d^4*f^4 - 960*A*a^4*c^7*d^3*f^5 + 800*A*a^5*c^6*d^2*f^6 - 40*A*a^4*c^7*e^6*f^2 - 80*A*a^5*c^6*e^4*f^4 + 24*A*a^6*c^5*e^2*f^6 - 8*A*b^2*c^9*d^6*f^2 - 16*A*b^4*c^7*d^5*f^3 - A*b^6*c^5*d^4*f^4 + 6*A*b^8*c^3*d^3*f^5 + 16*B*a^5*c^6*e^5*f^3 - 56*B*a^6*c^5*e^3*f^5 + 4*B*b^3*c^8*d^6*f^2 + 12*B*b^5*c^6*d^5*f^3 + 4*B*b^7*c^4*d^4*f^4 - 4*B*b^9*c^2*d^3*f^5 + 120*A*a*b^2*c^8*d^5*f^3 + 60*A*a*b^4*c^6*d^4*f^4 - 36*A*a*b^6*c^4*d^3*f^5 + 8*A*a*b^8*c^2*d^2*f^6 + 20*A*a^3*b^6*c^2*d*f^7 - 80*A*a^4*b^4*c^3*d*f^7 + 216*A*a^5*b^2*c^4*d*f^7 + 4*A*a*b^6*c^4*e^6*f^2 - 6*A*a*b^7*c^3*e^5*f^3 + 4*A*a*b^8*c^2*e^4*f^4 + 9*A*a^2*b^3*c^6*e^7*f + 2*A*a^2*b^8*c*e^2*f^6 + 88*A*a^4*b*c^6*e^5*f^3 + 172*A*a^5*b*c^5*e^3*f^5 - 92*B*a*b^3*c^7*d

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^5*f^3 - 72*B*a*b^5*c^5*d^4*f^4 + 20*B*a*b^7*c^3*d^3*f^5 + 176*B*a^2*b*c^8*
d^5*f^3 - 544*B*a^3*b*c^7*d^4*f^4 + 736*B*a^4*b*c^6*d^3*f^5 + 4*B*a^4*b^5*c
^2*d*f^7 - 464*B*a^5*b*c^5*d^2*f^6 - 44*B*a^5*b^3*c^3*d*f^7 - 2*B*a^3*b^2*c
^6*e^7*f - B*a^3*b^7*c*e^2*f^6 - 28*B*a^4*b*c^6*e^6*f^2 - 56*B*a^5*b*c^5*e^
4*f^4 - 12*B*a^5*b^4*c^2*e*f^7 + 36*B*a^6*b*c^4*e^2*f^6 + 48*B*a^6*b^2*c^3*
e*f^7 - 16*A*a^2*c^9*d^3*e^4*f + 48*A*a^4*c^7*d*e^4*f^3 - 168*A*a^5*c^6*d*e
^2*f^5 + 2*A*b^2*c^9*d^5*e^2*f - 3*A*b^3*c^8*d^4*e^3*f + 12*A*b^3*c^8*d^5*e
*f^2 - 4*A*b^7*c^4*d*e^5*f^2 - 12*A*b^7*c^4*d^3*e*f^4 + 6*A*b^8*c^3*d*e^4*f
^3 - 4*A*b^9*c^2*d*e^3*f^4 + 8*A*b^9*c^2*d^2*e*f^5 + 8*B*a^2*c^9*d^4*e^3*f
- 32*B*a^2*c^9*d^5*e*f^2 + 16*B*a^3*c^8*d^2*e^5*f + 64*B*a^3*c^8*d^4*e*f^3
+ 64*B*a^4*c^7*d^3*e*f^4 + 96*B*a^5*c^6*d*e^3*f^4 - 256*B*a^5*c^6*d^2*e*f^5
 - B*b^3*c^8*d^5*e^2*f + 2*B*b^4*c^7*d^4*e^3*f - 8*B*b^4*c^7*d^5*e*f^2 - B*
b^6*c^5*d^2*e^5*f - 3*B*b^6*c^5*d^4*e*f^3 + 12*B*b^8*c^3*d^3*e*f^4 - 384*A*
a^2*b^2*c^7*d^4*f^4 - 32*A*a^2*b^4*c^5*d^3*f^5 - 14*A*a^2*b^6*c^3*d^2*f^6 +
 560*A*a^3*b^2*c^6*d^3*f^5 + 56*A*a^3*b^4*c^4*d^2*f^6 - 456*A*a^4*b^2*c^5*d
^2*f^6 - 38*A*a^2*b^4*c^5*e^6*f^2 + 58*A*a^2*b^5*c^4*e^5*f^3 - 36*A*a^2*b^6
*c^3*e^4*f^4 + 5*A*a^2*b^7*c^2*e^3*f^5 + 98*A*a^3*b^2*c^6*e^6*f^2 - 158*A*a
^3*b^3*c^5*e^5*f^3 + 80*A*a^3*b^4*c^4*e^4*f^4 + 22*A*a^3*b^5*c^3*e^3*f^5 -
22*A*a^3*b^6*c^2*e^2*f^6 + 20*A*a^4*b^2*c^5*e^4*f^4 - 147*A*a^4*b^3*c^4*e^3
*f^5 + 80*A*a^4*b^4*c^3*e^2*f^6 - 102*A*a^5*b^2*c^4*e^2*f^6 + 360*B*a^2*b^3
*c^6*d^4*f^4 + 64*B*a^2*b^5*c^4*d^3*f^5 - 504*B*a^3*b^3*c^5*d^3*f^5 - 40*B*
a^3*b^5*c^3*d^2*f^6 + 276*B*a^4*b^3*c^4*d^2*f^6 + 7*B*a^3*b^3*c^5*e^6*f^2 -
 8*B*a^3*b^4*c^4*e^5*f^3 + 2*B*a^3*b^5*c^3*e^4*f^4 + 2*B*a^3*b^6*c^2*e^3*f^
5 + 28*B*a^4*b^2*c^5*e^5*f^3 + 6*B*a^4*b^3*c^4*e^4*f^4 - 26*B*a^4*b^4*c^3*e
^3*f^5 + 11*B*a^4*b^5*c^2*e^2*f^6 + 86*B*a^5*b^2*c^4*e^3*f^5 - 37*B*a^5*b^3
*c^3*e^2*f^6 + 120*A*a^2*c^9*d^4*e^2*f^2 + 48*A*a^3*c^8*d^2*e^4*f^2 - 336*A
*a^3*c^8*d^3*e^2*f^3 + 368*A*a^4*c^7*d^2*e^2*f^4 + 4*A*b^4*c^7*d^4*e^2*f^2
- 5*A*b^6*c^5*d^2*e^4*f^2 + 6*A*b^6*c^5*d^3*e^2*f^3 + 16*A*b^7*c^4*d^2*e^3*
f^3 - 18*A*b^8*c^3*d^2*e^2*f^4 - 32*B*a^3*c^8*d^3*e^3*f^2 - 16*B*a^4*c^7*d^
2*e^3*f^3 - 3*B*b^5*c^6*d^4*e^2*f^2 + 4*B*b^6*c^5*d^3*e^3*f^2 + 4*B*b^7*c^4
*d^2*e^4*f^2 - 12*B*b^7*c^4*d^3*e^2*f^3 - 6*B*b^8*c^3*d^2*e^3*f^3 + 4*B*b^9
*c^2*d^2*e^2*f^4 - 2*A*a^2*b^8*c*d*f^7 - A*a*b^5*c^5*e^7*f - A*a*b^9*c*e^3*
f^5 - 20*A*a^3*b*c^7*e^7*f - 16*B*a*b*c^9*d^6*f^2 + 112*B*a^6*b*c^4*d*f^7 +
 B*a^4*b^6*c*e*f^7 - 8*A*a*c^10*d^5*e^2*f - 8*A*a^3*c^8*d*e^6*f + A*b^6*c^5
*d*e^6*f + A*b^10*c*d*e^2*f^5 + 224*B*a^6*c^5*d*e*f^6 - B*b^10*c*d^2*e*f^5
+ 12*A*a*b*c^9*d^4*e^3*f - 48*A*a*b*c^9*d^5*e*f^2 - 10*A*a*b^4*c^6*d*e^6*f
+ 80*A*a^5*b*c^5*d*e*f^6 + 4*B*a*b*c^9*d^5*e^2*f + B*a*b^5*c^5*d*e^6*f + B*
a*b^9*c*d*e^2*f^5 + 4*B*a^3*b*c^7*d*e^6*f - 92*A*a^2*b^2*c^7*d^2*e^4*f^2 +
132*A*a^2*b^2*c^7*d^3*e^2*f^3 + 446*A*a^2*b^3*c^6*d^2*e^3*f^3 - 604*A*a^2*b
^4*c^5*d^2*e^2*f^4 + 340*A*a^3*b^2*c^6*d^2*e^2*f^4 + 72*B*a^2*b^2*c^7*d^3*e
^3*f^2 + 134*B*a^2*b^3*c^6*d^2*e^4*f^2 - 306*B*a^2*b^3*c^6*d^3*e^2*f^3 - 26
4*B*a^2*b^4*c^5*d^2*e^3*f^3 + 188*B*a^2*b^5*c^4*d^2*e^2*f^4 + 292*B*a^3*b^2
*c^6*d^2*e^3*f^3 + 6*B*a^3*b^3*c^5*d^2*e^2*f^4 + 4*A*a*b^2*c^8*d^3*e^4*f +
2*A*a*b^3*c^7*d^2*e^5*f - 16*A*a*b^3*c^7*d^4*e*f^3 + 48*A*a*b^5*c^5*d*e^5*f
^2 + 72*A*a*b^5*c^5*d^3*e*f^4 - 84*A*a*b^6*c^4*d*e^4*f^3 + 64*A*a*b^7*c^3*d
*e^3*f^4 - 88*A*a*b^7*c^3*d^2*e*f^5 - 18*A*a*b^8*c^2*d*e^2*f^5 - 8*A*a^2*b*
c^8*d^2*e^5*f + 64*A*a^2*b*c^8*d^4*e*f^3 + 26*A*a^2*b^2*c^7*d*e^6*f + 16*A*
a^2*b^7*c^2*d*e*f^6 + 192*A*a^3*b*c^7*d*e^5*f^2 + 96*A*a^3*b*c^7*d^3*e*f^4
- 136*A*a^3*b^5*c^3*d*e*f^6 + 80*A*a^4*b*c^6*d*e^3*f^4 - 192*A*a^4*b*c^6*d^
2*e*f^5 + 268*A*a^4*b^3*c^4*d*e*f^6 - 10*B*a*b^2*c^8*d^4*e^3*f + 40*B*a*b^2
*c^8*d^5*e*f^2 - 2*B*a*b^3*c^7*d^3*e^4*f + 8*B*a*b^4*c^6*d^2*e^5*f + 36*B*a
*b^4*c^6*d^4*e*f^3 - 4*B*a*b^6*c^4*d*e^5*f^2 - 88*B*a*b^6*c^4*d^3*e*f^4 + 6
*B*a*b^7*c^3*d*e^4*f^3 - 4*B*a*b^8*c^2*d*e^3*f^4 + 16*B*a*b^8*c^2*d^2*e*f^5
 + 8*B*a^2*b*c^8*d^3*e^4*f - 5*B*a^2*b^3*c^6*d*e^6*f - 8*B*a^3*b^6*c^2*d*e*
f^6 + 40*B*a^4*b*c^6*d*e^4*f^3 + 104*B*a^4*b^4*c^3*d*e*f^6 + 148*B*a^5*b*c^
5*d*e^2*f^5 - 344*B*a^5*b^2*c^4*d*e*f^6 - 46*A*a*b^2*c^8*d^4*e^2*f^2 - 4*A*
a*b^3*c^7*d^3*e^3*f^2 + 40*A*a*b^4*c^6*d^2*e^4*f^2 - 36*A*a*b^4*c^6*d^3*e^2
*f^3 - 158*A*a*b^5*c^5*d^2*e^3*f^3 + 196*A*a*b^6*c^4*d^2*e^2*f^4 + 16*A*a^2
*b*c^8*d^3*e^3*f^2 - 176*A*a^2*b^3*c^6*d*e^5*f^2 - 120*A*a^2*b^3*c^6*d^3*e*
```

f^4 + 380*A*a^2*b^4*c^5*d*e^4*f^3 - 324*A*a^2*b^5*c^4*d*e^3*f^4 + 272*A*a^2
*b^5*c^4*d^2*e*f^5 + 80*A*a^2*b^6*c^3*d*e^2*f^5 - 280*A*a^3*b*c^7*d^2*e^3*f
^3 - 572*A*a^3*b^2*c^6*d*e^4*f^3 + 508*A*a^3*b^3*c^5*d*e^3*f^4 - 144*A*a^3*
b^3*c^5*d^2*e*f^5 - 4*A*a^3*b^4*c^4*d*e^2*f^5 - 326*A*a^4*b^2*c^5*d*e^2*f^5
 + 31*B*a*b^3*c^7*d^4*e^2*f^2 - 32*B*a*b^4*c^6*d^3*e^3*f^2 - 40*B*a*b^5*c^5
*d^2*e^4*f^2 + 102*B*a*b^5*c^5*d^3*e^2*f^3 + 72*B*a*b^6*c^4*d^2*e^3*f^3 - 5
6*B*a*b^7*c^3*d^2*e^2*f^4 - 76*B*a^2*b*c^8*d^4*e^2*f^2 - 20*B*a^2*b^2*c^7*d
^2*e^5*f - 112*B*a^2*b^2*c^7*d^4*e*f^3 + 24*B*a^2*b^4*c^5*d*e^5*f^2 + 192*B
*a^2*b^4*c^5*d^3*e*f^4 - 42*B*a^2*b^5*c^4*d*e^4*f^3 + 32*B*a^2*b^6*c^3*d*e^
3*f^4 - 38*B*a^2*b^6*c^3*d^2*e*f^5 - 9*B*a^2*b^7*c^2*d*e^2*f^5 - 152*B*a^3*
b*c^7*d^2*e^4*f^2 + 360*B*a^3*b*c^7*d^3*e^2*f^3 - 32*B*a^3*b^2*c^6*d*e^5*f^
2 - 144*B*a^3*b^2*c^6*d^3*e*f^4 + 62*B*a^3*b^3*c^5*d*e^4*f^3 - 40*B*a^3*b^4
*c^4*d*e^3*f^4 - 152*B*a^3*b^4*c^4*d^2*e*f^5 + 14*B*a^3*b^5*c^3*d*e^2*f^5 -
 472*B*a^4*b*c^6*d^2*e^2*f^4 - 120*B*a^4*b^2*c^5*d*e^3*f^4 + 512*B*a^4*b^2*
c^5*d^2*e*f^5 - 13*B*a^4*b^3*c^4*d*e^2*f^5)/(16*a^2*c^6*d^4 + a^4*b^4*f^4 +
 16*a^4*c^4*e^4 + b^4*c^4*d^4 + 16*a^6*c^2*f^4 + b^8*d^2*f^2 - 8*a*b^2*c^5*
d^4 - 8*a^5*b^2*c*f^4 + 2*a^2*b^6*d*f^3 - 2*a^3*b^5*e*f^3 - 64*a^3*c^5*d^3*
f - 64*a^5*c^3*d*f^3 - 2*b^5*c^3*d^3*e + 2*b^6*c^2*d^3*f + a^2*b^4*c^2*e^4
 - 8*a^3*b^2*c^3*e^4 + 32*a^3*c^5*d^2*e^2 + a^2*b^6*e^2*f^2 + 96*a^4*c^4*d^2
*f^2 + b^6*c^2*d^2*e^2 + 32*a^5*c^3*e^2*f^2 - 2*a*b^7*d*e*f^2 - 2*b^7*c*d^2
*e*f + 54*a^2*b^4*c^2*d^2*f^2 - 112*a^3*b^2*c^3*d^2*f^2 + 16*a*b^3*c^4*d^3*
e - 2*a*b^5*c^2*d*e^3 - 32*a^2*b*c^5*d^3*e - 32*a^3*b*c^4*d*e^3 - 20*a*b^4*
c^3*d^3*f - 12*a*b^6*c*d^2*f^2 - 20*a^3*b^4*c*d*f^3 - 2*a^2*b^5*c*e^3*f - 3
2*a^4*b*c^3*e^3*f + 16*a^4*b^3*c*e*f^3 - 32*a^5*b*c^2*e*f^3 - 64*a^4*c^4*d*
e^2*f - 6*a*b^4*c^3*d^2*e^2 + 16*a^2*b^3*c^3*d*e^3 + 64*a^2*b^2*c^4*d^3*f +
 64*a^4*b^2*c^2*d*f^3 + 16*a^3*b^3*c^2*e^3*f - 6*a^3*b^4*c*e^2*f^2 - 48*a^2
*b^3*c^3*d^2*e*f - 36*a^2*b^4*c^2*d*e^2*f + 96*a^3*b^2*c^3*d*e^2*f - 48*a^3
*b^3*c^2*d*e*f^2 + 4*a*b^6*c*d*e^2*f + 18*a*b^5*c^2*d^2*e*f + 18*a^2*b^5*c*
d*e*f^2 + 32*a^3*b*c^4*d^2*e*f + 32*a^4*b*c^3*d*e*f^2) + (x*(64*B*a^7*c^4*f
^8 + 4*A*a^3*b^7*c*f^8 - 256*A*a^6*b*c^4*f^8 - B*a^4*b^6*c*f^8 + 48*A*a^3*c
^8*e^7*f + 256*A*a^6*c^5*e*f^7 - 320*B*a^6*c^5*d*f^7 + 3*B*b^10*c*d^2*f^6 -
 48*A*a^4*b^5*c^2*f^8 + 192*A*a^5*b^3*c^3*f^8 + 12*B*a^5*b^4*c^2*f^8 - 48*B
*a^6*b^2*c^3*f^8 + 256*A*a^4*c^7*e^5*f^3 + 464*A*a^5*c^6*e^3*f^5 - 16*A*b^3
*c^8*d^5*f^3 - 48*A*b^5*c^6*d^4*f^4 - 36*A*b^7*c^4*d^3*f^5 - 4*A*b^9*c^2*d^
2*f^6 - 64*B*a^2*c^9*d^5*f^3 + 320*B*a^3*c^8*d^4*f^4 - 640*B*a^4*c^7*d^3*f^
5 + 640*B*a^5*c^6*d^2*f^6 - 16*B*a^4*c^7*e^6*f^2 - 64*B*a^5*c^6*e^4*f^4 + 1
6*B*a^6*c^5*e^2*f^6 + 4*B*b^4*c^7*d^5*f^3 + 23*B*b^6*c^5*d^4*f^4 + 22*B*b^8
*c^3*d^3*f^5 + 320*A*a*b^3*c^7*d^4*f^4 + 352*A*a*b^5*c^5*d^3*f^5 + 76*A*a*b
^7*c^3*d^2*f^6 - 512*A*a^2*b*c^8*d^4*f^4 - 60*A*a^2*b^7*c^2*d*f^7 + 1408*A*
a^3*b*c^7*d^3*f^5 + 352*A*a^3*b^5*c^3*d*f^7 - 1792*A*a^4*b*c^6*d^2*f^6 - 97
6*A*a^4*b^3*c^4*d*f^7 - 6*A*a*b^5*c^5*e^6*f^2 + 4*A*a*b^6*c^4*e^5*f^3 + 4*A
*a*b^7*c^3*e^4*f^4 - 6*A*a*b^8*c^2*e^3*f^5 - 20*A*a^2*b^2*c^7*e^7*f - 144*A
*a^3*b*c^7*e^6*f^2 + 68*A*a^3*b^6*c^2*e*f^7 - 640*A*a^4*b*c^6*e^4*f^4 - 240
*A*a^4*b^4*c^3*e*f^7 - 848*A*a^5*b*c^5*e^2*f^6 + 192*A*a^5*b^2*c^4*e*f^7 -
132*B*a*b^4*c^6*d^4*f^4 - 196*B*a*b^6*c^4*d^3*f^5 - 40*B*a*b^8*c^2*d^2*f^6
- 20*B*a^3*b^6*c^2*d*f^7 + 52*B*a^4*b^4*c^3*d*f^7 + 64*B*a^5*b^2*c^4*d*f^7
+ 2*B*a^2*b^3*c^6*e^7*f + B*a^2*b^8*c*e^2*f^6 + 16*B*a^4*b*c^6*e^5*f^3 + 12
0*B*a^5*b*c^5*e^3*f^5 + 64*A*a^2*c^9*d^2*e^5*f + 512*A*a^2*c^9*d^4*e*f^3 -
384*A*a^3*c^8*d*e^5*f^2 - 1408*A*a^3*c^8*d^3*e*f^4 - 1280*A*a^4*c^7*d*e^3*f
^4 + 1792*A*a^4*c^7*d^2*e*f^5 - 4*A*b^2*c^9*d^4*e^3*f + 16*A*b^2*c^9*d^5*e*
f^2 + 8*A*b^3*c^8*d^3*e^4*f - 2*A*b^4*c^7*d^2*e^5*f + 80*A*b^4*c^7*d^4*e*f^
3 + 6*A*b^6*c^5*d*e^5*f^2 + 76*A*b^6*c^5*d^3*e*f^4 - 4*A*b^7*c^4*d*e^4*f^3
- 4*A*b^8*c^3*d*e^3*f^4 - 2*A*b^8*c^3*d^2*e*f^5 + 6*A*b^9*c^2*d*e^2*f^5 - 3
2*B*a^2*c^9*d^3*e^4*f - 96*B*a^4*c^7*d*e^4*f^3 - 192*B*a^5*c^6*d*e^2*f^5 +
2*B*b^3*c^8*d^4*e^3*f - 8*B*b^3*c^8*d^5*e*f^2 - 6*B*b^4*c^7*d^3*e^4*f + 4*B
*b^5*c^6*d^2*e^5*f - 48*B*b^5*c^6*d^4*e*f^3 - 60*B*b^7*c^4*d^3*e*f^4 - 8*B*
b^9*c^2*d^2*e*f^5 + 4*A*a*b^9*c*d*f^7 - 2*A*b^10*c*d*e*f^6 - 1184*A*a^2*b^3
*c^6*d^3*f^5 - 544*A*a^2*b^5*c^4*d^2*f^6 + 1664*A*a^3*b^3*c^5*d^2*f^6 + 60*
A*a^2*b^3*c^6*e^6*f^2 - 30*A*a^2*b^4*c^5*e^5*f^3 - 64*A*a^2*b^5*c^4*e^4*f^4

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 + 72*A*a^2*b^6*c^3*e^3*f^5 - 12*A*a^2*b^7*c^2*e^2*f^6 - 8*A*a^3*b^2*c^6*e^
5*f^3 + 352*A*a^3*b^3*c^5*e^4*f^4 - 268*A*a^3*b^4*c^4*e^3*f^5 - 52*A*a^3*b^
5*c^3*e^2*f^6 + 188*A*a^4*b^2*c^5*e^3*f^5 + 484*A*a^4*b^3*c^4*e^2*f^6 + 80*
B*a^2*b^2*c^7*d^4*f^4 + 520*B*a^2*b^4*c^5*d^3*f^5 + 210*B*a^2*b^6*c^3*d^2*f
^6 - 192*B*a^3*b^2*c^6*d^3*f^5 - 456*B*a^3*b^4*c^4*d^2*f^6 + 96*B*a^4*b^2*c
^5*d^2*f^6 - 7*B*a^2*b^4*c^5*e^6*f^2 + 8*B*a^2*b^5*c^4*e^5*f^3 - 2*B*a^2*b^
6*c^3*e^4*f^4 - 2*B*a^2*b^7*c^2*e^3*f^5 + 32*B*a^3*b^2*c^6*e^6*f^2 - 36*B*a
^3*b^3*c^5*e^5*f^3 - 4*B*a^3*b^4*c^4*e^4*f^4 + 28*B*a^3*b^5*c^3*e^3*f^5 - 1
2*B*a^3*b^6*c^2*e^2*f^6 + 64*B*a^4*b^2*c^5*e^4*f^4 - 110*B*a^4*b^3*c^4*e^3*
f^5 + 47*B*a^4*b^4*c^3*e^2*f^6 - 64*B*a^5*b^2*c^4*e^2*f^6 - 384*A*a^2*c^9*d
^3*e^3*f^2 + 1184*A*a^3*c^8*d^2*e^3*f^3 - 28*A*b^3*c^8*d^4*e^2*f^2 - 12*A*b
^4*c^7*d^3*e^3*f^2 + 20*A*b^5*c^6*d^2*e^4*f^2 - 36*A*b^5*c^6*d^3*e^2*f^3 -
44*A*b^6*c^5*d^2*e^3*f^3 + 32*A*b^7*c^4*d^2*e^2*f^4 + 144*B*a^2*c^9*d^4*e^2
*f^2 + 192*B*a^3*c^8*d^2*e^4*f^2 - 448*B*a^3*c^8*d^3*e^2*f^3 + 480*B*a^4*c^
7*d^2*e^2*f^4 + 23*B*b^4*c^7*d^4*e^2*f^2 - 4*B*b^5*c^6*d^3*e^3*f^2 - 13*B*b
^6*c^5*d^2*e^4*f^2 + 48*B*b^6*c^5*d^3*e^2*f^3 + 12*B*b^7*c^4*d^2*e^3*f^3 +
2*B*b^8*c^3*d^2*e^2*f^4 + 64*A*a*b*c^9*d^5*f^3 + 1088*A*a^5*b*c^5*d*f^7 + 2
*A*a*b^4*c^6*e^7*f + 2*A*a*b^9*c*e^2*f^6 - 6*A*a^2*b^8*c*e*f^7 + 2*B*a^2*b^
8*c*d*f^7 - 8*B*a^3*b*c^7*e^7*f + 16*A*a*c^10*d^4*e^3*f - 64*A*a*c^10*d^5*e
*f^2 - 1088*A*a^5*c^6*d*e*f^6 - 2*A*b^5*c^6*d*e^6*f - 32*B*a^3*c^8*d*e^6*f
- 32*A*a*b*c^9*d^3*e^4*f + 24*A*a*b^3*c^7*d*e^6*f + 24*A*a*b^8*c^2*d*e*f^6
- 64*A*a^2*b*c^8*d*e^6*f - 8*B*a*b*c^9*d^4*e^3*f + 32*B*a*b*c^9*d^5*e*f^2 -
 6*B*a*b^4*c^6*d*e^6*f + 288*B*a^5*b*c^5*d*e*f^6 - 1496*A*a^2*b^2*c^7*d^2*e
^3*f^3 + 1496*A*a^2*b^3*c^6*d^2*e^2*f^4 - 272*B*a^2*b^2*c^7*d^2*e^4*f^2 + 6
40*B*a^2*b^2*c^7*d^3*e^2*f^3 + 636*B*a^2*b^3*c^6*d^2*e^3*f^3 - 286*B*a^2*b^
4*c^5*d^2*e^2*f^4 + 192*B*a^3*b^2*c^6*d^2*e^2*f^4 + 112*A*a*b*c^9*d^4*e^2*f
^2 - 8*A*a*b^2*c^8*d^2*e^5*f - 448*A*a*b^2*c^8*d^4*e*f^3 - 88*A*a*b^4*c^6*d
*e^5*f^2 - 576*A*a*b^4*c^6*d^3*e*f^4 + 84*A*a*b^5*c^5*d*e^4*f^3 + 32*A*a*b^
6*c^4*d*e^3*f^4 + 12*A*a*b^6*c^4*d^2*e*f^5 - 80*A*a*b^7*c^3*d*e^2*f^5 - 108
*A*a^2*b^6*c^3*d*e*f^6 + 736*A*a^3*b*c^7*d*e^4*f^3 + 192*A*a^3*b^4*c^4*d*e*
f^6 + 2048*A*a^4*b*c^6*d*e^2*f^5 + 208*A*a^4*b^2*c^5*d*e*f^6 + 32*B*a*b^2*c
^8*d^3*e^4*f - 20*B*a*b^3*c^7*d^2*e^5*f + 288*B*a*b^3*c^7*d^4*e*f^3 + 20*B*
a*b^5*c^5*d*e^5*f^2 + 480*B*a*b^5*c^5*d^3*e*f^4 - 20*B*a*b^6*c^4*d*e^4*f^3
+ 72*B*a*b^7*c^3*d^2*e*f^5 + 10*B*a*b^8*c^2*d*e^2*f^5 + 16*B*a^2*b*c^8*d^2*
e^5*f - 384*B*a^2*b*c^8*d^4*e*f^3 + 32*B*a^2*b^2*c^7*d*e^6*f + 44*B*a^2*b^7
*c^2*d*e*f^6 + 192*B*a^3*b*c^7*d*e^5*f^2 + 960*B*a^3*b*c^7*d^3*e*f^4 - 160*
B*a^3*b^5*c^3*d*e*f^6 + 544*B*a^4*b*c^6*d*e^3*f^4 - 896*B*a^4*b*c^6*d^2*e*f
^5 + 120*B*a^4*b^3*c^4*d*e*f^6 - 4*B*a*b^9*c*d*e*f^6 + 144*A*a*b^2*c^8*d^3*
e^3*f^2 - 144*A*a*b^3*c^7*d^2*e^4*f^2 + 112*A*a*b^3*c^7*d^3*e^2*f^3 + 476*A
*a*b^4*c^6*d^2*e^3*f^3 - 412*A*a*b^5*c^5*d^2*e^2*f^4 + 256*A*a^2*b*c^8*d^2*
e^4*f^2 + 128*A*a^2*b*c^8*d^3*e^2*f^3 + 352*A*a^2*b^2*c^7*d*e^5*f^2 + 1440*
A*a^2*b^2*c^7*d^3*e*f^4 - 456*A*a^2*b^3*c^6*d*e^4*f^3 - 116*A*a^2*b^4*c^5*d
*e^3*f^4 + 224*A*a^2*b^4*c^5*d^2*e*f^5 + 452*A*a^2*b^5*c^4*d*e^2*f^5 - 1440
*A*a^3*b*c^7*d^2*e^2*f^4 + 528*A*a^3*b^2*c^6*d*e^3*f^4 - 1408*A*a^3*b^2*c^6
*d^2*e*f^5 - 1424*A*a^3*b^3*c^5*d*e^2*f^5 - 128*B*a*b^2*c^8*d^4*e^2*f^2 + 8
*B*a*b^3*c^7*d^3*e^3*f^2 + 108*B*a*b^4*c^6*d^2*e^4*f^2 - 324*B*a*b^4*c^6*d^
3*e^2*f^3 - 164*B*a*b^5*c^5*d^2*e^3*f^3 + 44*B*a*b^6*c^4*d^2*e^2*f^4 + 32*B
*a^2*b*c^8*d^3*e^3*f^2 - 128*B*a^2*b^3*c^6*d*e^5*f^2 - 1200*B*a^2*b^3*c^6*d
^3*e*f^4 + 142*B*a^2*b^4*c^5*d*e^4*f^3 + 20*B*a^2*b^5*c^4*d*e^3*f^4 - 304*B
*a^2*b^5*c^4*d^2*e*f^5 - 112*B*a^2*b^6*c^3*d*e^2*f^5 - 688*B*a^3*b*c^7*d^2*
e^3*f^3 - 224*B*a^3*b^2*c^6*d*e^4*f^3 - 216*B*a^3*b^3*c^5*d*e^3*f^4 + 800*B
*a^3*b^3*c^5*d^2*e*f^5 + 460*B*a^3*b^4*c^4*d*e^2*f^5 - 640*B*a^4*b^2*c^5*d*
e^2*f^5))/(16*a^2*c^6*d^4 + a^4*b^4*f^4 + 16*a^4*c^4*e^4 + b^4*c^4*d^4 + 16
*a^6*c^2*f^4 + b^8*d^2*f^2 - 8*a*b^2*c^5*d^4 - 8*a^5*b^2*c*f^4 + 2*a^2*b^6*
d*f^3 - 2*a^3*b^5*e*f^3 - 64*a^3*c^5*d^3*f - 64*a^5*c^3*d*f^3 - 2*b^5*c^3*d
^3*e + 2*b^6*c^2*d^3*f + a^2*b^4*c^2*e^4 - 8*a^3*b^2*c^3*e^4 + 32*a^3*c^5*d
^2*e^2 + a^2*b^6*e^2*f^2 + 96*a^4*c^4*d^2*f^2 + b^6*c^2*d^2*e^2 + 32*a^5*c^
3*e^2*f^2 - 2*a*b^7*d*e*f^2 - 2*b^7*c*d^2*e*f + 54*a^2*b^4*c^2*d^2*f^2 - 11
2*a^3*b^2*c^3*d^2*f^2 + 16*a*b^3*c^4*d^3*e - 2*a*b^5*c^2*d*e^3 - 32*a^2*b*c
```

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^5*d^3*e - 32*a^3*b*c^4*d*e^3 - 20*a*b^4*c^3*d^3*f - 12*a*b^6*c*d^2*f^2 - 2
0*a^3*b^4*c*d*f^3 - 2*a^2*b^5*c*e^3*f - 32*a^4*b*c^3*e^3*f + 16*a^4*b^3*c*e
*f^3 - 32*a^5*b*c^2*e*f^3 - 64*a^4*c^4*d*e^2*f - 6*a*b^4*c^3*d^2*e^2 + 16*a
^2*b^3*c^3*d*e^3 + 64*a^2*b^2*c^4*d^3*f + 64*a^4*b^2*c^2*d*f^3 + 16*a^3*b^3
*c^2*e^3*f - 6*a^3*b^4*c*e^2*f^2 - 48*a^2*b^3*c^3*d^2*e*f - 36*a^2*b^4*c^2*
d*e^2*f + 96*a^3*b^2*c^3*d*e^2*f - 48*a^3*b^3*c^2*d*e*f^2 + 4*a*b^6*c*d*e^2
*f + 18*a*b^5*c^2*d^2*e*f + 18*a^2*b^5*c*d*e*f^2 + 32*a^3*b*c^4*d^2*e*f + 3
2*a^4*b*c^3*d*e*f^2)) - (56*A^2*a^3*b^3*c^3*f^7 - 13*A^2*a^2*b^5*c^2*f^7 -
24*A^2*a^2*c^7*e^5*f^2 - 16*A^2*b^3*c^6*d^3*f^4 - A^2*b^5*c^4*d^2*f^5 + 2*A
^2*b^4*c^5*e^5*f^2 - A^2*b^5*c^4*e^4*f^3 - A^2*b^6*c^3*e^3*f^4 + 2*A^2*b^7*
c^2*e^2*f^5 - 12*B^2*a^4*c^5*e^3*f^4 - 8*B^2*b^3*c^6*d^4*f^3 - 9*B^2*b^5*c^
4*d^3*f^4 + 64*A*B*a^5*c^4*f^7 + A^2*a*b^7*c*f^7 - A^2*b^8*c*e*f^6 - 80*A^2
*a^4*b*c^4*f^7 - 16*A^2*b*c^8*d^4*f^3 + 28*A^2*a^4*c^5*e*f^6 - 2*A^2*b^7*c^
2*d*f^6 - A^2*b^3*c^6*e^6*f - 16*B^2*a^5*c^4*e*f^6 - 4*A^2*c^9*d^3*e^3*f +
12*A^2*c^9*d^4*e*f^2 + 48*A^2*a*b*c^7*d^3*f^4 + 22*A^2*a*b^5*c^3*d*f^6 + 48
*A^2*a^3*b*c^5*d*f^6 + 12*A^2*a*b^6*c^2*e*f^6 + 16*B^2*a*b*c^7*d^4*f^3 + 64
*B^2*a^4*b*c^4*d*f^6 - 16*A^2*a*c^8*d^3*e*f^3 + 80*A^2*a^3*c^6*d*e*f^5 + 4*
A^2*b*c^8*d^2*e^4*f + A^2*b^2*c^7*d*e^5*f + 2*A^2*b^6*c^3*d*e*f^5 + 4*B^2*a
^2*c^7*d*e^5*f - 64*B^2*a^4*c^5*d*e*f^5 + A*B*b^8*c*d*f^6 + 8*A^2*a*b^3*c^5
*d^2*f^5 - 64*A^2*a^2*b^3*c^4*d*f^6 - 2*A^2*a*b^2*c^6*e^5*f^2 - 10*A^2*a*b^
3*c^5*e^4*f^3 + 20*A^2*a*b^4*c^4*e^3*f^4 - 25*A^2*a*b^5*c^3*e^2*f^5 + 56*A^
2*a^2*b*c^6*e^4*f^3 - 44*A^2*a^2*b^4*c^3*e*f^6 - 76*A^2*a^3*b*c^5*e^2*f^5 +
 40*A^2*a^3*b^2*c^4*e*f^6 + 56*B^2*a*b^3*c^5*d^3*f^4 - 2*B^2*a*b^5*c^3*d^2*
f^5 - 96*B^2*a^2*b*c^6*d^3*f^4 + 11*B^2*a^2*b^5*c^2*d*f^6 + 16*B^2*a^3*b*c^
5*d^2*f^5 - 40*B^2*a^3*b^3*c^3*d*f^6 + 16*B^2*a^4*b*c^4*e^2*f^5 + 3*B^2*a^4
*b^2*c^3*e*f^6 + 24*A^2*a*c^8*d^2*e^3*f^2 + 92*A^2*a^2*c^7*d*e^3*f^3 - 104*
A^2*a^2*c^7*d^2*e*f^4 - 4*A^2*b*c^8*d^3*e^2*f^2 + 20*A^2*b^2*c^7*d^3*e*f^3
 - A^2*b^4*c^5*d*e^3*f^3 - 8*A^2*b^4*c^5*d^2*e*f^4 + 32*B^2*a^2*c^7*d^3*e*f^
3 - 8*B^2*a^3*c^6*d*e^3*f^3 + 48*B^2*a^3*c^6*d^2*e*f^4 - B^2*b^2*c^7*d^3*e^
3*f + 3*B^2*b^2*c^7*d^4*e*f^2 + B^2*b^3*c^6*d^2*e^4*f + 8*B^2*b^4*c^5*d^3*e
*f^3 - 3*B^2*b^6*c^3*d^2*e*f^4 - A*B*a^2*b^6*c*f^7 - 32*A*B*a*c^8*d^4*f^3 -
 32*A*B*a^4*c^5*d*f^6 - 8*A*B*a^2*c^7*e^6*f + 4*A^2*a*b*c^7*e^6*f - B^2*a*b
^7*c*d*f^6 - 8*A^2*a*c^8*d*e^5*f - 65*A^2*a^2*b^2*c^5*e^3*f^4 + 88*A^2*a^2*
b^3*c^4*e^2*f^5 + 8*B^2*a^2*b^3*c^4*d^2*f^5 + 2*B^2*a^3*b^2*c^4*e^3*f^4 - 3
*B^2*a^3*b^3*c^3*e^2*f^5 - 13*A^2*b^2*c^7*d^2*e^3*f^2 + 16*A^2*b^3*c^6*d^2*
e^2*f^3 - 28*B^2*a^2*c^7*d^2*e^3*f^2 + B^2*b^3*c^6*d^3*e^2*f^2 - 5*B^2*b^4*
c^5*d^2*e^3*f^2 + 7*B^2*b^5*c^4*d^2*e^2*f^3 + 12*A*B*a^3*b^4*c^2*f^7 - 48*A
*B*a^4*b^2*c^3*f^7 + 160*A*B*a^2*c^7*d^3*f^4 - 160*A*B*a^3*c^6*d^2*f^5 - 16
*A*B*a^3*c^6*e^4*f^3 + 48*A*B*a^4*c^5*e^2*f^5 + 24*A*B*b^2*c^7*d^4*f^3 + 24
*A*B*b^4*c^5*d^3*f^4 - A*B*b^6*c^3*d^2*f^5 + 20*B^2*a*b^2*c^6*d^2*e^3*f^2 -
 49*B^2*a*b^3*c^5*d^2*e^2*f^3 + 96*B^2*a^2*b*c^6*d^2*e^2*f^3 + 19*B^2*a^2*b
^2*c^5*d*e^3*f^3 - 102*B^2*a^2*b^2*c^5*d^2*e*f^4 + 3*B^2*a^2*b^3*c^4*d*e^2*
f^4 - 120*A*B*a*b^2*c^6*d^3*f^4 + 4*A*B*a*b^4*c^4*d^2*f^5 + 24*A*B*a^2*b^4*
c^3*d*f^6 - 8*A*B*a^3*b^2*c^4*d*f^6 - 7*A*B*a*b^3*c^5*e^5*f^2 + 8*A*B*a*b^4
*c^4*e^4*f^3 - 2*A*B*a*b^5*c^3*e^3*f^4 - 2*A*B*a*b^6*c^2*e^2*f^5 + 28*A*B*a
^2*b*c^6*e^5*f^2 - 11*A*B*a^2*b^5*c^2*e*f^6 + 72*A*B*a^3*b*c^5*e^3*f^4 + 34
*A*B*a^3*b^3*c^3*e*f^6 + 16*A*B*a*c^8*d^3*e^2*f^2 + 80*A*B*a^2*c^7*d*e^4*f^
2 + 16*A*B*a^3*c^6*d*e^2*f^4 - 4*A*B*b^2*c^7*d^2*e^4*f - 22*A*B*b^3*c^6*d^3
*e*f^3 + 3*A*B*b^4*c^5*d*e^4*f^2 - 6*A*B*b^5*c^4*d*e^3*f^3 + 15*A*B*b^5*c^4
*d^2*e*f^4 + 4*A*B*b^6*c^3*d*e^2*f^4 + 12*A^2*a*b*c^7*d*e^4*f^2 - 28*A^2*a*
b^4*c^4*d*e*f^5 - 2*B^2*a*b^2*c^6*d*e^5*f + 2*B^2*a*b^6*c^2*d*e*f^5 + A*B*a
*b^7*c*e*f^6 + 24*A*B*a^2*b^2*c^5*d^2*f^5 - 28*A*B*a^2*b^2*c^5*e^4*f^3 - 9*
A*B*a^2*b^3*c^4*e^3*f^4 + 29*A*B*a^2*b^4*c^3*e^2*f^5 - 100*A*B*a^3*b^2*c^4*
e^2*f^5 - 208*A*B*a^2*c^7*d^2*e^2*f^3 + 13*A*B*b^3*c^6*d^2*e^3*f^2 - 23*A*B
*b^4*c^5*d^2*e^2*f^3 - 52*A^2*a*b*c^7*d^2*e^2*f^3 - 34*A^2*a*b^2*c^6*d*e^3*
f^3 + 48*A^2*a*b^2*c^6*d^2*e*f^4 + 40*A^2*a*b^3*c^5*d*e^2*f^4 - 108*A^2*a^2
*b*c^6*d*e^2*f^4 + 36*A^2*a^2*b^2*c^5*d*e*f^5 - 8*B^2*a*b*c^7*d^3*e^2*f^2 -
 24*B^2*a*b^2*c^6*d^3*e*f^3 + 7*B^2*a*b^3*c^5*d*e^4*f^2 - 8*B^2*a*b^4*c^4*d
*e^3*f^3 + 32*B^2*a*b^4*c^4*d^2*e*f^4 + 2*B^2*a*b^5*c^3*d*e^2*f^4 - 16*B^2*
```

a^2*b*c^6*d*e^4*f^2 - 20*B^2*a^2*b^4*c^3*d*e*f^5 + 8*B^2*a^3*b*c^5*d*e^2*f^
4 + 40*B^2*a^3*b^2*c^4*d*e*f^5 - 10*A*B*a*b^6*c^2*d*f^6 + 2*A*B*a*b^2*c^6*e
^6*f - 20*A*B*a^4*b*c^4*e*f^6 + 4*A*B*b*c^8*d^3*e^3*f - 12*A*B*b*c^8*d^4*e*
f^2 - 2*A*B*b^7*c^2*d*e*f^5 + 132*A*B*a*b^2*c^6*d^2*e^2*f^3 + 96*A*B*a^2*b^
2*c^5*d*e^2*f^4 + 24*A*B*a*b*c^7*d^3*e*f^3 + 20*A*B*a*b^5*c^3*d*e*f^5 - 24*
A*B*a^3*b*c^5*d*e*f^5 - 24*A*B*a*b*c^7*d^2*e^3*f^2 - 44*A*B*a*b^2*c^6*d*e^4
*f^2 + 84*A*B*a*b^3*c^5*d*e^3*f^3 - 122*A*B*a*b^3*c^5*d^2*e*f^4 - 54*A*B*a*
b^4*c^4*d*e^2*f^4 - 180*A*B*a^2*b*c^6*d*e^3*f^3 + 288*A*B*a^2*b*c^6*d^2*e*f
^4 - 18*A*B*a^2*b^3*c^4*d*e*f^5 + 4*A*B*a*b*c^7*d*e^5*f)/(16*a^2*c^6*d^4 +
a^4*b^4*f^4 + 16*a^4*c^4*e^4 + b^4*c^4*d^4 + 16*a^6*c^2*f^4 + b^8*d^2*f^2 -
8*a*b^2*c^5*d^4 - 8*a^5*b^2*c*f^4 + 2*a^2*b^6*d*f^3 - 2*a^3*b^5*e*f^3 - 64
*a^3*c^5*d^3*f - 64*a^5*c^3*d*f^3 - 2*b^5*c^3*d^3*e + 2*b^6*c^2*d^3*f + a^2
*b^4*c^2*e^4 - 8*a^3*b^2*c^3*e^4 + 32*a^3*c^5*d^2*e^2 + a^2*b^6*e^2*f^2 + 9
6*a^4*c^4*d^2*f^2 + b^6*c^2*d^2*e^2 + 32*a^5*c^3*e^2*f^2 - 2*a*b^7*d*e*f^2
- 2*b^7*c*d^2*e*f + 54*a^2*b^4*c^2*d^2*f^2 - 112*a^3*b^2*c^3*d^2*f^2 + 16*a
*b^3*c^4*d^3*e - 2*a*b^5*c^2*d*e^3 - 32*a^2*b*c^5*d^3*e - 32*a^3*b*c^4*d*e^
3 - 20*a*b^4*c^3*d^3*f - 12*a*b^6*c*d^2*f^2 - 20*a^3*b^4*c*d*f^3 - 2*a^2*b^
5*c*e^3*f - 32*a^4*b*c^3*e^3*f + 16*a^4*b^3*c*e*f^3 - 32*a^5*b*c^2*e*f^3 -
64*a^4*c^4*d*e^2*f - 6*a*b^4*c^3*d^2*e^2 + 16*a^2*b^3*c^3*d*e^3 + 64*a^2*b^
2*c^4*d^3*f + 64*a^4*b^2*c^2*d*f^3 + 16*a^3*b^3*c^2*e^3*f - 6*a^3*b^4*c*e^2
*f^2 - 48*a^2*b^3*c^3*d^2*e*f - 36*a^2*b^4*c^2*d*e^2*f + 96*a^3*b^2*c^3*d*e
^2*f - 48*a^3*b^3*c^2*d*e*f^2 + 4*a*b^6*c*d*e^2*f + 18*a*b^5*c^2*d^2*e*f +
18*a^2*b^5*c*d*e*f^2 + 32*a^3*b*c^4*d^2*e*f + 32*a^4*b*c^3*d*e*f^2) + (x*(1
04*A^2*a^4*c^5*f^7 - 32*B^2*a^5*c^4*f^7 + 8*A^2*c^9*d^4*f^3 + A^2*b^8*c*f^7
+ 50*A^2*a^2*b^4*c^3*f^7 - 96*A^2*a^3*b^2*c^4*f^7 - 12*B^2*a^3*b^4*c^2*f^7
+ 42*B^2*a^4*b^2*c^3*f^7 + 208*A^2*a^2*c^7*d^2*f^5 + 36*A^2*a^2*c^7*e^4*f^
3 + 72*A^2*a^3*c^6*e^2*f^5 + 8*A^2*b^2*c^7*d^3*f^4 + 18*A^2*b^4*c^5*d^2*f^5
- 32*B^2*a^2*c^7*d^3*f^4 + 32*B^2*a^3*c^6*d^2*f^5 - 2*A^2*b^3*c^6*e^5*f^2
+ A^2*b^4*c^5*e^4*f^3 + A^2*b^6*c^3*e^2*f^5 + 24*B^2*a^3*c^6*e^4*f^3 + 56*B
^2*a^4*c^5*e^2*f^5 + 2*B^2*b^2*c^7*d^4*f^3 - 6*B^2*b^4*c^5*d^3*f^4 + 9*B^2*
b^6*c^3*d^2*f^5 - 16*A^2*c^9*d^3*e^2*f^2 - 12*A^2*a*b^6*c^2*f^7 + B^2*a^2*b
^6*c*f^7 - 64*A^2*a*c^8*d^3*f^4 - 256*A^2*a^3*c^6*d*f^6 + 2*A^2*b^6*c^3*d*f
^6 + 32*B^2*a^4*c^5*d*f^6 + A^2*b^2*c^7*e^6*f - 2*A^2*b^7*c^2*e*f^6 + 4*B^2
*a^2*c^7*e^6*f + 4*A^2*c^9*d^2*e^4*f - 36*A^2*a*b^4*c^4*d*f^6 - 4*A^2*a*b*c
^7*e^5*f^2 + 22*A^2*a*b^5*c^3*e*f^6 - 16*A^2*a^3*b*c^5*e*f^6 - 2*B^2*a*b^6*
c^2*d*f^6 - 40*B^2*a^4*b*c^4*e*f^6 + 8*A^2*a*c^8*d*e^4*f^2 + 16*A^2*b*c^8*d
^3*e*f^3 + 6*A^2*b^5*c^4*d*e*f^5 - 2*A*B*a*b^7*c*f^7 - 144*A^2*a*b^2*c^6*d^
2*f^5 + 168*A^2*a^2*b^2*c^5*d*f^6 + 2*A^2*a*b^2*c^6*e^4*f^3 + 10*A^2*a*b^3*
c^5*e^3*f^4 - 18*A^2*a*b^4*c^4*e^2*f^5 - 80*A^2*a^2*b*c^6*e^3*f^4 - 56*A^2*
a^2*b^3*c^4*e*f^6 + 24*B^2*a*b^2*c^6*d^3*f^4 - 64*B^2*a*b^4*c^4*d^2*f^5 + 2
6*B^2*a^2*b^4*c^3*d*f^6 - 88*B^2*a^3*b^2*c^4*d*f^6 - 12*B^2*a^2*b*c^6*e^5*f
^2 - 40*B^2*a^3*b*c^5*e^3*f^4 + 6*B^2*a^3*b^3*c^3*e*f^6 + 8*A^2*a*c^8*d^2*e
^2*f^3 - 128*A^2*a^2*c^7*d*e^2*f^4 + 8*A^2*b*c^8*d^2*e^3*f^2 + 4*A^2*b^2*c^
7*d*e^4*f^2 + 10*A^2*b^3*c^6*d*e^3*f^3 - 18*A^2*b^4*c^5*d*e^2*f^4 - 32*B^2*
a^2*c^7*d*e^4*f^2 - 80*B^2*a^3*c^6*d*e^2*f^4 + B^2*b^2*c^7*d^2*e^4*f + 10*B
^2*b^3*c^6*d^3*e*f^3 - 12*B^2*b^5*c^4*d^2*e*f^4 + 72*A*B*a^4*b*c^4*f^7 - 8*
A*B*b*c^8*d^4*f^3 - 176*A*B*a^4*c^5*e*f^6 + 2*A*B*b^7*c^2*d*f^6 - 4*A^2*b*c
^8*d*e^5*f + 54*A^2*a^2*b^2*c^5*e^2*f^5 + 84*B^2*a^2*b^2*c^5*d^2*f^5 + 9*B^
2*a^2*b^2*c^5*e^4*f^3 + 2*B^2*a^3*b^2*c^4*e^2*f^5 - 26*A^2*b^2*c^7*d^2*e^2*
f^3 + 88*B^2*a^2*c^7*d^2*e^2*f^3 - 4*B^2*b^2*c^7*d^3*e^2*f^2 - 4*B^2*b^3*c^
6*d^2*e^3*f^2 + 8*B^2*b^4*c^5*d^2*e^2*f^3 + 24*A*B*a^2*b^5*c^2*f^7 - 84*A*B
*a^3*b^3*c^3*f^7 + 8*A*B*a^2*c^7*e^5*f^2 - 32*A*B*a^3*c^6*e^3*f^4 + 4*A*B*b
^3*c^6*d^3*f^4 - 20*A*B*b^5*c^4*d^2*f^5 - 4*B^2*a*b*c^7*d*e^5*f - 78*B^2*a*
b^2*c^6*d^2*e^2*f^3 - 48*B^2*a^2*b^2*c^5*d*e^2*f^4 + 148*A*B*a*b^3*c^5*d^2*
f^5 - 192*A*B*a^2*b*c^6*d^2*f^5 - 4*A*B*a^2*b^3*c^4*d*f^6 + 10*A*B*a*b^2*c^
6*e^5*f^2 - 2*A*B*a*b^3*c^5*e^4*f^3 - 12*A*B*a*b^4*c^4*e^3*f^4 + 8*A*B*a*b^
5*c^3*e^2*f^5 - 52*A*B*a^2*b*c^6*e^4*f^3 - 44*A*B*a^2*b^4*c^3*e*f^6 - 48*A*
B*a^3*b*c^5*e^2*f^5 + 204*A*B*a^3*b^2*c^4*e*f^6 - 48*A*B*a*c^8*d^2*e^3*f^2
- 48*A*B*a^2*c^7*d*e^3*f^3 - 16*A*B*a^2*c^7*d^2*e*f^4 + 16*A*B*b*c^8*d^3*e^

```
2*f^2 - 28*A*B*b^2*c^7*d^3*e*f^3 - 6*A*B*b^3*c^6*d*e^4*f^2 + 8*A*B*b^4*c^5*
d^2*e*f^4 + 12*A*B*b^5*c^4*d*e^2*f^4 - 40*A^2*a*b*c^7*d*e^3*f^3 + 80*A^2*a*
b*c^7*d^2*e*f^4 - 24*A^2*a*b^3*c^5*d*e*f^5 + 48*A^2*a^2*b*c^6*d*e*f^5 - 24*
B^2*a*b*c^7*d^3*e*f^3 - 8*B^2*a*b^5*c^3*d*e*f^5 + 56*B^2*a^3*b*c^5*d*e*f^5
 - 4*A*B*a*b*c^7*e^6*f + 8*A*B*a*c^8*d*e^5*f + 96*A*B*a^2*b^2*c^5*e^3*f^4 -
36*A*B*a^2*b^3*c^4*e^2*f^5 + 4*A*B*b^2*c^7*d^2*e^3*f^2 + 8*A*B*b^3*c^6*d^2*
e^2*f^3 + 84*A^2*a*b^2*c^6*d*e^2*f^4 + 24*B^2*a*b*c^7*d^2*e^3*f^2 + 14*B^2*
a*b^2*c^6*d*e^4*f^2 - 16*B^2*a*b^3*c^5*d*e^3*f^3 + 98*B^2*a*b^3*c^5*d^2*e*f
^4 + 12*B^2*a*b^4*c^4*d*e^2*f^4 + 64*B^2*a^2*b*c^6*d*e^3*f^3 - 120*B^2*a^2*
b*c^6*d^2*e*f^4 + 30*B^2*a^2*b^3*c^4*d*e*f^5 + 16*A*B*a*b*c^7*d^3*f^4 - 12*
A*B*a*b^5*c^3*d*f^6 + 112*A*B*a^3*b*c^5*d*f^6 + 2*A*B*a*b^6*c^2*e*f^6 + 48*
A*B*a*c^8*d^3*e*f^3 + 144*A*B*a^3*c^6*d*e*f^5 - 4*A*B*b*c^8*d^2*e^4*f + 2*A
*B*b^2*c^7*d*e^5*f - 10*A*B*b^6*c^3*d*e*f^5 + 100*A*B*a*b^4*c^4*d*e*f^5 + 6
4*A*B*a*b*c^7*d^2*e^2*f^3 + 12*A*B*a*b^2*c^6*d*e^3*f^3 - 108*A*B*a*b^2*c^6*
d^2*e*f^4 - 100*A*B*a*b^3*c^5*d*e^2*f^4 + 288*A*B*a^2*b*c^6*d*e^2*f^4 - 324
*A*B*a^2*b^2*c^5*d*e*f^5))/(16*a^2*c^6*d^4 + a^4*b^4*f^4 + 16*a^4*c^4*e^4 +
 b^4*c^4*d^4 + 16*a^6*c^2*f^4 + b^8*d^2*f^2 - 8*a*b^2*c^5*d^4 - 8*a^5*b^2*c
*f^4 + 2*a^2*b^6*d*f^3 - 2*a^3*b^5*e*f^3 - 64*a^3*c^5*d^3*f - 64*a^5*c^3*d*
f^3 - 2*b^5*c^3*d^3*e + 2*b^6*c^2*d^3*f + a^2*b^4*c^2*e^4 - 8*a^3*b^2*c^3*e
^4 + 32*a^3*c^5*d^2*e^2 + a^2*b^6*e^2*f^2 + 96*a^4*c^4*d^2*f^2 + b^6*c^2*d^
2*e^2 + 32*a^5*c^3*e^2*f^2 - 2*a*b^7*d*e*f^2 - 2*b^7*c*d^2*e*f + 54*a^2*b^4
*c^2*d^2*f^2 - 112*a^3*b^2*c^3*d^2*f^2 + 16*a*b^3*c^4*d^3*e - 2*a*b^5*c^2*d
*e^3 - 32*a^2*b*c^5*d^3*e - 32*a^3*b*c^4*d*e^3 - 20*a*b^4*c^3*d^3*f - 12*a*
b^6*c*d^2*f^2 - 20*a^3*b^4*c*d*f^3 - 2*a^2*b^5*c*e^3*f - 32*a^4*b*c^3*e^3*f
 + 16*a^4*b^3*c*e*f^3 - 32*a^5*b*c^2*e*f^3 - 64*a^4*c^4*d*e^2*f - 6*a*b^4*c
^3*d^2*e^2 + 16*a^2*b^3*c^3*d*e^3 + 64*a^2*b^2*c^4*d^3*f + 64*a^4*b^2*c^2*d
*f^3 + 16*a^3*b^3*c^2*e^3*f - 6*a^3*b^4*c*e^2*f^2 - 48*a^2*b^3*c^3*d^2*e*f
- 36*a^2*b^4*c^2*d*e^2*f + 96*a^3*b^2*c^3*d*e^2*f - 48*a^3*b^3*c^2*d*e*f^2
+ 4*a*b^6*c*d*e^2*f + 18*a*b^5*c^2*d^2*e*f + 18*a^2*b^5*c*d*e*f^2 + 32*a^3*
b*c^4*d^2*e*f + 32*a^4*b*c^3*d*e*f^2)) - (3*A^3*b^2*c^5*e^2*f^4 - 4*A^3*c^7
*d^2*f^4 - 12*A^3*a^2*c^5*f^6 - B^3*b^3*c^4*d^2*f^4 + 16*A^3*a*c^6*d*f^5 -
16*A*B^2*a^3*c^4*f^6 + 2*A^3*a*b^2*c^4*f^6 - 16*A^3*a*c^6*e^2*f^4 - 6*A^3*b
^2*c^5*d*f^5 - 3*A^3*b^3*c^4*e*f^5 + 4*B^3*a*b*c^5*d^2*f^4 + 3*B^3*a*b^3*c^
3*d*f^5 - 12*B^3*a^2*b*c^4*d*f^5 + 8*B^3*a^2*c^5*d*e*f^4 + 3*A*B^2*a^2*b^2*
c^3*f^6 - 12*A*B^2*a^2*c^5*e^2*f^4 + A*B^2*b^2*c^5*d^2*f^4 - 3*A^2*B*b^3*c^
4*e^2*f^4 + 16*A^3*a*b*c^5*e*f^5 + 4*A^3*b*c^6*d*e*f^4 - 3*A^2*B*a*b^3*c^3*
f^6 + 16*A^2*B*a^2*b*c^4*f^6 - 8*A*B^2*a*c^6*d^2*f^4 + 24*A*B^2*a^2*c^5*d*f
^5 - 3*A*B^2*b^4*c^3*d*f^5 + 4*A^2*B*b*c^6*d^2*f^4 - 8*A^2*B*a^2*c^5*e*f^5
+ 9*A^2*B*b^3*c^4*d*f^5 + 3*A^2*B*b^4*c^3*e*f^5 + 4*A*B^2*a*b^2*c^4*d*f^5 -
 3*A*B^2*a*b^3*c^3*e*f^5 + 16*A*B^2*a^2*b*c^4*e*f^5 + 16*A^2*B*a*b*c^5*e^2*
f^4 - 14*A^2*B*a*b^2*c^4*e*f^5 + 4*A*B^2*b^3*c^4*d*e*f^4 - 8*A^2*B*b^2*c^5*
d*e*f^4 - 2*B^3*a*b^2*c^4*d*e*f^4 + 2*A*B^2*a*b^2*c^4*e^2*f^4 - 28*A^2*B*a*
b*c^5*d*f^5 + 16*A^2*B*a*c^6*d*e*f^4 - 12*A*B^2*a*b*c^5*d*e*f^4)/(16*a^2*c^
6*d^4 + a^4*b^4*f^4 + 16*a^4*c^4*e^4 + b^4*c^4*d^4 + 16*a^6*c^2*f^4 + b^8*d
^2*f^2 - 8*a*b^2*c^5*d^4 - 8*a^5*b^2*c*f^4 + 2*a^2*b^6*d*f^3 - 2*a^3*b^5*e*
f^3 - 64*a^3*c^5*d^3*f - 64*a^5*c^3*d*f^3 - 2*b^5*c^3*d^3*e + 2*b^6*c^2*d^3
*f + a^2*b^4*c^2*e^4 - 8*a^3*b^2*c^3*e^4 + 32*a^3*c^5*d^2*e^2 + a^2*b^6*e^2
*f^2 + 96*a^4*c^4*d^2*f^2 + b^6*c^2*d^2*e^2 + 32*a^5*c^3*e^2*f^2 - 2*a*b^7*
d*e*f^2 - 2*b^7*c*d^2*e*f + 54*a^2*b^4*c^2*d^2*f^2 - 112*a^3*b^2*c^3*d^2*f^
2 + 16*a*b^3*c^4*d^3*e - 2*a*b^5*c^2*d*e^3 - 32*a^2*b*c^5*d^3*e - 32*a^3*b*
c^4*d*e^3 - 20*a*b^4*c^3*d^3*f - 12*a*b^6*c*d^2*f^2 - 20*a^3*b^4*c*d*f^3 -
2*a^2*b^5*c*e^3*f - 32*a^4*b*c^3*e^3*f + 16*a^4*b^3*c*e*f^3 - 32*a^5*b*c^2*
e*f^3 - 64*a^4*c^4*d*e^2*f - 6*a*b^4*c^3*d^2*e^2 + 16*a^2*b^3*c^3*d*e^3 + 6
4*a^2*b^2*c^4*d^3*f + 64*a^4*b^2*c^2*d*f^3 + 16*a^3*b^3*c^2*e^3*f - 6*a^3*b
^4*c*e^2*f^2 - 48*a^2*b^3*c^3*d^2*e*f - 36*a^2*b^4*c^2*d*e^2*f + 96*a^3*b^2
*c^3*d*e^2*f - 48*a^3*b^3*c^2*d*e*f^2 + 4*a*b^6*c*d*e^2*f + 18*a*b^5*c^2*d^
2*e*f + 18*a^2*b^5*c*d*e*f^2 + 32*a^3*b*c^4*d^2*e*f + 32*a^4*b*c^3*d*e*f^2)
)*root(48416*a^6*b^2*c^6*d^4*e^2*f^4*z^4 - 41544*a^5*b^4*c^5*d^4*e^2*f^4*z^
4 - 31872*a^7*b^2*c^5*d^3*e^2*f^5*z^4 - 31872*a^5*b^2*c^7*d^5*e^2*f^3*z^4 -
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 29184*a^6*b^2*c^6*d^3*e^4*f^3*z^4 + 28800*a^5*b^4*c^5*d^3*e^4*f^3*z^4 + 21
510*a^4*b^6*c^4*d^4*e^2*f^4*z^4 + 21408*a^6*b^4*c^4*d^3*e^2*f^5*z^4 + 21408
*a^4*b^4*c^6*d^5*e^2*f^3*z^4 - 18112*a^7*b^3*c^4*d^2*e^3*f^5*z^4 - 18112*a^
4*b^3*c^7*d^5*e^3*f^2*z^4 - 15600*a^5*b^5*c^4*d^3*e^3*f^4*z^4 - 15600*a^4*b
^5*c^5*d^4*e^3*f^3*z^4 + 15296*a^6*b^3*c^5*d^3*e^3*f^4*z^4 + 15296*a^5*b^3*
c^6*d^4*e^3*f^3*z^4 + 14016*a^7*b^2*c^5*d^2*e^4*f^4*z^4 + 14016*a^5*b^2*c^7
*d^4*e^4*f^2*z^4 - 13920*a^4*b^6*c^4*d^3*e^4*f^3*z^4 - 11648*a^6*b^3*c^5*d^
2*e^5*f^3*z^4 - 11648*a^5*b^3*c^6*d^3*e^5*f^2*z^4 + 10432*a^6*b^2*c^6*d^2*e
^6*f^2*z^4 + 9008*a^6*b^5*c^3*d^2*e^3*f^5*z^4 + 9008*a^3*b^5*c^6*d^5*e^3*f^
2*z^4 + 8544*a^5*b^5*c^4*d^2*e^5*f^3*z^4 + 8544*a^4*b^5*c^5*d^3*e^5*f^2*z^4
 - 8496*a^5*b^4*c^5*d^2*e^6*f^2*z^4 + 7488*a^8*b^2*c^4*d^2*e^2*f^6*z^4 + 74
88*a^4*b^2*c^8*d^6*e^2*f^2*z^4 + 7380*a^4*b^7*c^3*d^3*e^3*f^4*z^4 + 7380*a^
3*b^7*c^4*d^4*e^3*f^3*z^4 - 6720*a^3*b^8*c^3*d^4*e^2*f^4*z^4 - 5784*a^5*b^6
*c^3*d^3*e^2*f^5*z^4 - 5784*a^3*b^6*c^5*d^5*e^2*f^3*z^4 - 3440*a^6*b^4*c^4*
d^2*e^4*f^4*z^4 - 3440*a^4*b^4*c^6*d^4*e^4*f^2*z^4 + 3360*a^3*b^8*c^3*d^3*e
^4*f^3*z^4 + 3140*a^4*b^6*c^4*d^2*e^6*f^2*z^4 - 2760*a^4*b^7*c^3*d^2*e^5*f^
3*z^4 - 2760*a^3*b^7*c^4*d^3*e^5*f^2*z^4 - 1764*a^5*b^7*c^2*d^2*e^3*f^5*z^4
 - 1764*a^2*b^7*c^5*d^5*e^3*f^2*z^4 - 1640*a^3*b^9*c^2*d^3*e^3*f^4*z^4 - 16
40*a^2*b^9*c^3*d^4*e^3*f^3*z^4 - 1604*a^6*b^6*c^2*d^2*e^2*f^6*z^4 - 1604*a^
2*b^6*c^6*d^6*e^2*f^2*z^4 - 1500*a^5*b^6*c^3*d^2*e^4*f^4*z^4 - 1500*a^3*b^6
*c^5*d^4*e^4*f^2*z^4 + 1140*a^2*b^10*c^2*d^4*e^2*f^4*z^4 + 810*a^4*b^8*c^2*
d^2*e^4*f^4*z^4 + 810*a^2*b^8*c^4*d^4*e^4*f^2*z^4 - 544*a^3*b^8*c^3*d^2*e^6
*f^2*z^4 + 416*a^3*b^9*c^2*d^2*e^5*f^3*z^4 + 416*a^2*b^9*c^3*d^3*e^5*f^2*z^
4 - 384*a^2*b^10*c^2*d^3*e^4*f^3*z^4 + 180*a^4*b^8*c^2*d^3*e^2*f^5*z^4 + 18
0*a^2*b^8*c^4*d^5*e^2*f^3*z^4 + 48*a^7*b^4*c^3*d^2*e^2*f^6*z^4 + 48*a^3*b^4
*c^7*d^6*e^2*f^2*z^4 + 36*a^2*b^10*c^2*d^2*e^6*f^2*z^4 - 1024*a^10*b*c^3*d*
e*f^8*z^4 - 1024*a^3*b*c^10*d^8*e*f*z^4 - 192*a^8*b^5*c*d*e*f^8*z^4 - 192*a
*b^5*c^8*d^8*e*f*z^4 + 16128*a^7*b^3*c^4*d^3*e*f^6*z^4 + 16128*a^4*b^3*c^7*
d^6*e*f^3*z^4 - 11712*a^6*b^5*c^3*d^3*e*f^6*z^4 - 11712*a^3*b^5*c^6*d^6*e*f
^3*z^4 + 11520*a^8*b*c^5*d^2*e^3*f^5*z^4 + 11520*a^5*b*c^8*d^5*e^3*f^2*z^4
 - 9984*a^6*b^3*c^5*d^4*e*f^5*z^4 - 9984*a^5*b^3*c^6*d^5*e*f^4*z^4 + 8640*a^
5*b^5*c^4*d^4*e*f^5*z^4 + 8640*a^4*b^5*c^5*d^5*e*f^4*z^4 - 7424*a^7*b*c^6*d
^3*e^3*f^4*z^4 - 7424*a^6*b*c^7*d^4*e^3*f^3*z^4 - 6912*a^8*b^3*c^3*d^2*e*f^
7*z^4 - 6912*a^3*b^3*c^8*d^7*e*f^2*z^4 + 4800*a^7*b^3*c^4*d*e^5*f^4*z^4 + 4
800*a^4*b^3*c^7*d^4*e^5*f*z^4 + 4608*a^7*b*c^6*d^2*e^5*f^3*z^4 + 4608*a^6*b
*c^7*d^3*e^5*f^2*z^4 - 4560*a^4*b^7*c^3*d^4*e*f^5*z^4 - 4560*a^3*b^7*c^4*d^
5*e*f^4*z^4 + 4176*a^5*b^7*c^2*d^3*e*f^6*z^4 + 4176*a^2*b^7*c^5*d^6*e*f^3*z
^4 + 3264*a^7*b^5*c^2*d^2*e*f^7*z^4 + 3264*a^2*b^5*c^7*d^7*e*f^2*z^4 + 3008
*a^8*b^3*c^3*d*e^3*f^6*z^4 + 3008*a^3*b^3*c^8*d^6*e^3*f*z^4 + 2880*a^6*b^3*
c^5*d*e^7*f^2*z^4 + 2880*a^5*b^3*c^6*d^2*e^7*f*z^4 - 2240*a^7*b^4*c^3*d*e^4
*f^5*z^4 - 2240*a^3*b^4*c^7*d^5*e^4*f*z^4 - 1488*a^5*b^5*c^4*d*e^7*f^2*z^4
 - 1488*a^4*b^5*c^5*d^2*e^7*f*z^4 + 1440*a^3*b^9*c^2*d^4*e*f^5*z^4 + 1440*a^
2*b^9*c^3*d^5*e*f^4*z^4 - 1328*a^6*b^5*c^3*d*e^5*f^4*z^4 - 1328*a^3*b^5*c^6
*d^4*e^5*f*z^4 - 1152*a^7*b^2*c^5*d*e^6*f^3*z^4 - 1152*a^5*b^2*c^7*d^3*e^6*
f*z^4 - 1120*a^6*b^4*c^4*d*e^6*f^3*z^4 - 1120*a^4*b^4*c^6*d^3*e^6*f*z^4 + 9
12*a^6*b^6*c^2*d*e^4*f^5*z^4 + 912*a^2*b^6*c^6*d^5*e^4*f*z^4 + 872*a^5*b^6*
c^3*d*e^6*f^3*z^4 + 872*a^3*b^6*c^5*d^3*e^6*f*z^4 + 768*a^8*b^2*c^4*d*e^4*f
^5*z^4 + 768*a^4*b^2*c^8*d^5*e^4*f*z^4 - 672*a^8*b^4*c^2*d*e^2*f^7*z^4 - 67
2*a^2*b^4*c^8*d^7*e^2*f*z^4 - 624*a^7*b^5*c^2*d*e^3*f^6*z^4 - 624*a^2*b^5*c
^7*d^6*e^3*f*z^4 + 480*a^5*b^8*c*d^2*e^2*f^6*z^4 + 480*a*b^8*c^5*d^6*e^2*f^
2*z^4 + 316*a^4*b^7*c^3*d*e^7*f^2*z^4 + 316*a^3*b^7*c^4*d^2*e^7*f*z^4 - 204
*a^4*b^8*c^2*d*e^6*f^3*z^4 - 204*a^2*b^8*c^4*d^3*e^6*f*z^4 + 168*a^3*b^10*c
*d^3*e^2*f^5*z^4 + 168*a*b^10*c^3*d^5*e^2*f^3*z^4 + 156*a^2*b^11*c*d^3*e^3*
f^4*z^4 + 156*a*b^11*c^2*d^4*e^3*f^3*z^4 + 128*a^9*b^2*c^3*d*e^2*f^7*z^4 +
128*a^3*b^2*c^9*d^7*e^2*f*z^4 - 124*a^3*b^10*c*d^2*e^4*f^4*z^4 - 124*a*b^10
*c^3*d^4*e^4*f^2*z^4 + 100*a^4*b^9*c*d^2*e^3*f^5*z^4 + 100*a*b^9*c^4*d^5*e^
3*f^2*z^4 + 36*a^5*b^7*c^2*d*e^5*f^4*z^4 + 36*a^2*b^7*c^5*d^4*e^5*f*z^4 - 2
4*a^3*b^9*c^2*d*e^7*f^2*z^4 - 24*a^2*b^11*c*d^2*e^5*f^3*z^4 - 24*a^2*b^9*c^
3*d^2*e^7*f*z^4 - 24*a*b^11*c^2*d^3*e^5*f^2*z^4 - 9216*a^8*b*c^5*d^3*e*f^6*
```

z^4 - 9216*a^5*b*c^8*d^6*e*f^3*z^4 - 5376*a^8*b*c^5*d*e^5*f^4*z^4 - 5376*a^
5*b*c^8*d^4*e^5*f*z^4 + 5120*a^9*b*c^4*d^2*e*f^7*z^4 + 5120*a^7*b*c^6*d^4*e
*f^5*z^4 + 5120*a^6*b*c^7*d^5*e*f^4*z^4 + 5120*a^4*b*c^9*d^7*e*f^2*z^4 - 43
52*a^9*b*c^4*d*e^3*f^6*z^4 - 4352*a^4*b*c^9*d^6*e^3*f*z^4 - 1792*a^7*b*c^6*
d*e^7*f^2*z^4 - 1792*a^6*b*c^7*d^2*e^7*f*z^4 - 1600*a^6*b^2*c^6*d*e^8*f*z^4
 + 912*a^5*b^4*c^5*d*e^8*f*z^4 + 768*a^9*b^3*c^2*d*e*f^8*z^4 + 768*a^2*b^3*
c^9*d^8*e*f*z^4 - 720*a^4*b^9*c*d^3*e*f^6*z^4 - 720*a*b^9*c^4*d^6*e*f^3*z^4
 - 656*a^6*b^7*c*d^2*e*f^7*z^4 - 656*a*b^7*c^6*d^7*e*f^2*z^4 - 240*a^2*b^11
*c*d^4*e*f^5*z^4 - 240*a*b^11*c^2*d^5*e*f^4*z^4 + 216*a^7*b^6*c*d*e^2*f^7*z
^4 + 216*a*b^6*c^7*d^7*e^2*f*z^4 - 204*a^4*b^6*c^4*d*e^8*f*z^4 - 144*a^5*b^
8*c*d*e^4*f^5*z^4 - 144*a*b^8*c^5*d^5*e^4*f*z^4 - 84*a*b^12*c*d^4*e^2*f^4*z
^4 + 36*a^4*b^9*c*d*e^5*f^4*z^4 + 36*a*b^9*c^4*d^4*e^5*f*z^4 + 20*a^6*b^7*c
*d*e^3*f^6*z^4 + 20*a*b^7*c^6*d^6*e^3*f*z^4 + 16*a^3*b^10*c*d*e^6*f^3*z^4 +
 16*a^3*b^8*c^3*d*e^8*f*z^4 + 16*a*b^12*c*d^3*e^4*f^3*z^4 + 16*a*b^10*c^3*d
^3*e^6*f*z^4 + 48*b^11*c^3*d^6*e*f^3*z^4 + 48*b^9*c^5*d^7*e*f^2*z^4 - 20*b^
8*c^6*d^7*e^2*f*z^4 + 8*b^10*c^4*d^5*e^4*f*z^4 - 4*b^13*c*d^4*e^3*f^3*z^4 -
 4*b^11*c^3*d^4*e^5*f*z^4 + 4*b^9*c^5*d^6*e^3*f*z^4 + 3072*a^9*c^5*d*e^4*f^
5*z^4 + 3072*a^5*c^9*d^5*e^4*f*z^4 + 2560*a^8*c^6*d*e^6*f^3*z^4 + 2560*a^6*
c^8*d^3*e^6*f*z^4 + 1536*a^10*c^4*d*e^2*f^7*z^4 + 1536*a^4*c^10*d^7*e^2*f*z
^4 + 48*a^5*b^9*d^2*e*f^7*z^4 + 48*a^3*b^11*d^3*e*f^6*z^4 - 20*a^6*b^8*d*e^
2*f^7*z^4 + 8*a^4*b^10*d*e^4*f^5*z^4 + 4*a^5*b^9*d*e^3*f^6*z^4 - 4*a^3*b^11
*d*e^5*f^4*z^4 - 4*a*b^13*d^3*e^3*f^4*z^4 + 768*a^9*b*c^4*e^5*f^5*z^4 + 768
*a^8*b*c^5*e^7*f^3*z^4 + 256*a^10*b*c^3*e^3*f^7*z^4 - 192*a^6*b^3*c^5*e^9*f
*z^4 - 68*a^7*b^6*c*e^4*f^6*z^4 + 48*a^8*b^5*c*e^3*f^7*z^4 + 48*a^5*b^5*c^4
*e^9*f*z^4 + 36*a^6*b^7*c*e^5*f^5*z^4 - 12*a^9*b^4*c*e^2*f^8*z^4 - 4*a^4*b^
9*c*e^7*f^3*z^4 - 4*a^4*b^7*c^3*e^9*f*z^4 + 384*a^5*b^8*c*d^3*f^7*z^4 + 384
*a*b^8*c^5*d^7*f^3*z^4 + 288*a^3*b^10*c*d^4*f^6*z^4 + 288*a*b^10*c^3*d^6*f^
4*z^4 + 224*a^7*b^6*c*d^2*f^8*z^4 + 224*a*b^6*c^7*d^8*f^2*z^4 - 192*a^10*b^
2*c^2*d*f^9*z^4 - 192*a^2*b^2*c^10*d^9*f*z^4 + 768*a^5*b*c^8*d^3*e^7*z^4 +
768*a^4*b*c^9*d^5*e^5*z^4 + 256*a^3*b*c^10*d^7*e^3*z^4 - 192*a^5*b^3*c^6*d*
e^9*z^4 - 68*a*b^6*c^7*d^6*e^4*z^4 + 48*a^4*b^5*c^5*d*e^9*z^4 + 48*a*b^5*c^
8*d^7*e^3*z^4 + 36*a*b^7*c^6*d^5*e^5*z^4 - 12*a*b^4*c^9*d^8*e^2*z^4 - 4*a^3
*b^7*c^4*d*e^9*z^4 - 4*a*b^9*c^4*d^3*e^7*z^4 + 16*b^13*c*d^5*e*f^4*z^4 + 16
*b^7*c^7*d^8*e*f*z^4 + 768*a^7*c^7*d*e^8*f*z^4 + 16*a^7*b^7*d*e*f^8*z^4 + 1
6*a*b^13*d^4*e*f^5*z^4 + 256*a^7*b*c^6*e^9*f*z^4 + 80*a*b^12*c*d^5*f^5*z^4
 + 48*a^9*b^4*c*d*f^9*z^4 + 48*a*b^4*c^9*d^9*f*z^4 + 256*a^6*b*c^7*d*e^9*z^4
 - 42*b^10*c^4*d^6*e^2*f^2*z^4 - 20*b^12*c^2*d^5*e^2*f^3*z^4 + 6*b^12*c^2*d
^4*e^4*f^2*z^4 + 4*b^11*c^3*d^5*e^3*f^2*z^4 - 24960*a^7*c^7*d^4*e^2*f^4*z^4
 + 18944*a^8*c^6*d^3*e^2*f^5*z^4 + 18944*a^6*c^8*d^5*e^2*f^3*z^4 + 14336*a^
7*c^7*d^3*e^4*f^3*z^4 - 9984*a^8*c^6*d^2*e^4*f^4*z^4 - 9984*a^6*c^8*d^4*e^4
*f^2*z^4 - 7936*a^9*c^5*d^2*e^2*f^6*z^4 - 7936*a^5*c^9*d^6*e^2*f^2*z^4 - 43
52*a^7*c^7*d^2*e^6*f^2*z^4 - 42*a^4*b^10*d^2*e^2*f^6*z^4 - 20*a^2*b^12*d^3*
e^2*f^5*z^4 + 6*a^2*b^12*d^2*e^4*f^4*z^4 + 4*a^3*b^11*d^2*e^3*f^5*z^4 - 480
*a^8*b^2*c^4*e^6*f^4*z^4 + 440*a^7*b^4*c^3*e^6*f^4*z^4 - 320*a^8*b^3*c^3*e^
5*f^5*z^4 - 320*a^7*b^3*c^4*e^7*f^3*z^4 + 240*a^8*b^4*c^2*e^4*f^6*z^4 + 240
*a^6*b^4*c^4*e^8*f^2*z^4 - 192*a^9*b^3*c^2*e^3*f^7*z^4 - 192*a^9*b^2*c^3*e^
4*f^6*z^4 - 192*a^7*b^2*c^5*e^8*f^2*z^4 - 90*a^6*b^6*c^2*e^6*f^4*z^4 - 68*a
^5*b^6*c^3*e^8*f^2*z^4 + 48*a^10*b^2*c^2*e^2*f^8*z^4 - 48*a^7*b^5*c^2*e^5*f
^5*z^4 - 48*a^6*b^5*c^3*e^7*f^3*z^4 + 36*a^5*b^7*c^2*e^7*f^3*z^4 + 6*a^4*b^
8*c^2*e^8*f^2*z^4 - 33920*a^6*b^2*c^6*d^5*f^5*z^4 + 27936*a^5*b^4*c^5*d^5*f
^5*z^4 + 26112*a^7*b^2*c^5*d^4*f^6*z^4 + 26112*a^5*b^2*c^7*d^6*f^4*z^4 - 20
352*a^6*b^4*c^4*d^4*f^6*z^4 - 20352*a^4*b^4*c^6*d^6*f^4*z^4 - 13080*a^4*b^6
*c^4*d^5*f^5*z^4 - 11520*a^8*b^2*c^4*d^3*f^7*z^4 - 11520*a^4*b^2*c^8*d^7*f^
3*z^4 + 8736*a^5*b^6*c^3*d^4*f^6*z^4 + 8736*a^3*b^6*c^5*d^6*f^4*z^4 + 7488*
a^7*b^4*c^3*d^3*f^7*z^4 + 7488*a^3*b^4*c^7*d^7*f^3*z^4 + 3840*a^3*b^8*c^3*d
^5*f^5*z^4 + 2560*a^9*b^2*c^3*d^2*f^8*z^4 + 2560*a^3*b^2*c^9*d^8*f^2*z^4 -
2416*a^6*b^6*c^2*d^3*f^7*z^4 - 2416*a^2*b^6*c^6*d^7*f^3*z^4 - 2160*a^4*b^8*
c^2*d^4*f^6*z^4 - 2160*a^2*b^8*c^4*d^6*f^4*z^4 - 1152*a^8*b^4*c^2*d^2*f^8*z
^4 - 1152*a^2*b^4*c^8*d^8*f^2*z^4 - 720*a^2*b^10*c^2*d^5*f^5*z^4 - 480*a^4*

b^2*c^8*d^4*e^6*z^4 + 440*a^3*b^4*c^7*d^4*e^6*z^4 - 320*a^4*b^3*c^7*d^3*e^7
*z^4 - 320*a^3*b^3*c^8*d^5*e^5*z^4 + 240*a^4*b^4*c^6*d^2*e^8*z^4 + 240*a^2*
b^4*c^8*d^6*e^4*z^4 - 192*a^5*b^2*c^7*d^2*e^8*z^4 - 192*a^3*b^2*c^9*d^6*e^4
*z^4 - 192*a^2*b^3*c^9*d^7*e^3*z^4 - 90*a^2*b^6*c^6*d^4*e^6*z^4 - 68*a^3*b^
6*c^5*d^2*e^8*z^4 - 48*a^3*b^5*c^6*d^3*e^7*z^4 - 48*a^2*b^5*c^7*d^5*e^5*z^4
 + 48*a^2*b^2*c^10*d^8*e^2*z^4 + 36*a^2*b^7*c^5*d^3*e^7*z^4 + 6*a^2*b^8*c^4
*d^2*e^8*z^4 - 4*b^6*c^8*d^9*f*z^4 + 256*a^11*c^3*d*f^9*z^4 + 256*a^3*c^11*
d^9*f*z^4 - 4*a^8*b^6*d*f^9*z^4 - 384*a^9*c^5*e^6*f^4*z^4 - 256*a^10*c^4*e^
4*f^6*z^4 - 256*a^8*c^6*e^8*f^2*z^4 - 64*a^11*c^3*e^2*f^8*z^4 - 24*b^10*c^4
*d^7*f^3*z^4 - 16*b^12*c^2*d^6*f^4*z^4 - 16*b^8*c^6*d^8*f^2*z^4 + 17920*a^7
*c^7*d^5*f^5*z^4 - 14336*a^8*c^6*d^4*f^6*z^4 - 14336*a^6*c^8*d^6*f^4*z^4 +
7168*a^9*c^5*d^3*f^7*z^4 + 7168*a^5*c^9*d^7*f^3*z^4 - 2048*a^10*c^4*d^2*f^8
*z^4 - 2048*a^4*c^10*d^8*f^2*z^4 + 6*b^8*c^6*d^6*e^4*z^4 + 6*a^6*b^8*e^4*f^
6*z^4 - 4*b^9*c^5*d^5*e^5*z^4 - 4*b^7*c^7*d^7*e^3*z^4 - 4*a^7*b^7*e^3*f^7*z
^4 - 4*a^5*b^9*e^5*f^5*z^4 - 384*a^5*c^9*d^4*e^6*z^4 - 256*a^6*c^8*d^2*e^8*
z^4 - 256*a^4*c^10*d^6*e^4*z^4 - 64*a^3*c^11*d^8*e^2*z^4 - 24*a^4*b^10*d^3*
f^7*z^4 - 16*a^6*b^8*d^2*f^8*z^4 - 16*a^2*b^12*d^4*f^6*z^4 + 48*a^6*b^2*c^6
*e^10*z^4 - 12*a^5*b^4*c^5*e^10*z^4 - 4*b^14*d^5*f^5*z^4 - 64*a^7*c^7*e^10*
z^4 + b^14*d^4*e^2*f^4*z^4 + b^10*c^4*d^4*e^6*z^4 + b^6*c^8*d^8*e^2*z^4 + a
^8*b^6*e^2*f^8*z^4 + a^4*b^10*e^6*f^4*z^4 + a^4*b^6*c^4*e^10*z^4 - 4820*A*B
*a^4*b*c^5*d^2*e^2*f^4*z^2 + 2976*A*B*a^3*b*c^6*d^3*e^2*f^3*z^2 - 2328*A*B*
a^3*b*c^6*d^2*e^4*f^2*z^2 + 1848*A*B*a^2*b^4*c^4*d^3*e*f^4*z^2 - 1768*A*B*a
^3*b^4*c^3*d^2*e*f^5*z^2 + 1528*A*B*a^4*b^2*c^4*d^2*e*f^5*z^2 - 1136*A*B*a^
3*b^2*c^5*d^3*e*f^4*z^2 - 974*A*B*a^4*b^3*c^3*d*e^2*f^5*z^2 + 692*A*B*a^2*b
*c^7*d^4*e^2*f^2*z^2 + 588*A*B*a*b^6*c^3*d^2*e^3*f^3*z^2 - 580*A*B*a^3*b^3*
c^4*d*e^4*f^3*z^2 + 488*A*B*a^3*b^4*c^3*d*e^3*f^4*z^2 - 444*A*B*a^2*b^2*c^6
*d^2*e^5*f*z^2 - 412*A*B*a*b^5*c^4*d^2*e^4*f^2*z^2 + 366*A*B*a^2*b^6*c^2*d^
2*e*f^5*z^2 - 352*A*B*a^2*b^2*c^6*d^4*e*f^3*z^2 + 326*A*B*a^2*b^4*c^4*d*e^5
*f^2*z^2 + 324*A*B*a*b^5*c^4*d^3*e^2*f^3*z^2 - 302*A*B*a*b^3*c^6*d^4*e^2*f^
2*z^2 - 296*A*B*a*b^7*c^2*d^2*e^2*f^4*z^2 + 122*A*B*a^4*b^2*c^4*d*e^3*f^4*z
^2 - 122*A*B*a^2*b^6*c^2*d*e^3*f^4*z^2 - 84*A*B*a^3*b^2*c^5*d*e^5*f^2*z^2 +
 72*A*B*a*b^4*c^5*d^3*e^3*f^2*z^2 - 64*A*B*a^2*b^5*c^3*d*e^4*f^3*z^2 + 60*A
*B*a^3*b^5*c^2*d*e^2*f^5*z^2 + 1312*A*B*a^5*b*c^4*d*e^2*f^5*z^2 + 1040*A*B*
a^4*b*c^5*d*e^4*f^3*z^2 - 500*A*B*a*b^6*c^3*d^3*e*f^4*z^2 - 376*A*B*a*b^2*c
^7*d^5*e*f^2*z^2 + 276*A*B*a^4*b^4*c^2*d*e*f^6*z^2 - 262*A*B*a^2*b^3*c^5*d*
e^6*f*z^2 + 238*A*B*a*b^2*c^7*d^4*e^3*f*z^2 + 232*A*B*a^5*b^2*c^3*d*e*f^6*z
^2 - 176*A*B*a^2*b*c^7*d^3*e^4*f*z^2 - 120*A*B*a*b^6*c^3*d*e^5*f^2*z^2 - 10
8*A*B*a*b^4*c^5*d^4*e*f^3*z^2 + 68*A*B*a*b^7*c^2*d*e^4*f^3*z^2 + 68*A*B*a*b
^4*c^5*d^2*e^5*f*z^2 + 46*A*B*a^2*b^7*c*d*e^2*f^5*z^2 - 36*A*B*a*b^3*c^6*d^
3*e^4*f*z^2 - 1932*A*B*a^2*b^3*c^5*d^3*e^2*f^3*z^2 - 1818*A*B*a^2*b^4*c^4*d
^2*e^3*f^3*z^2 + 1620*A*B*a^3*b^3*c^4*d^2*e^2*f^4*z^2 + 1560*A*B*a^2*b^3*c^
5*d^2*e^4*f^2*z^2 + 1244*A*B*a^3*b^2*c^5*d^2*e^3*f^3*z^2 + 820*A*B*a^2*b^2*
c^6*d^3*e^3*f^2*z^2 + 480*A*B*a^2*b^5*c^3*d^2*e^2*f^4*z^2 + 352*A*B*a^3*b*c
^6*d*e^6*f*z^2 - 108*A*B*a^3*b^6*c*d*e*f^6*z^2 + 82*A*B*a*b^5*c^4*d*e^6*f*z
^2 - 64*A*B*a*b*c^8*d^5*e^2*f*z^2 + 16*A*B*a*b^8*c*d^2*e*f^5*z^2 - 4*A*B*a*
b^8*c*d*e^3*f^4*z^2 + 16*B^2*a*b*c^8*d^6*e*f*z^2 + 56*A*B*b^2*c^8*d^6*e*f*z
^2 - 8*A*B*b^9*c*d*e^4*f^3*z^2 - 8*A*B*b^7*c^3*d*e^6*f*z^2 - 800*A*B*a^6*c^
4*d*e*f^6*z^2 + 10*A*B*a^2*b^8*d*e*f^6*z^2 - 6*A*B*a*b^9*d*e^2*f^5*z^2 - 12
*A*B*a^5*b^4*c*e*f^7*z^2 + 912*A*B*a^6*b*c^3*d*f^7*z^2 + 192*A*B*a^4*b^5*c*
d*f^7*z^2 + 192*A*B*a*b*c^8*d^6*f^2*z^2 - 20*A*B*a*b^4*c^5*d*e^7*z^2 + 4*A*
B*a*b*c^8*d^4*e^4*z^2 + 2144*B^2*a^4*b*c^5*d^3*e*f^4*z^2 - 1120*B^2*a^3*b*c
^6*d^4*e*f^3*z^2 - 688*B^2*a^5*b*c^4*d^2*e*f^5*z^2 - 256*B^2*a^3*b*c^6*d^2*
e^5*f*z^2 + 152*B^2*a*b^3*c^6*d^5*e*f^2*z^2 + 120*B^2*a^5*b^3*c^2*d*e*f^6*z
^2 - 116*B^2*a^5*b*c^4*d*e^3*f^4*z^2 + 110*B^2*a*b^7*c^2*d^3*e*f^4*z^2 - 80
*B^2*a^2*b*c^7*d^5*e*f^2*z^2 - 72*B^2*a*b^5*c^4*d^4*e*f^3*z^2 - 48*B^2*a^4*
b*c^5*d*e^5*f^2*z^2 - 46*B^2*a*b^3*c^6*d^4*e^3*f*z^2 - 44*B^2*a*b^4*c^5*d^3
*e^4*f*z^2 - 34*B^2*a*b^5*c^4*d^2*e^5*f*z^2 + 20*B^2*a^2*b*c^7*d^4*e^3*f*z^
2 - 10*B^2*a^3*b^6*c*d*e^2*f^5*z^2 - 10*B^2*a^2*b^7*c*d^2*e*f^5*z^2 - 10*B^
2*a*b^2*c^7*d^5*e^2*f*z^2 - 7*B^2*a^2*b^4*c^4*d*e^6*f*z^2 - 6*B^2*a^3*b^2*c

^5*d*e^6*f*z^2 + 4*B^2*a*b^8*c*d^2*e^2*f^4*z^2 - 2*B^2*a^2*b^7*c*d*e^3*f^4*
z^2 + 3196*A^2*a^4*b*c^5*d*e^3*f^4*z^2 - 3184*A^2*a^4*b*c^5*d^2*e*f^5*z^2 +
1568*A^2*a^3*b*c^6*d^3*e*f^4*z^2 + 1504*A^2*a^3*b*c^6*d*e^5*f^2*z^2 - 656*
A^2*a^4*b^3*c^3*d*e*f^6*z^2 - 400*A^2*a*b^6*c^3*d*e^4*f^3*z^2 + 314*A^2*a*b
^5*c^4*d*e^5*f^2*z^2 - 264*A^2*a^3*b^5*c^2*d*e*f^6*z^2 + 240*A^2*a^2*b^2*c^
6*d*e^6*f*z^2 - 224*A^2*a^2*b*c^7*d^4*e*f^3*z^2 + 216*A^2*a*b^5*c^4*d^3*e*f
^4*z^2 - 192*A^2*a^2*b*c^7*d^2*e^5*f*z^2 + 178*A^2*a*b^7*c^2*d*e^3*f^4*z^2
- 154*A^2*a*b^7*c^2*d^2*e*f^5*z^2 + 128*A^2*a*b^3*c^6*d^4*e*f^3*z^2 + 106*A
^2*a*b^3*c^6*d^2*e^5*f*z^2 - 12*A^2*a*b^2*c^7*d^3*e^4*f*z^2 - 58*A*B*b^8*c^
2*d^2*e^3*f^3*z^2 + 40*A*B*b^7*c^3*d^2*e^4*f^2*z^2 - 28*A*B*b^7*c^3*d^3*e^2
*f^3*z^2 - 24*A*B*b^5*c^5*d^4*e^2*f^2*z^2 - 20*A*B*b^6*c^4*d^3*e^3*f^2*z^2
+ 2768*A*B*a^4*c^6*d^2*e^3*f^3*z^2 - 1712*A*B*a^3*c^7*d^3*e^3*f^2*z^2 - 156
*A*B*a^4*b^2*c^4*e^5*f^3*z^2 + 146*A*B*a^4*b^3*c^3*e^4*f^4*z^2 - 106*A*B*a^
5*b^2*c^3*e^3*f^5*z^2 + 90*A*B*a^5*b^3*c^2*e^2*f^6*z^2 + 38*A*B*a^3*b^3*c^4
*e^6*f^2*z^2 - 36*A*B*a^3*b^5*c^2*e^4*f^4*z^2 + 16*A*B*a^3*b^4*c^3*e^5*f^3*
z^2 - 9*A*B*a^4*b^4*c^2*e^3*f^5*z^2 - 8*A*B*a^2*b^5*c^3*e^6*f^2*z^2 + 2*A*B
*a^2*b^6*c^2*e^5*f^3*z^2 + 920*A*B*a^4*b^3*c^3*d^2*f^6*z^2 - 480*A*B*a^2*b^
5*c^3*d^3*f^5*z^2 - 336*A*B*a^2*b^3*c^5*d^4*f^4*z^2 - 272*A*B*a^3*b^3*c^4*d
^3*f^5*z^2 + 240*A*B*a^3*b^5*c^2*d^2*f^6*z^2 - 32*A*B*a*c^9*d^6*e*f*z^2 - 7
92*B^2*a^2*b^3*c^5*d^3*e^3*f^2*z^2 + 714*B^2*a^2*b^4*c^4*d^3*e^2*f^3*z^2 -
572*B^2*a^3*b^2*c^5*d^3*e^2*f^3*z^2 - 475*B^2*a^2*b^2*c^6*d^4*e^2*f^2*z^2 +
265*B^2*a^4*b^2*c^4*d^2*e^2*f^4*z^2 + 260*B^2*a^3*b^3*c^4*d^2*e^3*f^3*z^2
- 212*B^2*a^3*b^4*c^3*d^2*e^2*f^4*z^2 + 180*B^2*a^3*b^2*c^5*d^2*e^4*f^2*z^2
- 158*B^2*a^2*b^4*c^4*d^2*e^4*f^2*z^2 + 47*B^2*a^2*b^6*c^2*d^2*e^2*f^4*z^2
+ 16*B^2*a^2*b^5*c^3*d^2*e^3*f^3*z^2 + 2752*A^2*a^3*b^2*c^5*d^2*e^2*f^4*z^
2 - 2148*A^2*a^2*b^4*c^4*d^2*e^2*f^4*z^2 + 2064*A^2*a^2*b^3*c^5*d^2*e^3*f^3
*z^2 - 424*A^2*a^2*b^2*c^6*d^3*e^2*f^3*z^2 - 198*A^2*a^2*b^2*c^6*d^2*e^4*f^
2*z^2 - 272*B^2*a^6*b*c^3*d*e*f^6*z^2 - 24*B^2*a^4*b^5*c*d*e*f^6*z^2 + 1808
*A^2*a^5*b*c^4*d*e*f^6*z^2 - 244*A^2*a*b*c^8*d^4*e^3*f*z^2 + 208*A^2*a*b*c^
8*d^5*e*f^2*z^2 + 134*A^2*a^2*b^7*c*d*e*f^6*z^2 - 76*A^2*a*b^4*c^5*d*e^6*f*
z^2 + 4*A^2*a*b^8*c*d*e^2*f^5*z^2 + 148*A*B*b^4*c^6*d^5*e*f^2*z^2 + 65*A*B*
b^6*c^4*d^4*e*f^3*z^2 + 46*A*B*b^8*c^2*d^3*e*f^4*z^2 - 38*A*B*b^3*c^7*d^5*e
^2*f*z^2 + 34*A*B*b^9*c*d^2*e^2*f^4*z^2 - 29*A*B*b^4*c^6*d^4*e^3*f*z^2 + 20
*A*B*b^5*c^5*d^3*e^4*f*z^2 + 12*A*B*b^8*c^2*d*e^5*f^2*z^2 - 7*A*B*b^6*c^4*d
^2*e^5*f*z^2 - 2880*A*B*a^4*c^6*d^3*e*f^4*z^2 + 2784*A*B*a^5*c^5*d^2*e*f^5*
z^2 - 1112*A*B*a^5*c^5*d*e^3*f^4*z^2 + 896*A*B*a^3*c^7*d^4*e*f^3*z^2 + 848*
A*B*a^3*c^7*d^2*e^5*f*z^2 - 560*A*B*a^4*c^6*d*e^5*f^2*z^2 + 96*A*B*a^2*c^8*
d^5*e*f^2*z^2 - 88*A*B*a^2*c^8*d^4*e^3*f*z^2 - 100*A*B*a^6*b*c^3*e^2*f^6*z^
2 - 76*A*B*a^5*b*c^4*e^4*f^4*z^2 + 48*A*B*a^6*b^2*c^2*e*f^7*z^2 - 42*A*B*a^
3*b^2*c^5*e^7*f*z^2 + 36*A*B*a^4*b*c^5*e^6*f^2*z^2 - 24*A*B*a^4*b^5*c*e^2*f
^6*z^2 + 10*A*B*a^3*b^6*c*e^3*f^5*z^2 + 7*A*B*a^2*b^4*c^4*e^7*f*z^2 + 2*A*B
*a^2*b^7*c*e^4*f^4*z^2 - 2496*A*B*a^5*b*c^4*d^2*f^6*z^2 + 1872*A*B*a^4*b*c^
5*d^3*f^5*z^2 - 744*A*B*a^5*b^3*c^2*d*f^7*z^2 - 720*A*B*a^2*b*c^7*d^5*f^3*z
^2 + 504*A*B*a*b^3*c^6*d^5*f^3*z^2 + 256*A*B*a^3*b*c^6*d^4*f^4*z^2 + 168*A*
B*a*b^7*c^2*d^3*f^5*z^2 - 144*A*B*a^2*b^7*c*d^2*f^6*z^2 + 144*A*B*a*b^5*c^4
*d^4*f^4*z^2 + 66*A*B*a^2*b^2*c^6*d*e^7*z^2 - 36*A*B*a*b^2*c^7*d^3*e^5*z^2
+ 20*A*B*a*b^3*c^6*d^2*e^6*z^2 + 12*A*B*a^2*b*c^7*d^2*e^6*z^2 + 1208*B^2*a^
3*b*c^6*d^3*e^3*f^2*z^2 - 848*B^2*a^3*b^3*c^4*d^3*e*f^4*z^2 + 672*B^2*a^2*b
^3*c^5*d^4*e*f^3*z^2 - 632*B^2*a^4*b*c^5*d^2*e^3*f^3*z^2 + 432*B^2*a^4*b^3*
c^3*d^2*e*f^5*z^2 + 276*B^2*a^2*b^2*c^6*d^3*e^4*f*z^2 - 196*B^2*a*b^6*c^3*d
^3*e^2*f^3*z^2 - 168*B^2*a^2*b^5*c^3*d^3*e*f^4*z^2 + 154*B^2*a^2*b^3*c^5*d^
2*e^5*f*z^2 + 148*B^2*a*b^5*c^4*d^3*e^3*f^2*z^2 + 96*B^2*a*b^4*c^5*d^4*e^2*
f^2*z^2 - 72*B^2*a^3*b^5*c^2*d^2*e*f^5*z^2 + 70*B^2*a^5*b^2*c^3*d*e^2*f^5*z
^2 - 60*B^2*a^4*b^3*c^3*d*e^3*f^4*z^2 + 52*B^2*a*b^6*c^3*d^2*e^4*f^2*z^2 +
36*B^2*a^4*b^2*c^4*d*e^4*f^3*z^2 - 32*B^2*a*b^7*c^2*d^2*e^3*f^3*z^2 + 24*B^
2*a^3*b^5*c^2*d*e^3*f^4*z^2 + 15*B^2*a^4*b^4*c^2*d*e^2*f^5*z^2 - 8*B^2*a^3*
b^4*c^3*d*e^4*f^3*z^2 + 8*B^2*a^2*b^5*c^3*d*e^5*f^2*z^2 - 2*B^2*a^3*b^3*c^4
*d*e^5*f^2*z^2 - 2*B^2*a^2*b^6*c^2*d*e^4*f^3*z^2 - 3176*A^2*a^3*b*c^6*d^2*e
^3*f^3*z^2 - 2252*A^2*a^4*b^2*c^4*d*e^2*f^5*z^2 + 1952*A^2*a^3*b^4*c^3*d*e^

```
2*f^5*z^2 - 1496*A^2*a^3*b^3*c^4*d*e^3*f^4*z^2 + 1378*A^2*a^2*b^4*c^4*d*e^4
*f^3*z^2 + 1184*A^2*a^3*b^3*c^4*d^2*e*f^5*z^2 - 1166*A^2*a^2*b^3*c^5*d*e^5*
f^2*z^2 - 1164*A^2*a^3*b^2*c^5*d*e^4*f^3*z^2 - 1152*A^2*a^2*b^3*c^5*d^3*e*f
^4*z^2 + 578*A^2*a*b^6*c^3*d^2*e^2*f^4*z^2 - 548*A^2*a*b^5*c^4*d^2*e^3*f^3*
z^2 + 440*A^2*a*b^2*c^7*d^4*e^2*f^2*z^2 - 412*A^2*a^2*b^6*c^2*d*e^2*f^5*z^2
 - 360*A^2*a*b^3*c^6*d^3*e^3*f^2*z^2 + 312*A^2*a*b^4*c^5*d^3*e^2*f^3*z^2 +
248*A^2*a^2*b*c^7*d^3*e^3*f^2*z^2 - 224*A^2*a^2*b^5*c^3*d*e^3*f^4*z^2 + 216
*A^2*a^2*b^5*c^3*d^2*e*f^5*z^2 + 52*A^2*a*b^4*c^5*d^2*e^4*f^2*z^2 - 16*B^2*
b^3*c^7*d^6*e*f*z^2 - 14*B^2*b^9*c*d^3*e*f^4*z^2 + 32*B^2*a^4*c^6*d*e^6*f*z
^2 - 20*A^2*b^9*c*d*e^3*f^4*z^2 + 18*A^2*b^9*c*d^2*e*f^5*z^2 + 8*A^2*b^6*c^
4*d*e^6*f*z^2 - 360*A^2*a^3*c^7*d*e^6*f*z^2 + 136*A^2*a*c^9*d^5*e^2*f*z^2 +
 2*B^2*a^3*b^7*d*e*f^6*z^2 + 2*B^2*a*b^9*d^2*e*f^5*z^2 + 12*B^2*a^4*b*c^5*e
^7*f*z^2 - 204*A^2*a^3*b*c^6*e^7*f*z^2 - 128*A^2*a^6*b*c^3*e*f^7*z^2 - 48*A
^2*a*b^5*c^4*e^7*f*z^2 - 36*B^2*a^5*b^4*c*d*f^7*z^2 - 24*A^2*a^4*b^5*c*e*f^
7*z^2 - 16*B^2*a*b^8*c*d^3*f^5*z^2 - 164*A^2*a^3*b^6*c*d*f^7*z^2 - 16*A^2*a
*b^8*c*d^2*f^6*z^2 + 4*B^2*a^3*b*c^6*d*e^7*z^2 - 4*B^2*a*b*c^8*d^5*e^3*z^2
+ 48*A^2*a*b*c^8*d^3*e^5*z^2 + 36*A^2*a^2*b*c^7*d*e^7*z^2 - 6*A^2*a*b^3*c^6
*d*e^7*z^2 + 136*A*B*a^6*c^4*e^3*f^5*z^2 - 96*A*B*b^5*c^5*d^5*f^3*z^2 + 80*
A*B*a^5*c^5*e^5*f^3*z^2 - 72*A*B*b^3*c^7*d^6*f^2*z^2 - 24*A*B*b^7*c^3*d^4*f
^4*z^2 + 14*A*B*b^3*c^7*d^4*e^4*z^2 - 14*A*B*b^2*c^8*d^5*e^3*z^2 - 2*A*B*b^
5*c^5*d^2*e^6*z^2 - 2*A*B*b^4*c^6*d^3*e^5*z^2 + 2*A*B*a^3*b^7*e^2*f^6*z^2 -
 A*B*a^2*b^8*e^3*f^5*z^2 + 16*A*B*a^2*c^8*d^3*e^5*z^2 - 2*A*B*a^2*b^3*c^5*e
^8*z^2 + 22*B^2*b^8*c^2*d^3*e^2*f^3*z^2 - 12*B^2*b^7*c^3*d^3*e^3*f^2*z^2 +
12*B^2*b^6*c^4*d^4*e^2*f^2*z^2 - 6*B^2*b^8*c^2*d^2*e^4*f^2*z^2 - 864*B^2*a^
4*c^6*d^3*e^2*f^3*z^2 + 496*B^2*a^3*c^7*d^4*e^2*f^2*z^2 + 224*B^2*a^5*c^5*d
^2*e^2*f^4*z^2 + 136*B^2*a^4*c^6*d^2*e^4*f^2*z^2 - 53*A^2*b^8*c^2*d^2*e^2*f
^4*z^2 + 52*A^2*b^7*c^3*d^2*e^3*f^3*z^2 + 52*A^2*b^5*c^5*d^3*e^3*f^2*z^2 -
36*A^2*b^6*c^4*d^3*e^2*f^3*z^2 - 12*A^2*b^4*c^6*d^4*e^2*f^2*z^2 - 9*A^2*b^6
*c^4*d^2*e^4*f^2*z^2 + 836*A^2*a^4*c^6*d^2*e^2*f^4*z^2 - 668*A^2*a^2*c^8*d^
4*e^2*f^2*z^2 + 656*A^2*a^3*c^7*d^2*e^4*f^2*z^2 + 368*A^2*a^3*c^7*d^3*e^2*f
^3*z^2 - 45*B^2*a^6*b^2*c^2*e^2*f^6*z^2 - 18*B^2*a^5*b^2*c^3*e^4*f^4*z^2 -
9*B^2*a^4*b^2*c^4*e^6*f^2*z^2 - 6*B^2*a^5*b^3*c^2*e^3*f^5*z^2 + 3*B^2*a^4*b
^4*c^2*e^4*f^4*z^2 - 2*B^2*a^4*b^3*c^3*e^5*f^3*z^2 - 580*B^2*a^4*b^2*c^4*d^
3*f^5*z^2 + 536*B^2*a^3*b^4*c^3*d^3*f^5*z^2 + 471*A^2*a^4*b^2*c^4*e^4*f^4*z
^2 - 436*A^2*a^3*b^4*c^3*e^4*f^4*z^2 - 348*B^2*a^4*b^4*c^2*d^2*f^6*z^2 + 31
6*B^2*a^2*b^2*c^6*d^5*f^3*z^2 + 310*A^2*a^3*b^3*c^4*e^5*f^3*z^2 + 232*A^2*a
^5*b^2*c^3*e^2*f^6*z^2 - 229*A^2*a^2*b^4*c^4*e^6*f^2*z^2 - 216*A^2*a^4*b^4*
c^2*e^2*f^6*z^2 + 204*A^2*a^4*b^3*c^3*e^3*f^5*z^2 + 200*B^2*a^5*b^2*c^3*d^2
*f^6*z^2 + 150*A^2*a^3*b^2*c^5*e^6*f^2*z^2 - 120*B^2*a^2*b^4*c^4*d^4*f^4*z^
2 + 91*A^2*a^2*b^6*c^2*e^4*f^4*z^2 + 72*A^2*a^3*b^5*c^2*e^3*f^5*z^2 - 66*B^
2*a^2*b^6*c^2*d^3*f^5*z^2 + 44*A^2*a^2*b^5*c^3*e^5*f^3*z^2 - 16*B^2*a^3*b^2
*c^5*d^4*f^4*z^2 + 1952*A^2*a^4*b^2*c^4*d^2*f^6*z^2 - 1792*A^2*a^3*b^2*c^5*
d^3*f^5*z^2 - 1272*A^2*a^3*b^4*c^3*d^2*f^6*z^2 + 976*A^2*a^2*b^2*c^6*d^4*f^
4*z^2 + 960*A^2*a^2*b^4*c^4*d^3*f^5*z^2 + 282*A^2*a^2*b^6*c^2*d^2*f^6*z^2 -
 45*B^2*a^2*b^2*c^6*d^2*e^6*z^2 - 48*A^2*b*c^9*d^6*e*f*z^2 - 14*A^2*a*b^9*d
*e*f^6*z^2 - 7*A*B*b^10*d^2*e*f^5*z^2 + 2*A*B*b^10*d*e^3*f^4*z^2 - 64*A*B*a
^7*c^3*e*f^7*z^2 - 16*A*B*b^9*c*d^3*f^5*z^2 + 8*A*B*a^4*c^6*e^7*f*z^2 + 4*A
*B*b*c^9*d^6*e^2*z^2 + 2*A*B*b^6*c^4*d*e^7*z^2 - 120*A*B*a^3*c^7*d*e^7*z^2
- 16*A*B*a^3*b^7*d*f^7*z^2 + 16*A*B*a*b^9*d^2*f^6*z^2 + 8*A*B*a*c^9*d^5*e^3
*z^2 + 12*A*B*a^3*b*c^6*e^8*z^2 - 48*B^2*b^5*c^5*d^5*e*f^2*z^2 + 15*B^2*b^4
*c^6*d^5*e^2*f*z^2 - 14*B^2*b^7*c^3*d^4*e*f^3*z^2 + 4*B^2*b^9*c*d^2*e^3*f^3
*z^2 + 4*B^2*b^7*c^3*d^2*e^5*f*z^2 + 4*B^2*b^5*c^5*d^4*e^3*f*z^2 - B^2*b^6*
c^4*d^3*e^4*f*z^2 - 336*B^2*a^3*c^7*d^3*e^4*f*z^2 + 112*B^2*a^5*c^5*d*e^4*f
^3*z^2 - 112*A^2*b^3*c^7*d^5*e*f^2*z^2 + 80*B^2*a^6*c^4*d*e^2*f^5*z^2 - 48*
A^2*b^5*c^5*d^4*e*f^3*z^2 + 36*A^2*b^8*c^2*d*e^4*f^3*z^2 + 36*A^2*b^3*c^7*d
^4*e^3*f*z^2 - 28*A^2*b^7*c^3*d*e^5*f^2*z^2 + 20*A^2*b^2*c^8*d^5*e^2*f*z^2
+ 16*B^2*a^2*c^8*d^5*e^2*f*z^2 - 14*A^2*b^7*c^3*d^3*e*f^4*z^2 - 14*A^2*b^4*
c^6*d^3*e^4*f*z^2 - 10*A^2*b^5*c^5*d^2*e^5*f*z^2 - 1008*A^2*a^4*c^6*d*e^4*f
^3*z^2 - 760*A^2*a^5*c^5*d*e^2*f^5*z^2 + 272*A^2*a^2*c^8*d^3*e^4*f*z^2 + 48
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*B^2*a^5*b*c^4*e^5*f^3*z^2 + 36*B^2*a^6*b*c^3*e^3*f^5*z^2 + 12*B^2*a^5*b^4*
c*e^2*f^6*z^2 - 624*A^2*a^4*b*c^5*e^5*f^3*z^2 - 548*A^2*a^5*b*c^4*e^3*f^5*z
^2 + 182*A^2*a^2*b^3*c^5*e^7*f*z^2 - 180*B^2*a*b^4*c^5*d^5*f^3*z^2 + 132*B^
2*a^6*b^2*c^2*d*f^7*z^2 + 108*B^2*a^3*b^6*c*d^2*f^6*z^2 + 96*A^2*a^5*b^3*c^
2*e*f^7*z^2 + 68*A^2*a*b^6*c^3*e^6*f^2*z^2 + 58*A^2*a^3*b^6*c*e^2*f^6*z^2 -
 56*B^2*a*b^2*c^7*d^6*f^2*z^2 - 38*A^2*a^2*b^7*c*e^3*f^5*z^2 - 36*A^2*a*b^7
*c^2*e^5*f^3*z^2 + 20*B^2*a*b^6*c^3*d^4*f^4*z^2 - 736*A^2*a^5*b^2*c^3*d*f^7
*z^2 + 624*A^2*a^4*b^4*c^2*d*f^7*z^2 - 416*A^2*a*b^2*c^7*d^5*f^3*z^2 - 276*
A^2*a*b^4*c^5*d^4*f^4*z^2 - 196*A^2*a*b^6*c^3*d^3*f^5*z^2 + 8*B^2*a*b^4*c^5
*d^2*e^6*z^2 + 6*B^2*a*b^2*c^7*d^4*e^4*z^2 + 2*B^2*a^2*b^3*c^5*d*e^7*z^2 +
2*B^2*a*b^3*c^6*d^3*e^5*z^2 - 18*A^2*a*b^2*c^7*d^2*e^6*z^2 - 16*A*B*b*c^9*d
^7*f*z^2 - B^2*b^10*d^2*e^2*f^4*z^2 + 48*B^2*a^7*c^3*e^2*f^6*z^2 - 36*B^2*a
^6*c^4*e^4*f^4*z^2 + 31*B^2*b^6*c^4*d^5*f^3*z^2 - 24*B^2*a^5*c^5*e^6*f^2*z^
2 + 20*B^2*b^4*c^6*d^6*f^2*z^2 - 6*A^2*b^8*c^2*e^6*f^2*z^2 + 2*B^2*b^8*c^2*
d^4*f^4*z^2 - 768*B^2*a^5*c^5*d^3*f^5*z^2 + 512*B^2*a^6*c^4*d^2*f^6*z^2 + 5
12*B^2*a^4*c^6*d^4*f^4*z^2 + 232*A^2*a^5*c^5*e^4*f^4*z^2 + 188*A^2*a^4*c^6*
e^6*f^2*z^2 - 128*B^2*a^3*c^7*d^5*f^3*z^2 + 92*A^2*a^6*c^4*e^2*f^6*z^2 + 80
*A^2*b^4*c^6*d^5*f^3*z^2 + 64*A^2*b^2*c^8*d^6*f^2*z^2 + 31*A^2*b^6*c^4*d^4*
f^4*z^2 + 14*A^2*b^8*c^2*d^3*f^5*z^2 - 5*B^2*b^4*c^6*d^4*e^4*z^2 + 4*B^2*b^
3*c^7*d^5*e^3*z^2 + 2*B^2*b^5*c^5*d^3*e^5*z^2 - B^2*b^6*c^4*d^2*e^6*z^2 - B
^2*b^2*c^8*d^6*e^2*z^2 - B^2*a^4*b^6*e^2*f^6*z^2 - 1152*A^2*a^3*c^7*d^4*f^4
*z^2 + 1008*A^2*a^4*c^6*d^3*f^5*z^2 + 624*A^2*a^2*c^8*d^5*f^3*z^2 - 288*A^2
*a^5*c^5*d^2*f^6*z^2 + 56*B^2*a^3*c^7*d^2*e^6*z^2 - 10*B^2*a^2*b^8*d^2*f^6*
z^2 - 9*A^2*b^2*c^8*d^4*e^4*z^2 - 5*A^2*a^2*b^8*e^2*f^6*z^2 - 4*B^2*a^2*c^8
*d^4*e^4*z^2 + 3*A^2*b^4*c^6*d^2*e^6*z^2 - 2*A^2*b^3*c^7*d^3*e^5*z^2 - 36*A
^2*a^2*c^8*d^2*e^6*z^2 - 48*A^2*a^6*b^2*c^2*f^8*z^2 - 45*A^2*a^2*b^2*c^6*e^
8*z^2 + 4*A^2*b^10*d*e^2*f^5*z^2 + 4*B^2*b^2*c^8*d^7*f*z^2 + 4*A^2*b^9*c*e^
5*f^3*z^2 + 4*A^2*b^7*c^3*e^7*f*z^2 - 128*B^2*a^7*c^3*d*f^7*z^2 - 160*A^2*a
*c^9*d^6*f^2*z^2 - 112*A^2*a^6*c^4*d*f^7*z^2 + 12*A^2*b*c^9*d^5*e^3*z^2 + 4
*A^2*a*b^9*e^3*f^5*z^2 + 3*B^2*a^4*b^6*d*f^7*z^2 + 2*A^2*a^3*b^7*e*f^7*z^2
- 24*A^2*a*c^9*d^4*e^4*z^2 + 14*A^2*a^2*b^8*d*f^7*z^2 + 12*A^2*a^5*b^4*c*f^
8*z^2 + 12*A^2*a*b^4*c^5*e^8*z^2 + A*B*a^4*b^6*e*f^7*z^2 + B^2*a^2*b^8*d*e^
2*f^5*z^2 + 16*A^2*c^10*d^7*f*z^2 + 3*B^2*b^10*d^3*f^5*z^2 - A^2*b^10*e^4*f
^4*z^2 - 4*A^2*c^10*d^6*e^2*z^2 - A^2*b^10*d^2*f^6*z^2 + 64*A^2*a^7*c^3*f^8
*z^2 - 4*B^2*a^4*c^6*e^8*z^2 - A^2*b^6*c^4*e^8*z^2 + 48*A^2*a^3*c^7*e^8*z^2
 - A^2*a^4*b^6*f^8*z^2 + 720*A^2*B*a*b^2*c^5*d^2*e^2*f^3*z - 600*A^2*B*a^2*
b^2*c^4*d*e^2*f^4*z + 576*A*B^2*a^2*b^2*c^4*d^2*e*f^4*z + 348*A*B^2*a*b^2*c
^5*d^2*e^3*f^2*z - 336*A*B^2*a^2*b*c^5*d^2*e^2*f^3*z - 260*A*B^2*a*b^3*c^4*
d^2*e^2*f^3*z - 240*A*B^2*a^2*b^2*c^4*d*e^3*f^3*z + 196*A*B^2*a^2*b^3*c^3*d
*e^2*f^4*z + 172*A^2*B*a*b*c^6*d*e^5*f*z + 20*A*B^2*a*b^6*c*d*e*f^5*z - 912
*A^2*B*a^2*b*c^5*d^2*e*f^4*z - 644*A^2*B*a*b*c^6*d^2*e^3*f^2*z - 432*A*B^2*
a*b^2*c^5*d^3*e*f^3*z + 372*A^2*B*a^2*b*c^5*d*e^3*f^3*z - 330*A^2*B*a*b^2*c
^5*d*e^4*f^2*z + 312*A*B^2*a*b*c^6*d^3*e^2*f^2*z - 208*A*B^2*a^3*b^2*c^3*d*
e*f^5*z + 192*A^2*B*a^2*b^3*c^3*d*e*f^5*z + 172*A^2*B*a*b^3*c^4*d*e^3*f^3*z
 + 108*A*B^2*a^2*b*c^5*d*e^4*f^2*z + 104*A*B^2*a^3*b*c^4*d*e^2*f^4*z - 80*A
^2*B*a*b^3*c^4*d^2*e*f^4*z + 68*A^2*B*a*b^4*c^3*d*e^2*f^4*z - 60*A*B^2*a*b^
5*c^2*d*e^2*f^4*z + 58*A*B^2*a*b^3*c^4*d*e^4*f^2*z - 36*A*B^2*a*b^4*c^3*d^2
*e*f^4*z - 24*A*B^2*a^2*b^4*c^2*d*e*f^5*z + 24*A*B^2*a*b^4*c^3*d*e^3*f^3*z
+ 592*A^2*B*a*b*c^6*d^3*e*f^3*z + 240*A^2*B*a^3*b*c^4*d*e*f^5*z - 132*A*B^2
*a*b*c^6*d^2*e^4*f*z - 60*A*B^2*a*b^2*c^5*d*e^5*f*z - 48*A^2*B*a*b^5*c^2*d*
e*f^5*z + 20*B^3*a*b*c^6*d^3*e^3*f*z + 16*B^3*a^4*b*c^3*d*e*f^5*z - 16*B^3*
a*b*c^6*d^4*e*f^2*z + 12*B^3*a^2*b*c^5*d*e^5*f*z + 320*A^3*a*b*c^6*d*e^4*f^
2*z + 40*A^3*a*b^4*c^3*d*e*f^5*z - 48*A^2*B*b*c^7*d^4*e*f^2*z - 44*A^2*B*b^
3*c^5*d*e^5*f*z - 20*A*B^2*b*c^7*d^4*e^2*f*z + 14*A*B^2*b^4*c^4*d*e^5*f*z +
 12*A^2*B*b*c^7*d^3*e^3*f*z + 4*A*B^2*b^7*c*d*e^2*f^4*z + 160*A*B^2*a^4*c^4
*d*e*f^5*z + 152*A^2*B*a*c^7*d^2*e^4*f*z - 40*A*B^2*a*c^7*d^3*e^3*f*z + 32*
A*B^2*a*c^7*d^4*e*f^2*z - 16*A*B^2*a^2*c^6*d*e^5*f*z + 128*A^2*B*a^4*b*c^3*
e*f^6*z + 42*A^2*B*a*b^2*c^5*e^6*f*z + 24*A^2*B*a^2*b^5*c*e*f^6*z - 12*A*B^
2*a^3*b^4*c*e*f^6*z - 12*A*B^2*a^2*b*c^5*e^6*f*z - 10*A^2*B*a*b^6*c*e^2*f^5
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*z - 160*A*B^2*a*b*c^6*d^4*f^3*z + 112*A*B^2*a^4*b*c^3*d*f^6*z - 24*A*B^2*a
^2*b^5*c*d*f^6*z - 84*B^3*a*b^2*c^5*d^3*e^2*f^2*z - 80*B^3*a^2*b^3*c^3*d^2*
e*f^4*z - 60*B^3*a^2*b*c^5*d^2*e^3*f^2*z - 20*B^3*a^3*b^2*c^3*d*e^2*f^4*z -
 20*B^3*a*b^3*c^4*d^2*e^3*f^2*z - 9*B^3*a^2*b^2*c^4*d*e^4*f^2*z - 8*B^3*a*b
^4*c^3*d^2*e^2*f^3*z + 6*B^3*a^2*b^4*c^2*d*e^2*f^4*z - 4*B^3*a^2*b^3*c^3*d*
e^3*f^3*z - 216*A^2*B*b^4*c^4*d^2*e^2*f^3*z + 196*A^2*B*b^3*c^5*d^2*e^3*f^2
*z - 108*A*B^2*b^3*c^5*d^3*e^2*f^2*z - 94*A*B^2*b^4*c^4*d^2*e^3*f^2*z + 88*
A^2*B*b^2*c^6*d^3*e^2*f^2*z + 80*A*B^2*b^5*c^3*d^2*e^2*f^3*z + 360*A^2*B*a^
2*c^6*d^2*e^2*f^3*z + 8*A*B^2*a^2*c^6*d^2*e^3*f^2*z + 153*A^2*B*a^2*b^2*c^4
*e^4*f^3*z - 144*A^2*B*a^2*b^3*c^3*e^3*f^4*z + 80*A^2*B*a^3*b^2*c^3*e^2*f^5
*z + 36*A*B^2*a^3*b^2*c^3*e^3*f^4*z + 12*A^2*B*a^2*b^4*c^2*e^2*f^5*z + 12*A
*B^2*a^3*b^3*c^2*e^2*f^5*z + 9*A*B^2*a^2*b^2*c^4*e^5*f^2*z - 6*A*B^2*a^2*b^
4*c^2*e^3*f^4*z + 4*A*B^2*a^2*b^3*c^3*e^4*f^3*z + 480*A^2*B*a^2*b^2*c^4*d^2
*f^5*z - 176*A*B^2*a^2*b^3*c^3*d^2*f^5*z - 10*A^2*B*a*b^6*c*d*f^6*z + 16*A*
B^2*a*b*c^6*d*e^6*z + 80*B^3*a*b^3*c^4*d^3*e*f^3*z - 48*B^3*a^3*b*c^4*d^2*e
*f^4*z + 48*B^3*a^2*b*c^5*d^3*e*f^3*z + 44*B^3*a^3*b*c^4*d*e^3*f^3*z + 24*B
^3*a*b^5*c^2*d^2*e*f^4*z + 18*B^3*a*b^2*c^5*d^2*e^4*f*z + 696*A^3*a^2*b*c^5
*d*e^2*f^4*z - 504*A^3*a*b*c^6*d^2*e^2*f^3*z - 192*A^3*a*b^2*c^5*d*e^3*f^3*
z - 144*A^3*a^2*b^2*c^4*d*e*f^5*z + 96*A^3*a*b^2*c^5*d^2*e*f^4*z - 72*A^3*a
*b^3*c^4*d*e^2*f^4*z - 208*A^2*B*b^3*c^5*d^3*e*f^3*z + 152*A*B^2*b^4*c^4*d^
3*e*f^3*z + 80*A^2*B*b^5*c^3*d^2*e*f^4*z + 75*A^2*B*b^4*c^4*d*e^4*f^2*z - 5
9*A^2*B*b^2*c^6*d^2*e^4*f*z - 52*A^2*B*b^5*c^3*d*e^3*f^3*z + 42*A*B^2*b^3*c
^5*d^2*e^4*f*z - 21*A*B^2*b^6*c^2*d^2*e*f^4*z - 16*A*B^2*b^5*c^3*d*e^4*f^2*
z + 16*A*B^2*b^2*c^6*d^4*e*f^2*z + 16*A*B^2*b^2*c^6*d^3*e^3*f*z + 11*A^2*B*
b^6*c^2*d*e^2*f^4*z + 4*A*B^2*b^6*c^2*d*e^3*f^3*z - 256*A^2*B*a*c^7*d^3*e^2
*f^2*z - 96*A*B^2*a^3*c^5*d^2*e*f^4*z - 36*A^2*B*a^2*c^6*d*e^4*f^2*z - 32*A
^2*B*a^3*c^5*d*e^2*f^4*z - 32*A*B^2*a^2*c^6*d^3*e*f^3*z + 8*A*B^2*a^3*c^5*d
*e^3*f^3*z - 96*A^2*B*a^3*b^3*c^2*e*f^6*z + 68*A^2*B*a^3*b*c^4*e^3*f^4*z -
60*A*B^2*a^4*b*c^3*e^2*f^5*z - 60*A*B^2*a^3*b*c^4*e^4*f^3*z + 48*A*B^2*a^4*
b^2*c^2*e*f^6*z - 38*A^2*B*a*b^3*c^4*e^5*f^2*z - 36*A^2*B*a^2*b*c^5*e^5*f^2
*z + 36*A^2*B*a*b^5*c^2*e^3*f^4*z - 16*A^2*B*a*b^4*c^3*e^4*f^3*z + 384*A*B^
2*a^2*b*c^5*d^3*f^4*z - 352*A*B^2*a^3*b*c^4*d^2*f^5*z - 288*A^2*B*a*b^2*c^5
*d^3*f^4*z - 160*A^2*B*a^3*b^2*c^3*d*f^6*z - 148*A^2*B*a*b^4*c^3*d^2*f^5*z
+ 112*A*B^2*a*b^3*c^4*d^3*f^4*z + 72*A^2*B*a^2*b^4*c^2*d*f^6*z + 72*A*B^2*a
*b^5*c^2*d^2*f^5*z + 48*A*B^2*a^3*b^3*c^2*d*f^6*z + 102*B^3*a^2*b^2*c^4*d^2
*e^2*f^3*z - 32*B^3*b^5*c^3*d^3*e*f^3*z - 8*B^3*b^3*c^5*d^3*e^3*f*z - 7*B^3
*b^4*c^4*d^2*e^4*f*z + 5*B^3*b^2*c^6*d^4*e^2*f*z + 80*A^3*b^2*c^6*d^3*e*f^3
*z - 74*A^3*b^3*c^5*d*e^4*f^2*z - 64*A^3*b^4*c^4*d^2*e*f^4*z + 60*A^3*b^4*c
^4*d*e^3*f^3*z - 48*B^3*a^4*c^4*d*e^2*f^4*z - 24*B^3*a^3*c^5*d*e^4*f^2*z +
20*B^3*a^2*c^6*d^2*e^4*f*z - 16*A^3*b^5*c^3*d*e^2*f^4*z + 8*A^3*b*c^7*d^3*e
^2*f^2*z + 480*A^3*a^2*c^6*d^2*e*f^4*z - 392*A^3*a^2*c^6*d*e^3*f^3*z + 280*
A^3*a*c^7*d^2*e^3*f^2*z - 4*B^3*a^4*b*c^3*e^3*f^4*z - 200*A^3*a^3*b*c^4*e^2
*f^5*z - 144*A^3*a^2*b*c^5*e^4*f^3*z + 48*B^3*a*b^2*c^5*d^4*f^3*z + 42*A^3*
a*b^2*c^5*e^5*f^2*z - 36*B^3*a^4*b^2*c^2*d*f^6*z - 32*A^3*a^3*b^2*c^3*e*f^6
*z - 24*A^3*a^2*b^4*c^2*e*f^6*z - 24*A^3*a*b^5*c^2*e^2*f^5*z + 10*A^3*a*b^3
*c^4*e^4*f^3*z - 4*B^3*a*b^4*c^3*d^3*f^4*z - 4*A^3*a*b^4*c^3*e^3*f^4*z - 48
0*A^3*a^2*b*c^5*d^2*f^5*z - 160*A^3*a^2*b^3*c^3*d*f^6*z + 128*A^3*a*b^3*c^4
*d^2*f^5*z + 8*A^2*B*b^5*c^3*e^5*f^2*z - 2*A^2*B*b^6*c^2*e^4*f^3*z + 112*A^
2*B*b^4*c^4*d^3*f^4*z - 92*A^2*B*a^4*c^4*e^2*f^5*z - 64*A^2*B*a^3*c^5*e^4*f
^3*z - 64*A*B^2*b^5*c^3*d^3*f^4*z + 24*A*B^2*a^4*c^4*e^3*f^4*z + 24*A*B^2*a
^3*c^5*e^5*f^2*z + 16*A^2*B*b^2*c^6*d^4*f^3*z + 16*A*B^2*b^3*c^5*d^4*f^3*z
- A^2*B*b^6*c^2*d^2*f^5*z + 448*A^2*B*a^3*c^5*d^2*f^5*z - 352*A^2*B*a^2*c^6
*d^3*f^4*z - 5*A*B^2*b^2*c^6*d^2*e^5*z - 48*A^2*B*a^4*b^2*c^2*f^7*z - 2*B^3
*b^7*c*d^2*e*f^4*z + 34*A^3*b^2*c^6*d*e^5*f*z + 16*A^3*b*c^7*d^2*e^4*f*z +
2*A^3*b^6*c^2*d*e*f^5*z - 416*A^3*a^3*c^5*d*e*f^5*z - 224*A^3*a*c^7*d^3*e*f
^3*z + 12*B^3*a^3*b^4*c*d*f^6*z - 10*B^3*a*b^6*c*d^2*f^5*z + 416*A^3*a^3*b*
c^4*d*f^6*z + 224*A^3*a*b*c^6*d^3*f^4*z + 24*A^3*a*b^5*c^2*d*f^6*z - 4*B^3*
a*b*c^6*d^2*e^5*z + 20*A^2*B*c^8*d^4*e^2*f*z - 7*A^2*B*b^4*c^4*e^6*f*z - 2*
A^2*B*b^7*c*e^3*f^4*z - 64*A*B^2*a^5*c^3*e*f^6*z + 16*A*B^2*b*c^7*d^5*f^2*z

```
 - 8*A^2*B*a^2*c^6*e^6*f*z - 2*A*B^2*b^7*c*d^2*f^5*z - 272*A^2*B*a^4*c^4*d*
f^6*z + 128*A^2*B*a*c^7*d^4*f^3*z + 9*A^2*B*b^2*c^6*d*e^6*z - 4*A*B^2*b^3*c
^5*d*e^6*z + 4*A*B^2*b*c^7*d^3*e^4*z + 8*A*B^2*a*c^7*d^2*e^5*z + 12*A^2*B*a
^3*b^4*c*f^7*z + 30*B^3*b^4*c^4*d^3*e^2*f^2*z + 8*B^3*b^5*c^3*d^2*e^3*f^2*z
 - 2*B^3*b^6*c^2*d^2*e^2*f^3*z + 152*A^3*b^3*c^5*d^2*e^2*f^3*z - 108*A^3*b^
2*c^6*d^2*e^3*f^2*z + 48*B^3*a^3*c^5*d^2*e^2*f^3*z - 16*B^3*a^2*c^6*d^3*e^2
*f^2*z - 3*B^3*a^4*b^2*c^2*e^2*f^5*z - 120*B^3*a^2*b^2*c^4*d^3*f^4*z + 112*
B^3*a^3*b^2*c^3*d^2*f^5*z + 112*A^3*a^2*b^3*c^3*e^2*f^5*z + 12*A^3*a^2*b^2*
c^4*e^3*f^4*z - 120*A^3*a*c^7*d*e^5*f*z - 52*A^3*a*b*c^6*e^6*f*z + 10*A^3*a
*b^6*c*e*f^6*z - 2*A*B^2*b^8*d*e*f^5*z - 2*A^2*B*a*b^7*e*f^6*z - 24*A^2*B*a
*c^7*d*e^6*z + 2*A*B^2*a*b^7*d*f^6*z - 12*A^2*B*a*b*c^6*e^7*z - 2*A^3*b^7*c
*d*f^6*z - 4*A^3*b*c^7*d*e^6*z + 16*B^3*a^5*c^3*e^2*f^5*z + 11*B^3*b^6*c^2*
d^3*f^4*z - 11*A^3*b^4*c^4*e^5*f^2*z - 8*B^3*b^4*c^4*d^4*f^3*z - 4*B^3*b^2*
c^6*d^5*f^2*z + 4*B^3*a^4*c^4*e^4*f^3*z + 4*A^3*b^5*c^3*e^4*f^3*z - A^3*b^6
*c^2*e^3*f^4*z + 136*A^3*a^3*c^5*e^3*f^4*z + 68*A^3*a^2*c^6*e^5*f^2*z - 64*
A^3*b^3*c^5*d^3*f^4*z + 2*B^3*b^3*c^5*d^2*e^5*z - B^3*b^2*c^6*d^3*e^4*z + 9
6*A^3*a^3*b^3*c^2*f^7*z + A*B^2*a^2*b^6*e*f^6*z + 32*A^3*c^8*d^4*e*f^2*z -
24*A^3*c^8*d^3*e^3*f*z + 10*A^3*b^3*c^5*e^6*f*z + 2*A^3*b^7*c*e^2*f^5*z + 1
28*A^3*a^4*c^4*e*f^6*z - 32*A^3*b*c^7*d^4*f^3*z - 4*B^3*a^2*c^6*d*e^6*z - B
^3*a^2*b^6*d*f^6*z - 128*A^3*a^4*b*c^3*f^7*z - 24*A^3*a^2*b^5*c*f^7*z - 16*
A^2*B*c^8*d^5*f^2*z - 4*A^2*B*c^8*d^3*e^4*z + 64*A^2*B*a^5*c^3*f^7*z + 2*A^
2*B*b^3*c^5*e^7*z + 4*A*B^2*a^2*c^6*e^7*z - A^2*B*a^2*b^6*f^7*z + 4*A^3*c^8
*d^2*e^5*z - 3*A^3*b^2*c^6*e^7*z + A^2*B*b^8*d*f^6*z - A^3*b^8*e*f^6*z + 16
*A^3*a*c^7*e^7*z + 2*A^3*a*b^7*f^7*z + A^2*B*b^8*e^2*f^5*z + B^3*b^8*d^2*f^
5*z - 48*A^2*B^2*a*b*c^4*d*e*f^4 + 28*A*B^3*a*b^2*c^3*d*e*f^4 - 16*A*B^3*a*
b*c^4*d*e^2*f^3 + 16*A^3*B*a*c^5*d*e*f^4 + 32*A^3*B*a*b*c^4*d*f^5 + 12*A^2*
B^2*b^3*c^3*d*e*f^4 + 5*A*B^3*b^2*c^4*d^2*e*f^3 + 4*A*B^3*b^3*c^3*d*e^2*f^3
 + 24*A^2*B^2*a*c^5*d*e^2*f^3 + 24*A^2*B^2*a^2*b*c^3*e*f^5 + 12*A^2*B^2*a*b
*c^4*e^3*f^3 - 6*A^2*B^2*a*b^3*c^2*e*f^5 + 4*A*B^3*a^2*b*c^3*e^2*f^4 + 3*A*
B^3*a^2*b^2*c^2*e*f^5 - 18*A^2*B^2*a*b^2*c^3*d*f^5 - 4*B^4*a^2*b*c^3*d*e*f^
4 + 4*B^4*a*b*c^4*d^2*e*f^3 - 6*A*B^3*b^4*c^2*d*e*f^4 + 4*A^3*B*b*c^5*d*e^2
*f^3 - 2*A^3*B*b^2*c^4*d*e*f^4 - 8*A*B^3*a^2*c^4*d*e*f^4 - 8*A*B^3*a*c^5*d^
2*e*f^3 + 26*A^3*B*a*b^2*c^3*e*f^5 + 8*A^3*B*a*b*c^4*e^2*f^4 + 32*A*B^3*a*b
*c^4*d^2*f^4 - 28*A*B^3*a^2*b*c^3*d*f^5 + 6*A*B^3*a*b^3*c^2*d*f^5 - 9*A^2*B
^2*b^2*c^4*d*e^2*f^3 - 18*A^2*B^2*a*b^2*c^3*e^2*f^4 - 4*A^3*B*c^6*d^2*e*f^3
 - 3*A^3*B*b^4*c^2*e*f^5 - 44*A^3*B*a^2*c^4*e*f^5 - 16*A^3*B*a*c^5*e^3*f^3
- 16*A*B^3*a^3*c^3*e*f^5 - 10*A^3*B*b^3*c^3*d*f^5 - 4*A^3*B*b*c^5*d^2*f^4 -
 4*A*B^3*b*c^5*d^3*f^3 - 28*A^3*B*a^2*b*c^3*f^6 + 6*A^3*B*a*b^3*c^2*f^6 - 4
*A^4*b*c^5*d*e*f^4 - 20*A^4*a*b*c^4*e*f^5 + 3*A^2*B^2*b^4*c^2*e^2*f^4 - 2*A
^2*B^2*b^3*c^3*e^3*f^3 + 12*A^2*B^2*a^2*c^4*e^2*f^4 + 9*A^2*B^2*b^2*c^4*d^2
*f^4 - 3*A^2*B^2*a^2*b^2*c^2*f^6 - 2*B^4*b^3*c^3*d^2*e*f^3 + 4*B^4*a^2*c^4*
d*e^2*f^3 - 10*B^4*a*b^2*c^3*d^2*f^4 - 3*B^4*a^2*b^2*c^2*d*f^5 + 3*A^3*B*b^
2*c^4*e^3*f^3 - 2*A^3*B*b^3*c^3*e^2*f^4 - 10*A*B^3*b^3*c^3*d^2*f^4 - 4*A*B^
3*a^2*c^4*e^3*f^3 + 3*A^2*B^2*b^4*c^2*d*f^5 + 36*A^2*B^2*a^2*c^4*d*f^5 - 24
*A^2*B^2*a*c^5*d^2*f^4 + 4*A^2*B^2*c^6*d^3*f^3 + 16*A^2*B^2*a^3*c^3*f^6 + 4
*A^4*b^3*c^3*e*f^5 + 16*B^4*a^3*c^3*d*f^5 + 16*A^4*a*c^5*e^2*f^4 + 8*A^4*b^
2*c^4*d*f^5 - 8*A^4*a*b^2*c^3*f^6 - 24*A^4*a*c^5*d*f^5 + 3*B^4*b^4*c^2*d^2*
f^4 - 3*A^4*b^2*c^4*e^2*f^4 + 4*A^4*c^6*d^2*f^4 + 36*A^4*a^2*c^4*f^6 + B^4*
b^2*c^4*d^3*f^3, z, k), k, 1, 4) - ((A*b^3*f + 2*A*a*c^2*e + A*b*c^2*d - 2*
B*a*c^2*d - A*b^2*c*e - B*a*b^2*f + 2*B*a^2*c*f - 3*A*a*b*c*f + B*a*b*c*e)/
(a^2*b^2*f^2 - 4*a^3*c*f^2 - 4*a*c^3*d^2 - 4*a^2*c^2*e^2 + b^2*c^2*d^2 + b^
4*d*f - a*b^3*e*f - b^3*c*d*e + a*b^2*c*e^2 + 8*a^2*c^2*d*f + 4*a*b*c^2*d*e
 - 6*a*b^2*c*d*f + 4*a^2*b*c*e*f) - (x*(2*A*a*c^2*f - 2*A*c^3*d + A*b*c^2*e
 - 2*B*a*c^2*e + B*b*c^2*d - A*b^2*c*f + B*a*b*c*f))/(a^2*b^2*f^2 - 4*a^3*c
*f^2 - 4*a*c^3*d^2 - 4*a^2*c^2*e^2 + b^2*c^2*d^2 + b^4*d*f - a*b^3*e*f - b^
3*c*d*e + a*b^2*c*e^2 + 8*a^2*c^2*d*f + 4*a*b*c^2*d*e - 6*a*b^2*c*d*f + 4*a
^2*b*c*e*f))/(a + b*x + c*x^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(c*x**2+b*x+a)**2/(f*x**2+e*x+d),x)
```

```
[Out] Timed out
```

3.17 $\int \frac{g+hx}{(a+bx+cx^2)(ad+bdx+cdx^2)^2}\,dx$

Optimal. Leaf size=140

$$\frac{3(b+2cx)(2cg-bh)}{2d^2(b^2-4ac)^2(a+bx+cx^2)}-\frac{-2ah+x(2cg-bh)+bg}{2d^2(b^2-4ac)(a+bx+cx^2)^2}-\frac{6c(2cg-bh)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{d^2(b^2-4ac)^{5/2}}$$

```
[Out] 1/2*(-b*g+2*a*h-(-b*h+2*c*g)*x)/(-4*a*c+b^2)/d^2/(c*x^2+b*x+a)^2+3/2*(-b*h+
2*c*g)*(2*c*x+b)/(-4*a*c+b^2)^2/d^2/(c*x^2+b*x+a)-6*c*(-b*h+2*c*g)*arctanh(
(2*c*x+b)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(5/2)/d^2
```

Rubi [A] time = 0.13, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.147, Rules used = {998, 638, 614, 618, 206}

$$\frac{3(b+2cx)(2cg-bh)}{2d^2(b^2-4ac)^2(a+bx+cx^2)}-\frac{-2ah+x(2cg-bh)+bg}{2d^2(b^2-4ac)(a+bx+cx^2)^2}-\frac{6c(2cg-bh)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{d^2(b^2-4ac)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(g + h*x)/((a + b*x + c*x^2)*(a*d + b*d*x + c*d*x^2)^2),x]
```

```
[Out] -(b*g - 2*a*h + (2*c*g - b*h)*x)/(2*(b^2 - 4*a*c)*d^2*(a + b*x + c*x^2)^2)
+ (3*(2*c*g - b*h)*(b + 2*c*x))/(2*(b^2 - 4*a*c)^2*d^2*(a + b*x + c*x^2)) -
 (6*c*(2*c*g - b*h)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/((b^2 - 4*a*c)^
(5/2)*d^2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 638

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p +
 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a
*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 998

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_
) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Dist[(c/f)^p, Int[(g + h*
x)^m*(d + e*x + f*x^2)^(p + q), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, p,
 q}, x] && EqQ[c*d - a*f, 0] && EqQ[b*d - a*e, 0] && (IntegerQ[p] || GtQ[c/
f, 0]) && ( !IntegerQ[q] || LeafCount[d + e*x + f*x^2] <= LeafCount[a + b*x
 + c*x^2])
```

Rubi steps

$$\begin{aligned}
\int \frac{g+hx}{\left(a+bx+cx^2\right)\left(ad+bdx+cdx^2\right)^2}\,dx &= \frac{\int \frac{g+hx}{\left(a+bx+cx^2\right)^3}\,dx}{d^2} \\
&= -\frac{bg-2ah+(2cg-bh)x}{2\left(b^2-4ac\right)d^2\left(a+bx+cx^2\right)^2}-\frac{(3(2cg-bh))\int \frac{1}{\left(a+bx+cx^2\right)^2}\,dx}{2\left(b^2-4ac\right)d^2} \\
&= -\frac{bg-2ah+(2cg-bh)x}{2\left(b^2-4ac\right)d^2\left(a+bx+cx^2\right)^2}+\frac{3(2cg-bh)(b+2cx)}{2\left(b^2-4ac\right)^2d^2\left(a+bx+cx^2\right)} \\
&= -\frac{bg-2ah+(2cg-bh)x}{2\left(b^2-4ac\right)d^2\left(a+bx+cx^2\right)^2}+\frac{3(2cg-bh)(b+2cx)}{2\left(b^2-4ac\right)^2d^2\left(a+bx+cx^2\right)} \\
&= -\frac{bg-2ah+(2cg-bh)x}{2\left(b^2-4ac\right)d^2\left(a+bx+cx^2\right)^2}+\frac{3(2cg-bh)(b+2cx)}{2\left(b^2-4ac\right)^2d^2\left(a+bx+cx^2\right)}
\end{aligned}$$

Mathematica [A] time = 0.14, size = 131, normalized size = 0.94

$$\frac{\frac{\left(b^2-4ac\right)(2ah-bg+bhx-2cgx)}{(a+x(b+cx))^2}-\frac{12c(bh-2cg)\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}+\frac{3(b+2cx)(2cg-bh)}{a+x(b+cx)}}{2d^2\left(b^2-4ac\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g + h*x)/((a + b*x + c*x^2)*(a*d + b*d*x + c*d*x^2)^2),x]
```

```
[Out] (((b^2 - 4*a*c)*(-(b*g) + 2*a*h - 2*c*g*x + b*h*x))/(a + x*(b + c*x))^2 + (
3*(2*c*g - b*h)*(b + 2*c*x))/(a + x*(b + c*x)) - (12*c*(-2*c*g + b*h)*ArcTa
n[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c])/(2*(b^2 - 4*a*c)^2*d
^2)
```

fricas [B] time = 1.42, size = 1150, normalized size = 8.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)/(c*x^2+b*x+a)/(c*d*x^2+b*d*x+a*d)^2,x, algorithm="fricas"
)
```

```
[Out] [1/2*(6*(2*(b^2*c^3 - 4*a*c^4)*g - (b^3*c^2 - 4*a*b*c^3)*h)*x^3 + 9*(2*(b^3
*c^2 - 4*a*b*c^3)*g - (b^4*c - 4*a*b^2*c^2)*h)*x^2 - 6*(2*a^2*c^2*g - a^2*b
*c*h + (2*c^4*g - b*c^3*h)*x^4 + 2*(2*b*c^3*g - b^2*c^2*h)*x^3 + (2*(b^2*c^
2 + 2*a*c^3)*g - (b^3*c + 2*a*b*c^2)*h)*x^2 + 2*(2*a*b*c^2*g - a*b^2*c*h)*x
)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a
```

```
*c)*(2*c*x + b))/(c*x^2 + b*x + a)) - (b^5 - 14*a*b^3*c + 40*a^2*b*c^2)*g -
 (a*b^4 + 4*a^2*b^2*c - 32*a^3*c^2)*h + 2*(2*(b^4*c + a*b^2*c^2 - 20*a^2*c^
3)*g - (b^5 + a*b^3*c - 20*a^2*b*c^2)*h)*x)/((b^6*c^2 - 12*a*b^4*c^3 + 48*a
^2*b^2*c^4 - 64*a^3*c^5)*d^2*x^4 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3
 - 64*a^3*b*c^4)*d^2*x^3 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*
c^3 - 128*a^4*c^4)*d^2*x^2 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*
a^4*b*c^3)*d^2*x + (a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)*d
^2), 1/2*(6*(2*(b^2*c^3 - 4*a*c^4)*g - (b^3*c^2 - 4*a*b*c^3)*h)*x^3 + 9*(2*
(b^3*c^2 - 4*a*b*c^3)*g - (b^4*c - 4*a*b^2*c^2)*h)*x^2 - 12*(2*a^2*c^2*g -
a^2*b*c*h + (2*c^4*g - b*c^3*h)*x^4 + 2*(2*b*c^3*g - b^2*c^2*h)*x^3 + (2*(b
^2*c^2 + 2*a*c^3)*g - (b^3*c + 2*a*b*c^2)*h)*x^2 + 2*(2*a*b*c^2*g - a*b^2*c
*h)*x)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a
*c)) - (b^5 - 14*a*b^3*c + 40*a^2*b*c^2)*g - (a*b^4 + 4*a^2*b^2*c - 32*a^3*
c^2)*h + 2*(2*(b^4*c + a*b^2*c^2 - 20*a^2*c^3)*g - (b^5 + a*b^3*c - 20*a^2*
b*c^2)*h)*x)/((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^2*x^
4 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^2*x^3 + (b^8
 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d^2*x^2 + 2*
(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*d^2*x + (a^2*b^6 - 1
2*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)*d^2)]
```

giac [A] time = 0.17, size = 219, normalized size = 1.56

$$\frac{6\left(2\,c^2g - bch\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(b^4d^2 - 8\,ab^2cd^2 + 16\,a^2c^2d^2\right)\sqrt{-b^2+4\,ac}} + \frac{12\,c^3gx^3 - 6\,bc^2hx^3 + 18\,bc^2gx^2 - 9\,b^2chx^2 + 4\,b^2cgx + 20\,ac^2gx - 2}{2\left(b^4d^2 - 8\,ab^2cd^2 + 16\,a^2c^2d^2\right)(c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)/(c*x^2+b*x+a)/(c*d*x^2+b*d*x+a*d)^2,x, algorithm="giac")
```

```
[Out] 6*(2*c^2*g - b*c*h)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^4*d^2 - 8*a*
b^2*c*d^2 + 16*a^2*c^2*d^2)*sqrt(-b^2 + 4*a*c)) + 1/2*(12*c^3*g*x^3 - 6*b*c
^2*h*x^3 + 18*b*c^2*g*x^2 - 9*b^2*c*h*x^2 + 4*b^2*c*g*x + 20*a*c^2*g*x - 2*
b^3*h*x - 10*a*b*c*h*x - b^3*g + 10*a*b*c*g - a*b^2*h - 8*a^2*c*h)/((b^4*d^
2 - 8*a*b^2*c*d^2 + 16*a^2*c^2*d^2)*(c*x^2 + b*x + a)^2)
```

maple [B] time = 0.01, size = 340, normalized size = 2.43

$$-\frac{3bchx}{\left(4ac-b^2\right)^2\left(c\,x^2+bx+a\right)d^2} - \frac{6bch\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{\frac{5}{2}}d^2} + \frac{6c^2gx}{\left(4ac-b^2\right)^2\left(c\,x^2+bx+a\right)d^2} + \frac{12c^2g\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{\frac{5}{2}}d^2} -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)/(c*x^2+b*x+a)/(c*d*x^2+b*d*x+a*d)^2,x)
```

```
[Out] -1/2/d^2/(4*a*c-b^2)/(c*x^2+b*x+a)^2*x*b*h+1/d^2/(4*a*c-b^2)/(c*x^2+b*x+a)^
2*x*c*g-1/d^2/(4*a*c-b^2)/(c*x^2+b*x+a)^2*a*h+1/2/d^2/(4*a*c-b^2)/(c*x^2+b*
x+a)^2*b*g-3/d^2/(4*a*c-b^2)^2/(c*x^2+b*x+a)*c*x*b*h+6/d^2/(4*a*c-b^2)^2/(c
*x^2+b*x+a)*c^2*x*g-3/2/d^2/(4*a*c-b^2)^2/(c*x^2+b*x+a)*b^2*h+3/d^2/(4*a*c-
b^2)^2/(c*x^2+b*x+a)*b*c*g-6/d^2/(4*a*c-b^2)^(5/2)*c*arctan((2*c*x+b)/(4*a*
c-b^2)^(1/2))*b*h+12/d^2/(4*a*c-b^2)^(5/2)*c^2*arctan((2*c*x+b)/(4*a*c-b^2)
^(1/2))*g
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)/(c*x^2+b*x+a)/(c*d*x^2+b*d*x+a*d)^2,x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 0.42, size = 395, normalized size = 2.82

$$\frac{6\,c\,\mathrm{atan}\left(\frac{d^2\left(\frac{6\,c^2\,x\,(b\,h-2\,c\,g)}{d^2\,(4\,a\,c-b^2)^{5/2}}+\frac{3\,c\,(b\,h-2\,c\,g)\,(16\,a^2\,b\,c^2\,d^2-8\,a\,b^3\,c\,d^2+b^5\,d^2)}{d^4\,(4\,a\,c-b^2)^{5/2}\,(16\,a^2\,c^2-8\,a\,b^2\,c+b^4)}\right)(16\,a^2\,c^2-8\,a\,b^2\,c+b^4)}{6\,c^2\,g-3\,b\,c\,h}\right)(b\,h-2\,c\,g)}{d^2\,(4\,a\,c-b^2)^{5/2}}-\frac{\frac{8\,c\,h\,a^2+h\,a\,b^2-10\,c\,g\,a\,b+g\,b^3}{2\,(16\,a^2\,c^2-8\,a\,b^2\,c+b^4)}}{x^2\,(b^2\,d^2+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g + h*x)/((a*d + b*d*x + c*d*x^2)^2*(a + b*x + c*x^2)),x)
```

```
[Out] (6*c*atan((d^2*((6*c^2*x*(b*h - 2*c*g))/(d^2*(4*a*c - b^2)^(5/2)) + (3*c*(b
*h - 2*c*g)*(b^5*d^2 + 16*a^2*b*c^2*d^2 - 8*a*b^3*c*d^2))/(d^4*(4*a*c - b^2
)^(5/2)*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))*(b^4 + 16*a^2*c^2 - 8*a*b^2*c))/(6
*c^2*g - 3*b*c*h))*(b*h - 2*c*g))/(d^2*(4*a*c - b^2)^(5/2)) - ((b^3*g + a*b
^2*h + 8*a^2*c*h - 10*a*b*c*g)/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x*(5*a
*c + b^2)*(b*h - 2*c*g))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c) + (3*c^2*x^3*(b*h -
 2*c*g))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c) + (9*b*c*x^2*(b*h - 2*c*g))/(2*(b^4
 + 16*a^2*c^2 - 8*a*b^2*c)))/(x^2*(b^2*d^2 + 2*a*c*d^2) + a^2*d^2 + c^2*d^2
*x^4 + 2*a*b*d^2*x + 2*b*c*d^2*x^3)
```

sympy [B] time = 2.44, size = 709, normalized size = 5.06

$$\frac{3c\sqrt{-\frac{1}{(4ac-b^2)^5}}\,(bh-2cg)\log\left(x+\frac{-192a^3c^4\sqrt{-\frac{1}{(4ac-b^2)^5}}\,(bh-2cg)+144a^2b^2c^3\sqrt{-\frac{1}{(4ac-b^2)^5}}\,(bh-2cg)-36ab^4c^2\sqrt{-\frac{1}{(4ac-b^2)^5}}\,(bh-2cg)}{6bc^2h-12c^3g}\right.}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)/(c*x**2+b*x+a)/(c*d*x**2+b*d*x+a*d)**2,x)
```

```
[Out] 3*c*sqrt(-1/(4*a*c - b**2)**5)*(b*h - 2*c*g)*log(x + (-192*a**3*c**4*sqrt(-
1/(4*a*c - b**2)**5)*(b*h - 2*c*g) + 144*a**2*b**2*c**3*sqrt(-1/(4*a*c - b*
*2)**5)*(b*h - 2*c*g) - 36*a*b**4*c**2*sqrt(-1/(4*a*c - b**2)**5)*(b*h - 2*
c*g) + 3*b**6*c*sqrt(-1/(4*a*c - b**2)**5)*(b*h - 2*c*g) + 3*b**2*c*h - 6*b
*c**2*g)/(6*b*c**2*h - 12*c**3*g))/d**2 - 3*c*sqrt(-1/(4*a*c - b**2)**5)*(b
*h - 2*c*g)*log(x + (192*a**3*c**4*sqrt(-1/(4*a*c - b**2)**5)*(b*h - 2*c*g)
 - 144*a**2*b**2*c**3*sqrt(-1/(4*a*c - b**2)**5)*(b*h - 2*c*g) + 36*a*b**4*
c**2*sqrt(-1/(4*a*c - b**2)**5)*(b*h - 2*c*g) - 3*b**6*c*sqrt(-1/(4*a*c - b
**2)**5)*(b*h - 2*c*g) + 3*b**2*c*h - 6*b*c**2*g)/(6*b*c**2*h - 12*c**3*g))
/d**2 + (-8*a**2*c*h - a*b**2*h + 10*a*b*c*g - b**3*g + x**3*(-6*b*c**2*h +
 12*c**3*g) + x**2*(-9*b**2*c*h + 18*b*c**2*g) + x*(-10*a*b*c*h + 20*a*c**2
*g - 2*b**3*h + 4*b**2*c*g))/(32*a**4*c**2*d**2 - 16*a**3*b**2*c*d**2 + 2*a
**2*b**4*d**2 + x**4*(32*a**2*c**4*d**2 - 16*a*b**2*c**3*d**2 + 2*b**4*c**2
*d**2) + x**3*(64*a**2*b*c**3*d**2 - 32*a*b**3*c**2*d**2 + 4*b**5*c*d**2) +
 x**2*(64*a**3*c**3*d**2 - 12*a*b**4*c*d**2 + 2*b**6*d**2) + x*(64*a**3*b*c
**2*d**2 - 32*a**2*b**3*c*d**2 + 4*a*b**5*d**2))
```

3.18 $\int \frac{g+hx}{(a+bx+cx^2)^2(ad+bdx+cdx^2)}\,dx$

Optimal. Leaf size=140

$$\frac{3(b+2cx)(2cg-bh)}{2d\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)}-\frac{-2ah+x(2cg-bh)+bg}{2d\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}-\frac{6c(2cg-bh)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{d\left(b^2-4ac\right)^{5/2}}$$

```
[Out] 1/2*(-b*g+2*a*h-(-b*h+2*c*g)*x)/(-4*a*c+b^2)/d/(c*x^2+b*x+a)^2+3/2*(-b*h+2*
c*g)*(2*c*x+b)/(-4*a*c+b^2)^2/d/(c*x^2+b*x+a)-6*c*(-b*h+2*c*g)*arctanh((2*c
*x+b)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(5/2)/d
```

Rubi [A] time = 0.10, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.147, Rules used = {998, 638, 614, 618, 206}

$$\frac{3(b+2cx)(2cg-bh)}{2d\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)}-\frac{-2ah+x(2cg-bh)+bg}{2d\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}-\frac{6c(2cg-bh)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{d\left(b^2-4ac\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(g + h*x)/((a + b*x + c*x^2)^2*(a*d + b*d*x + c*d*x^2)),x]
```

```
[Out] -(b*g - 2*a*h + (2*c*g - b*h)*x)/(2*(b^2 - 4*a*c)*d*(a + b*x + c*x^2)^2) +
(3*(2*c*g - b*h)*(b + 2*c*x))/(2*(b^2 - 4*a*c)^2*d*(a + b*x + c*x^2)) - (6*
c*(2*c*g - b*h)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/((b^2 - 4*a*c)^(5/2
)*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 638

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p +
 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a
*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 998

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_
) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Dist[(c/f)^p, Int[(g + h*
x)^m*(d + e*x + f*x^2)^(p + q), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, p,
 q}, x] && EqQ[c*d - a*f, 0] && EqQ[b*d - a*e, 0] && (IntegerQ[p] || GtQ[c/
f, 0]) && ( !IntegerQ[q] || LeafCount[d + e*x + f*x^2] <= LeafCount[a + b*x
 + c*x^2])
```

Rubi steps

$$\int \frac{g+hx}{\left(a+bx+cx^2\right)^2\left(ad+bdx+cdx^2\right)}\,dx = \frac{\int \frac{g+hx}{\left(a+bx+cx^2\right)^3}\,dx}{d}$$

$$= -\frac{bg-2ah+(2cg-bh)x}{2\left(b^2-4ac\right)d\left(a+bx+cx^2\right)^2} - \frac{(3(2cg-bh))\int \frac{1}{\left(a+bx+cx^2\right)^2}\,dx}{2\left(b^2-4ac\right)d}$$

$$= -\frac{bg-2ah+(2cg-bh)x}{2\left(b^2-4ac\right)d\left(a+bx+cx^2\right)^2} + \frac{3(2cg-bh)(b+2cx)}{2\left(b^2-4ac\right)^2d\left(a+bx+cx^2\right)} +$$

$$= -\frac{bg-2ah+(2cg-bh)x}{2\left(b^2-4ac\right)d\left(a+bx+cx^2\right)^2} + \frac{3(2cg-bh)(b+2cx)}{2\left(b^2-4ac\right)^2d\left(a+bx+cx^2\right)} -$$

$$= -\frac{bg-2ah+(2cg-bh)x}{2\left(b^2-4ac\right)d\left(a+bx+cx^2\right)^2} + \frac{3(2cg-bh)(b+2cx)}{2\left(b^2-4ac\right)^2d\left(a+bx+cx^2\right)} -$$

Mathematica [A] time = 0.03, size = 131, normalized size = 0.94

$$\frac{\frac{\left(b^2-4ac\right)(2ah-bg+bhx-2cgx)}{(a+x(b+cx))^2} - \frac{12c(bh-2cg)\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}} + \frac{3(b+2cx)(2cg-bh)}{a+x(b+cx)}}{2d\left(b^2-4ac\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g + h*x)/((a + b*x + c*x^2)^2*(a*d + b*d*x + c*d*x^2)),x]
```

```
[Out] (((b^2 - 4*a*c)*(-(b*g) + 2*a*h - 2*c*g*x + b*h*x))/(a + x*(b + c*x))^2 + (
3*(2*c*g - b*h)*(b + 2*c*x))/(a + x*(b + c*x)) - (12*c*(-2*c*g + b*h)*ArcTa
n[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c])/(2*(b^2 - 4*a*c)^2*d
)
```

fricas [B] time = 0.89, size = 1130, normalized size = 8.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)/(c*x^2+b*x+a)^2/(c*d*x^2+b*d*x+a*d),x, algorithm="fricas"
)
```

```
[Out] [1/2*(6*(2*(b^2*c^3 - 4*a*c^4)*g - (b^3*c^2 - 4*a*b*c^3)*h)*x^3 + 9*(2*(b^3
*c^2 - 4*a*b*c^3)*g - (b^4*c - 4*a*b^2*c^2)*h)*x^2 - 6*(2*a^2*c^2*g - a^2*b
*c*h + (2*c^4*g - b*c^3*h)*x^4 + 2*(2*b*c^3*g - b^2*c^2*h)*x^3 + (2*(b^2*c^
2 + 2*a*c^3)*g - (b^3*c + 2*a*b*c^2)*h)*x^2 + 2*(2*a*b*c^2*g - a*b^2*c*h)*x
)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a
```

```
*c)*(2*c*x + b))/(c*x^2 + b*x + a)) - (b^5 - 14*a*b^3*c + 40*a^2*b*c^2)*g -
 (a*b^4 + 4*a^2*b^2*c - 32*a^3*c^2)*h + 2*(2*(b^4*c + a*b^2*c^2 - 20*a^2*c^
3)*g - (b^5 + a*b^3*c - 20*a^2*b*c^2)*h)*x)/((b^6*c^2 - 12*a*b^4*c^3 + 48*a
^2*b^2*c^4 - 64*a^3*c^5)*d*x^4 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 -
 64*a^3*b*c^4)*d*x^3 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3
- 128*a^4*c^4)*d*x^2 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*
c^3)*d*x + (a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)*d), 1/2*(
6*(2*(b^2*c^3 - 4*a*c^4)*g - (b^3*c^2 - 4*a*b*c^3)*h)*x^3 + 9*(2*(b^3*c^2 -
 4*a*b*c^3)*g - (b^4*c - 4*a*b^2*c^2)*h)*x^2 - 12*(2*a^2*c^2*g - a^2*b*c*h
+ (2*c^4*g - b*c^3*h)*x^4 + 2*(2*b*c^3*g - b^2*c^2*h)*x^3 + (2*(b^2*c^2 + 2
*a*c^3)*g - (b^3*c + 2*a*b*c^2)*h)*x^2 + 2*(2*a*b*c^2*g - a*b^2*c*h)*x)*sqr
t(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) - (b^
5 - 14*a*b^3*c + 40*a^2*b*c^2)*g - (a*b^4 + 4*a^2*b^2*c - 32*a^3*c^2)*h + 2
*(2*(b^4*c + a*b^2*c^2 - 20*a^2*c^3)*g - (b^5 + a*b^3*c - 20*a^2*b*c^2)*h)*
x)/((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d*x^4 + 2*(b^7*c
 - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d*x^3 + (b^8 - 10*a*b^6*c
+ 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d*x^2 + 2*(a*b^7 - 12*a^2*
b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*d*x + (a^2*b^6 - 12*a^3*b^4*c + 48*a
^4*b^2*c^2 - 64*a^5*c^3)*d)]
```

giac [A] time = 0.17, size = 207, normalized size = 1.48

$$\frac{6\left(2c^2g-bch\right)\arctan\left(\frac{2cx+b}{\sqrt{-b^2+4ac}}\right)}{\left(b^4d-8ab^2cd+16a^2c^2d\right)\sqrt{-b^2+4ac}}+\frac{12c^3gx^3-6bc^2hx^3+18bc^2gx^2-9b^2chx^2+4b^2cgx+20ac^2gx-2b^3h}{2\left(b^4d-8ab^2cd+16a^2c^2d\right)\left(cx^2+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)/(c*x^2+b*x+a)^2/(c*d*x^2+b*d*x+a*d),x, algorithm="giac")
```

```
[Out] 6*(2*c^2*g - b*c*h)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^4*d - 8*a*b^
2*c*d + 16*a^2*c^2*d)*sqrt(-b^2 + 4*a*c)) + 1/2*(12*c^3*g*x^3 - 6*b*c^2*h*x
^3 + 18*b*c^2*g*x^2 - 9*b^2*c*h*x^2 + 4*b^2*c*g*x + 20*a*c^2*g*x - 2*b^3*h*
x - 10*a*b*c*h*x - b^3*g + 10*a*b*c*g - a*b^2*h - 8*a^2*c*h)/((b^4*d - 8*a*
b^2*c*d + 16*a^2*c^2*d)*(c*x^2 + b*x + a)^2)
```

maple [B] time = 0.01, size = 340, normalized size = 2.43

$$-\frac{3bchx}{\left(4ac-b^2\right)^2\left(cx^2+bx+a\right)d}-\frac{6bch\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{\frac{5}{2}}d}+\frac{6c^2gx}{\left(4ac-b^2\right)^2\left(cx^2+bx+a\right)d}+\frac{12c^2g\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{\frac{5}{2}}d}-\frac{}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)/(c*x^2+b*x+a)^2/(c*d*x^2+b*d*x+a*d),x)
```

```
[Out] -1/2/d/(4*a*c-b^2)/(c*x^2+b*x+a)^2*x*b*h+1/d/(4*a*c-b^2)/(c*x^2+b*x+a)^2*x*
c*g-1/d/(4*a*c-b^2)/(c*x^2+b*x+a)^2*a*h+1/2/d/(4*a*c-b^2)/(c*x^2+b*x+a)^2*b
*g-3/d/(4*a*c-b^2)^2/(c*x^2+b*x+a)*c*x*b*h+6/d/(4*a*c-b^2)^2/(c*x^2+b*x+a)*
c^2*x*g-3/2/d/(4*a*c-b^2)^2/(c*x^2+b*x+a)*b^2*h+3/d/(4*a*c-b^2)^2/(c*x^2+b*
x+a)*b*c*g-6/d/(4*a*c-b^2)^(5/2)*c*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b*h+
12/d/(4*a*c-b^2)^(5/2)*c^2*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*g
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)/(c*x^2+b*x+a)^2/(c*d*x^2+b*d*x+a*d),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 4.02, size = 375, normalized size = 2.68

$$\frac{6\,c\,\mathrm{atan}\left(\frac{d\left(\frac{6\,c^2\,x\,\left(b\,h-2\,c\,g\right)}{d\,\left(4\,a\,c-b^2\right)^{5/2}}+\frac{3\,c\,\left(b\,h-2\,c\,g\right)\,\left(16\,d\,a^2\,b\,c^2-8\,d\,a\,b^3\,c+d\,b^5\right)}{d^2\,\left(4\,a\,c-b^2\right)^{5/2}\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}\right)\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}{6\,c^2\,g-3\,b\,c\,h}\right)\left(b\,h-2\,c\,g\right)}{d\,\left(4\,a\,c-b^2\right)^{5/2}}-\frac{\frac{8\,c\,h\,a^2+h\,a\,b^2-10\,c\,g\,a\,b+g\,b^3}{2\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}+\cdots}{a^2\,d+x^2\,\left(d\,b\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g + h*x)/((a*d + b*d*x + c*d*x^2)*(a + b*x + c*x^2)^2),x)
```

```
[Out] (6*c*atan((d*((6*c^2*x*(b*h - 2*c*g))/(d*(4*a*c - b^2)^(5/2)) + (3*c*(b*h -
 2*c*g)*(b^5*d - 8*a*b^3*c*d + 16*a^2*b*c^2*d))/(d^2*(4*a*c - b^2)^(5/2)*(b
^4 + 16*a^2*c^2 - 8*a*b^2*c)))*(b^4 + 16*a^2*c^2 - 8*a*b^2*c))/(6*c^2*g - 3
*b*c*h))*(b*h - 2*c*g))/(d*(4*a*c - b^2)^(5/2)) - ((b^3*g + a*b^2*h + 8*a^2
*c*h - 10*a*b*c*g)/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x*(5*a*c + b^2)*(b
*h - 2*c*g))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c) + (3*c^2*x^3*(b*h - 2*c*g))/(b^
4 + 16*a^2*c^2 - 8*a*b^2*c) + (9*b*c*x^2*(b*h - 2*c*g))/(2*(b^4 + 16*a^2*c^
2 - 8*a*b^2*c)))/(a^2*d + x^2*(b^2*d + 2*a*c*d) + c^2*d*x^4 + 2*b*c*d*x^3 +
 2*a*b*d*x)
```

sympy [B] time = 2.26, size = 680, normalized size = 4.86

$$\frac{3c\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}\left(bh-2cg\right)\log\left(x+\frac{-192a^3c^4\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}\left(bh-2cg\right)+144a^2b^2c^3\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}\left(bh-2cg\right)-36ab^4c^2\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}\left(bh-2cg\right)\cdots}{6bc^2h-12c^3g}\right)}{d}\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)/(c*x**2+b*x+a)**2/(c*d*x**2+b*d*x+a*d),x)
```

```
[Out] 3*c*sqrt(-1/(4*a*c - b**2)**5)*(b*h - 2*c*g)*log(x + (-192*a**3*c**4*sqrt(-
1/(4*a*c - b**2)**5)*(b*h - 2*c*g) + 144*a**2*b**2*c**3*sqrt(-1/(4*a*c - b*
*2)**5)*(b*h - 2*c*g) - 36*a*b**4*c**2*sqrt(-1/(4*a*c - b**2)**5)*(b*h - 2*
c*g) + 3*b**6*c*sqrt(-1/(4*a*c - b**2)**5)*(b*h - 2*c*g) + 3*b**2*c*h - 6*b
*c**2*g)/(6*b*c**2*h - 12*c**3*g))/d - 3*c*sqrt(-1/(4*a*c - b**2)**5)*(b*h
- 2*c*g)*log(x + (192*a**3*c**4*sqrt(-1/(4*a*c - b**2)**5)*(b*h - 2*c*g) -
144*a**2*b**2*c**3*sqrt(-1/(4*a*c - b**2)**5)*(b*h - 2*c*g) + 36*a*b**4*c**
2*sqrt(-1/(4*a*c - b**2)**5)*(b*h - 2*c*g) - 3*b**6*c*sqrt(-1/(4*a*c - b**2
)**5)*(b*h - 2*c*g) + 3*b**2*c*h - 6*b*c**2*g)/(6*b*c**2*h - 12*c**3*g))/d
+ (-8*a**2*c*h - a*b**2*h + 10*a*b*c*g - b**3*g + x**3*(-6*b*c**2*h + 12*c*
*3*g) + x**2*(-9*b**2*c*h + 18*b*c**2*g) + x*(-10*a*b*c*h + 20*a*c**2*g - 2
*b**3*h + 4*b**2*c*g))/(32*a**4*c**2*d - 16*a**3*b**2*c*d + 2*a**2*b**4*d +
 x**4*(32*a**2*c**4*d - 16*a*b**2*c**3*d + 2*b**4*c**2*d) + x**3*(64*a**2*b
*c**3*d - 32*a*b**3*c**2*d + 4*b**5*c*d) + x**2*(64*a**3*c**3*d - 12*a*b**4
*c*d + 2*b**6*d) + x*(64*a**3*b*c**2*d - 32*a**2*b**3*c*d + 4*a*b**5*d))
```

3.19 $\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{d+ex+fx^2}\,dx$

Optimal. Leaf size=617

$$\frac{\left(2f(Af(cd-af)-Bd(ce-bf))-\left(e-\sqrt{e^2-4df}\right)\left(B\left(f(be-af)-c\left(e^2-df\right)\right)+Af(ce-bf)\right)\right)\tanh^{-1}\left(\frac{4a}{2\sqrt{2}\sqrt{a}}\right.}{\sqrt{2}f^2\sqrt{e^2-4df}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}$$

```
[Out] -1/2*(-2*A*c*f-B*b*f+2*B*c*e)*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(
1/2))/f^2/c^(1/2)+B*(c*x^2+b*x+a)^(1/2)/f+1/2*arctanh(1/4*(4*a*f+2*x*(b*f-c
*(e-(-4*d*f+e^2)^(1/2)))-b*(e-(-4*d*f+e^2)^(1/2)))*2^(1/2)/(c*x^2+b*x+a)^(1
/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2-(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2))*(2*f
*(A*f*(-a*f+c*d)-B*d*(-b*f+c*e))-(A*f*(-b*f+c*e)+B*(f*(-a*f+b*e)-c*(-d*f+e^
2)))*(e-(-4*d*f+e^2)^(1/2)))/f^2*2^(1/2)/(-4*d*f+e^2)^(1/2)/(c*e^2-2*c*d*f-
b*e*f+2*a*f^2-(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2)-1/2*arctanh(1/4*(4*a*f-b
*(e+(-4*d*f+e^2)^(1/2))+2*x*(b*f-c*(e+(-4*d*f+e^2)^(1/2))))*2^(1/2)/(c*x^2+
b*x+a)^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2+(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1
/2))*(2*f*(A*f*(-a*f+c*d)-B*d*(-b*f+c*e))-(A*f*(-b*f+c*e)+B*(f*(-a*f+b*e)-c
*(-d*f+e^2)))*(e+(-4*d*f+e^2)^(1/2)))/f^2*2^(1/2)/(-4*d*f+e^2)^(1/2)/(c*e^2
-2*c*d*f-b*e*f+2*a*f^2+(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2)
```

Rubi [A] time = 9.00, antiderivative size = 615, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1019, 1076, 621, 206, 1032, 724}

$$\frac{\left(2f(Af(cd-af)-Bd(ce-bf))-\left(e-\sqrt{e^2-4df}\right)\left(Bf(be-af)+Af(ce-bf)-Bc\left(e^2-df\right)\right)\right)\tanh^{-1}\left(\frac{4a}{2\sqrt{2}\sqrt{a+}}\right.}{\sqrt{2}f^2\sqrt{e^2-4df}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x)*Sqrt[a + b*x + c*x^2])/(d + e*x + f*x^2),x]
```

```
[Out] (B*Sqrt[a + b*x + c*x^2])/f - ((2*B*c*e - b*B*f - 2*A*c*f)*ArcTanh[(b + 2*c
*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[c]*f^2) + ((2*f*(A*f*(c*d -
 a*f) - B*d*(c*e - b*f)) - (e - Sqrt[e^2 - 4*d*f])*(B*f*(b*e - a*f) + A*f*(
c*e - b*f) - B*c*(e^2 - d*f)))*ArcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) +
 2*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b
*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(S
qrt[2]*f^2*Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e
- b*f)*Sqrt[e^2 - 4*d*f]]) - ((2*f*(A*f*(c*d - a*f) - B*d*(c*e - b*f)) - (e
 + Sqrt[e^2 - 4*d*f])*(B*f*(b*e - a*f) + A*f*(c*e - b*f) - B*c*(e^2 - d*f))
)*ArcTanh[(4*a*f - b*(e + Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e + Sqrt[e^2 - 4
*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*
Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*f^2*Sqrt[e^2 - 4*d*f]*
Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
```

```
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1019

```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e
_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(h*(a + b*x + c*x^2)^p*(d
+ e*x + f*x^2)^(q + 1))/(2*f*(p + q + 1)), x] - Dist[1/(2*f*(p + q + 1)), I
nt[(a + b*x + c*x^2)^(p - 1)*(d + e*x + f*x^2)^q*Simp[h*p*(b*d - a*e) + a*(
h*e - 2*g*f)*(p + q + 1) + (2*h*p*(c*d - a*f) + b*(h*e - 2*g*f)*(p + q + 1)
)*x + (h*p*(c*e - b*f) + c*(h*e - 2*g*f)*(p + q + 1))*x^2, x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, g, h, q}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d
*f, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0]
```

Rule 1032

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1076

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqr
t[d + e*x + f*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + (B*c - b*C)*x)/((a
+ b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, A
, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]
```

Rubi steps

$$\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{d+ex+fx^2}\,dx = \frac{B\sqrt{a+bx+cx^2}}{f} - \frac{\int \frac{\frac{1}{2}(bBd-2aAf)-\frac{1}{2}(2Abf-B(2cd+be-2af))x+\frac{1}{2}(2Bce-bBf-2Acf)x^2}{\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)}dx}{f}$$

$$= \frac{B\sqrt{a+bx+cx^2}}{f} - \frac{\int \frac{\frac{1}{2}f(bBd-2aAf)-\frac{1}{2}d(2Bce-bBf-2Acf)+\left(-\frac{1}{2}e(2Bce-bBf-2Acf)+\frac{1}{2}f(-2Abf\cdots\right.}{\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)}}{f^2}$$

$$= \frac{B\sqrt{a+bx+cx^2}}{f} - \frac{(2Bce-bBf-2Acf)\,\text{Subst}\left(\int \frac{1}{4c-x^2}dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{f^2} + \frac{(2f\cdots}{\cdots}$$

$$= \frac{B\sqrt{a+bx+cx^2}}{f} - \frac{(2Bce-bBf-2Acf)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}f^2} - \frac{\left(2\left(2f(Af(c\cdots\right.\right.}{\cdots}$$

$$= \frac{B\sqrt{a+bx+cx^2}}{f} - \frac{(2Bce-bBf-2Acf)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}f^2} + \frac{\left(2f(Af(cd\cdots\right.}{\cdots}$$

Mathematica [A] time = 2.09, size = 517, normalized size = 0.84

$$\frac{-\sqrt{2}\left(B\left(\sqrt{e^2-4df}+e\right)-2Af\right)\sqrt{f\left(2af-b\left(\sqrt{e^2-4df}+e\right)\right)+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}\tanh^{-1}\left(\frac{4\cdots}{2\sqrt{2}\sqrt{a+x(b\cdots}}\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x)*Sqrt[a + b*x + c*x^2])/(d + e*x + f*x^2),x]
```

```
[Out] ((-2*B*c*e + b*B*f + 2*A*c*f)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b
+ c*x)])])/(2*Sqrt[c]*f^2) + (4*B*f*Sqrt[e^2 - 4*d*f]*Sqrt[a + x*(b + c*x)]
 - Sqrt[2]*(-2*A*f + B*(e + Sqrt[e^2 - 4*d*f]))*Sqrt[c*(e^2 - 2*d*f + e*Sqr
t[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))]*ArcTanh[(4*a*f - 2
*c*(e + Sqrt[e^2 - 4*d*f])*x - b*(e + Sqrt[e^2 - 4*d*f] - 2*f*x))/(2*Sqrt[2
]*Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 -
 4*d*f]))]*Sqrt[a + x*(b + c*x)])] - Sqrt[2]*(2*A*f + B*(-e + Sqrt[e^2 - 4*
d*f]))*Sqrt[c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f + b*(-e + Sqrt
[e^2 - 4*d*f]))]*ArcTanh[(4*a*f + 2*c*(-e + Sqrt[e^2 - 4*d*f])*x + b*(-e +
Sqrt[e^2 - 4*d*f] + 2*f*x))/(2*Sqrt[2]*Sqrt[c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4
*d*f]) + f*(2*a*f + b*(-e + Sqrt[e^2 - 4*d*f]))]*Sqrt[a + x*(b + c*x)])])/(
4*f^2*Sqrt[e^2 - 4*d*f])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.05, size = 16209, normalized size = 26.27

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x+A)*(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x)
```

```
[Out] result too large to display
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for mo
re details)Is 4*d*f-e^2 positive, negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\,x)\,\sqrt{c\,x^2+b\,x+a}}{f\,x^2+e\,x+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x)*(a + b*x + c*x^2)^(1/2))/(d + e*x + f*x^2),x)
```

```
[Out] int(((A + B*x)*(a + b*x + c*x^2)^(1/2))/(d + e*x + f*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{d+ex+fx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(c*x**2+b*x+a)**(1/2)/(f*x**2+e*x+d),x)
```

```
[Out] Integral((A + B*x)*sqrt(a + b*x + c*x**2)/(d + e*x + f*x**2), x)
```

3.20 $\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{3/2}}{d+ex+fx^2}\,dx$

Optimal. Leaf size=1092

$$\frac{B\left(cx^2+bx+a\right)^{3/2}}{3f}-\frac{\left(2Acf(4ce-5bf)-B\left(8\left(e^2-df\right)c^2-2f(5be-4af)c+b^2f^2\right)+2cf(2Bce-bBf-2Acf)x\right.}{8cf^3}$$

```
[Out] 1/3*B*(c*x^2+b*x+a)^(3/2)/f+1/16*(2*A*c*f*(3*b^2*f^2-12*c*f*(-a*f+b*e)+8*c^
2*(-d*f+e^2))-B*(b^3*f^3+6*b*c*f^2*(-2*a*f+b*e)-24*c^2*f*(-a*e*f-b*d*f+b*e^
2)+16*c^3*(-2*d*e*f+e^3)))*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2
))/c^(3/2)/f^4-1/8*(2*A*c*f*(-5*b*f+4*c*e)-B*(b^2*f^2-2*c*f*(-4*a*f+5*b*e)+
8*c^2*(-d*f+e^2))+2*c*f*(-2*A*c*f-B*b*f+2*B*c*e)*x)*(c*x^2+b*x+a)^(1/2)/c/f
^3-1/2*arctanh(1/4*(4*a*f+2*x*(b*f-c*(e-(-4*d*f+e^2)^(1/2)))-b*(e-(-4*d*f+e
^2)^(1/2)))*2^(1/2)/(c*x^2+b*x+a)^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2-(-b*f+
c*e)*(-4*d*f+e^2)^(1/2))^(1/2))*(2*c*f*(B*d*(-b*f+c*e)*(2*a*f^2-b*e*f-2*c*d
*f+c*e^2)+A*f*(2*c*d*f*(-a*f+b*e)-f^2*(-a^2*f+b^2*d)-c^2*d*(-d*f+e^2)))-c*(
A*f*(-b*f+c*e)*(f*(-2*a*f+b*e)-c*(-2*d*f+e^2))+B*(c^2*(d^2*f^2-3*d*e^2*f+e^
4)-f^2*(2*a*b*e*f-a^2*f^2-b^2*(-d*f+e^2))+2*c*f*(a*f*(-d*f+e^2)-b*(-2*d*e*f
+e^3))))*(e-(-4*d*f+e^2)^(1/2)))/c/f^4*2^(1/2)/(-4*d*f+e^2)^(1/2)/(c*e^2-2*
c*d*f-b*e*f+2*a*f^2-(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2)+1/2*arctanh(1/4*(4
*a*f-b*(e+(-4*d*f+e^2)^(1/2))+2*x*(b*f-c*(e+(-4*d*f+e^2)^(1/2))))*2^(1/2)/(
c*x^2+b*x+a)^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2+(-b*f+c*e)*(-4*d*f+e^2)^(1/
2))^(1/2))*(2*f*(B*d*(-b*f+c*e)*(2*a*f^2-b*e*f-2*c*d*f+c*e^2)+A*f*(2*c*d*f*
(-a*f+b*e)-f^2*(-a^2*f+b^2*d)-c^2*d*(-d*f+e^2)))-(A*f*(-b*f+c*e)*(f*(-2*a*f
+b*e)-c*(-2*d*f+e^2))+B*(c^2*(d^2*f^2-3*d*e^2*f+e^4)-f^2*(2*a*b*e*f-a^2*f^2
-b^2*(-d*f+e^2))+2*c*f*(a*f*(-d*f+e^2)-b*(-2*d*e*f+e^3))))*(e+(-4*d*f+e^2)^
(1/2)))/f^4*2^(1/2)/(-4*d*f+e^2)^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2+(-b*f+c
*e)*(-4*d*f+e^2)^(1/2))^(1/2)
```

Rubi [A] time = 18.87, antiderivative size = 1092, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.219$, Rules used = {1019, 1066, 1076, 621, 206, 1032, 724}

$$\frac{B\left(cx^2+bx+a\right)^{3/2}}{3f}-\frac{\left(2Acf(4ce-5bf)-B\left(8\left(e^2-df\right)c^2-2f(5be-4af)c+b^2f^2\right)+2cf(2Bce-bBf-2Acf)x\right.}{8cf^3}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x)*(a + b*x + c*x^2)^(3/2))/(d + e*x + f*x^2),x]
```

```
[Out] -((2*A*c*f*(4*c*e - 5*b*f) - B*(b^2*f^2 - 2*c*f*(5*b*e - 4*a*f) + 8*c^2*(e^
2 - d*f)) + 2*c*f*(2*B*c*e - b*B*f - 2*A*c*f)*x)*Sqrt[a + b*x + c*x^2])/(8*
c*f^3) + (B*(a + b*x + c*x^2)^(3/2))/(3*f) + ((2*A*c*f*(3*b^2*f^2 - 12*c*f*
(b*e - a*f) + 8*c^2*(e^2 - d*f)) - B*(b^3*f^3 + 6*b*c*f^2*(b*e - 2*a*f) - 2
4*c^2*f*(b*e^2 - b*d*f - a*e*f) + 16*c^3*(e^3 - 2*d*e*f)))*ArcTanh[(b + 2*c
*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(16*c^(3/2)*f^4) - ((2*c*f*(B*d*(c*
e - b*f)*(c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2) + A*f*(2*c*d*f*(b*e - a*f) - f
^2*(b^2*d - a^2*f) - c^2*d*(e^2 - d*f))) - c*(e - Sqrt[e^2 - 4*d*f])*(A*f*(
c*e - b*f)*(f*(b*e - 2*a*f) - c*(e^2 - 2*d*f)) + B*(c^2*(e^4 - 3*d*e^2*f +
d^2*f^2) - f^2*(2*a*b*e*f - a^2*f^2 - b^2*(e^2 - d*f)) + 2*c*f*(a*f*(e^2 -
d*f) - b*(e^3 - 2*d*e*f)))))*ArcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) + 2
*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e
*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqr
t[2]*c*f^4*Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e
```

```
- b*f)*Sqrt[e^2 - 4*d*f]]) + ((2*f*(B*d*(c*e - b*f)*(c*e^2 - 2*c*d*f - b*e*
f + 2*a*f^2) + A*f*(2*c*d*f*(b*e - a*f) - f^2*(b^2*d - a^2*f) - c^2*d*(e^2
- d*f))) - (e + Sqrt[e^2 - 4*d*f])*(A*f*(c*e - b*f)*(f*(b*e - 2*a*f) - c*(e
^2 - 2*d*f)) + B*(c^2*(e^4 - 3*d*e^2*f + d^2*f^2) - f^2*(2*a*b*e*f - a^2*f^
2 - b^2*(e^2 - d*f)) + 2*c*f*(a*f*(e^2 - d*f) - b*(e^3 - 2*d*e*f)))))*ArcTa
nh[(4*a*f - b*(e + Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e + Sqrt[e^2 - 4*d*f]))
*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^
2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*f^4*Sqrt[e^2 - 4*d*f]*Sqrt[c*
e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1019

```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e
_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(h*(a + b*x + c*x^2)^p*(d
+ e*x + f*x^2)^(q + 1))/(2*f*(p + q + 1)), x] - Dist[1/(2*f*(p + q + 1)), I
nt[(a + b*x + c*x^2)^(p - 1)*(d + e*x + f*x^2)^q*Simp[h*p*(b*d - a*e) + a*(
h*e - 2*g*f)*(p + q + 1) + (2*h*p*(c*d - a*f) + b*(h*e - 2*g*f)*(p + q + 1)
)*x + (h*p*(c*e - b*f) + c*(h*e - 2*g*f)*(p + q + 1))*x^2, x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, g, h, q}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d
*f, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0]
```

Rule 1032

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1066

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((B*c*f*(2*
p + 2*q + 3) + C*(b*f*p - c*e*(2*p + q + 2)) + 2*c*C*f*(p + q + 1)*x)*(a +
b*x + c*x^2)^p*(d + e*x + f*x^2)^(q + 1))/(2*c*f^2*(p + q + 1)*(2*p + 2*q +
 3)), x] - Dist[1/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3)), Int[(a + b*x + c*x
^2)^(p - 1)*(d + e*x + f*x^2)^q*Simp[p*(b*d - a*e)*(C*(c*e - b*f)*(q + 1) -
 c*(C*e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(b^2*C*d*f*p + a*c*(C*(2*d*f
- e^2*(2*p + q + 2)) + f*(B*e - 2*A*f)*(2*p + 2*q + 3))) + (2*p*(c*d - a*f)
*(C*(c*e - b*f)*(q + 1) - c*(C*e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(C*e
```

```
*f*p*(b^2 - 4*a*c) - b*c*(C*(e^2 - 4*d*f)*(2*p + q + 2) + f*(2*C*d - B*e +
2*A*f)*(2*p + 2*q + 3))))*x + (p*(c*e - b*f)*(C*(c*e - b*f)*(q + 1) - c*(C*
e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(C*f^2*p*(b^2 - 4*a*c) - c^2*(C*(e^
2 - 4*d*f)*(2*p + q + 2) + f*(2*C*d - B*e + 2*A*f)*(2*p + 2*q + 3))))*x^2,
x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] && NeQ[b^2 - 4*a*c,
0] && NeQ[e^2 - 4*d*f, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0] && NeQ[2*p + 2*
q + 3, 0] &&  !IGtQ[p, 0] &&  !IGtQ[q, 0]
```

Rule 1076

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqr
t[d + e*x + f*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + (B*c - b*C)*x)/((a
+ b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, A
, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]
```

Rubi steps

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{3/2}}{d+ex+fx^2}\,dx = \frac{B\left(a+bx+cx^2\right)^{3/2}}{3f} - \frac{\int \frac{\sqrt{a+bx+cx^2}\left(\frac{3}{2}(bBd-2aAf)-\frac{3}{2}(2Abf-B(2cd+be-2af))x+\frac{3}{2}(2Bce-bBf}{d+ex+fx^2}}{3f}$$

$$= -\frac{\left(2Acf(4ce-5bf)-B\left(b^2f^2-2cf(5be-4af)+8c^2\left(e^2-df\right)\right)+2cf(2Bce}{8cf^3}$$

$$= -\frac{\left(2Acf(4ce-5bf)-B\left(b^2f^2-2cf(5be-4af)+8c^2\left(e^2-df\right)\right)+2cf(2Bce}{8cf^3}$$

$$= -\frac{\left(2Acf(4ce-5bf)-B\left(b^2f^2-2cf(5be-4af)+8c^2\left(e^2-df\right)\right)+2cf(2Bce}{8cf^3}$$

$$= -\frac{\left(2Acf(4ce-5bf)-B\left(b^2f^2-2cf(5be-4af)+8c^2\left(e^2-df\right)\right)+2cf(2Bce}{8cf^3}$$

$$= -\frac{\left(2Acf(4ce-5bf)-B\left(b^2f^2-2cf(5be-4af)+8c^2\left(e^2-df\right)\right)+2cf(2Bce}{8cf^3}$$

Mathematica [A] time = 6.56, size = 1627, normalized size = 1.49

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x)*(a + b*x + c*x^2)^(3/2))/(d + e*x + f*x^2),x]
```

```
[Out] ((B - (B*e - 2*A*f)/Sqrt[e^2 - 4*d*f])*(a + x*(b + c*x))^(3/2))/(6*f) + ((B
 + (B*e - 2*A*f)/Sqrt[e^2 - 4*d*f])*(a + x*(b + c*x))^(3/2))/(6*f) - ((B +
(-(B*e) + 2*A*f)/Sqrt[e^2 - 4*d*f])*(a + x*(b + c*x))^(3/2)*(((4*c*f*(-4*a*
f + b*(e - Sqrt[e^2 - 4*d*f])) + 2*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*(-(b*f
) + 2*c*(e - Sqrt[e^2 - 4*d*f])) - 4*c*f*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*
x)*Sqrt[a + b*x + c*x^2])/(8*c*f^2) - ((-2*(b*f - c*(e - Sqrt[e^2 - 4*d*f])
)*(b^2*f^2 - 4*c^2*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) - 4*c*f*(3*a*f - b*(
```

```
e - Sqrt[e^2 - 4*d*f])))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^
2])])/(Sqrt[c]*f) - (2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - c*e
*Sqrt[e^2 - 4*d*f] + b*f*Sqrt[e^2 - 4*d*f]]*(4*(e - Sqrt[e^2 - 4*d*f])*(b*f
 - c*(e - Sqrt[e^2 - 4*d*f]))*(b^2*f^2 - 4*c^2*(e^2 - 2*d*f - e*Sqrt[e^2 -
4*d*f]) - 4*c*f*(3*a*f - b*(e - Sqrt[e^2 - 4*d*f]))) + 4*f*(2*c*f*(4*a*f -
b*(e - Sqrt[e^2 - 4*d*f]))^2 - (e - Sqrt[e^2 - 4*d*f])*(b*f - c*(e - Sqrt[e
^2 - 4*d*f]))*(b^2*f + 4*a*c*f - 2*b*c*(e - Sqrt[e^2 - 4*d*f]))))*ArcTanh[(
4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) - (-2*b*f + 2*c*(e - Sqrt[e^2 - 4*d*f]))*
x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - c*e*Sqrt[e^2 - 4*d*f
] + b*f*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(f*(16*a*f^2 - 8*b*f*(e
 - Sqrt[e^2 - 4*d*f]) + 4*c*(e - Sqrt[e^2 - 4*d*f])^2)))/(16*c*f^2)))/(4*f*
(a + b*x + c*x^2)^(3/2)) - ((B - (-(B*e) + 2*A*f)/Sqrt[e^2 - 4*d*f])*(a + x
*(b + c*x))^(3/2)*(((4*c*f*(-4*a*f + b*(e + Sqrt[e^2 - 4*d*f])) + 2*(b*f -
c*(e + Sqrt[e^2 - 4*d*f]))*(-(b*f) + 2*c*(e + Sqrt[e^2 - 4*d*f])) - 4*c*f*(
b*f - c*(e + Sqrt[e^2 - 4*d*f]))*x)*Sqrt[a + b*x + c*x^2])/(8*c*f^2) - ((-2
*(b*f - c*(e + Sqrt[e^2 - 4*d*f]))*(b^2*f^2 - 4*c^2*(e^2 - 2*d*f + e*Sqrt[e
^2 - 4*d*f]) - 4*c*f*(3*a*f - b*(e + Sqrt[e^2 - 4*d*f])))*ArcTanh[(b + 2*c*
x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(Sqrt[c]*f) - (2*Sqrt[2]*Sqrt[c*e^2
- 2*c*d*f - b*e*f + 2*a*f^2 + c*e*Sqrt[e^2 - 4*d*f] - b*f*Sqrt[e^2 - 4*d*f]
]*(4*(e + Sqrt[e^2 - 4*d*f])*(b*f - c*(e + Sqrt[e^2 - 4*d*f]))*(b^2*f^2 - 4
*c^2*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) - 4*c*f*(3*a*f - b*(e + Sqrt[e^2 -
 4*d*f]))) + 4*f*(2*c*f*(4*a*f - b*(e + Sqrt[e^2 - 4*d*f]))^2 - (e + Sqrt[e
^2 - 4*d*f])*(b*f - c*(e + Sqrt[e^2 - 4*d*f]))*(b^2*f + 4*a*c*f - 2*b*c*(e
+ Sqrt[e^2 - 4*d*f]))))*ArcTanh[(4*a*f - b*(e + Sqrt[e^2 - 4*d*f]) - (-2*b*
f + 2*c*(e + Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f
 + 2*a*f^2 + c*e*Sqrt[e^2 - 4*d*f] - b*f*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x +
c*x^2])])/(f*(16*a*f^2 - 8*b*f*(e + Sqrt[e^2 - 4*d*f]) + 4*c*(e + Sqrt[e^2
- 4*d*f])^2)))/(16*c*f^2)))/(4*f*(a + b*x + c*x^2)^(3/2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [B] time = 0.03, size = 59465, normalized size = 54.46

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x+A)*(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x)
```

```
[Out] result too large to display
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for mo
re details)Is 4*d*f-e^2 positive, negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+Bx)\left(cx^2+bx+a\right)^{3/2}}{fx^2+ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x)*(a + b*x + c*x^2)^(3/2))/(d + e*x + f*x^2),x)
```

```
[Out] int(((A + B*x)*(a + b*x + c*x^2)^(3/2))/(d + e*x + f*x^2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(c*x**2+b*x+a)**(3/2)/(f*x**2+e*x+d),x)
```

```
[Out] Timed out
```

3.21 $\int \frac{A+Bx}{(a+bx+cx^2)\sqrt{d+ex+fx^2}}\,dx$

Optimal. Leaf size=416

$$\frac{\left(-B\sqrt{b^2-4ac}-2Ac+bB\right)\tanh^{-1}\left(\frac{2x\left(ce-f\left(b-\sqrt{b^2-4ac}\right)\right)-e\left(b-\sqrt{b^2-4ac}\right)+4cd}{2\sqrt{2}\sqrt{d+ex+fx^2}\sqrt{\sqrt{b^2-4ac}(ce-bf)-2acf+b^2f-bce+2c^2d}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}(ce-bf)-2acf+b^2f-bce+2c^2d}}+\frac{\left(2Ac-B\left(\sqrt{b^2-4ac}+\right.\right.}{\sqrt{2}\sqrt{b^2-4ac}}$$

```
[Out] 1/2*arctanh(1/4*(4*c*d-e*(b+(-4*a*c+b^2)^(1/2))+2*x*(c*e-f*(b+(-4*a*c+b^2)^
(1/2))))*2^(1/2)/(f*x^2+e*x+d)^(1/2)/(2*c^2*d-b*c*e+b^2*f-2*a*c*f-(-b*f+c*e
)*(-4*a*c+b^2)^(1/2))^(1/2))*(2*A*c-B*(b+(-4*a*c+b^2)^(1/2)))*2^(1/2)/(-4*a
*c+b^2)^(1/2)/(2*c^2*d-b*c*e+b^2*f-2*a*c*f-(-b*f+c*e)*(-4*a*c+b^2)^(1/2))^(
1/2)+1/2*arctanh(1/4*(4*c*d+2*x*(c*e-f*(b-(-4*a*c+b^2)^(1/2)))-e*(b-(-4*a*c
+b^2)^(1/2)))*2^(1/2)/(f*x^2+e*x+d)^(1/2)/(2*c^2*d-b*c*e+b^2*f-2*a*c*f+(-b*
f+c*e)*(-4*a*c+b^2)^(1/2))^(1/2))*(b*B-2*A*c-B*(-4*a*c+b^2)^(1/2))*2^(1/2)/
(-4*a*c+b^2)^(1/2)/(2*c^2*d-b*c*e+b^2*f-2*a*c*f+(-b*f+c*e)*(-4*a*c+b^2)^(1/
2))^(1/2)
```

Rubi [A] time = 2.70, antiderivative size = 416, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {1032, 724, 206}

$$\frac{\left(-B\sqrt{b^2-4ac}-2Ac+bB\right)\tanh^{-1}\left(\frac{2x\left(ce-f\left(b-\sqrt{b^2-4ac}\right)\right)-e\left(b-\sqrt{b^2-4ac}\right)+4cd}{2\sqrt{2}\sqrt{d+ex+fx^2}\sqrt{\sqrt{b^2-4ac}(ce-bf)-2acf+b^2f-bce+2c^2d}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}(ce-bf)-2acf+b^2f-bce+2c^2d}}+\frac{\left(2Ac-B\left(\sqrt{b^2-4ac}+\right.\right.}{\sqrt{2}\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]),x]
```

```
[Out] ((b*B - 2*A*c - B*Sqrt[b^2 - 4*a*c])*ArcTanh[(4*c*d - (b - Sqrt[b^2 - 4*a*c
])*e + 2*(c*e - (b - Sqrt[b^2 - 4*a*c])*f)*x)/(2*Sqrt[2]*Sqrt[2*c^2*d - b*c
*e + b^2*f - 2*a*c*f + Sqrt[b^2 - 4*a*c]*(c*e - b*f)]*Sqrt[d + e*x + f*x^2]
)])/(Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt[2*c^2*d - b*c*e + b^2*f - 2*a*c*f + Sqr
t[b^2 - 4*a*c]*(c*e - b*f)]) + ((2*A*c - B*(b + Sqrt[b^2 - 4*a*c]))*ArcTanh
[(4*c*d - (b + Sqrt[b^2 - 4*a*c])*e + 2*(c*e - (b + Sqrt[b^2 - 4*a*c])*f)*x
)/(2*Sqrt[2]*Sqrt[2*c^2*d - b*c*e + b^2*f - 2*a*c*f - Sqrt[b^2 - 4*a*c]*(c*
e - b*f)]*Sqrt[d + e*x + f*x^2])])/(Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt[2*c^2*d
- b*c*e + b^2*f - 2*a*c*f - Sqrt[b^2 - 4*a*c]*(c*e - b*f)])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1032

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
```

```
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{A+Bx}{\left(a+bx+cx^2\right)\sqrt{d+ex+fx^2}}\,dx = \frac{\left(2Ac-B\left(b-\sqrt{b^2-4ac}\right)\right)\int \frac{1}{\left(b-\sqrt{b^2-4ac}+2cx\right)\sqrt{d+ex+fx^2}}\,dx}{\sqrt{b^2-4ac}} - \frac{\left(2Ac-B\left(b\right.\right.}{}$$

$$= \frac{\left(2\left(bB-2Ac-B\sqrt{b^2-4ac}\right)\right)\text{Subst}\left(\int \frac{1}{16c^2d-8c\left(b-\sqrt{b^2-4ac}\right)e+4\left(b-\sqrt{b^2-4ac}\right)^2 f}\right.}{\sqrt{b^2-4ac}}$$

$$= \frac{\left(bB-2Ac-B\sqrt{b^2-4ac}\right)\tanh^{-1}\left(\frac{4cd-\left(b-\sqrt{b^2-4ac}\right)e+2\left(ce-\left(b-\sqrt{b^2-4ac}\right)f\right)}{2\sqrt{2}\sqrt{2c^2d-bce+b^2f-2acf+\sqrt{b^2-4ac}\,(ce-bf)}\sqrt{d}}\right.}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{2c^2d-bce+b^2f-2acf+\sqrt{b^2-4ac}\,(ce-bf)}}$$

Mathematica [A] time = 4.18, size = 393, normalized size = 0.94

$$\frac{-\frac{\left(B\sqrt{b^2-4ac}+2Ac-bB\right)\tanh^{-1}\left(\frac{\left(\sqrt{b^2-4ac}-b\right)(e+2fx)+2c(2d+ex)}{2\sqrt{2}\sqrt{d+x(e+fx)}\sqrt{c\left(e\sqrt{b^2-4ac}-2af-be\right)+bf\left(b-\sqrt{b^2-4ac}\right)+2c^2d}}\right)}{\sqrt{c\left(e\sqrt{b^2-4ac}-2af-be\right)+bf\left(b-\sqrt{b^2-4ac}\right)+2c^2d}}-\frac{\left(B\sqrt{b^2-4ac}-2Ac+bB\right)\tanh^{-1}\left(\frac{2c(2d+ex}{2\sqrt{d+x(e+fx)}\sqrt{-2c\left(e\sqrt{b^2\right.}}}\right.}{\sqrt{-c\left(e\sqrt{b^2-4ac}+2af+be\right)+bf\left(\sqrt{b}\right.}}}{\sqrt{2}\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]),x]
```

```
[Out] (-(((-(b*B) + 2*A*c + B*Sqrt[b^2 - 4*a*c])*ArcTanh[(2*c*(2*d + e*x) + (-b +
 Sqrt[b^2 - 4*a*c])*(e + 2*f*x))/(2*Sqrt[2]*Sqrt[2*c^2*d + b*(b - Sqrt[b^2
- 4*a*c])*f + c*(-(b*e) + Sqrt[b^2 - 4*a*c]*e - 2*a*f)]*Sqrt[d + x*(e + f*x
)])])/Sqrt[2*c^2*d + b*(b - Sqrt[b^2 - 4*a*c])*f + c*(-(b*e) + Sqrt[b^2 - 4
*a*c]*e - 2*a*f)]) - ((b*B - 2*A*c + B*Sqrt[b^2 - 4*a*c])*ArcTanh[(2*c*(2*d
 + e*x) - (b + Sqrt[b^2 - 4*a*c])*(e + 2*f*x))/(2*Sqrt[4*c^2*d + 2*b*(b + S
qrt[b^2 - 4*a*c])*f - 2*c*(b*e + Sqrt[b^2 - 4*a*c]*e + 2*a*f)]*Sqrt[d + x*(
e + f*x)])])/Sqrt[2*c^2*d + b*(b + Sqrt[b^2 - 4*a*c])*f - c*(b*e + Sqrt[b^2
 - 4*a*c]*e + 2*a*f)])/(Sqrt[2]*Sqrt[b^2 - 4*a*c])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(c*x^2+b*x+a)/(f*x^2+e*x+d)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(c*x^2+b*x+a)/(f*x^2+e*x+d)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to divide, perhaps due to rounding er
ror%%%{%%{poly1[%%%{-4,[3,2,0]%%%}+%%%{16,[1,3,1]%%%},%%%{4,[4,2,0]%%%}+%%%
{-24,[2,3,1]%%%}+%%%{32,[0,4,2]%%%}]:[1,0,%%%{-1,[2,0,0]%%%}+%%%{4,[0,1,1]%
%%}]%%},[2,1,0,0,0]%%%}+%%%{%%{[%%%{-4,[5,0,0]%%%}+%%%{24,[3,1,1]%%%}+%%%{-
32,[1,2,2]%%%},%%%{4,[6,0,0]%%%}+%%%{-32,[4,1,1]%%%}+%%%{72,[2,2,2]%%%}+%%%
{-32,[0,3,3]%%%}]:[1,0,%%%{-1,[2,0,0]%%%}+%%%{4,[0,1,1]%%%}]%%},[2,0,0,1,0]
%%%}+%%%{%%{[%%%{4,[4,1,0]%%%}+%%%{-20,[2,2,1]%%%}+%%%{16,[0,3,2]%%%},%%%{-
4,[5,1,0]%%%}+%%%{28,[3,2,1]%%%}+%%%{-48,[1,3,2]%%%}]:[1,0,%%%{-1,[2,0,0]%%
%}+%%%{4,[0,1,1]%%%}]%%},[2,0,0,0,1]%%%}+%%%{%%{poly1[%%%{8,[2,3,0]%%%}+%%%
{-32,[0,4,1]%%%},%%%{-8,[3,3,0]%%%}+%%%{32,[1,4,1]%%%}]:[1,0,%%%{-1,[2,0,0]
%%%}+%%%{4,[0,1,1]%%%}]%%},[1,1,1,0,0]%%%}+%%%{%%{[%%%{8,[4,1,0]%%%}+%%%{-4
0,[2,2,1]%%%}+%%%{32,[0,3,2]%%%},%%%{-8,[5,1,0]%%%}+%%%{56,[3,2,1]%%%}+%%%{
-96,[1,3,2]%%%}]:[1,0,%%%{-1,[2,0,0]%%%}+%%%{4,[0,1,1]%%%}]%%},[1,0,1,1,0]%
%%}+%%%{%%{[%%%{-8,[3,2,0]%%%}+%%%{32,[1,3,1]%%%},%%%{8,[4,2,0]%%%}+%%%{-48
,[2,3,1]%%%}+%%%{64,[0,4,2]%%%}]:[1,0,%%%{-1,[2,0,0]%%%}+%%%{4,[0,1,1]%%%}]
%%},[1,0,1,0,1]%%%}+%%%{%%%{8,[2,4,0]%%%}+%%%{-32,[0,5,1]%%%},[0,1,2,0,0]%%
%}+%%%{%%{poly1[%%%{-4,[3,2,0]%%%}+%%%{16,[1,3,1]%%%},%%%{4,[4,2,0]%%%}+%%%
{-24,[2,3,1]%%%}+%%%{32,[0,4,2]%%%}]:[1,0,%%%{-1,[2,0,0]%%%}+%%%{4,[0,1,1]%
%%}]%%},[0,0,2,1,0]%%%}+%%%{%%{poly1[%%%{4,[2,3,0]%%%}+%%%{-16,[0,4,1]%%%},
%%%{-4,[3,3,0]%%%}+%%%{16,[1,4,1]%%%}]:[1,0,%%%{-1,[2,0,0]%%%}+%%%{4,[0,1,1
]%%%}]%%},[0,0,2,0,1]%%%} / %%%{%%{[%%%{4,[3,2,0]%%%}+%%%{-16,[1,3,1]%%%},%
%%{-4,[4,2,0]%%%}+%%%{24,[2,3,1]%%%}+%%%{-32,[0,4,2]%%%}]:[1,0,%%%{-1,[2,0,
0]%%%}+%%%{4,[0,1,1]%%%}]%%},[0,1,0,0,0]%%%}+%%%{%%{[%%%{4,[5,0,0]%%%}+%%%{
-24,[3,1,1]%%%}+%%%{32,[1,2,2]%%%},%%%{-4,[6,0,0]%%%}+%%%{32,[4,1,1]%%%}+%%
%{-72,[2,2,2]%%%}+%%%{32,[0,3,3]%%%}]:[1,0,%%%{-1,[2,0,0]%%%}+%%%{4,[0,1,1]
%%%}]%%},[0,0,0,1,0]%%%}+%%%{%%{[%%%{-4,[4,1,0]%%%}+%%%{20,[2,2,1]%%%}+%%%{
-16,[0,3,2]%%%},%%%{4,[5,1,0]%%%}+%%%{-28,[3,2,1]%%%}+%%%{48,[1,3,2]%%%}]:[
1,0,%%%{-1,[2,0,0]%%%}+%%%{4,[0,1,1]%%%}]%%},[0,0,0,0,1]%%%} Error: Bad Arg
ument ValueEvaluation time: 0.92Unable to divide, perhaps due to rounding e
rror%%%{%%{[%%%{1,[3,0,0]%%%}+%%%{-4,[1,1,1]%%%},%%%{1,[4,0,0]%%%}+%%%{-6,[
2,1,1]%%%}+%%%{8,[0,2,2]%%%}]:[1,0,%%%{-1,[2,0,0]%%%}+%%%{4,[0,1,1]%%%}]%%}
/%%%{4,[0,0,2]%%%},[2,1,0,0,0]%%%}+%%%{%%%{1,[2,0,0]%%%}+%%%{-4,[0,1,1]%%%}
/2,[2,0,0,1,0]%%%}+%%%{%%{[%%%{-1,[2,0,0]%%%}+%%%{4,[0,1,1]%%%},%%%{-1,[3,0
,0]%%%}+%%%{4,[1,1,1]%%%}]:[1,0,%%%{-1,[2,0,0]%%%}+%%%{4,[0,1,1]%%%}]%%}/%%
%{4,[0,0,1]%%%},[2,0,0,0,1]%%%}+%%%{%%{[%%%{-1,[4,0,0]%%%}+%%%{5,[2,1,1]%%%
}+%%%{-4,[0,2,2]%%%},%%%{-1,[5,0,0]%%%}+%%%{7,[3,1,1]%%%}+%%%{-12,[1,2,2]%%
%}]:[1,0,%%%{-1,[2,0,0]%%%}+%%%{4,[0,1,1]%%%}]%%}/%%%{2,[0,0,3]%%%},[1,1,1,
0,0]%%%}+%%%{%%{[%%%{-1,[2,0,0]%%%}+%%%{4,[0,1,1]%%%},%%%{-1,[3,0,0]%%%}+%%
%{4,[1,1,1]%%%}]:[1,0,%%%{-1,[2,0,0]%%%}+%%%{4,[0,1,1]%%%}]%%}/%%%{2,[0,0,1
]%%%},[1,0,1,1,0]%%%}+%%%{%%{[%%%{1,[3,0,0]%%%}+%%%{-4,[1,1,1]%%%},%%%{1,[4
,0,0]%%%}+%%%{-6,[2,1,1]%%%}+%%%{8,[0,2,2]%%%}]:[1,0,%%%{-1,[2,0,0]%%%}+%%%
{4,[0,1,1]%%%}]%%}/%%%{2,[0,0,2]%%%},[1,0,1,0,1]%%%}+%%%{%%{[%%%{1,[5,0,0]%
%%}+%%%{-6,[3,1,1]%%%}+%%%{8,[1,2,2]%%%},%%%{1,[6,0,0]%%%}+%%%{-8,[4,1,1]%%
%}+%%%{18,[2,2,2]%%%}+%%%{-8,[0,3,3]%%%}]:[1,0,%%%{-1,[2,0,0]%%%}+%%%{4,[0,
1,1]%%%}]%%}/%%%{4,[0,0,4]%%%},[0,1,2,0,0]%%%}+%%%{%%{[%%%{1,[3,0,0]%%%}+%%
%{-4,[1,1,1]%%%},%%%{1,[4,0,0]%%%}+%%%{-6,[2,1,1]%%%}+%%%{8,[0,2,2]%%%}]:[1
,0,%%%{-1,[2,0,0]%%%}+%%%{4,[0,1,1]%%%}]%%}/%%%{4,[0,0,2]%%%},[0,0,2,1,0]%%
%}+%%%{%%{[%%%{-1,[4,0,0]%%%}+%%%{5,[2,1,1]%%%}+%%%{-4,[0,2,2]%%%},%%%{-1,[
5,0,0]%%%}+%%%{7,[3,1,1]%%%}+%%%{-12,[1,2,2]%%%}]:[1,0,%%%{-1,[2,0,0]%%%}+%
%%{4,[0,1,1]%%%}]%%}/%%%{4,[0,0,3]%%%},[0,0,2,0,1]%%%} / %%%{%%{[%%%{-1,[3,
0,0]%%%}+%%%{4,[1,1,1]%%%},%%%{-1,[4,0,0]%%%}+%%%{6,[2,1,1]%%%}+%%%{-8,[0,2
,2]%%%}]:[1,0,%%%{-1,[2,0,0]%%%}+%%%{4,[0,1,1]%%%}]%%}/%%%{4,[0,0,4]%%%},[0
,1,0,0,0]%%%}+%%%{%%%{-1,[2,0,0]%%%}+%%%{4,[0,1,1]%%%}/%%%{2,[0,0,2]%%%},[0
,0,0,1,0]%%%}+%%%{%%{[%%%{1,[2,0,0]%%%}+%%%{-4,[0,1,1]%%%},%%%{1,[3,0,0]%%%
}+%%%{-4,[1,1,1]%%%}]:[1,0,%%%{-1,[2,0,0]%%%}+%%%{4,[0,1,1]%%%}]%%}/%%%{4,[
0,0,3]%%%},[0,0,0,0,1]%%%} Error: Bad Argument Value
```

maple [B] time = 0.04, size = 2269, normalized size = 5.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x+A)/(c*x^2+b*x+a)/(f*x^2+e*x+d)^(1/2),x)
```

```
[Out] 2/(-4*a*c+b^2)^(1/2)/(-2*(-(-4*a*c+b^2)^(1/2)*b*f+(-4*a*c+b^2)^(1/2)*c*e+2*
a*c*f-b^2*f+b*c*e-2*c^2*d)/c^2)^(1/2)*ln((-(-(-4*a*c+b^2)^(1/2)*b*f+(-4*a*c
+b^2)^(1/2)*c*e+2*a*c*f-b^2*f+b*c*e-2*c^2*d)/c^2-(f*(-4*a*c+b^2)^(1/2)+b*f-
c*e)/c*(x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)+1/2*(-2*(-(-4*a*c+b^2)^(1/2)*b*f+(-
4*a*c+b^2)^(1/2)*c*e+2*a*c*f-b^2*f+b*c*e-2*c^2*d)/c^2)^(1/2)*(4*(x+1/2*(b+(
-4*a*c+b^2)^(1/2))/c)^2*f-4*(f*(-4*a*c+b^2)^(1/2)+b*f-c*e)/c*(x+1/2*(b+(-4*
a*c+b^2)^(1/2))/c)-2*(-(-4*a*c+b^2)^(1/2)*b*f+(-4*a*c+b^2)^(1/2)*c*e+2*a*c*
f-b^2*f+b*c*e-2*c^2*d)/c^2)^(1/2))/(x+1/2*(b+(-4*a*c+b^2)^(1/2))/c))*A-1/c/
(-2*(-(-4*a*c+b^2)^(1/2)*b*f+(-4*a*c+b^2)^(1/2)*c*e+2*a*c*f-b^2*f+b*c*e-2*c
^2*d)/c^2)^(1/2)*ln((-(-(-4*a*c+b^2)^(1/2)*b*f+(-4*a*c+b^2)^(1/2)*c*e+2*a*c
*f-b^2*f+b*c*e-2*c^2*d)/c^2-(f*(-4*a*c+b^2)^(1/2)+b*f-c*e)/c*(x+1/2*(b+(-4*
a*c+b^2)^(1/2))/c)+1/2*(-2*(-(-4*a*c+b^2)^(1/2)*b*f+(-4*a*c+b^2)^(1/2)*c*e+
2*a*c*f-b^2*f+b*c*e-2*c^2*d)/c^2)^(1/2)*(4*(x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)
^2*f-4*(f*(-4*a*c+b^2)^(1/2)+b*f-c*e)/c*(x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)-2*
(-(-4*a*c+b^2)^(1/2)*b*f+(-4*a*c+b^2)^(1/2)*c*e+2*a*c*f-b^2*f+b*c*e-2*c^2*d
)/c^2)^(1/2))/(x+1/2*(b+(-4*a*c+b^2)^(1/2))/c))*B-1/(-4*a*c+b^2)^(1/2)/c/(-
2*(-(-4*a*c+b^2)^(1/2)*b*f+(-4*a*c+b^2)^(1/2)*c*e+2*a*c*f-b^2*f+b*c*e-2*c^2
*d)/c^2)^(1/2)*ln((-(-(-4*a*c+b^2)^(1/2)*b*f+(-4*a*c+b^2)^(1/2)*c*e+2*a*c*f
-b^2*f+b*c*e-2*c^2*d)/c^2-(f*(-4*a*c+b^2)^(1/2)+b*f-c*e)/c*(x+1/2*(b+(-4*a*
c+b^2)^(1/2))/c)+1/2*(-2*(-(-4*a*c+b^2)^(1/2)*b*f+(-4*a*c+b^2)^(1/2)*c*e+2*
a*c*f-b^2*f+b*c*e-2*c^2*d)/c^2)^(1/2)*(4*(x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)^2
*f-4*(f*(-4*a*c+b^2)^(1/2)+b*f-c*e)/c*(x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)-2*(-
(-4*a*c+b^2)^(1/2)*b*f+(-4*a*c+b^2)^(1/2)*c*e+2*a*c*f-b^2*f+b*c*e-2*c^2*d)/
c^2)^(1/2))/(x+1/2*(b+(-4*a*c+b^2)^(1/2))/c))*b*B-2/(-4*a*c+b^2)^(1/2)/(-2*
((-4*a*c+b^2)^(1/2)*b*f-(-4*a*c+b^2)^(1/2)*c*e+2*a*c*f-b^2*f+b*c*e-2*c^2*d)
/c^2)^(1/2)*ln((-((-4*a*c+b^2)^(1/2)*b*f-(-4*a*c+b^2)^(1/2)*c*e+2*a*c*f-b^2
*f+b*c*e-2*c^2*d)/c^2-(-f*(-4*a*c+b^2)^(1/2)+b*f-c*e)/c*(x-1/2*(-b+(-4*a*c+
b^2)^(1/2))/c)+1/2*(-2*((-4*a*c+b^2)^(1/2)*b*f-(-4*a*c+b^2)^(1/2)*c*e+2*a*c
*f-b^2*f+b*c*e-2*c^2*d)/c^2)^(1/2)*(4*(x-1/2*(-b+(-4*a*c+b^2)^(1/2))/c)^2*f
-4*(-f*(-4*a*c+b^2)^(1/2)+b*f-c*e)/c*(x-1/2*(-b+(-4*a*c+b^2)^(1/2))/c)-2*((
-4*a*c+b^2)^(1/2)*b*f-(-4*a*c+b^2)^(1/2)*c*e+2*a*c*f-b^2*f+b*c*e-2*c^2*d)/c
^2)^(1/2))/(x-1/2*(-b+(-4*a*c+b^2)^(1/2))/c))*A-1/c/(-2*((-4*a*c+b^2)^(1/2)
*b*f-(-4*a*c+b^2)^(1/2)*c*e+2*a*c*f-b^2*f+b*c*e-2*c^2*d)/c^2)^(1/2)*ln((-((
-4*a*c+b^2)^(1/2)*b*f-(-4*a*c+b^2)^(1/2)*c*e+2*a*c*f-b^2*f+b*c*e-2*c^2*d)/c
^2-(-f*(-4*a*c+b^2)^(1/2)+b*f-c*e)/c*(x-1/2*(-b+(-4*a*c+b^2)^(1/2))/c)+1/2*
(-2*((-4*a*c+b^2)^(1/2)*b*f-(-4*a*c+b^2)^(1/2)*c*e+2*a*c*f-b^2*f+b*c*e-2*c^
2*d)/c^2)^(1/2)*(4*(x-1/2*(-b+(-4*a*c+b^2)^(1/2))/c)^2*f-4*(-f*(-4*a*c+b^2)
^(1/2)+b*f-c*e)/c*(x-1/2*(-b+(-4*a*c+b^2)^(1/2))/c)-2*((-4*a*c+b^2)^(1/2)*b
*f-(-4*a*c+b^2)^(1/2)*c*e+2*a*c*f-b^2*f+b*c*e-2*c^2*d)/c^2)^(1/2))/(x-1/2*(
-b+(-4*a*c+b^2)^(1/2))/c))*B+1/(-4*a*c+b^2)^(1/2)/c/(-2*((-4*a*c+b^2)^(1/2)
*b*f-(-4*a*c+b^2)^(1/2)*c*e+2*a*c*f-b^2*f+b*c*e-2*c^2*d)/c^2)^(1/2)*ln((-((
-4*a*c+b^2)^(1/2)*b*f-(-4*a*c+b^2)^(1/2)*c*e+2*a*c*f-b^2*f+b*c*e-2*c^2*d)/c
^2-(-f*(-4*a*c+b^2)^(1/2)+b*f-c*e)/c*(x-1/2*(-b+(-4*a*c+b^2)^(1/2))/c)+1/2*
(-2*((-4*a*c+b^2)^(1/2)*b*f-(-4*a*c+b^2)^(1/2)*c*e+2*a*c*f-b^2*f+b*c*e-2*c^
2*d)/c^2)^(1/2)*(4*(x-1/2*(-b+(-4*a*c+b^2)^(1/2))/c)^2*f-4*(-f*(-4*a*c+b^2)
^(1/2)+b*f-c*e)/c*(x-1/2*(-b+(-4*a*c+b^2)^(1/2))/c)-2*((-4*a*c+b^2)^(1/2)*b
*f-(-4*a*c+b^2)^(1/2)*c*e+2*a*c*f-b^2*f+b*c*e-2*c^2*d)/c^2)^(1/2))/(x-1/2*(
-b+(-4*a*c+b^2)^(1/2))/c))*b*B
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(c*x^2+b*x+a)/(f*x^2+e*x+d)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+B\,x}{\left(c\,x^2+b\,x+a\right)\,\sqrt{f\,x^2+e\,x+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x)/((a + b*x + c*x^2)*(d + e*x + f*x^2)^(1/2)),x)
```

```
[Out] int((A + B*x)/((a + b*x + c*x^2)*(d + e*x + f*x^2)^(1/2)), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A+Bx}{\left(a+bx+cx^2\right)\sqrt{d+ex+fx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(c*x**2+b*x+a)/(f*x**2+e*x+d)**(1/2),x)
```

```
[Out] Integral((A + B*x)/((a + b*x + c*x**2)*sqrt(d + e*x + f*x**2)), x)
```

3.22 $\int \frac{A+Bx}{(a+cx^2)\sqrt{d+ex+fx^2}}\,dx$

Optimal. Leaf size=780

$$\frac{\sqrt{A\left(-\sqrt{a^2f^2+ac\left(e^2-2df\right)+c^2d^2}-af+cd\right)+aBe}\sqrt{B\left(\sqrt{a^2f^2+ac\left(e^2-2df\right)+c^2d^2}-af+cd\right)-Ace}\ \tan}{\sqrt{2}\sqrt{a}\sqrt{c}\sqrt{e}\sqrt{a^2f^2+ac\left(e^2-\right.}}$$

```
[Out] -1/2*arctanh(1/2*e^(1/2)*(a*(A*c*e-B*(c*d-a*f-(c^2*d^2+a^2*f^2+a*c*(-2*d*f+
e^2))^(1/2)))-c*x*(a*B*e+A*(c*d-a*f+(c^2*d^2+a^2*f^2+a*c*(-2*d*f+e^2))^(1/2
))))*2^(1/2)/a^(1/2)/c^(1/2)/(f*x^2+e*x+d)^(1/2)/(-A*c*e+B*(c*d-a*f-(c^2*d^
2+a^2*f^2+a*c*(-2*d*f+e^2))^(1/2)))^(1/2)/(a*B*e+A*(c*d-a*f+(c^2*d^2+a^2*f^
2+a*c*(-2*d*f+e^2))^(1/2)))^(1/2))*(-A*c*e+B*(c*d-a*f-(c^2*d^2+a^2*f^2+a*c*
(-2*d*f+e^2))^(1/2)))^(1/2)*(a*B*e+A*(c*d-a*f+(c^2*d^2+a^2*f^2+a*c*(-2*d*f+
e^2))^(1/2)))^(1/2)*2^(1/2)/a^(1/2)/c^(1/2)/e^(1/2)/(c^2*d^2+a^2*f^2+a*c*(-
2*d*f+e^2))^(1/2)+1/2*arctanh(1/2*e^(1/2)*(-c*x*(a*B*e+A*(c*d-a*f-(c^2*d^2+
a^2*f^2+a*c*(-2*d*f+e^2))^(1/2)))+a*(A*c*e-B*(c*d-a*f+(c^2*d^2+a^2*f^2+a*c*
(-2*d*f+e^2))^(1/2))))*2^(1/2)/a^(1/2)/c^(1/2)/(f*x^2+e*x+d)^(1/2)/(a*B*e+A
*(c*d-a*f-(c^2*d^2+a^2*f^2+a*c*(-2*d*f+e^2))^(1/2)))^(1/2)/(-A*c*e+B*(c*d-a
*f+(c^2*d^2+a^2*f^2+a*c*(-2*d*f+e^2))^(1/2)))^(1/2))*(a*B*e+A*(c*d-a*f-(c^2
*d^2+a^2*f^2+a*c*(-2*d*f+e^2))^(1/2)))^(1/2)*(-A*c*e+B*(c*d-a*f+(c^2*d^2+a^
2*f^2+a*c*(-2*d*f+e^2))^(1/2)))^(1/2)*2^(1/2)/a^(1/2)/c^(1/2)/e^(1/2)/(c^2*
d^2+a^2*f^2+a*c*(-2*d*f+e^2))^(1/2)
```

Rubi [A] time = 5.16, antiderivative size = 780, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {1036, 1030, 208}

$$\frac{\sqrt{A\left(-\sqrt{a^2f^2+ac\left(e^2-2df\right)+c^2d^2}-af+cd\right)+aBe}\sqrt{B\left(\sqrt{a^2f^2+ac\left(e^2-2df\right)+c^2d^2}-af+cd\right)-Ace}\ \tan}{\sqrt{2}\sqrt{a}\sqrt{c}\sqrt{e}\sqrt{a^2f^2+ac\left(e^2-\right.}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x)/((a + c*x^2)*Sqrt[d + e*x + f*x^2]),x]
```

```
[Out] (Sqrt[a*B*e + A*(c*d - a*f - Sqrt[c^2*d^2 + a^2*f^2 + a*c*(e^2 - 2*d*f)])]*
Sqrt[-(A*c*e) + B*(c*d - a*f + Sqrt[c^2*d^2 + a^2*f^2 + a*c*(e^2 - 2*d*f)])
]*ArcTanh[(Sqrt[e]*(a*(A*c*e - B*(c*d - a*f + Sqrt[c^2*d^2 + a^2*f^2 + a*c*
(e^2 - 2*d*f)])) - c*(a*B*e + A*(c*d - a*f - Sqrt[c^2*d^2 + a^2*f^2 + a*c*(
e^2 - 2*d*f)]))*x))/(Sqrt[2]*Sqrt[a]*Sqrt[c]*Sqrt[a*B*e + A*(c*d - a*f - Sq
rt[c^2*d^2 + a^2*f^2 + a*c*(e^2 - 2*d*f)])]*Sqrt[-(A*c*e) + B*(c*d - a*f +
Sqrt[c^2*d^2 + a^2*f^2 + a*c*(e^2 - 2*d*f)])]*Sqrt[d + e*x + f*x^2])])/(Sqr
t[2]*Sqrt[a]*Sqrt[c]*Sqrt[e]*Sqrt[c^2*d^2 + a^2*f^2 + a*c*(e^2 - 2*d*f)]) -
 (Sqrt[-(A*c*e) + B*(c*d - a*f - Sqrt[c^2*d^2 + a^2*f^2 + a*c*(e^2 - 2*d*f)
])]*Sqrt[a*B*e + A*(c*d - a*f + Sqrt[c^2*d^2 + a^2*f^2 + a*c*(e^2 - 2*d*f)]
)]*ArcTanh[(Sqrt[e]*(a*(A*c*e - B*(c*d - a*f - Sqrt[c^2*d^2 + a^2*f^2 + a*c
*(e^2 - 2*d*f)])) - c*(a*B*e + A*(c*d - a*f + Sqrt[c^2*d^2 + a^2*f^2 + a*c*
(e^2 - 2*d*f)]))*x))/(Sqrt[2]*Sqrt[a]*Sqrt[c]*Sqrt[-(A*c*e) + B*(c*d - a*f
- Sqrt[c^2*d^2 + a^2*f^2 + a*c*(e^2 - 2*d*f)])]*Sqrt[a*B*e + A*(c*d - a*f +
 Sqrt[c^2*d^2 + a^2*f^2 + a*c*(e^2 - 2*d*f)])]*Sqrt[d + e*x + f*x^2])])/(Sq
rt[2]*Sqrt[a]*Sqrt[c]*Sqrt[e]*Sqrt[c^2*d^2 + a^2*f^2 + a*c*(e^2 - 2*d*f)])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1030

```
Int[((g_) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f
_.)*(x_)^2]), x_Symbol] :> Dist[-2*a*g*h, Subst[Int[1/Simp[2*a^2*g*h*c + a*
e*x^2, x], x], x, Simp[a*h - g*c*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[
{a, c, d, e, f, g, h}, x] && EqQ[a*h^2*e + 2*g*h*(c*d - a*f) - g^2*c*e, 0]
```

Rule 1036

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 + a*c*e^2, 2]}, Dist
[1/(2*q), Int[Simp[-(a*h*e) - g*(c*d - a*f - q) + (h*(c*d - a*f + q) - g*c*
e)*x, x]/((a + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Si
mp[-(a*h*e) - g*(c*d - a*f + q) + (h*(c*d - a*f - q) - g*c*e)*x, x]/((a + c
*x^2)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f, g, h}, x] &&
 NeQ[e^2 - 4*d*f, 0] && NegQ[-(a*c)]
```

Rubi steps

$$\int \frac{A+Bx}{\left(a+cx^2\right)\sqrt{d+ex+fx^2}}\,dx = -\frac{\int \frac{-aBe-A\left(cd-af+\sqrt{c^2d^2+a^2f^2+ac\left(e^2-2df\right)}\right)+\left(-Ace+B\left(cd-af-\sqrt{c^2d^2+a^2f^2+ac\left(e^2-2df\right)}\right)\right)x}{\left(a+cx^2\right)\sqrt{d+ex+fx^2}}}{2\sqrt{c^2d^2+a^2f^2+ac\left(e^2-2df\right)}}$$

$$= \frac{\left(a\left(Ace-B\left(cd-af-\sqrt{c^2d^2+a^2f^2+ac\left(e^2-2df\right)}\right)\right)\right)\left(aBe+A\left(cd-af\right.\right.}{}$$

$$= \frac{\sqrt{aBe+A\left(cd-af-\sqrt{c^2d^2+a^2f^2+ac\left(e^2-2df\right)}\right)}\sqrt{-Ace+B\left(cd-af\right.}}{}$$

Mathematica [A] time = 0.41, size = 254, normalized size = 0.33

$$\frac{\frac{\left(A\sqrt{c}-\sqrt{-a}B\right)\tanh^{-1}\left(\frac{\sqrt{c}(2d+ex)-\sqrt{-a}(e+2fx)}{2\sqrt{d+x(e+fx)}\sqrt{-\sqrt{-a}\sqrt{c}e-af+cd}}\right)}{\sqrt{-\sqrt{-a}\sqrt{c}e-af+cd}}-\frac{\left(\sqrt{-a}B+A\sqrt{c}\right)\tanh^{-1}\left(\frac{\sqrt{-a}(e+2fx)+\sqrt{c}(2d+ex)}{2\sqrt{d+x(e+fx)}\sqrt{\sqrt{-a}\sqrt{c}e-af+cd}}\right)}{\sqrt{\sqrt{-a}\sqrt{c}e-af+cd}}}{2\sqrt{-a}\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x)/((a + c*x^2)*Sqrt[d + e*x + f*x^2]),x]
```

```
[Out] (((-(Sqrt[-a]*B) + A*Sqrt[c])*ArcTanh[(Sqrt[c]*(2*d + e*x) - Sqrt[-a]*(e +
2*f*x))/(2*Sqrt[c*d - Sqrt[-a]*Sqrt[c]*e - a*f]*Sqrt[d + x*(e + f*x)])]])/Sq
rt[c*d - Sqrt[-a]*Sqrt[c]*e - a*f] - ((Sqrt[-a]*B + A*Sqrt[c])*ArcTanh[(Sqr
t[c]*(2*d + e*x) + Sqrt[-a]*(e + 2*f*x))/(2*Sqrt[c*d + Sqrt[-a]*Sqrt[c]*e -
 a*f]*Sqrt[d + x*(e + f*x)])]])/Sqrt[c*d + Sqrt[-a]*Sqrt[c]*e - a*f])/(2*Sqr
t[-a]*Sqrt[c])
```

fricas [B] time = 134.78, size = 6861, normalized size = 8.80

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(c*x^2+a)/(f*x^2+e*x+d)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(-(2*A*B*a*c*e - (B^2*a*c - A^2*c^2)*d + (B^2*a^2 - A^2*a*c)*f + (
a*c^3*d^2 + a^2*c^2*e^2 - 2*a^2*c^2*d*f + a^3*c*f^2)*sqrt(-(4*A^2*B^2*c^2*d
^2 + 4*A^2*B^2*a^2*f^2 + 4*(A*B^3*a*c - A^3*B*c^2)*d*e + (B^4*a^2 - 2*A^2*B
^2*a*c + A^4*c^2)*e^2 - 4*(2*A^2*B^2*a*c*d + (A*B^3*a^2 - A^3*B*a*c)*e)*f)/
(a*c^5*d^4 + 2*a^2*c^4*d^2*e^2 + a^3*c^3*e^4 - 4*a^4*c^2*d*f^3 + a^5*c*f^4
+ 2*(3*a^3*c^3*d^2 + a^4*c^2*e^2)*f^2 - 4*(a^2*c^4*d^3 + a^3*c^3*d*e^2)*f))
)/(a*c^3*d^2 + a^2*c^2*e^2 - 2*a^2*c^2*d*f + a^3*c*f^2))*log(-(2*(A*B^3*a*c
 + A^3*B*c^2)*d*e + (B^4*a^2 - A^4*c^2)*e^2 - 2*(A*B^3*a^2 + A^3*B*a*c)*e*f
 - 2*(2*(A*B^3*a^2 + A^3*B*a*c)*f^2 - (2*(A*B^3*a*c + A^3*B*c^2)*d + (B^4*a
^2 - A^4*c^2)*e)*f)*x + 2*(2*A^2*B*c^3*d^2 + 2*A^2*B*a^2*c*f^2 + (3*A*B^2*a
*c^2 - A^3*c^3)*d*e + (B^3*a^2*c - A^2*B*a*c^2)*e^2 - (4*A^2*B*a*c^2*d + (3
*A*B^2*a^2*c - A^3*a*c^2)*e)*f - (B*a*c^4*d^3 - A*a*c^4*d^2*e + B*a^2*c^3*d
*e^2 - A*a^2*c^3*e^3 - B*a^4*c*f^3 + (3*B*a^3*c^2*d - A*a^3*c^2*e)*f^2 - (3
*B*a^2*c^3*d^2 - 2*A*a^2*c^3*d*e + B*a^3*c^2*e^2)*f)*sqrt(-(4*A^2*B^2*c^2*d
^2 + 4*A^2*B^2*a^2*f^2 + 4*(A*B^3*a*c - A^3*B*c^2)*d*e + (B^4*a^2 - 2*A^2*B
^2*a*c + A^4*c^2)*e^2 - 4*(2*A^2*B^2*a*c*d + (A*B^3*a^2 - A^3*B*a*c)*e)*f)/
(a*c^5*d^4 + 2*a^2*c^4*d^2*e^2 + a^3*c^3*e^4 - 4*a^4*c^2*d*f^3 + a^5*c*f^4
+ 2*(3*a^3*c^3*d^2 + a^4*c^2*e^2)*f^2 - 4*(a^2*c^4*d^3 + a^3*c^3*d*e^2)*f))
)*sqrt(f*x^2 + e*x + d)*sqrt(-(2*A*B*a*c*e - (B^2*a*c - A^2*c^2)*d + (B^2*a
^2 - A^2*a*c)*f + (a*c^3*d^2 + a^2*c^2*e^2 - 2*a^2*c^2*d*f + a^3*c*f^2)*sqr
t(-(4*A^2*B^2*c^2*d^2 + 4*A^2*B^2*a^2*f^2 + 4*(A*B^3*a*c - A^3*B*c^2)*d*e +
 (B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)*e^2 - 4*(2*A^2*B^2*a*c*d + (A*B^3*a^2
- A^3*B*a*c)*e)*f)/(a*c^5*d^4 + 2*a^2*c^4*d^2*e^2 + a^3*c^3*e^4 - 4*a^4*c^2
*d*f^3 + a^5*c*f^4 + 2*(3*a^3*c^3*d^2 + a^4*c^2*e^2)*f^2 - 4*(a^2*c^4*d^3 +
 a^3*c^3*d*e^2)*f)))/(a*c^3*d^2 + a^2*c^2*e^2 - 2*a^2*c^2*d*f + a^3*c*f^2))
 - (2*(B^2*a*c^3 + A^2*c^4)*d^3 + 2*(B^2*a^2*c^2 + A^2*a*c^3)*d*e^2 - 4*(B^
2*a^2*c^2 + A^2*a*c^3)*d^2*f + 2*(B^2*a^3*c + A^2*a^2*c^2)*d*f^2 + ((B^2*a*
c^3 + A^2*c^4)*d^2*e + (B^2*a^2*c^2 + A^2*a*c^3)*e^3 - 2*(B^2*a^2*c^2 + A^2
*a*c^3)*d*e*f + (B^2*a^3*c + A^2*a^2*c^2)*e*f^2)*x)*sqrt(-(4*A^2*B^2*c^2*d^
2 + 4*A^2*B^2*a^2*f^2 + 4*(A*B^3*a*c - A^3*B*c^2)*d*e + (B^4*a^2 - 2*A^2*B^
2*a*c + A^4*c^2)*e^2 - 4*(2*A^2*B^2*a*c*d + (A*B^3*a^2 - A^3*B*a*c)*e)*f)/(
a*c^5*d^4 + 2*a^2*c^4*d^2*e^2 + a^3*c^3*e^4 - 4*a^4*c^2*d*f^3 + a^5*c*f^4 +
 2*(3*a^3*c^3*d^2 + a^4*c^2*e^2)*f^2 - 4*(a^2*c^4*d^3 + a^3*c^3*d*e^2)*f)))
/x) + 1/4*sqrt(-(2*A*B*a*c*e - (B^2*a*c - A^2*c^2)*d + (B^2*a^2 - A^2*a*c)*
f + (a*c^3*d^2 + a^2*c^2*e^2 - 2*a^2*c^2*d*f + a^3*c*f^2)*sqrt(-(4*A^2*B^2*
c^2*d^2 + 4*A^2*B^2*a^2*f^2 + 4*(A*B^3*a*c - A^3*B*c^2)*d*e + (B^4*a^2 - 2*
A^2*B^2*a*c + A^4*c^2)*e^2 - 4*(2*A^2*B^2*a*c*d + (A*B^3*a^2 - A^3*B*a*c)*e
)*f)/(a*c^5*d^4 + 2*a^2*c^4*d^2*e^2 + a^3*c^3*e^4 - 4*a^4*c^2*d*f^3 + a^5*c
*f^4 + 2*(3*a^3*c^3*d^2 + a^4*c^2*e^2)*f^2 - 4*(a^2*c^4*d^3 + a^3*c^3*d*e^2
)*f)))/(a*c^3*d^2 + a^2*c^2*e^2 - 2*a^2*c^2*d*f + a^3*c*f^2))*log(-(2*(A*B^
3*a*c + A^3*B*c^2)*d*e + (B^4*a^2 - A^4*c^2)*e^2 - 2*(A*B^3*a^2 + A^3*B*a*c
)*e*f - 2*(2*(A*B^3*a^2 + A^3*B*a*c)*f^2 - (2*(A*B^3*a*c + A^3*B*c^2)*d + (
B^4*a^2 - A^4*c^2)*e)*f)*x - 2*(2*A^2*B*c^3*d^2 + 2*A^2*B*a^2*c*f^2 + (3*A*
B^2*a*c^2 - A^3*c^3)*d*e + (B^3*a^2*c - A^2*B*a*c^2)*e^2 - (4*A^2*B*a*c^2*d
 + (3*A*B^2*a^2*c - A^3*a*c^2)*e)*f - (B*a*c^4*d^3 - A*a*c^4*d^2*e + B*a^2*
c^3*d*e^2 - A*a^2*c^3*e^3 - B*a^4*c*f^3 + (3*B*a^3*c^2*d - A*a^3*c^2*e)*f^2
 - (3*B*a^2*c^3*d^2 - 2*A*a^2*c^3*d*e + B*a^3*c^2*e^2)*f)*sqrt(-(4*A^2*B^2*
c^2*d^2 + 4*A^2*B^2*a^2*f^2 + 4*(A*B^3*a*c - A^3*B*c^2)*d*e + (B^4*a^2 - 2*
A^2*B^2*a*c + A^4*c^2)*e^2 - 4*(2*A^2*B^2*a*c*d + (A*B^3*a^2 - A^3*B*a*c)*e
)*f)/(a*c^5*d^4 + 2*a^2*c^4*d^2*e^2 + a^3*c^3*e^4 - 4*a^4*c^2*d*f^3 + a^5*c
*f^4 + 2*(3*a^3*c^3*d^2 + a^4*c^2*e^2)*f^2 - 4*(a^2*c^4*d^3 + a^3*c^3*d*e^2
)*f)))*sqrt(f*x^2 + e*x + d)*sqrt(-(2*A*B*a*c*e - (B^2*a*c - A^2*c^2)*d + (
B^2*a^2 - A^2*a*c)*f + (a*c^3*d^2 + a^2*c^2*e^2 - 2*a^2*c^2*d*f + a^3*c*f^2
)*sqrt(-(4*A^2*B^2*c^2*d^2 + 4*A^2*B^2*a^2*f^2 + 4*(A*B^3*a*c - A^3*B*c^2)*
d*e + (B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)*e^2 - 4*(2*A^2*B^2*a*c*d + (A*B^3
*a^2 - A^3*B*a*c)*e)*f)/(a*c^5*d^4 + 2*a^2*c^4*d^2*e^2 + a^3*c^3*e^4 - 4*a^
```

```
4*c^2*d*f^3 + a^5*c*f^4 + 2*(3*a^3*c^3*d^2 + a^4*c^2*e^2)*f^2 - 4*(a^2*c^4*
d^3 + a^3*c^3*d*e^2)*f)))/(a*c^3*d^2 + a^2*c^2*e^2 - 2*a^2*c^2*d*f + a^3*c*
f^2)) - (2*(B^2*a*c^3 + A^2*c^4)*d^3 + 2*(B^2*a^2*c^2 + A^2*a*c^3)*d*e^2 -
4*(B^2*a^2*c^2 + A^2*a*c^3)*d^2*f + 2*(B^2*a^3*c + A^2*a^2*c^2)*d*f^2 + ((B
^2*a*c^3 + A^2*c^4)*d^2*e + (B^2*a^2*c^2 + A^2*a*c^3)*e^3 - 2*(B^2*a^2*c^2
+ A^2*a*c^3)*d*e*f + (B^2*a^3*c + A^2*a^2*c^2)*e*f^2)*x)*sqrt(-(4*A^2*B^2*c
^2*d^2 + 4*A^2*B^2*a^2*f^2 + 4*(A*B^3*a*c - A^3*B*c^2)*d*e + (B^4*a^2 - 2*A
^2*B^2*a*c + A^4*c^2)*e^2 - 4*(2*A^2*B^2*a*c*d + (A*B^3*a^2 - A^3*B*a*c)*e)
*f)/(a*c^5*d^4 + 2*a^2*c^4*d^2*e^2 + a^3*c^3*e^4 - 4*a^4*c^2*d*f^3 + a^5*c*
f^4 + 2*(3*a^3*c^3*d^2 + a^4*c^2*e^2)*f^2 - 4*(a^2*c^4*d^3 + a^3*c^3*d*e^2)
*f)))/x) - 1/4*sqrt(-(2*A*B*a*c*e - (B^2*a*c - A^2*c^2)*d + (B^2*a^2 - A^2*
a*c)*f - (a*c^3*d^2 + a^2*c^2*e^2 - 2*a^2*c^2*d*f + a^3*c*f^2)*sqrt(-(4*A^2
*B^2*c^2*d^2 + 4*A^2*B^2*a^2*f^2 + 4*(A*B^3*a*c - A^3*B*c^2)*d*e + (B^4*a^2
 - 2*A^2*B^2*a*c + A^4*c^2)*e^2 - 4*(2*A^2*B^2*a*c*d + (A*B^3*a^2 - A^3*B*a
*c)*e)*f)/(a*c^5*d^4 + 2*a^2*c^4*d^2*e^2 + a^3*c^3*e^4 - 4*a^4*c^2*d*f^3 +
a^5*c*f^4 + 2*(3*a^3*c^3*d^2 + a^4*c^2*e^2)*f^2 - 4*(a^2*c^4*d^3 + a^3*c^3*
d*e^2)*f)))/(a*c^3*d^2 + a^2*c^2*e^2 - 2*a^2*c^2*d*f + a^3*c*f^2))*log(-(2*
(A*B^3*a*c + A^3*B*c^2)*d*e + (B^4*a^2 - A^4*c^2)*e^2 - 2*(A*B^3*a^2 + A^3*
B*a*c)*e*f - 2*(2*(A*B^3*a^2 + A^3*B*a*c)*f^2 - (2*(A*B^3*a*c + A^3*B*c^2)*
d + (B^4*a^2 - A^4*c^2)*e)*f)*x + 2*(2*A^2*B*c^3*d^2 + 2*A^2*B*a^2*c*f^2 +
(3*A*B^2*a*c^2 - A^3*c^3)*d*e + (B^3*a^2*c - A^2*B*a*c^2)*e^2 - (4*A^2*B*a*
c^2*d + (3*A*B^2*a^2*c - A^3*a*c^2)*e)*f + (B*a*c^4*d^3 - A*a*c^4*d^2*e + B
*a^2*c^3*d*e^2 - A*a^2*c^3*e^3 - B*a^4*c*f^3 + (3*B*a^3*c^2*d - A*a^3*c^2*e
)*f^2 - (3*B*a^2*c^3*d^2 - 2*A*a^2*c^3*d*e + B*a^3*c^2*e^2)*f)*sqrt(-(4*A^2
*B^2*c^2*d^2 + 4*A^2*B^2*a^2*f^2 + 4*(A*B^3*a*c - A^3*B*c^2)*d*e + (B^4*a^2
 - 2*A^2*B^2*a*c + A^4*c^2)*e^2 - 4*(2*A^2*B^2*a*c*d + (A*B^3*a^2 - A^3*B*a
*c)*e)*f)/(a*c^5*d^4 + 2*a^2*c^4*d^2*e^2 + a^3*c^3*e^4 - 4*a^4*c^2*d*f^3 +
a^5*c*f^4 + 2*(3*a^3*c^3*d^2 + a^4*c^2*e^2)*f^2 - 4*(a^2*c^4*d^3 + a^3*c^3*
d*e^2)*f)))*sqrt(f*x^2 + e*x + d)*sqrt(-(2*A*B*a*c*e - (B^2*a*c - A^2*c^2)*
d + (B^2*a^2 - A^2*a*c)*f - (a*c^3*d^2 + a^2*c^2*e^2 - 2*a^2*c^2*d*f + a^3*
c*f^2)*sqrt(-(4*A^2*B^2*c^2*d^2 + 4*A^2*B^2*a^2*f^2 + 4*(A*B^3*a*c - A^3*B*
c^2)*d*e + (B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)*e^2 - 4*(2*A^2*B^2*a*c*d + (
A*B^3*a^2 - A^3*B*a*c)*e)*f)/(a*c^5*d^4 + 2*a^2*c^4*d^2*e^2 + a^3*c^3*e^4 -
 4*a^4*c^2*d*f^3 + a^5*c*f^4 + 2*(3*a^3*c^3*d^2 + a^4*c^2*e^2)*f^2 - 4*(a^2
*c^4*d^3 + a^3*c^3*d*e^2)*f)))/(a*c^3*d^2 + a^2*c^2*e^2 - 2*a^2*c^2*d*f + a
^3*c*f^2)) + (2*(B^2*a*c^3 + A^2*c^4)*d^3 + 2*(B^2*a^2*c^2 + A^2*a*c^3)*d*e
^2 - 4*(B^2*a^2*c^2 + A^2*a*c^3)*d^2*f + 2*(B^2*a^3*c + A^2*a^2*c^2)*d*f^2
+ ((B^2*a*c^3 + A^2*c^4)*d^2*e + (B^2*a^2*c^2 + A^2*a*c^3)*e^3 - 2*(B^2*a^2
*c^2 + A^2*a*c^3)*d*e*f + (B^2*a^3*c + A^2*a^2*c^2)*e*f^2)*x)*sqrt(-(4*A^2*
B^2*c^2*d^2 + 4*A^2*B^2*a^2*f^2 + 4*(A*B^3*a*c - A^3*B*c^2)*d*e + (B^4*a^2
- 2*A^2*B^2*a*c + A^4*c^2)*e^2 - 4*(2*A^2*B^2*a*c*d + (A*B^3*a^2 - A^3*B*a*
c)*e)*f)/(a*c^5*d^4 + 2*a^2*c^4*d^2*e^2 + a^3*c^3*e^4 - 4*a^4*c^2*d*f^3 + a
^5*c*f^4 + 2*(3*a^3*c^3*d^2 + a^4*c^2*e^2)*f^2 - 4*(a^2*c^4*d^3 + a^3*c^3*d
*e^2)*f)))/x) + 1/4*sqrt(-(2*A*B*a*c*e - (B^2*a*c - A^2*c^2)*d + (B^2*a^2 -
 A^2*a*c)*f - (a*c^3*d^2 + a^2*c^2*e^2 - 2*a^2*c^2*d*f + a^3*c*f^2)*sqrt(-(
4*A^2*B^2*c^2*d^2 + 4*A^2*B^2*a^2*f^2 + 4*(A*B^3*a*c - A^3*B*c^2)*d*e + (B^
4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)*e^2 - 4*(2*A^2*B^2*a*c*d + (A*B^3*a^2 - A^
3*B*a*c)*e)*f)/(a*c^5*d^4 + 2*a^2*c^4*d^2*e^2 + a^3*c^3*e^4 - 4*a^4*c^2*d*f
^3 + a^5*c*f^4 + 2*(3*a^3*c^3*d^2 + a^4*c^2*e^2)*f^2 - 4*(a^2*c^4*d^3 + a^3
*c^3*d*e^2)*f)))/(a*c^3*d^2 + a^2*c^2*e^2 - 2*a^2*c^2*d*f + a^3*c*f^2))*log
(-(2*(A*B^3*a*c + A^3*B*c^2)*d*e + (B^4*a^2 - A^4*c^2)*e^2 - 2*(A*B^3*a^2 +
 A^3*B*a*c)*e*f - 2*(2*(A*B^3*a^2 + A^3*B*a*c)*f^2 - (2*(A*B^3*a*c + A^3*B*
c^2)*d + (B^4*a^2 - A^4*c^2)*e)*f)*x - 2*(2*A^2*B*c^3*d^2 + 2*A^2*B*a^2*c*f
^2 + (3*A*B^2*a*c^2 - A^3*c^3)*d*e + (B^3*a^2*c - A^2*B*a*c^2)*e^2 - (4*A^2
*B*a*c^2*d + (3*A*B^2*a^2*c - A^3*a*c^2)*e)*f + (B*a*c^4*d^3 - A*a*c^4*d^2*
e + B*a^2*c^3*d*e^2 - A*a^2*c^3*e^3 - B*a^4*c*f^3 + (3*B*a^3*c^2*d - A*a^3*
c^2*e)*f^2 - (3*B*a^2*c^3*d^2 - 2*A*a^2*c^3*d*e + B*a^3*c^2*e^2)*f)*sqrt(-(
4*A^2*B^2*c^2*d^2 + 4*A^2*B^2*a^2*f^2 + 4*(A*B^3*a*c - A^3*B*c^2)*d*e + (B^
4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)*e^2 - 4*(2*A^2*B^2*a*c*d + (A*B^3*a^2 - A^
```

```
3*B*a*c)*e)*f)/(a*c^5*d^4 + 2*a^2*c^4*d^2*e^2 + a^3*c^3*e^4 - 4*a^4*c^2*d*f
^3 + a^5*c*f^4 + 2*(3*a^3*c^3*d^2 + a^4*c^2*e^2)*f^2 - 4*(a^2*c^4*d^3 + a^3
*c^3*d*e^2)*f)))*sqrt(f*x^2 + e*x + d)*sqrt(-(2*A*B*a*c*e - (B^2*a*c - A^2*
c^2)*d + (B^2*a^2 - A^2*a*c)*f - (a*c^3*d^2 + a^2*c^2*e^2 - 2*a^2*c^2*d*f +
 a^3*c*f^2)*sqrt(-(4*A^2*B^2*c^2*d^2 + 4*A^2*B^2*a^2*f^2 + 4*(A*B^3*a*c - A
^3*B*c^2)*d*e + (B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)*e^2 - 4*(2*A^2*B^2*a*c*
d + (A*B^3*a^2 - A^3*B*a*c)*e)*f)/(a*c^5*d^4 + 2*a^2*c^4*d^2*e^2 + a^3*c^3*
e^4 - 4*a^4*c^2*d*f^3 + a^5*c*f^4 + 2*(3*a^3*c^3*d^2 + a^4*c^2*e^2)*f^2 - 4
*(a^2*c^4*d^3 + a^3*c^3*d*e^2)*f)))/(a*c^3*d^2 + a^2*c^2*e^2 - 2*a^2*c^2*d*
f + a^3*c*f^2)) + (2*(B^2*a*c^3 + A^2*c^4)*d^3 + 2*(B^2*a^2*c^2 + A^2*a*c^3
)*d*e^2 - 4*(B^2*a^2*c^2 + A^2*a*c^3)*d^2*f + 2*(B^2*a^3*c + A^2*a^2*c^2)*d
*f^2 + ((B^2*a*c^3 + A^2*c^4)*d^2*e + (B^2*a^2*c^2 + A^2*a*c^3)*e^3 - 2*(B^
2*a^2*c^2 + A^2*a*c^3)*d*e*f + (B^2*a^3*c + A^2*a^2*c^2)*e*f^2)*x)*sqrt(-(4
*A^2*B^2*c^2*d^2 + 4*A^2*B^2*a^2*f^2 + 4*(A*B^3*a*c - A^3*B*c^2)*d*e + (B^4
*a^2 - 2*A^2*B^2*a*c + A^4*c^2)*e^2 - 4*(2*A^2*B^2*a*c*d + (A*B^3*a^2 - A^3
*B*a*c)*e)*f)/(a*c^5*d^4 + 2*a^2*c^4*d^2*e^2 + a^3*c^3*e^4 - 4*a^4*c^2*d*f^
3 + a^5*c*f^4 + 2*(3*a^3*c^3*d^2 + a^4*c^2*e^2)*f^2 - 4*(a^2*c^4*d^3 + a^3*
c^3*d*e^2)*f)))/x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(c*x^2+a)/(f*x^2+e*x+d)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument ValueConj Error: Bad Argument Typ
e
```

maple [A] time = 0.04, size = 784, normalized size = 1.01

$$-\frac{A \ln\left(\frac{-\frac{2\left(af-cd-\sqrt{-ac}\,e\right)}{c}+\frac{\left(ce+2\sqrt{-ac}\,f\right)\left(x-\frac{\sqrt{-ac}}{c}\right)}{c}+2\sqrt{-\frac{af-cd-\sqrt{-ac}\,e}{c}}\,\sqrt{\left(x-\frac{\sqrt{-ac}}{c}\right)^{2} f+\frac{\left(ce+2\sqrt{-ac}\,f\right)\left(x-\frac{\sqrt{-ac}}{c}\right)}{c}-\frac{af-cd-\sqrt{-ac}\,e}{c}}}{x-\frac{\sqrt{-ac}}{c}}\right)}{2\sqrt{-ac}\,\sqrt{-\frac{af-cd-\sqrt{-ac}\,e}{c}}}+\frac{A \ln\left(\frac{-\frac{2\left(af-cd+\cdots\right.}{c}}{\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x+A)/(c*x^2+a)/(f*x^2+e*x+d)^(1/2),x)
```

```
[Out] -1/2/(-a*c)^(1/2)/(-(-e*(-a*c)^(1/2)+a*f-c*d)/c)^(1/2)*ln((-2*(-e*(-a*c)^(1
/2)+a*f-c*d)/c+(2*f*(-a*c)^(1/2)+c*e)/c*(x-(-a*c)^(1/2)/c)+2*(-(-e*(-a*c)^(
1/2)+a*f-c*d)/c)^(1/2)*((x-(-a*c)^(1/2)/c)^2*f+(2*f*(-a*c)^(1/2)+c*e)/c*(x-
(-a*c)^(1/2)/c)-(-e*(-a*c)^(1/2)+a*f-c*d)/c)^(1/2))/(x-(-a*c)^(1/2)/c))*A-1
/2/c/(-(-e*(-a*c)^(1/2)+a*f-c*d)/c)^(1/2)*ln((-2*(-e*(-a*c)^(1/2)+a*f-c*d)/
c+(2*f*(-a*c)^(1/2)+c*e)/c*(x-(-a*c)^(1/2)/c)+2*(-(-e*(-a*c)^(1/2)+a*f-c*d)
/c)^(1/2)*((x-(-a*c)^(1/2)/c)^2*f+(2*f*(-a*c)^(1/2)+c*e)/c*(x-(-a*c)^(1/2)/
c)-(-e*(-a*c)^(1/2)+a*f-c*d)/c)^(1/2))/(x-(-a*c)^(1/2)/c))*B+1/2/(-a*c)^(1/
2)/(-(e*(-a*c)^(1/2)+a*f-c*d)/c)^(1/2)*ln((-2*(e*(-a*c)^(1/2)+a*f-c*d)/c+1/
c*(-2*f*(-a*c)^(1/2)+c*e)*(x+(-a*c)^(1/2)/c)+2*(-(e*(-a*c)^(1/2)+a*f-c*d)/c
)^(1/2)*((x+(-a*c)^(1/2)/c)^2*f+1/c*(-2*f*(-a*c)^(1/2)+c*e)*(x+(-a*c)^(1/2)
/c)-(e*(-a*c)^(1/2)+a*f-c*d)/c)^(1/2))/(x+(-a*c)^(1/2)/c))*A-1/2/c/(-(e*(-a
*c)^(1/2)+a*f-c*d)/c)^(1/2)*ln((-2*(e*(-a*c)^(1/2)+a*f-c*d)/c+1/c*(-2*f*(-a
*c)^(1/2)+c*e)*(x+(-a*c)^(1/2)/c)+2*(-(e*(-a*c)^(1/2)+a*f-c*d)/c)^(1/2)*((x
+(-a*c)^(1/2)/c)^2*f+1/c*(-2*f*(-a*c)^(1/2)+c*e)*(x+(-a*c)^(1/2)/c)-(e*(-a*
c)^(1/2)+a*f-c*d)/c)^(1/2))/(x+(-a*c)^(1/2)/c))*B
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Bx + A}{(cx^2 + a)\sqrt{fx^2 + ex + d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(c*x^2+a)/(f*x^2+e*x+d)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x + A)/((c*x^2 + a)*sqrt(f*x^2 + e*x + d)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + B\,x}{\left(c\,x^2 + a\right)\,\sqrt{f\,x^2 + e\,x + d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x)/((a + c*x^2)*(d + e*x + f*x^2)^(1/2)),x)
```

```
[Out] int((A + B*x)/((a + c*x^2)*(d + e*x + f*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + Bx}{\left(a + cx^2\right)\sqrt{d + ex + fx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(c*x**2+a)/(f*x**2+e*x+d)**(1/2),x)
```

```
[Out] Integral((A + B*x)/((a + c*x**2)*sqrt(d + e*x + f*x**2)), x)
```

3.23 $\int \frac{A+Bx}{\left(a+bx+cx^2\right)\sqrt{d+fx^2}}\,dx$

Optimal. Leaf size=302

$$\frac{\left(-B\sqrt{b^2-4ac}-2Ac+bB\right)\tanh^{-1}\left(\frac{2cd-fx\left(b-\sqrt{b^2-4ac}\right)}{\sqrt{2}\sqrt{d+fx^2}\sqrt{bf\left(b-\sqrt{b^2-4ac}\right)-2acf+2c^2d}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{bf\left(b-\sqrt{b^2-4ac}\right)-2acf+2c^2d}}+\frac{\left(2Ac-B\left(\sqrt{b^2-4ac}+b\right)\right)\tanh^{-1}\left(\cdots\right.}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{bf\left(\sqrt{b^2-4\cdots\right.}}}$$

```
[Out] 1/2*arctanh(1/2*(2*c*d-f*x*(b-(-4*a*c+b^2)^(1/2)))*2^(1/2)/(f*x^2+d)^(1/2)/
(2*c^2*d-2*a*c*f+b*f*(b-(-4*a*c+b^2)^(1/2)))^(1/2))*(b*B-2*A*c-B*(-4*a*c+b^
2)^(1/2))*2^(1/2)/(-4*a*c+b^2)^(1/2)/(2*c^2*d-2*a*c*f+b*f*(b-(-4*a*c+b^2)^(
1/2)))^(1/2)+1/2*arctanh(1/2*(2*c*d-f*x*(b+(-4*a*c+b^2)^(1/2)))*2^(1/2)/(f*
x^2+d)^(1/2)/(2*c^2*d-2*a*c*f+b*f*(b+(-4*a*c+b^2)^(1/2)))^(1/2))*(2*A*c-B*(
b+(-4*a*c+b^2)^(1/2)))*2^(1/2)/(-4*a*c+b^2)^(1/2)/(2*c^2*d-2*a*c*f+b*f*(b+(
-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A] time = 0.84, antiderivative size = 302, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {1034, 725, 206}

$$\frac{\left(-B\sqrt{b^2-4ac}-2Ac+bB\right)\tanh^{-1}\left(\frac{2cd-fx\left(b-\sqrt{b^2-4ac}\right)}{\sqrt{2}\sqrt{d+fx^2}\sqrt{bf\left(b-\sqrt{b^2-4ac}\right)-2acf+2c^2d}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{bf\left(b-\sqrt{b^2-4ac}\right)-2acf+2c^2d}}+\frac{\left(2Ac-B\left(\sqrt{b^2-4ac}+b\right)\right)\tanh^{-1}\left(\cdots\right.}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{bf\left(\sqrt{b^2-4\cdots\right.}}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x)/((a + b*x + c*x^2)*Sqrt[d + f*x^2]),x]
```

```
[Out] ((b*B - 2*A*c - B*Sqrt[b^2 - 4*a*c])*ArcTanh[(2*c*d - (b - Sqrt[b^2 - 4*a*c
])*f*x)/(Sqrt[2]*Sqrt[2*c^2*d - 2*a*c*f + b*(b - Sqrt[b^2 - 4*a*c])*f]*Sqrt
[d + f*x^2])])/(Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt[2*c^2*d - 2*a*c*f + b*(b - S
qrt[b^2 - 4*a*c])*f]) + ((2*A*c - B*(b + Sqrt[b^2 - 4*a*c]))*ArcTanh[(2*c*d
 - (b + Sqrt[b^2 - 4*a*c])*f*x)/(Sqrt[2]*Sqrt[2*c^2*d - 2*a*c*f + b*(b + Sq
rt[b^2 - 4*a*c])*f]*Sqrt[d + f*x^2])])/(Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt[2*c^
2*d - 2*a*c*f + b*(b + Sqrt[b^2 - 4*a*c])*f])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 1034

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f
_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(
b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Dist[(2*c*g -
h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a,
```

```
b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{A+Bx}{\left(a+bx+cx^2\right)\sqrt{d+fx^2}}\,dx = \frac{\left(2Ac-B\left(b-\sqrt{b^2-4ac}\right)\right)\int \frac{1}{\left(b-\sqrt{b^2-4ac}+2cx\right)\sqrt{d+fx^2}}\,dx}{\sqrt{b^2-4ac}} - \frac{\left(2Ac-B\left(b+\sqrt{b\right.\right.}}{}$$

$$= -\frac{\left(2Ac-B\left(b-\sqrt{b^2-4ac}\right)\right)\text{Subst}\left(\int \frac{1}{4c^2d+\left(b-\sqrt{b^2-4ac}\right)^2f-x^2}\,dx, x, \frac{2cd-\left(b-\sqrt{b^2\right.}}{\sqrt{d+f}}\right.}{\sqrt{b^2-4ac}}$$

$$= \frac{\left(bB-2Ac-B\sqrt{b^2-4ac}\right)\tanh^{-1}\left(\frac{2cd-\left(b-\sqrt{b^2-4ac}\right)fx}{\sqrt{2}\sqrt{2c^2d-2acf+b\left(b-\sqrt{b^2-4ac}\right)f}\sqrt{d+fx^2}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{2c^2d-2acf+b\left(b-\sqrt{b^2-4ac}\right)f}} + \frac{\left(2\right.}{}$$

Mathematica [A] time = 0.43, size = 283, normalized size = 0.94

$$\frac{\sqrt{2}\left(-\frac{\left(B\sqrt{b^2-4ac}+2Ac-bB\right)\tanh^{-1}\left(\frac{fx\left(\sqrt{b^2-4ac}-b\right)+2cd}{\sqrt{d+fx^2}\sqrt{2bf\left(b-\sqrt{b^2-4ac}\right)-4acf+4c^2d}}\right)}{2\sqrt{bf\left(b-\sqrt{b^2-4ac}\right)-2acf+2c^2d}} - \frac{\left(B\sqrt{b^2-4ac}-2Ac+bB\right)\tanh^{-1}\left(\frac{2cd-fx\left(\sqrt{b^2-4ac}+b\right)}{\sqrt{d+fx^2}\sqrt{2bf\left(\sqrt{b^2-4ac}+b\right)-4acf+4\right.}}\right.}{2\sqrt{bf\left(\sqrt{b^2-4ac}+b\right)-2acf+2c^2d}}\right.}{\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x)/((a + b*x + c*x^2)*Sqrt[d + f*x^2]),x]
```

```
[Out] (Sqrt[2]*(-1/2*((-(b*B) + 2*A*c + B*Sqrt[b^2 - 4*a*c])*ArcTanh[(2*c*d + (-b
 + Sqrt[b^2 - 4*a*c])*f*x)/(Sqrt[4*c^2*d - 4*a*c*f + 2*b*(b - Sqrt[b^2 - 4*
a*c])*f]*Sqrt[d + f*x^2])])/Sqrt[2*c^2*d - 2*a*c*f + b*(b - Sqrt[b^2 - 4*a*
c])*f] - ((b*B - 2*A*c + B*Sqrt[b^2 - 4*a*c])*ArcTanh[(2*c*d - (b + Sqrt[b^
2 - 4*a*c])*f*x)/(Sqrt[4*c^2*d - 4*a*c*f + 2*b*(b + Sqrt[b^2 - 4*a*c])*f]*S
qrt[d + f*x^2])])/(2*Sqrt[2*c^2*d - 2*a*c*f + b*(b + Sqrt[b^2 - 4*a*c])*f])
))/Sqrt[b^2 - 4*a*c]
```

fricas [B] time = 124.91, size = 8977, normalized size = 29.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(c*x^2+b*x+a)/(f*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*sqrt(((B^2*b^2 + 2*A^2*c^2 - 2*(B^2*a + A*B*b)*c)*d + (2*B^2*a^
2 - 2*A*B*a*b + A^2*b^2 - 2*A^2*a*c)*f + ((b^2*c^2 - 4*a*c^3)*d^2 + (b^4 -
6*a*b^2*c + 8*a^2*c^2)*d*f + (a^2*b^2 - 4*a^3*c)*f^2)*sqrt(((B^4*b^2 - 4*A*
B^3*b*c + 4*A^2*B^2*c^2)*d^2 + 2*(2*A*B^3*a*b - A^2*B^2*b^2 - 2*(2*A^2*B^2*
a - A^3*B*b)*c)*d*f + (4*A^2*B^2*a^2 - 4*A^3*B*a*b + A^4*b^2)*f^2)/((b^2*c^
4 - 4*a*c^5)*d^4 + 2*(b^4*c^2 - 6*a*b^2*c^3 + 8*a^2*c^4)*d^3*f + (b^6 - 8*a
*b^4*c + 22*a^2*b^2*c^2 - 24*a^3*c^3)*d^2*f^2 + 2*(a^2*b^4 - 6*a^3*b^2*c +
8*a^4*c^2)*d*f^3 + (a^4*b^2 - 4*a^5*c)*f^4)))/((b^2*c^2 - 4*a*c^3)*d^2 + (b
^4 - 6*a*b^2*c + 8*a^2*c^2)*d*f + (a^2*b^2 - 4*a^3*c)*f^2))*log((2*(B^4*a*b
```

```
^2 - A*B^3*b^3 - 2*A^3*B*b*c^2 - (2*A*B^3*a*b - 3*A^2*B^2*b^2)*c)*d^2 + 2*(
2*A*B^3*a^2*b - 3*A^2*B^2*a*b^2 + A^3*B*b^3 + (2*A^3*B*a*b - A^4*b^2)*c)*d*
f + sqrt(2)*((B^3*b^4 - 8*A^2*B*a*c^3 + 2*(6*A*B^2*a*b + A^2*B*b^2)*c^2 - (
4*B^3*a*b^2 + 3*A*B^2*b^3)*c)*d^2 + (3*A*B^2*a*b^3 - A^2*B*b^4 + 4*(4*A^2*B
*a^2 - A^3*a*b)*c^2 - (12*A*B^2*a^2*b - A^3*b^3)*c)*d*f + (2*A^2*B*a^2*b^2
- A^3*a*b^3 - 4*(2*A^2*B*a^3 - A^3*a^2*b)*c)*f^2 - ((B*b^4*c^2 + 4*(2*B*a^2
 + A*a*b)*c^4 - (6*B*a*b^2 + A*b^3)*c^3)*d^3 + (B*b^6 - 4*(6*B*a^3 + A*a^2*
b)*c^3 + (22*B*a^2*b^2 + 5*A*a*b^3)*c^2 - (8*B*a*b^4 + A*b^5)*c)*d^2*f + (3
*B*a^2*b^4 - A*a*b^5 + 4*(6*B*a^4 - A*a^3*b)*c^2 - (18*B*a^3*b^2 - 5*A*a^2*
b^3)*c)*d*f^2 + (2*B*a^4*b^2 - A*a^3*b^3 - 4*(2*B*a^5 - A*a^4*b)*c)*f^3)*sq
rt(((B^4*b^2 - 4*A*B^3*b*c + 4*A^2*B^2*c^2)*d^2 + 2*(2*A*B^3*a*b - A^2*B^2*
b^2 - 2*(2*A^2*B^2*a - A^3*B*b)*c)*d*f + (4*A^2*B^2*a^2 - 4*A^3*B*a*b + A^4
*b^2)*f^2)/((b^2*c^4 - 4*a*c^5)*d^4 + 2*(b^4*c^2 - 6*a*b^2*c^3 + 8*a^2*c^4)
*d^3*f + (b^6 - 8*a*b^4*c + 22*a^2*b^2*c^2 - 24*a^3*c^3)*d^2*f^2 + 2*(a^2*b
^4 - 6*a^3*b^2*c + 8*a^4*c^2)*d*f^3 + (a^4*b^2 - 4*a^5*c)*f^4)))*sqrt(f*x^2
 + d)*sqrt(((B^2*b^2 + 2*A^2*c^2 - 2*(B^2*a + A*B*b)*c)*d + (2*B^2*a^2 - 2*
A*B*a*b + A^2*b^2 - 2*A^2*a*c)*f + ((b^2*c^2 - 4*a*c^3)*d^2 + (b^4 - 6*a*b^
2*c + 8*a^2*c^2)*d*f + (a^2*b^2 - 4*a^3*c)*f^2)*sqrt(((B^4*b^2 - 4*A*B^3*b*
c + 4*A^2*B^2*c^2)*d^2 + 2*(2*A*B^3*a*b - A^2*B^2*b^2 - 2*(2*A^2*B^2*a - A^
3*B*b)*c)*d*f + (4*A^2*B^2*a^2 - 4*A^3*B*a*b + A^4*b^2)*f^2)/((b^2*c^4 - 4*
a*c^5)*d^4 + 2*(b^4*c^2 - 6*a*b^2*c^3 + 8*a^2*c^4)*d^3*f + (b^6 - 8*a*b^4*c
 + 22*a^2*b^2*c^2 - 24*a^3*c^3)*d^2*f^2 + 2*(a^2*b^4 - 6*a^3*b^2*c + 8*a^4*
c^2)*d*f^3 + (a^4*b^2 - 4*a^5*c)*f^4)))/((b^2*c^2 - 4*a*c^3)*d^2 + (b^4 - 6
*a*b^2*c + 8*a^2*c^2)*d*f + (a^2*b^2 - 4*a^3*c)*f^2)) - 4*((B^4*a^2*b - A*B
^3*a*b^2 - 2*A^3*B*a*c^2 - (2*A*B^3*a^2 - 3*A^2*B^2*a*b)*c)*d*f + (2*A*B^3*
a^3 - 3*A^2*B^2*a^2*b + A^3*B*a*b^2 + (2*A^3*B*a^2 - A^4*a*b)*c)*f^2)*x + 2
*((4*A^2*a*c^4 + (4*B^2*a^2 - 4*A*B*a*b - A^2*b^2)*c^3 - (B^2*a*b^2 - A*B*b
^3)*c^2)*d^3 - (B^2*a*b^4 - A*B*b^5 + 8*A^2*a^2*c^3 + 2*(4*B^2*a^3 - 4*A*B*
a^2*b - 3*A^2*a*b^2)*c^2 - (6*B^2*a^2*b^2 - 6*A*B*a*b^3 - A^2*b^4)*c)*d^2*f
 - (B^2*a^3*b^2 - A*B*a^2*b^3 - 4*A^2*a^3*c^2 - (4*B^2*a^4 - 4*A*B*a^3*b -
A^2*a^2*b^2)*c)*d*f^2)*sqrt(((B^4*b^2 - 4*A*B^3*b*c + 4*A^2*B^2*c^2)*d^2 +
2*(2*A*B^3*a*b - A^2*B^2*b^2 - 2*(2*A^2*B^2*a - A^3*B*b)*c)*d*f + (4*A^2*B^
2*a^2 - 4*A^3*B*a*b + A^4*b^2)*f^2)/((b^2*c^4 - 4*a*c^5)*d^4 + 2*(b^4*c^2 -
 6*a*b^2*c^3 + 8*a^2*c^4)*d^3*f + (b^6 - 8*a*b^4*c + 22*a^2*b^2*c^2 - 24*a^
3*c^3)*d^2*f^2 + 2*(a^2*b^4 - 6*a^3*b^2*c + 8*a^4*c^2)*d*f^3 + (a^4*b^2 - 4
*a^5*c)*f^4)))/x) - 1/4*sqrt(2)*sqrt(((B^2*b^2 + 2*A^2*c^2 - 2*(B^2*a + A*B
*b)*c)*d + (2*B^2*a^2 - 2*A*B*a*b + A^2*b^2 - 2*A^2*a*c)*f + ((b^2*c^2 - 4*
a*c^3)*d^2 + (b^4 - 6*a*b^2*c + 8*a^2*c^2)*d*f + (a^2*b^2 - 4*a^3*c)*f^2)*s
qrt(((B^4*b^2 - 4*A*B^3*b*c + 4*A^2*B^2*c^2)*d^2 + 2*(2*A*B^3*a*b - A^2*B^2
*b^2 - 2*(2*A^2*B^2*a - A^3*B*b)*c)*d*f + (4*A^2*B^2*a^2 - 4*A^3*B*a*b + A^
4*b^2)*f^2)/((b^2*c^4 - 4*a*c^5)*d^4 + 2*(b^4*c^2 - 6*a*b^2*c^3 + 8*a^2*c^4
)*d^3*f + (b^6 - 8*a*b^4*c + 22*a^2*b^2*c^2 - 24*a^3*c^3)*d^2*f^2 + 2*(a^2*
b^4 - 6*a^3*b^2*c + 8*a^4*c^2)*d*f^3 + (a^4*b^2 - 4*a^5*c)*f^4)))/((b^2*c^2
 - 4*a*c^3)*d^2 + (b^4 - 6*a*b^2*c + 8*a^2*c^2)*d*f + (a^2*b^2 - 4*a^3*c)*f
^2))*log((2*(B^4*a*b^2 - A*B^3*b^3 - 2*A^3*B*b*c^2 - (2*A*B^3*a*b - 3*A^2*B
^2*b^2)*c)*d^2 + 2*(2*A*B^3*a^2*b - 3*A^2*B^2*a*b^2 + A^3*B*b^3 + (2*A^3*B*
a*b - A^4*b^2)*c)*d*f - sqrt(2)*((B^3*b^4 - 8*A^2*B*a*c^3 + 2*(6*A*B^2*a*b
+ A^2*B*b^2)*c^2 - (4*B^3*a*b^2 + 3*A*B^2*b^3)*c)*d^2 + (3*A*B^2*a*b^3 - A^
2*B*b^4 + 4*(4*A^2*B*a^2 - A^3*a*b)*c^2 - (12*A*B^2*a^2*b - A^3*b^3)*c)*d*f
 + (2*A^2*B*a^2*b^2 - A^3*a*b^3 - 4*(2*A^2*B*a^3 - A^3*a^2*b)*c)*f^2 - ((B*
b^4*c^2 + 4*(2*B*a^2 + A*a*b)*c^4 - (6*B*a*b^2 + A*b^3)*c^3)*d^3 + (B*b^6 -
 4*(6*B*a^3 + A*a^2*b)*c^3 + (22*B*a^2*b^2 + 5*A*a*b^3)*c^2 - (8*B*a*b^4 +
A*b^5)*c)*d^2*f + (3*B*a^2*b^4 - A*a*b^5 + 4*(6*B*a^4 - A*a^3*b)*c^2 - (18*
B*a^3*b^2 - 5*A*a^2*b^3)*c)*d*f^2 + (2*B*a^4*b^2 - A*a^3*b^3 - 4*(2*B*a^5 -
 A*a^4*b)*c)*f^3)*sqrt(((B^4*b^2 - 4*A*B^3*b*c + 4*A^2*B^2*c^2)*d^2 + 2*(2*
A*B^3*a*b - A^2*B^2*b^2 - 2*(2*A^2*B^2*a - A^3*B*b)*c)*d*f + (4*A^2*B^2*a^2
 - 4*A^3*B*a*b + A^4*b^2)*f^2)/((b^2*c^4 - 4*a*c^5)*d^4 + 2*(b^4*c^2 - 6*a*
b^2*c^3 + 8*a^2*c^4)*d^3*f + (b^6 - 8*a*b^4*c + 22*a^2*b^2*c^2 - 24*a^3*c^3
)*d^2*f^2 + 2*(a^2*b^4 - 6*a^3*b^2*c + 8*a^4*c^2)*d*f^3 + (a^4*b^2 - 4*a^5*
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c)*f^4)))*sqrt(f*x^2 + d)*sqrt(((B^2*b^2 + 2*A^2*c^2 - 2*(B^2*a + A*B*b)*c)
*d + (2*B^2*a^2 - 2*A*B*a*b + A^2*b^2 - 2*A^2*a*c)*f + ((b^2*c^2 - 4*a*c^3)
*d^2 + (b^4 - 6*a*b^2*c + 8*a^2*c^2)*d*f + (a^2*b^2 - 4*a^3*c)*f^2)*sqrt(((
B^4*b^2 - 4*A*B^3*b*c + 4*A^2*B^2*c^2)*d^2 + 2*(2*A*B^3*a*b - A^2*B^2*b^2 -
 2*(2*A^2*B^2*a - A^3*B*b)*c)*d*f + (4*A^2*B^2*a^2 - 4*A^3*B*a*b + A^4*b^2)
*f^2)/((b^2*c^4 - 4*a*c^5)*d^4 + 2*(b^4*c^2 - 6*a*b^2*c^3 + 8*a^2*c^4)*d^3*
f + (b^6 - 8*a*b^4*c + 22*a^2*b^2*c^2 - 24*a^3*c^3)*d^2*f^2 + 2*(a^2*b^4 -
6*a^3*b^2*c + 8*a^4*c^2)*d*f^3 + (a^4*b^2 - 4*a^5*c)*f^4)))/((b^2*c^2 - 4*a
*c^3)*d^2 + (b^4 - 6*a*b^2*c + 8*a^2*c^2)*d*f + (a^2*b^2 - 4*a^3*c)*f^2)) -
 4*((B^4*a^2*b - A*B^3*a*b^2 - 2*A^3*B*a*c^2 - (2*A*B^3*a^2 - 3*A^2*B^2*a*b
)*c)*d*f + (2*A*B^3*a^3 - 3*A^2*B^2*a^2*b + A^3*B*a*b^2 + (2*A^3*B*a^2 - A^
4*a*b)*c)*f^2)*x + 2*((4*A^2*a*c^4 + (4*B^2*a^2 - 4*A*B*a*b - A^2*b^2)*c^3
- (B^2*a*b^2 - A*B*b^3)*c^2)*d^3 - (B^2*a*b^4 - A*B*b^5 + 8*A^2*a^2*c^3 + 2
*(4*B^2*a^3 - 4*A*B*a^2*b - 3*A^2*a*b^2)*c^2 - (6*B^2*a^2*b^2 - 6*A*B*a*b^3
 - A^2*b^4)*c)*d^2*f - (B^2*a^3*b^2 - A*B*a^2*b^3 - 4*A^2*a^3*c^2 - (4*B^2*
a^4 - 4*A*B*a^3*b - A^2*a^2*b^2)*c)*d*f^2)*sqrt(((B^4*b^2 - 4*A*B^3*b*c + 4
*A^2*B^2*c^2)*d^2 + 2*(2*A*B^3*a*b - A^2*B^2*b^2 - 2*(2*A^2*B^2*a - A^3*B*b
)*c)*d*f + (4*A^2*B^2*a^2 - 4*A^3*B*a*b + A^4*b^2)*f^2)/((b^2*c^4 - 4*a*c^5
)*d^4 + 2*(b^4*c^2 - 6*a*b^2*c^3 + 8*a^2*c^4)*d^3*f + (b^6 - 8*a*b^4*c + 22
*a^2*b^2*c^2 - 24*a^3*c^3)*d^2*f^2 + 2*(a^2*b^4 - 6*a^3*b^2*c + 8*a^4*c^2)*
d*f^3 + (a^4*b^2 - 4*a^5*c)*f^4)))/x) + 1/4*sqrt(2)*sqrt(((B^2*b^2 + 2*A^2*
c^2 - 2*(B^2*a + A*B*b)*c)*d + (2*B^2*a^2 - 2*A*B*a*b + A^2*b^2 - 2*A^2*a*c
)*f - ((b^2*c^2 - 4*a*c^3)*d^2 + (b^4 - 6*a*b^2*c + 8*a^2*c^2)*d*f + (a^2*b
^2 - 4*a^3*c)*f^2)*sqrt(((B^4*b^2 - 4*A*B^3*b*c + 4*A^2*B^2*c^2)*d^2 + 2*(2
*A*B^3*a*b - A^2*B^2*b^2 - 2*(2*A^2*B^2*a - A^3*B*b)*c)*d*f + (4*A^2*B^2*a^
2 - 4*A^3*B*a*b + A^4*b^2)*f^2)/((b^2*c^4 - 4*a*c^5)*d^4 + 2*(b^4*c^2 - 6*a
*b^2*c^3 + 8*a^2*c^4)*d^3*f + (b^6 - 8*a*b^4*c + 22*a^2*b^2*c^2 - 24*a^3*c^
3)*d^2*f^2 + 2*(a^2*b^4 - 6*a^3*b^2*c + 8*a^4*c^2)*d*f^3 + (a^4*b^2 - 4*a^5
*c)*f^4)))/((b^2*c^2 - 4*a*c^3)*d^2 + (b^4 - 6*a*b^2*c + 8*a^2*c^2)*d*f + (
a^2*b^2 - 4*a^3*c)*f^2))*log((2*(B^4*a*b^2 - A*B^3*b^3 - 2*A^3*B*b*c^2 - (2
*A*B^3*a*b - 3*A^2*B^2*b^2)*c)*d^2 + 2*(2*A*B^3*a^2*b - 3*A^2*B^2*a*b^2 + A
^3*B*b^3 + (2*A^3*B*a*b - A^4*b^2)*c)*d*f + sqrt(2)*((B^3*b^4 - 8*A^2*B*a*c
^3 + 2*(6*A*B^2*a*b + A^2*B*b^2)*c^2 - (4*B^3*a*b^2 + 3*A*B^2*b^3)*c)*d^2 +
 (3*A*B^2*a*b^3 - A^2*B*b^4 + 4*(4*A^2*B*a^2 - A^3*a*b)*c^2 - (12*A*B^2*a^2
*b - A^3*b^3)*c)*d*f + (2*A^2*B*a^2*b^2 - A^3*a*b^3 - 4*(2*A^2*B*a^3 - A^3*
a^2*b)*c)*f^2 + ((B*b^4*c^2 + 4*(2*B*a^2 + A*a*b)*c^4 - (6*B*a*b^2 + A*b^3)
*c^3)*d^3 + (B*b^6 - 4*(6*B*a^3 + A*a^2*b)*c^3 + (22*B*a^2*b^2 + 5*A*a*b^3)
*c^2 - (8*B*a*b^4 + A*b^5)*c)*d^2*f + (3*B*a^2*b^4 - A*a*b^5 + 4*(6*B*a^4 -
 A*a^3*b)*c^2 - (18*B*a^3*b^2 - 5*A*a^2*b^3)*c)*d*f^2 + (2*B*a^4*b^2 - A*a^
3*b^3 - 4*(2*B*a^5 - A*a^4*b)*c)*f^3)*sqrt(((B^4*b^2 - 4*A*B^3*b*c + 4*A^2*
B^2*c^2)*d^2 + 2*(2*A*B^3*a*b - A^2*B^2*b^2 - 2*(2*A^2*B^2*a - A^3*B*b)*c)*
d*f + (4*A^2*B^2*a^2 - 4*A^3*B*a*b + A^4*b^2)*f^2)/((b^2*c^4 - 4*a*c^5)*d^4
 + 2*(b^4*c^2 - 6*a*b^2*c^3 + 8*a^2*c^4)*d^3*f + (b^6 - 8*a*b^4*c + 22*a^2*
b^2*c^2 - 24*a^3*c^3)*d^2*f^2 + 2*(a^2*b^4 - 6*a^3*b^2*c + 8*a^4*c^2)*d*f^3
 + (a^4*b^2 - 4*a^5*c)*f^4)))*sqrt(f*x^2 + d)*sqrt(((B^2*b^2 + 2*A^2*c^2 -
2*(B^2*a + A*B*b)*c)*d + (2*B^2*a^2 - 2*A*B*a*b + A^2*b^2 - 2*A^2*a*c)*f -
((b^2*c^2 - 4*a*c^3)*d^2 + (b^4 - 6*a*b^2*c + 8*a^2*c^2)*d*f + (a^2*b^2 - 4
*a^3*c)*f^2)*sqrt(((B^4*b^2 - 4*A*B^3*b*c + 4*A^2*B^2*c^2)*d^2 + 2*(2*A*B^3
*a*b - A^2*B^2*b^2 - 2*(2*A^2*B^2*a - A^3*B*b)*c)*d*f + (4*A^2*B^2*a^2 - 4*
A^3*B*a*b + A^4*b^2)*f^2)/((b^2*c^4 - 4*a*c^5)*d^4 + 2*(b^4*c^2 - 6*a*b^2*c
^3 + 8*a^2*c^4)*d^3*f + (b^6 - 8*a*b^4*c + 22*a^2*b^2*c^2 - 24*a^3*c^3)*d^2
*f^2 + 2*(a^2*b^4 - 6*a^3*b^2*c + 8*a^4*c^2)*d*f^3 + (a^4*b^2 - 4*a^5*c)*f^
4)))/((b^2*c^2 - 4*a*c^3)*d^2 + (b^4 - 6*a*b^2*c + 8*a^2*c^2)*d*f + (a^2*b^
2 - 4*a^3*c)*f^2)) - 4*((B^4*a^2*b - A*B^3*a*b^2 - 2*A^3*B*a*c^2 - (2*A*B^3
*a^2 - 3*A^2*B^2*a*b)*c)*d*f + (2*A*B^3*a^3 - 3*A^2*B^2*a^2*b + A^3*B*a*b^2
 + (2*A^3*B*a^2 - A^4*a*b)*c)*f^2)*x - 2*((4*A^2*a*c^4 + (4*B^2*a^2 - 4*A*B
*a*b - A^2*b^2)*c^3 - (B^2*a*b^2 - A*B*b^3)*c^2)*d^3 - (B^2*a*b^4 - A*B*b^5
 + 8*A^2*a^2*c^3 + 2*(4*B^2*a^3 - 4*A*B*a^2*b - 3*A^2*a*b^2)*c^2 - (6*B^2*a
^2*b^2 - 6*A*B*a*b^3 - A^2*b^4)*c)*d^2*f - (B^2*a^3*b^2 - A*B*a^2*b^3 - 4*A
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^2*a^3*c^2 - (4*B^2*a^4 - 4*A*B*a^3*b - A^2*a^2*b^2)*c)*d*f^2)*sqrt(((B^4*b
^2 - 4*A*B^3*b*c + 4*A^2*B^2*c^2)*d^2 + 2*(2*A*B^3*a*b - A^2*B^2*b^2 - 2*(2
*A^2*B^2*a - A^3*B*b)*c)*d*f + (4*A^2*B^2*a^2 - 4*A^3*B*a*b + A^4*b^2)*f^2)
/((b^2*c^4 - 4*a*c^5)*d^4 + 2*(b^4*c^2 - 6*a*b^2*c^3 + 8*a^2*c^4)*d^3*f + (
b^6 - 8*a*b^4*c + 22*a^2*b^2*c^2 - 24*a^3*c^3)*d^2*f^2 + 2*(a^2*b^4 - 6*a^3
*b^2*c + 8*a^4*c^2)*d*f^3 + (a^4*b^2 - 4*a^5*c)*f^4)))/x) - 1/4*sqrt(2)*sqr
t(((B^2*b^2 + 2*A^2*c^2 - 2*(B^2*a + A*B*b)*c)*d + (2*B^2*a^2 - 2*A*B*a*b +
 A^2*b^2 - 2*A^2*a*c)*f - ((b^2*c^2 - 4*a*c^3)*d^2 + (b^4 - 6*a*b^2*c + 8*a
^2*c^2)*d*f + (a^2*b^2 - 4*a^3*c)*f^2)*sqrt(((B^4*b^2 - 4*A*B^3*b*c + 4*A^2
*B^2*c^2)*d^2 + 2*(2*A*B^3*a*b - A^2*B^2*b^2 - 2*(2*A^2*B^2*a - A^3*B*b)*c)
*d*f + (4*A^2*B^2*a^2 - 4*A^3*B*a*b + A^4*b^2)*f^2)/((b^2*c^4 - 4*a*c^5)*d^
4 + 2*(b^4*c^2 - 6*a*b^2*c^3 + 8*a^2*c^4)*d^3*f + (b^6 - 8*a*b^4*c + 22*a^2
*b^2*c^2 - 24*a^3*c^3)*d^2*f^2 + 2*(a^2*b^4 - 6*a^3*b^2*c + 8*a^4*c^2)*d*f^
3 + (a^4*b^2 - 4*a^5*c)*f^4)))/((b^2*c^2 - 4*a*c^3)*d^2 + (b^4 - 6*a*b^2*c
+ 8*a^2*c^2)*d*f + (a^2*b^2 - 4*a^3*c)*f^2))*log((2*(B^4*a*b^2 - A*B^3*b^3
- 2*A^3*B*b*c^2 - (2*A*B^3*a*b - 3*A^2*B^2*b^2)*c)*d^2 + 2*(2*A*B^3*a^2*b -
 3*A^2*B^2*a*b^2 + A^3*B*b^3 + (2*A^3*B*a*b - A^4*b^2)*c)*d*f - sqrt(2)*((B
^3*b^4 - 8*A^2*B*a*c^3 + 2*(6*A*B^2*a*b + A^2*B*b^2)*c^2 - (4*B^3*a*b^2 + 3
*A*B^2*b^3)*c)*d^2 + (3*A*B^2*a*b^3 - A^2*B*b^4 + 4*(4*A^2*B*a^2 - A^3*a*b)
*c^2 - (12*A*B^2*a^2*b - A^3*b^3)*c)*d*f + (2*A^2*B*a^2*b^2 - A^3*a*b^3 - 4
*(2*A^2*B*a^3 - A^3*a^2*b)*c)*f^2 + ((B*b^4*c^2 + 4*(2*B*a^2 + A*a*b)*c^4 -
 (6*B*a*b^2 + A*b^3)*c^3)*d^3 + (B*b^6 - 4*(6*B*a^3 + A*a^2*b)*c^3 + (22*B*
a^2*b^2 + 5*A*a*b^3)*c^2 - (8*B*a*b^4 + A*b^5)*c)*d^2*f + (3*B*a^2*b^4 - A*
a*b^5 + 4*(6*B*a^4 - A*a^3*b)*c^2 - (18*B*a^3*b^2 - 5*A*a^2*b^3)*c)*d*f^2 +
 (2*B*a^4*b^2 - A*a^3*b^3 - 4*(2*B*a^5 - A*a^4*b)*c)*f^3)*sqrt(((B^4*b^2 -
4*A*B^3*b*c + 4*A^2*B^2*c^2)*d^2 + 2*(2*A*B^3*a*b - A^2*B^2*b^2 - 2*(2*A^2*
B^2*a - A^3*B*b)*c)*d*f + (4*A^2*B^2*a^2 - 4*A^3*B*a*b + A^4*b^2)*f^2)/((b^
2*c^4 - 4*a*c^5)*d^4 + 2*(b^4*c^2 - 6*a*b^2*c^3 + 8*a^2*c^4)*d^3*f + (b^6 -
 8*a*b^4*c + 22*a^2*b^2*c^2 - 24*a^3*c^3)*d^2*f^2 + 2*(a^2*b^4 - 6*a^3*b^2*
c + 8*a^4*c^2)*d*f^3 + (a^4*b^2 - 4*a^5*c)*f^4)))*sqrt(f*x^2 + d)*sqrt(((B^
2*b^2 + 2*A^2*c^2 - 2*(B^2*a + A*B*b)*c)*d + (2*B^2*a^2 - 2*A*B*a*b + A^2*b
^2 - 2*A^2*a*c)*f - ((b^2*c^2 - 4*a*c^3)*d^2 + (b^4 - 6*a*b^2*c + 8*a^2*c^2
)*d*f + (a^2*b^2 - 4*a^3*c)*f^2)*sqrt(((B^4*b^2 - 4*A*B^3*b*c + 4*A^2*B^2*c
^2)*d^2 + 2*(2*A*B^3*a*b - A^2*B^2*b^2 - 2*(2*A^2*B^2*a - A^3*B*b)*c)*d*f +
 (4*A^2*B^2*a^2 - 4*A^3*B*a*b + A^4*b^2)*f^2)/((b^2*c^4 - 4*a*c^5)*d^4 + 2*
(b^4*c^2 - 6*a*b^2*c^3 + 8*a^2*c^4)*d^3*f + (b^6 - 8*a*b^4*c + 22*a^2*b^2*c
^2 - 24*a^3*c^3)*d^2*f^2 + 2*(a^2*b^4 - 6*a^3*b^2*c + 8*a^4*c^2)*d*f^3 + (a
^4*b^2 - 4*a^5*c)*f^4)))/((b^2*c^2 - 4*a*c^3)*d^2 + (b^4 - 6*a*b^2*c + 8*a^
2*c^2)*d*f + (a^2*b^2 - 4*a^3*c)*f^2)) - 4*((B^4*a^2*b - A*B^3*a*b^2 - 2*A^
3*B*a*c^2 - (2*A*B^3*a^2 - 3*A^2*B^2*a*b)*c)*d*f + (2*A*B^3*a^3 - 3*A^2*B^2
*a^2*b + A^3*B*a*b^2 + (2*A^3*B*a^2 - A^4*a*b)*c)*f^2)*x - 2*((4*A^2*a*c^4
+ (4*B^2*a^2 - 4*A*B*a*b - A^2*b^2)*c^3 - (B^2*a*b^2 - A*B*b^3)*c^2)*d^3 -
(B^2*a*b^4 - A*B*b^5 + 8*A^2*a^2*c^3 + 2*(4*B^2*a^3 - 4*A*B*a^2*b - 3*A^2*a
*b^2)*c^2 - (6*B^2*a^2*b^2 - 6*A*B*a*b^3 - A^2*b^4)*c)*d^2*f - (B^2*a^3*b^2
 - A*B*a^2*b^3 - 4*A^2*a^3*c^2 - (4*B^2*a^4 - 4*A*B*a^3*b - A^2*a^2*b^2)*c)
*d*f^2)*sqrt(((B^4*b^2 - 4*A*B^3*b*c + 4*A^2*B^2*c^2)*d^2 + 2*(2*A*B^3*a*b
- A^2*B^2*b^2 - 2*(2*A^2*B^2*a - A^3*B*b)*c)*d*f + (4*A^2*B^2*a^2 - 4*A^3*B
*a*b + A^4*b^2)*f^2)/((b^2*c^4 - 4*a*c^5)*d^4 + 2*(b^4*c^2 - 6*a*b^2*c^3 +
8*a^2*c^4)*d^3*f + (b^6 - 8*a*b^4*c + 22*a^2*b^2*c^2 - 24*a^3*c^3)*d^2*f^2
+ 2*(a^2*b^4 - 6*a^3*b^2*c + 8*a^4*c^2)*d*f^3 + (a^4*b^2 - 4*a^5*c)*f^4)))/
x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(c*x^2+b*x+a)/(f*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
```

```
UT:sage2:=int(sage0,x):;OUTPUT:Unable to divide, perhaps due to rounding er
ror%%%{%%{poly1[%%%{-4,[3,2,0]%%%}+%%%{16,[1,3,1]%%%},%%%{4,[4,2,0]%%%}+%%%
{-24,[2,3,1]%%%}+%%%{32,[0,4,2]%%%}]:[1,0,%%%{-1,[2,0,0]%%%}+%%%{4,[0,1,1]%
%%}]%%},[2,1,0,0]%%%}+%%%{%%{[%%%{-4,[5,0,0]%%%}+%%%{24,[3,1,1]%%%}+%%%{-32
,[1,2,2]%%%},%%%{4,[6,0,0]%%%}+%%%{-32,[4,1,1]%%%}+%%%{72,[2,2,2]%%%}+%%%{-
32,[0,3,3]%%%}]:[1,0,%%%{-1,[2,0,0]%%%}+%%%{4,[0,1,1]%%%}]%%},[2,0,0,1]%%%}
+%%%{%%{poly1[%%%{8,[2,3,0]%%%}+%%%{-32,[0,4,1]%%%},%%%{-8,[3,3,0]%%%}+%%%{
32,[1,4,1]%%%}]:[1,0,%%%{-1,[2,0,0]%%%}+%%%{4,[0,1,1]%%%}]%%},[1,1,1,0]%%%}
+%%%{%%{[%%%{8,[4,1,0]%%%}+%%%{-40,[2,2,1]%%%}+%%%{32,[0,3,2]%%%},%%%{-8,[5
,1,0]%%%}+%%%{56,[3,2,1]%%%}+%%%{-96,[1,3,2]%%%}]:[1,0,%%%{-1,[2,0,0]%%%}+%
%%{4,[0,1,1]%%%}]%%},[1,0,1,1]%%%}+%%%{%%%{8,[2,4,0]%%%}+%%%{-32,[0,5,1]%%%
},[0,1,2,0]%%%}+%%%{%%{poly1[%%%{-4,[3,2,0]%%%}+%%%{16,[1,3,1]%%%},%%%{4,[4
,2,0]%%%}+%%%{-24,[2,3,1]%%%}+%%%{32,[0,4,2]%%%}]:[1,0,%%%{-1,[2,0,0]%%%}+%
%%{4,[0,1,1]%%%}]%%},[0,0,2,1]%%%} / %%%{%%{[%%%{-4,[3,2,0]%%%}+%%%{16,[1,3
,1]%%%},%%%{4,[4,2,0]%%%}+%%%{-24,[2,3,1]%%%}+%%%{32,[0,4,2]%%%}]:[1,0,%%%{
-1,[2,0,0]%%%}+%%%{4,[0,1,1]%%%}]%%},[0,1,0,0]%%%}+%%%{%%{[%%%{-4,[5,0,0]%%
%}+%%%{24,[3,1,1]%%%}+%%%{-32,[1,2,2]%%%},%%%{4,[6,0,0]%%%}+%%%{-32,[4,1,1]
%%%}+%%%{72,[2,2,2]%%%}+%%%{-32,[0,3,3]%%%}]:[1,0,%%%{-1,[2,0,0]%%%}+%%%{4,
[0,1,1]%%%}]%%},[0,0,0,1]%%%} Error: Bad Argument ValueUnable to divide, pe
rhaps due to rounding error%%%{%%{[%%%{1,[3,0,0]%%%}+%%%{-4,[1,1,1]%%%},%%%
{1,[4,0,0]%%%}+%%%{-6,[2,1,1]%%%}+%%%{8,[0,2,2]%%%}]:[1,0,%%%{-1,[2,0,0]%%%
}+%%%{4,[0,1,1]%%%}]%%}/%%%{4,[0,0,2]%%%},[2,1,0,0]%%%}+%%%{%%%{1,[2,0,0]%%
%}+%%%{-4,[0,1,1]%%%}/2,[2,0,0,1]%%%}+%%%{%%{[%%%{-1,[4,0,0]%%%}+%%%{5,[2,1
,1]%%%}+%%%{-4,[0,2,2]%%%},%%%{-1,[5,0,0]%%%}+%%%{7,[3,1,1]%%%}+%%%{-12,[1,
2,2]%%%}]:[1,0,%%%{-1,[2,0,0]%%%}+%%%{4,[0,1,1]%%%}]%%}/%%%{2,[0,0,3]%%%},[
1,1,1,0]%%%}+%%%{%%{[%%%{-1,[2,0,0]%%%}+%%%{4,[0,1,1]%%%},%%%{-1,[3,0,0]%%%
}+%%%{4,[1,1,1]%%%}]:[1,0,%%%{-1,[2,0,0]%%%}+%%%{4,[0,1,1]%%%}]%%}/%%%{2,[0
,0,1]%%%},[1,0,1,1]%%%}+%%%{%%{[%%%{1,[5,0,0]%%%}+%%%{-6,[3,1,1]%%%}+%%%{8,
[1,2,2]%%%},%%%{1,[6,0,0]%%%}+%%%{-8,[4,1,1]%%%}+%%%{18,[2,2,2]%%%}+%%%{-8,
[0,3,3]%%%}]:[1,0,%%%{-1,[2,0,0]%%%}+%%%{4,[0,1,1]%%%}]%%}/%%%{4,[0,0,4]%%%
},[0,1,2,0]%%%}+%%%{%%{[%%%{1,[3,0,0]%%%}+%%%{-4,[1,1,1]%%%},%%%{1,[4,0,0]%
%%}+%%%{-6,[2,1,1]%%%}+%%%{8,[0,2,2]%%%}]:[1,0,%%%{-1,[2,0,0]%%%}+%%%{4,[0,
1,1]%%%}]%%}/%%%{4,[0,0,2]%%%},[0,0,2,1]%%%} / %%%{%%{[%%%{1,[3,0,0]%%%}+%%
%{-4,[1,1,1]%%%},%%%{1,[4,0,0]%%%}+%%%{-6,[2,1,1]%%%}+%%%{8,[0,2,2]%%%}]:[1
,0,%%%{-1,[2,0,0]%%%}+%%%{4,[0,1,1]%%%}]%%}/%%%{4,[0,0,4]%%%},[0,1,0,0]%%%}
+%%%{%%%{1,[2,0,0]%%%}+%%%{-4,[0,1,1]%%%}/%%%{2,[0,0,2]%%%},[0,0,0,1]%%%} E
rror: Bad Argument Value
```

maple [B] time = 0.03, size = 1771, normalized size = 5.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x+A)/(c*x^2+b*x+a)/(f*x^2+d)^(1/2),x)
```

```
[Out] 2/(-4*a*c+b^2)^(1/2)/(-2*(-(-4*a*c+b^2)^(1/2)*b*f+2*a*c*f-b^2*f-2*c^2*d)/c^
2)^(1/2)*ln((-(-(-4*a*c+b^2)^(1/2)*b*f+2*a*c*f-b^2*f-2*c^2*d)/c^2-f*(b+(-4*
a*c+b^2)^(1/2))/c*(x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)+1/2*(-2*(-(-4*a*c+b^2)^(
1/2)*b*f+2*a*c*f-b^2*f-2*c^2*d)/c^2)^(1/2)*(4*(x+1/2*(b+(-4*a*c+b^2)^(1/2))
/c)^2*f-4*f*(b+(-4*a*c+b^2)^(1/2))/c*(x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)-2*(-(
-4*a*c+b^2)^(1/2)*b*f+2*a*c*f-b^2*f-2*c^2*d)/c^2)^(1/2))/(x+1/2*(b+(-4*a*c+
b^2)^(1/2))/c))*A-1/c/(-2*(-(-4*a*c+b^2)^(1/2)*b*f+2*a*c*f-b^2*f-2*c^2*d)/c
^2)^(1/2)*ln((-(-(-4*a*c+b^2)^(1/2)*b*f+2*a*c*f-b^2*f-2*c^2*d)/c^2-f*(b+(-4
*a*c+b^2)^(1/2))/c*(x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)+1/2*(-2*(-(-4*a*c+b^2)^
(1/2)*b*f+2*a*c*f-b^2*f-2*c^2*d)/c^2)^(1/2)*(4*(x+1/2*(b+(-4*a*c+b^2)^(1/2)
)/c)^2*f-4*f*(b+(-4*a*c+b^2)^(1/2))/c*(x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)-2*(-
(-4*a*c+b^2)^(1/2)*b*f+2*a*c*f-b^2*f-2*c^2*d)/c^2)^(1/2))/(x+1/2*(b+(-4*a*c
+b^2)^(1/2))/c))*B-1/(-4*a*c+b^2)^(1/2)/c/(-2*(-(-4*a*c+b^2)^(1/2)*b*f+2*a*
c*f-b^2*f-2*c^2*d)/c^2)^(1/2)*ln((-(-(-4*a*c+b^2)^(1/2)*b*f+2*a*c*f-b^2*f-2
*c^2*d)/c^2-f*(b+(-4*a*c+b^2)^(1/2))/c*(x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)+1/2
```

```
*(-2*(-(-4*a*c+b^2)^(1/2)*b*f+2*a*c*f-b^2*f-2*c^2*d)/c^2)^(1/2)*(4*(x+1/2*(
b+(-4*a*c+b^2)^(1/2))/c)^2*f-4*f*(b+(-4*a*c+b^2)^(1/2))/c*(x+1/2*(b+(-4*a*c
+b^2)^(1/2))/c)-2*(-(-4*a*c+b^2)^(1/2)*b*f+2*a*c*f-b^2*f-2*c^2*d)/c^2)^(1/2
))/(x+1/2*(b+(-4*a*c+b^2)^(1/2))/c))*b*B-2/(-4*a*c+b^2)^(1/2)/(-2*((-4*a*c+
b^2)^(1/2)*b*f+2*a*c*f-b^2*f-2*c^2*d)/c^2)^(1/2)*ln((-((-4*a*c+b^2)^(1/2)*b
*f+2*a*c*f-b^2*f-2*c^2*d)/c^2-f*(b-(-4*a*c+b^2)^(1/2))/c*(x-1/2*(-b+(-4*a*c
+b^2)^(1/2))/c)+1/2*(-2*((-4*a*c+b^2)^(1/2)*b*f+2*a*c*f-b^2*f-2*c^2*d)/c^2)
^(1/2)*(4*(x-1/2*(-b+(-4*a*c+b^2)^(1/2))/c)^2*f-4*f*(b-(-4*a*c+b^2)^(1/2))/
c*(x-1/2*(-b+(-4*a*c+b^2)^(1/2))/c)-2*((-4*a*c+b^2)^(1/2)*b*f+2*a*c*f-b^2*f
-2*c^2*d)/c^2)^(1/2))/(x-1/2*(-b+(-4*a*c+b^2)^(1/2))/c))*A-1/c/(-2*((-4*a*c
+b^2)^(1/2)*b*f+2*a*c*f-b^2*f-2*c^2*d)/c^2)^(1/2)*ln((-((-4*a*c+b^2)^(1/2)*
b*f+2*a*c*f-b^2*f-2*c^2*d)/c^2-f*(b-(-4*a*c+b^2)^(1/2))/c*(x-1/2*(-b+(-4*a*
c+b^2)^(1/2))/c)+1/2*(-2*((-4*a*c+b^2)^(1/2)*b*f+2*a*c*f-b^2*f-2*c^2*d)/c^2
)^(1/2)*(4*(x-1/2*(-b+(-4*a*c+b^2)^(1/2))/c)^2*f-4*f*(b-(-4*a*c+b^2)^(1/2))
/c*(x-1/2*(-b+(-4*a*c+b^2)^(1/2))/c)-2*((-4*a*c+b^2)^(1/2)*b*f+2*a*c*f-b^2*
f-2*c^2*d)/c^2)^(1/2))/(x-1/2*(-b+(-4*a*c+b^2)^(1/2))/c))*B+1/(-4*a*c+b^2)^
(1/2)/c/(-2*((-4*a*c+b^2)^(1/2)*b*f+2*a*c*f-b^2*f-2*c^2*d)/c^2)^(1/2)*ln((-
((-4*a*c+b^2)^(1/2)*b*f+2*a*c*f-b^2*f-2*c^2*d)/c^2-f*(b-(-4*a*c+b^2)^(1/2))
/c*(x-1/2*(-b+(-4*a*c+b^2)^(1/2))/c)+1/2*(-2*((-4*a*c+b^2)^(1/2)*b*f+2*a*c*
f-b^2*f-2*c^2*d)/c^2)^(1/2)*(4*(x-1/2*(-b+(-4*a*c+b^2)^(1/2))/c)^2*f-4*f*(b
-(-4*a*c+b^2)^(1/2))/c*(x-1/2*(-b+(-4*a*c+b^2)^(1/2))/c)-2*((-4*a*c+b^2)^(1
/2)*b*f+2*a*c*f-b^2*f-2*c^2*d)/c^2)^(1/2))/(x-1/2*(-b+(-4*a*c+b^2)^(1/2))/c
))*b*B
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(c*x^2+b*x+a)/(f*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+B\,x}{\sqrt{f\,x^2+d}\,\left(c\,x^2+b\,x+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x)/((d + f*x^2)^(1/2)*(a + b*x + c*x^2)),x)
```

```
[Out] int((A + B*x)/((d + f*x^2)^(1/2)*(a + b*x + c*x^2)), x)
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A+Bx}{\sqrt{d+fx^2}\,\left(a+bx+cx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(c*x**2+b*x+a)/(f*x**2+d)**(1/2),x)
```

```
[Out] Integral((A + B*x)/(sqrt(d + f*x**2)*(a + b*x + c*x**2)), x)
```

3.24 $\int \frac{A+Bx}{(a+cx^2)\sqrt{d+fx^2}}\,dx$

Optimal. Leaf size=101

$$\frac{A\tan^{-1}\left(\frac{x\sqrt{cd-af}}{\sqrt{a}\sqrt{d+fx^2}}\right)}{\sqrt{a}\sqrt{cd-af}}-\frac{B\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{d+fx^2}}{\sqrt{cd-af}}\right)}{\sqrt{c}\sqrt{cd-af}}$$

```
[Out] A*arctan(x*(-a*f+c*d)^(1/2)/a^(1/2)/(f*x^2+d)^(1/2))/a^(1/2)/(-a*f+c*d)^(1/
2)-B*arctanh(c^(1/2)*(f*x^2+d)^(1/2)/(-a*f+c*d)^(1/2))/c^(1/2)/(-a*f+c*d)^(
1/2)
```

Rubi [A] time = 0.13, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1010, 377, 205, 444, 63, 208}

$$\frac{A\tan^{-1}\left(\frac{x\sqrt{cd-af}}{\sqrt{a}\sqrt{d+fx^2}}\right)}{\sqrt{a}\sqrt{cd-af}}-\frac{B\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{d+fx^2}}{\sqrt{cd-af}}\right)}{\sqrt{c}\sqrt{cd-af}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x)/((a + c*x^2)*Sqrt[d + f*x^2]),x]
```

```
[Out] (A*ArcTan[(Sqrt[c*d - a*f]*x)/(Sqrt[a]*Sqrt[d + f*x^2])])/(Sqrt[a]*Sqrt[c*d
 - a*f]) - (B*ArcTanh[(Sqrt[c]*Sqrt[d + f*x^2])/Sqrt[c*d - a*f]])/(Sqrt[c]*
Sqrt[c*d - a*f])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 1010

```
Int[((g_) + (h_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_)*((d_) + (f_.)*(x_)^2)^(q
_), x_Symbol] :> Dist[g, Int[(a + c*x^2)^p*(d + f*x^2)^q, x], x] + Dist[h,
Int[x*(a + c*x^2)^p*(d + f*x^2)^q, x], x] /; FreeQ[{a, c, d, f, g, h, p, q}
, x]
```

Rubi steps

$$\begin{aligned}\int \frac{A+Bx}{(a+cx^2)\sqrt{d+fx^2}}\,dx &= A\int \frac{1}{(a+cx^2)\sqrt{d+fx^2}}\,dx + B\int \frac{x}{(a+cx^2)\sqrt{d+fx^2}}\,dx\\ &= A\,\text{Subst}\left(\int \frac{1}{a-(-cd+af)x^2}\,dx, x, \frac{x}{\sqrt{d+fx^2}}\right) + \frac{1}{2}B\,\text{Subst}\left(\int \frac{1}{(a+cx)\sqrt{d+fx}}\,d\right.\\ &= \frac{A\tan^{-1}\left(\frac{\sqrt{cd-af}\,x}{\sqrt{a}\sqrt{d+fx^2}}\right)}{\sqrt{a}\sqrt{cd-af}} + \frac{B\,\text{Subst}\left(\int \frac{1}{a-\frac{cd}{f}+\frac{cx^2}{f}}\,dx, x, \sqrt{d+fx^2}\right)}{f}\\ &= \frac{A\tan^{-1}\left(\frac{\sqrt{cd-af}\,x}{\sqrt{a}\sqrt{d+fx^2}}\right)}{\sqrt{a}\sqrt{cd-af}} - \frac{B\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{d+fx^2}}{\sqrt{cd-af}}\right)}{\sqrt{c}\sqrt{cd-af}}\end{aligned}$$

Mathematica [A] time = 0.17, size = 154, normalized size = 1.52

$$\frac{\left(A\sqrt{c}-\sqrt{-a}B\right)\tanh^{-1}\left(\frac{\sqrt{c}d-\sqrt{-a}fx}{\sqrt{d+fx^2}\sqrt{cd-af}}\right)-\left(\sqrt{-a}B+A\sqrt{c}\right)\tanh^{-1}\left(\frac{\sqrt{-a}fx+\sqrt{c}d}{\sqrt{d+fx^2}\sqrt{cd-af}}\right)}{2\sqrt{-a}\sqrt{c}\sqrt{cd-af}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x)/((a + c*x^2)*Sqrt[d + f*x^2]),x]
```

```
[Out] ((-(Sqrt[-a]*B) + A*Sqrt[c])*ArcTanh[(Sqrt[c]*d - Sqrt[-a]*f*x)/(Sqrt[c*d -
 a*f]*Sqrt[d + f*x^2])] - (Sqrt[-a]*B + A*Sqrt[c])*ArcTanh[(Sqrt[c]*d + Sqr
t[-a]*f*x)/(Sqrt[c*d - a*f]*Sqrt[d + f*x^2])])/(2*Sqrt[-a]*Sqrt[c]*Sqrt[c*d
 - a*f])
```

fricas [B] time = 1.20, size = 1515, normalized size = 15.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(c*x^2+a)/(f*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt((B^2*a - A^2*c + 2*(a*c^2*d - a^2*c*f)*sqrt(-A^2*B^2/(a*c^3*d^2 -
 2*a^2*c^2*d*f + a^3*c*f^2)))/(a*c^2*d - a^2*c*f))*log(((A*B^3*a + A^3*B*c)
*f*x + (A^2*B*c^2*d - A^2*B*a*c*f + (B*a*c^3*d^2 - 2*B*a^2*c^2*d*f + B*a^3*
c*f^2)*sqrt(-A^2*B^2/(a*c^3*d^2 - 2*a^2*c^2*d*f + a^3*c*f^2)))*sqrt(f*x^2 +
 d)*sqrt((B^2*a - A^2*c + 2*(a*c^2*d - a^2*c*f)*sqrt(-A^2*B^2/(a*c^3*d^2 -
2*a^2*c^2*d*f + a^3*c*f^2)))/(a*c^2*d - a^2*c*f)) + sqrt(-A^2*B^2/(a*c^3*d^
2 - 2*a^2*c^2*d*f + a^3*c*f^2))*((B^2*a*c^2 + A^2*c^3)*d^2 - (B^2*a^2*c + A
^2*a*c^2)*d*f))/x) + 1/4*sqrt((B^2*a - A^2*c + 2*(a*c^2*d - a^2*c*f)*sqrt(-
A^2*B^2/(a*c^3*d^2 - 2*a^2*c^2*d*f + a^3*c*f^2)))/(a*c^2*d - a^2*c*f))*log(
((A*B^3*a + A^3*B*c)*f*x - (A^2*B*c^2*d - A^2*B*a*c*f + (B*a*c^3*d^2 - 2*B*
a^2*c^2*d*f + B*a^3*c*f^2)*sqrt(-A^2*B^2/(a*c^3*d^2 - 2*a^2*c^2*d*f + a^3*c
*f^2)))*sqrt(f*x^2 + d)*sqrt((B^2*a - A^2*c + 2*(a*c^2*d - a^2*c*f)*sqrt(-A
```

```
^2*B^2/(a*c^3*d^2 - 2*a^2*c^2*d*f + a^3*c*f^2)))/(a*c^2*d - a^2*c*f)) + sqr
t(-A^2*B^2/(a*c^3*d^2 - 2*a^2*c^2*d*f + a^3*c*f^2))*((B^2*a*c^2 + A^2*c^3)*
d^2 - (B^2*a^2*c + A^2*a*c^2)*d*f))/x) - 1/4*sqrt((B^2*a - A^2*c - 2*(a*c^2
*d - a^2*c*f)*sqrt(-A^2*B^2/(a*c^3*d^2 - 2*a^2*c^2*d*f + a^3*c*f^2)))/(a*c^
2*d - a^2*c*f))*log(((A*B^3*a + A^3*B*c)*f*x + (A^2*B*c^2*d - A^2*B*a*c*f -
 (B*a*c^3*d^2 - 2*B*a^2*c^2*d*f + B*a^3*c*f^2)*sqrt(-A^2*B^2/(a*c^3*d^2 - 2
*a^2*c^2*d*f + a^3*c*f^2)))*sqrt(f*x^2 + d)*sqrt((B^2*a - A^2*c - 2*(a*c^2*
d - a^2*c*f)*sqrt(-A^2*B^2/(a*c^3*d^2 - 2*a^2*c^2*d*f + a^3*c*f^2)))/(a*c^2
*d - a^2*c*f)) - sqrt(-A^2*B^2/(a*c^3*d^2 - 2*a^2*c^2*d*f + a^3*c*f^2))*((B
^2*a*c^2 + A^2*c^3)*d^2 - (B^2*a^2*c + A^2*a*c^2)*d*f))/x) + 1/4*sqrt((B^2*
a - A^2*c - 2*(a*c^2*d - a^2*c*f)*sqrt(-A^2*B^2/(a*c^3*d^2 - 2*a^2*c^2*d*f
+ a^3*c*f^2)))/(a*c^2*d - a^2*c*f))*log(((A*B^3*a + A^3*B*c)*f*x - (A^2*B*c
^2*d - A^2*B*a*c*f - (B*a*c^3*d^2 - 2*B*a^2*c^2*d*f + B*a^3*c*f^2)*sqrt(-A^
2*B^2/(a*c^3*d^2 - 2*a^2*c^2*d*f + a^3*c*f^2)))*sqrt(f*x^2 + d)*sqrt((B^2*a
 - A^2*c - 2*(a*c^2*d - a^2*c*f)*sqrt(-A^2*B^2/(a*c^3*d^2 - 2*a^2*c^2*d*f +
 a^3*c*f^2)))/(a*c^2*d - a^2*c*f)) - sqrt(-A^2*B^2/(a*c^3*d^2 - 2*a^2*c^2*d
*f + a^3*c*f^2))*((B^2*a*c^2 + A^2*c^3)*d^2 - (B^2*a^2*c + A^2*a*c^2)*d*f))
/x)
```

giac [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(c*x^2+a)/(f*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [**B**] time = 0.02, size = 608, normalized size = 6.02

$$-\frac{A\ln\left(\frac{\frac{2\sqrt{-ac}\left(x-\frac{\sqrt{-ac}}{c}\right)f}{c}-\frac{2(af-cd)}{c}+2\sqrt{-\frac{af-cd}{c}}\sqrt{\left(x-\frac{\sqrt{-ac}}{c}\right)^2 f+\frac{2\sqrt{-ac}\left(x-\frac{\sqrt{-ac}}{c}\right)f}{c}-\frac{af-cd}{c}}}{x-\frac{\sqrt{-ac}}{c}}\right)}{2\sqrt{-ac}\sqrt{-\frac{af-cd}{c}}}+\frac{A\ln\left(\frac{-\frac{2\sqrt{-ac}\left(x+\frac{\sqrt{-ac}}{c}\right)f}{c}-\frac{2(af-cd)}{c}+2\sqrt{-\frac{af-}{c}}\cdots}{x}\right)}{2\sqrt{-ac}\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x+A)/(c*x^2+a)/(f*x^2+d)^(1/2),x)
```

```
[Out] -1/2/(-a*c)^(1/2)/(-(a*f-c*d)/c)^(1/2)*ln((-2*(a*f-c*d)/c+2*f*(-a*c)^(1/2)/
c*(x-(-a*c)^(1/2)/c)+2*(-(a*f-c*d)/c)^(1/2)*((x-(-a*c)^(1/2)/c)^2*f+2*f*(-a
*c)^(1/2)/c*(x-(-a*c)^(1/2)/c)-(a*f-c*d)/c)^(1/2))/(x-(-a*c)^(1/2)/c))*A-1/
2/c/(-(a*f-c*d)/c)^(1/2)*ln((-2*(a*f-c*d)/c+2*f*(-a*c)^(1/2)/c*(x-(-a*c)^(1
/2)/c)+2*(-(a*f-c*d)/c)^(1/2)*((x-(-a*c)^(1/2)/c)^2*f+2*f*(-a*c)^(1/2)/c*(x
-(-a*c)^(1/2)/c)-(a*f-c*d)/c)^(1/2))/(x-(-a*c)^(1/2)/c))*B+1/2/(-a*c)^(1/2)
/(-(a*f-c*d)/c)^(1/2)*ln((-2*(a*f-c*d)/c-2*f*(-a*c)^(1/2)/c*(x+(-a*c)^(1/2)
/c)+2*(-(a*f-c*d)/c)^(1/2)*((x+(-a*c)^(1/2)/c)^2*f-2*f*(-a*c)^(1/2)/c*(x+(-
a*c)^(1/2)/c)-(a*f-c*d)/c)^(1/2))/(x+(-a*c)^(1/2)/c))*A-1/2/c/(-(a*f-c*d)/c
)^(1/2)*ln((-2*(a*f-c*d)/c-2*f*(-a*c)^(1/2)/c*(x+(-a*c)^(1/2)/c)+2*(-(a*f-c
*d)/c)^(1/2)*((x+(-a*c)^(1/2)/c)^2*f-2*f*(-a*c)^(1/2)/c*(x+(-a*c)^(1/2)/c)-
(a*f-c*d)/c)^(1/2))/(x+(-a*c)^(1/2)/c))*B
```

maxima [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Bx + A}{\left(cx^2 + a\right)\sqrt{fx^2 + d}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(c*x^2+a)/(f*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x + A)/((c*x^2 + a)*sqrt(f*x^2 + d)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\begin{cases} \frac{B\,\mathrm{atan}\left(\frac{c\,\sqrt{f\,x^2+d}}{\sqrt{a\,c\,f-c^2\,d}}\right)}{\sqrt{a\,c\,f-c^2\,d}}+\frac{A\,\mathrm{atan}\left(\frac{x\,\sqrt{c\,d-a\,f}}{\sqrt{a}\,\sqrt{f\,x^2+d}}\right)}{\sqrt{-a\,\left(a\,f-c\,d\right)}} & \text{if } 0<c\,d-a\,f \\ \frac{A\,\ln\left(\frac{\sqrt{a\,\left(f\,x^2+d\right)}+x\,\sqrt{a\,f-c\,d}}{\sqrt{a\,\left(f\,x^2+d\right)}-x\,\sqrt{a\,f-c\,d}}\right)}{2\,\sqrt{a\,\left(a\,f-c\,d\right)}}+\frac{B\,\mathrm{atan}\left(\frac{c\,\sqrt{f\,x^2+d}}{\sqrt{a\,c\,f-c^2\,d}}\right)}{\sqrt{a\,c\,f-c^2\,d}} & \text{if } c\,d-a\,f<0 \\ \int \frac{A+B\,x}{\left(c\,x^2+a\right)\,\sqrt{f\,x^2+d}}\,dx & \text{if } c\,d-a\,f\notin \mathbb{R}\vee a\,f=c\,d \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x)/((a + c*x^2)*(d + f*x^2)^(1/2)),x)
```

```
[Out] piecewise(0 < - a*f + c*d, (B*atan((c*(d + f*x^2)^(1/2))/(- c^2*d + a*c*f)^
(1/2)))/(- c^2*d + a*c*f)^(1/2) + (A*atan((x*(- a*f + c*d)^(1/2))/(a^(1/2)*
(d + f*x^2)^(1/2))))/(-a*(a*f - c*d))^(1/2), - a*f + c*d < 0, (A*log(((a*(d
 + f*x^2))^(1/2) + x*(a*f - c*d)^(1/2))/((a*(d + f*x^2))^(1/2) - x*(a*f - c
*d)^(1/2))))/(2*(a*(a*f - c*d))^(1/2)) + (B*atan((c*(d + f*x^2)^(1/2))/(- c
^2*d + a*c*f)^(1/2)))/(- c^2*d + a*c*f)^(1/2), ~in(- a*f + c*d, 'real') | a
*f == c*d, int((A + B*x)/((a + c*x^2)*(d + f*x^2)^(1/2)), x))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + Bx}{\left(a + cx^{2}\right)\sqrt{d + fx^{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(c*x**2+a)/(f*x**2+d)**(1/2),x)
```

```
[Out] Integral((A + B*x)/((a + c*x**2)*sqrt(d + f*x**2)), x)
```

3.25 $\int \frac{2+x}{\left(2+4x-3x^2\right)\sqrt{1+3x-2x^2}}\,dx$

Optimal. Leaf size=139

$$\frac{1}{2}\sqrt{\sqrt{10}-\frac{13}{5}}\ \tan^{-1}\left(\frac{\left(1+4\sqrt{10}\right)x+3\left(4-\sqrt{10}\right)}{2\sqrt{1+\sqrt{10}}\ \sqrt{-2x^2+3x+1}}\right)+\frac{1}{2}\sqrt{\frac{13}{5}+\sqrt{10}}\ \tanh^{-1}\left(\frac{\left(1-4\sqrt{10}\right)x+3\left(4+\sqrt{10}\right)}{2\sqrt{\sqrt{10}-1}\ \sqrt{-2x^2+3x+1}}\right)$$

```
[Out] 1/10*arctan(1/2*(12-3*10^(1/2)+x*(1+4*10^(1/2)))/(-2*x^2+3*x+1)^(1/2)/(1+10
^(1/2))^(1/2))*(-65+25*10^(1/2))^(1/2)+1/10*arctanh(1/2*(x*(1-4*10^(1/2))+1
2+3*10^(1/2))/(-2*x^2+3*x+1)^(1/2)/(-1+10^(1/2))^(1/2))*(65+25*10^(1/2))^(1
/2)
```

Rubi [A] time = 0.22, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1032, 724, 204, 206}

$$\frac{1}{2}\sqrt{\sqrt{10}-\frac{13}{5}}\ \tan^{-1}\left(\frac{\left(1+4\sqrt{10}\right)x+3\left(4-\sqrt{10}\right)}{2\sqrt{1+\sqrt{10}}\ \sqrt{-2x^2+3x+1}}\right)+\frac{1}{2}\sqrt{\frac{13}{5}+\sqrt{10}}\ \tanh^{-1}\left(\frac{\left(1-4\sqrt{10}\right)x+3\left(4+\sqrt{10}\right)}{2\sqrt{\sqrt{10}-1}\ \sqrt{-2x^2+3x+1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x)/((2 + 4*x - 3*x^2)*Sqrt[1 + 3*x - 2*x^2]),x]
```

```
[Out] (Sqrt[-13/5 + Sqrt[10]]*ArcTan[(3*(4 - Sqrt[10]) + (1 + 4*Sqrt[10])*x)/(2*S
qrt[1 + Sqrt[10]]*Sqrt[1 + 3*x - 2*x^2])])/2 + (Sqrt[13/5 + Sqrt[10]]*ArcTa
nh[(3*(4 + Sqrt[10]) + (1 - 4*Sqrt[10])*x)/(2*Sqrt[-1 + Sqrt[10]]*Sqrt[1 +
3*x - 2*x^2])])/2
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1032

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{2+x}{\left(2+4x-3x^2\right)\sqrt{1+3x-2x^2}}\,dx = \frac{1}{5}\left(5-4\sqrt{10}\right)\int \frac{1}{\left(4-2\sqrt{10}-6x\right)\sqrt{1+3x-2x^2}}\,dx + \frac{1}{5}\left(5+4\sqrt{10}\right)\int$$

$$= -\left(\frac{1}{5}\left(2\left(5-4\sqrt{10}\right)\right)\text{Subst}\left(\int \frac{1}{144+72\left(4-2\sqrt{10}\right)-8\left(4-2\sqrt{10}\right)^2-x\right.\right.$$

$$= \frac{1}{10}\sqrt{-65+25\sqrt{10}}\,\tan^{-1}\left(\frac{3\left(4-\sqrt{10}\right)+\left(1+4\sqrt{10}\right)x}{2\sqrt{1+\sqrt{10}}\sqrt{1+3x-2x^2}}\right)+\frac{1}{10}\sqrt{65+25\sqrt{}}$$

Mathematica [A] time = 0.30, size = 140, normalized size = 1.01

$$\frac{\left(4\sqrt{10}-5\right)\tan^{-1}\left(\frac{4\sqrt{10}x+x-3\sqrt{10}+12}{2\sqrt{1+\sqrt{10}}\sqrt{-2x^2+3x+1}}\right)+3\sqrt{5\left(7+2\sqrt{10}\right)}\tanh^{-1}\left(\frac{-4\sqrt{10}x+x+3\left(4+\sqrt{10}\right)}{2\sqrt{\sqrt{10}-1}\sqrt{-2x^2+3x+1}}\right)}{10\sqrt{1+\sqrt{10}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(2 + x)/((2 + 4*x - 3*x^2)*Sqrt[1 + 3*x - 2*x^2]),x]
```

```
[Out] ((-5 + 4*Sqrt[10])*ArcTan[(12 - 3*Sqrt[10] + x + 4*Sqrt[10]*x)/(2*Sqrt[1 +
Sqrt[10]]*Sqrt[1 + 3*x - 2*x^2])] + 3*Sqrt[5*(7 + 2*Sqrt[10])]*ArcTanh[(3*(
4 + Sqrt[10]) + x - 4*Sqrt[10]*x)/(2*Sqrt[-1 + Sqrt[10]]*Sqrt[1 + 3*x - 2*x
^2])])/(10*Sqrt[1 + Sqrt[10]])
```

fricas [B] time = 0.93, size = 322, normalized size = 2.32

$$\frac{2}{5}\sqrt{5}\sqrt{5\sqrt{5}\sqrt{2}-13}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{5}x-\sqrt{2}x\right)\sqrt{5\sqrt{5}\sqrt{2}-13}\sqrt{\frac{\sqrt{5}\sqrt{2}\left(3x^2+2x\right)+6x^2-2\left(\sqrt{5}\sqrt{2}x+2x+2\right)\sqrt{-2x^2+3x+1}}{x^2}}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(-3*x^2+4*x+2)/(-2*x^2+3*x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/5*sqrt(5)*sqrt(5*sqrt(5)*sqrt(2) - 13)*arctan(1/18*(sqrt(2)*(2*sqrt(5)*x
- sqrt(2)*x)*sqrt(5*sqrt(5)*sqrt(2) - 13)*sqrt((sqrt(5)*sqrt(2)*(3*x^2 + 2*
x) + 6*x^2 - 2*(sqrt(5)*sqrt(2)*x + 2*x + 2)*sqrt(-2*x^2 + 3*x + 1) + 10*x
+ 4)/x^2) + 2*(sqrt(2)*(4*x - 1) + sqrt(5)*(x + 2) - sqrt(-2*x^2 + 3*x + 1)
*(2*sqrt(5) - sqrt(2)))*sqrt(5*sqrt(5)*sqrt(2) - 13))/x) - 1/10*sqrt(5)*sqr
t(5*sqrt(5)*sqrt(2) + 13)*log((9*sqrt(5)*sqrt(2)*x + (4*sqrt(5)*x - 7*sqrt(
2)*x)*sqrt(5*sqrt(5)*sqrt(2) + 13) - 18*x + 18*sqrt(-2*x^2 + 3*x + 1) - 18)
/x) + 1/10*sqrt(5)*sqrt(5*sqrt(5)*sqrt(2) + 13)*log((9*sqrt(5)*sqrt(2)*x -
(4*sqrt(5)*x - 7*sqrt(2)*x)*sqrt(5*sqrt(5)*sqrt(2) + 13) - 18*x + 18*sqrt(-
2*x^2 + 3*x + 1) - 18)/x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(-3*x^2+4*x+2)/(-2*x^2+3*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.07, size = 324, normalized size = 2.33

$$\frac{2\sqrt{10}\,\operatorname{arctanh}\left(\frac{-1+\sqrt{10}+\frac{9\left(\frac{1}{3}-\frac{4\sqrt{10}}{3}\right)\left(x-\frac{2}{3}-\frac{\sqrt{10}}{3}\right)}{2}}{\sqrt{-1+\sqrt{10}}\,\sqrt{-18\left(x-\frac{2}{3}-\frac{\sqrt{10}}{3}\right)^{2}+9\left(\frac{1}{3}-\frac{4\sqrt{10}}{3}\right)\left(x-\frac{2}{3}-\frac{\sqrt{10}}{3}\right)-1+\sqrt{10}}}\right)}{5\sqrt{-1+\sqrt{10}}}+\frac{\operatorname{arctanh}\left(\frac{-1+\sqrt{10}+\frac{9\left(\frac{1}{3}-\frac{4\sqrt{1}}{3}\right.}{}}{\sqrt{-1+\sqrt{10}}\,\sqrt{-18\left(x-\frac{2}{3}-\frac{\sqrt{10}}{3}\right)^{2}+9\left(\right.}}\right.}{2\sqrt{-1+\sqrt{10}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2+x)/(-3*x^2+4*x+2)/(-2*x^2+3*x+1)^(1/2),x)
```

```
[Out] 2/5*10^(1/2)/(1+10^(1/2))^(1/2)*arctan(9/2*(-2/9-2/9*10^(1/2)+(1/3+4/3*10^(
1/2))*(x-2/3+1/3*10^(1/2)))/(1+10^(1/2))^(1/2)/(-18*(x-2/3+1/3*10^(1/2))^2+
9*(1/3+4/3*10^(1/2))*(x-2/3+1/3*10^(1/2))-1-10^(1/2))^(1/2))-1/2/(1+10^(1/2
))^(1/2)*arctan(9/2*(-2/9-2/9*10^(1/2)+(1/3+4/3*10^(1/2))*(x-2/3+1/3*10^(1/
2)))/(1+10^(1/2))^(1/2)/(-18*(x-2/3+1/3*10^(1/2))^2+9*(1/3+4/3*10^(1/2))*(x
-2/3+1/3*10^(1/2))-1-10^(1/2))^(1/2))+2/5*10^(1/2)/(-1+10^(1/2))^(1/2)*arct
anh(9/2*(-2/9+2/9*10^(1/2)+(1/3-4/3*10^(1/2))*(x-2/3-1/3*10^(1/2)))/(-1+10^
(1/2))^(1/2)/(-18*(x-2/3-1/3*10^(1/2))^2+9*(1/3-4/3*10^(1/2))*(x-2/3-1/3*10
^(1/2))-1+10^(1/2))^(1/2))+1/2/(-1+10^(1/2))^(1/2)*arctanh(9/2*(-2/9+2/9*10
^(1/2)+(1/3-4/3*10^(1/2))*(x-2/3-1/3*10^(1/2)))/(-1+10^(1/2))^(1/2)/(-18*(x
-2/3-1/3*10^(1/2))^2+9*(1/3-4/3*10^(1/2))*(x-2/3-1/3*10^(1/2))-1+10^(1/2))^
(1/2))
```

maxima [B] time = 1.07, size = 361, normalized size = 2.60

$$-\frac{1}{20}\sqrt{10}\left(\frac{\sqrt{10}\arcsin\left(\frac{8\sqrt{17}\sqrt{10}x}{17\left|6x+2\sqrt{10}-4\right|}+\frac{2\sqrt{17}x}{17\left|6x+2\sqrt{10}-4\right|}-\frac{6\sqrt{17}\sqrt{10}}{17\left|6x+2\sqrt{10}-4\right|}+\frac{24\sqrt{17}}{17\left|6x+2\sqrt{10}-4\right|}\right)}{\sqrt{\sqrt{10}+1}}-\frac{\sqrt{10}\log\left(-\frac{2}{9}\sqrt{10}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(-3*x^2+4*x+2)/(-2*x^2+3*x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/20*sqrt(10)*(sqrt(10)*arcsin(8/17*sqrt(17)*sqrt(10)*x/abs(6*x + 2*sqrt(1
0) - 4) + 2/17*sqrt(17)*x/abs(6*x + 2*sqrt(10) - 4) - 6/17*sqrt(17)*sqrt(10
)/abs(6*x + 2*sqrt(10) - 4) + 24/17*sqrt(17)/abs(6*x + 2*sqrt(10) - 4))/sqr
t(sqrt(10) + 1) - sqrt(10)*log(-2/9*sqrt(10) + 2/3*sqrt(-2*x^2 + 3*x + 1)*s
qrt(sqrt(10) - 1)/abs(6*x - 2*sqrt(10) - 4) + 2/9*sqrt(10)/abs(6*x - 2*sqrt
(10) - 4) - 2/9/abs(6*x - 2*sqrt(10) - 4) + 1/18)/sqrt(sqrt(10) - 1) - 8*ar
csin(8/17*sqrt(17)*sqrt(10)*x/abs(6*x + 2*sqrt(10) - 4) + 2/17*sqrt(17)*x/a
bs(6*x + 2*sqrt(10) - 4) - 6/17*sqrt(17)*sqrt(10)/abs(6*x + 2*sqrt(10) - 4)
 + 24/17*sqrt(17)/abs(6*x + 2*sqrt(10) - 4))/sqrt(sqrt(10) + 1) - 8*log(-2/
9*sqrt(10) + 2/3*sqrt(-2*x^2 + 3*x + 1)*sqrt(sqrt(10) - 1)/abs(6*x - 2*sqrt
(10) - 4) + 2/9*sqrt(10)/abs(6*x - 2*sqrt(10) - 4) - 2/9/abs(6*x - 2*sqrt(1
0) - 4) + 1/18)/sqrt(sqrt(10) - 1))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x+2}{\sqrt{-2\,x^2+3\,x+1}\,\left(-3\,x^2+4\,x+2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 2)/((3*x - 2*x^2 + 1)^(1/2)*(4*x - 3*x^2 + 2)),x)
```

```
[Out] int((x + 2)/((3*x - 2*x^2 + 1)^(1/2)*(4*x - 3*x^2 + 2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{3x^2\sqrt{-2x^2+3x+1}-4x\sqrt{-2x^2+3x+1}-2\sqrt{-2x^2+3x+1}}\,dx-\int \frac{2}{3x^2\sqrt{-2x^2+3x+1}-4x\sqrt{-2x^2+3x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(-3*x**2+4*x+2)/(-2*x**2+3*x+1)**(1/2),x)
```

```
[Out] -Integral(x/(3*x**2*sqrt(-2*x**2 + 3*x + 1) - 4*x*sqrt(-2*x**2 + 3*x + 1) -
 2*sqrt(-2*x**2 + 3*x + 1)), x) - Integral(2/(3*x**2*sqrt(-2*x**2 + 3*x + 1
) - 4*x*sqrt(-2*x**2 + 3*x + 1) - 2*sqrt(-2*x**2 + 3*x + 1)), x)
```

3.26 $\int \frac{2+x}{\left(2+4x-3x^2\right)\left(1+3x-2x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=166

$$-\frac{2(14x+15)}{17\sqrt{-2x^2+3x+1}}-\frac{9}{2}\sqrt{\frac{1}{5}\left(\sqrt{10}-3\right)}\tan^{-1}\left(\frac{\left(1+4\sqrt{10}\right)x+3\left(4-\sqrt{10}\right)}{2\sqrt{1+\sqrt{10}}\sqrt{-2x^2+3x+1}}\right)+\frac{9}{2}\sqrt{\frac{1}{5}\left(3+\sqrt{10}\right)}\tanh^{-1}\left(\frac{(1-}{2\sqrt{}}\right.$$

```
[Out] -2/17*(15+14*x)/(-2*x^2+3*x+1)^(1/2)-9/10*arctan(1/2*(12-3*10^(1/2)+x*(1+4*
10^(1/2)))/(-2*x^2+3*x+1)^(1/2)/(1+10^(1/2))^(1/2))*(-15+5*10^(1/2))^(1/2)+
9/10*arctanh(1/2*(x*(1-4*10^(1/2))+12+3*10^(1/2))/(-2*x^2+3*x+1)^(1/2)/(-1+
10^(1/2))^(1/2))*(15+5*10^(1/2))^(1/2)
```

Rubi [A] time = 0.22, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1016, 12, 1032, 724, 204, 206}

$$-\frac{2(14x+15)}{17\sqrt{-2x^2+3x+1}}-\frac{9}{2}\sqrt{\frac{1}{5}\left(\sqrt{10}-3\right)}\tan^{-1}\left(\frac{\left(1+4\sqrt{10}\right)x+3\left(4-\sqrt{10}\right)}{2\sqrt{1+\sqrt{10}}\sqrt{-2x^2+3x+1}}\right)+\frac{9}{2}\sqrt{\frac{1}{5}\left(3+\sqrt{10}\right)}\tanh^{-1}\left(\frac{(1-}{2\sqrt{}}\right.$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x)/((2 + 4*x - 3*x^2)*(1 + 3*x - 2*x^2)^(3/2)),x]
```

```
[Out] (-2*(15 + 14*x))/(17*Sqrt[1 + 3*x - 2*x^2]) - (9*Sqrt[(-3 + Sqrt[10])/5]*Ar
cTan[(3*(4 - Sqrt[10]) + (1 + 4*Sqrt[10])*x)/(2*Sqrt[1 + Sqrt[10]]*Sqrt[1 +
 3*x - 2*x^2])])/2 + (9*Sqrt[(3 + Sqrt[10])/5]*ArcTanh[(3*(4 + Sqrt[10]) +
(1 - 4*Sqrt[10])*x)/(2*Sqrt[-1 + Sqrt[10]]*Sqrt[1 + 3*x - 2*x^2])])/2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1016

```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e
_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((a + b*x + c*x^2)^(p + 1)
*(d + e*x + f*x^2)^(q + 1)*(g*c*(2*a*c*e - b*(c*d + a*f)) + (g*b - a*h)*(2*
```

```
c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(g*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))
 - h*(b*c*d - 2*a*c*e + a*b*f))*x))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d -
a*e)*(c*e - b*f))*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*
d - a*e)*(c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*
x^2)^q*Simp[(b*h - 2*g*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)
 + (b^2*(g*f) - b*(h*c*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) - a*(-(h*c*e
))))*(a*f*(p + 1) - c*d*(p + 2)) - e*((g*c)*(2*a*c*e - b*(c*d + a*f)) + (g*
b - a*h)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (2*f*((g*c)*(2*
a*c*e - b*(c*d + a*f)) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(
p + q + 2) - (b^2*g*f - b*(h*c*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) - a*
(-(h*c*e))))*(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(g*f) - b*(h*c
*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) + a*h*c*e))*(2*p + 2*q + 5)*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, q}, x] && NeQ[b^2 - 4*a*c, 0] &
& NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (b*d - a*e)*(c*e
 - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1])
```

Rule 1032

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{2+x}{\left(2+4x-3x^2\right)\left(1+3x-2x^2\right)^{3/2}}\,dx = -\frac{2(15+14x)}{17\sqrt{1+3x-2x^2}}+\frac{2}{17}\int \frac{153x}{2\left(2+4x-3x^2\right)\sqrt{1+3x-2x^2}}\,dx$$

$$= -\frac{2(15+14x)}{17\sqrt{1+3x-2x^2}}+9\int \frac{x}{\left(2+4x-3x^2\right)\sqrt{1+3x-2x^2}}\,dx$$

$$= -\frac{2(15+14x)}{17\sqrt{1+3x-2x^2}}+\frac{1}{5}\left(9\left(5-\sqrt{10}\right)\right)\int \frac{1}{\left(4-2\sqrt{10}-6x\right)\sqrt{1+3x-2}}$$

$$= -\frac{2(15+14x)}{17\sqrt{1+3x-2x^2}}-\frac{1}{5}\left(18\left(5-\sqrt{10}\right)\right)\text{Subst}\left(\int \frac{}{144+72\left(4-2\sqrt{10}\right)}\right.$$

$$= -\frac{2(15+14x)}{17\sqrt{1+3x-2x^2}}-\frac{9}{2}\sqrt{\frac{1}{5}\left(-3+\sqrt{10}\right)}\tan^{-1}\left(\frac{3\left(4-\sqrt{10}\right)+\left(1+4\sqrt{}\right.}{2\sqrt{1+\sqrt{10}}\sqrt{1+3x-}}\right.$$

Mathematica [A] time = 0.38, size = 167, normalized size = 1.01

$$\frac{1}{170}\left(153\sqrt{5\left(3+\sqrt{10}\right)}\tanh^{-1}\left(\frac{-4\sqrt{10}x+x+3\left(4+\sqrt{10}\right)}{2\sqrt{\sqrt{10}-1}\sqrt{-2x^2+3x+1}}\right)-\frac{153\sqrt{5\left(\sqrt{10}-3\right)}\sqrt{-2x^2+3x+1}\tan^{-1}\left(\frac{4\sqrt{}}{2\sqrt{1}}\right.}{\sqrt{-2x^2+3x+1}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x)/((2 + 4*x - 3*x^2)*(1 + 3*x - 2*x^2)^(3/2)),x]
```

```
[Out] (-((300 + 280*x + 153*Sqrt[5*(-3 + Sqrt[10])]*Sqrt[1 + 3*x - 2*x^2]*ArcTan[
(12 - 3*Sqrt[10] + x + 4*Sqrt[10]*x)/(2*Sqrt[1 + Sqrt[10]]*Sqrt[1 + 3*x - 2
```

```
*x^2])])/Sqrt[1 + 3*x - 2*x^2]) + 153*Sqrt[5*(3 + Sqrt[10])]*ArcTanh[(3*(4
+ Sqrt[10]) + x - 4*Sqrt[10]*x)/(2*Sqrt[-1 + Sqrt[10]]*Sqrt[1 + 3*x - 2*x^2
])])])/170
```

fricas [B] time = 0.90, size = 344, normalized size = 2.07

$$-\frac{612\sqrt{5}\left(2x^2-3x-1\right)\sqrt{\sqrt{10}-3}\arctan\left(\frac{\sqrt{10}\sqrt{5}\sqrt{2}x\sqrt{\sqrt{10}-3}\sqrt{\frac{6x^2+\sqrt{10}\left(3x^2+2x\right)-2\sqrt{-2x^2+3x+1}\left(\sqrt{10}x+2x+2\right)+10x+4}{x^2}}+2\left(\sqrt{1\cdots\right.}{10x}\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(-3*x^2+4*x+2)/(-2*x^2+3*x+1)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/170*(612*sqrt(5)*(2*x^2 - 3*x - 1)*sqrt(sqrt(10) - 3)*arctan(1/10*(sqrt(
10)*sqrt(5)*sqrt(2)*x*sqrt(sqrt(10) - 3)*sqrt((6*x^2 + sqrt(10)*(3*x^2 + 2*
x) - 2*sqrt(-2*x^2 + 3*x + 1)*(sqrt(10)*x + 2*x + 2) + 10*x + 4)/x^2) + 2*(
sqrt(10)*sqrt(5)*(x + 1) - sqrt(10)*sqrt(5)*sqrt(-2*x^2 + 3*x + 1) + 5*sqrt
(5)*x)*sqrt(sqrt(10) - 3))/x) + 153*sqrt(5)*(2*x^2 - 3*x - 1)*sqrt(sqrt(10)
 + 3)*log(9*(5*sqrt(10)*x + (3*sqrt(10)*sqrt(5)*x - 10*sqrt(5)*x)*sqrt(sqrt
(10) + 3) - 10*x + 10*sqrt(-2*x^2 + 3*x + 1) - 10)/x) - 153*sqrt(5)*(2*x^2
- 3*x - 1)*sqrt(sqrt(10) + 3)*log(9*(5*sqrt(10)*x - (3*sqrt(10)*sqrt(5)*x -
 10*sqrt(5)*x)*sqrt(sqrt(10) + 3) - 10*x + 10*sqrt(-2*x^2 + 3*x + 1) - 10)/
x) + 600*x^2 - 20*sqrt(-2*x^2 + 3*x + 1)*(14*x + 15) - 900*x - 300)/(2*x^2
- 3*x - 1)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(-3*x^2+4*x+2)/(-2*x^2+3*x+1)^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.03, size = 760, normalized size = 4.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2+x)/(-3*x^2+4*x+2)/(-2*x^2+3*x+1)^(3/2),x)
```

```
[Out] 26/255*10^(1/2)/(-1/9-1/9*10^(1/2))/(-2*(x-2/3+1/3*10^(1/2))^2+(1/3+4/3*10^
(1/2))*(x-2/3+1/3*10^(1/2))-1/9-1/9*10^(1/2))^(1/2)+32/765/(-1/9-1/9*10^(1/
2))/(-2*(x-2/3+1/3*10^(1/2))^2+(1/3+4/3*10^(1/2))*(x-2/3+1/3*10^(1/2))-1/9-
1/9*10^(1/2))^(1/2)*10^(1/2)*x-62/153/(-1/9-1/9*10^(1/2))/(-2*(x-2/3+1/3*10
^(1/2))^2+(1/3+4/3*10^(1/2))*(x-2/3+1/3*10^(1/2))-1/9-1/9*10^(1/2))^(1/2)*x
+7/51/(-1/9-1/9*10^(1/2))/(-2*(x-2/3+1/3*10^(1/2))^2+(1/3+4/3*10^(1/2))*(x-
2/3+1/3*10^(1/2))-1/9-1/9*10^(1/2))^(1/2)+2/5*10^(1/2)/(-1/9-1/9*10^(1/2))/
(1+10^(1/2))^(1/2)*arctan(9/2*(-2/9-2/9*10^(1/2)+(1/3+4/3*10^(1/2))*(x-2/3+
1/3*10^(1/2)))/(1+10^(1/2))^(1/2)/(-18*(x-2/3+1/3*10^(1/2))^2+9*(1/3+4/3*10
^(1/2))*(x-2/3+1/3*10^(1/2))-1-10^(1/2))^(1/2))-1/2/(-1/9-1/9*10^(1/2))/(1+
10^(1/2))^(1/2)*arctan(9/2*(-2/9-2/9*10^(1/2)+(1/3+4/3*10^(1/2))*(x-2/3+1/3
*10^(1/2)))/(1+10^(1/2))^(1/2)/(-18*(x-2/3+1/3*10^(1/2))^2+9*(1/3+4/3*10^(1
/2))*(x-2/3+1/3*10^(1/2))-1-10^(1/2))^(1/2))-26/255*10^(1/2)/(-1/9+1/9*10^(
1/2))/(-2*(x-2/3-1/3*10^(1/2))^2+(1/3-4/3*10^(1/2))*(x-2/3-1/3*10^(1/2))-1/
9+1/9*10^(1/2))^(1/2)-32/765/(-1/9+1/9*10^(1/2))/(-2*(x-2/3-1/3*10^(1/2))^2
+(1/3-4/3*10^(1/2))*(x-2/3-1/3*10^(1/2))-1/9+1/9*10^(1/2))^(1/2)*10^(1/2)*x
```

```
-62/153/(-1/9+1/9*10^(1/2))/(-2*(x-2/3-1/3*10^(1/2))^2+(1/3-4/3*10^(1/2))*(
x-2/3-1/3*10^(1/2))-1/9+1/9*10^(1/2))^(1/2)*x+7/51/(-1/9+1/9*10^(1/2))/(-2*
(x-2/3-1/3*10^(1/2))^2+(1/3-4/3*10^(1/2))*(x-2/3-1/3*10^(1/2))-1/9+1/9*10^(
1/2))^(1/2)+2/5*10^(1/2)/(-1/9+1/9*10^(1/2))/(-1+10^(1/2))^(1/2)*arctanh(9/
2*(-2/9+2/9*10^(1/2)+(1/3-4/3*10^(1/2))*(x-2/3-1/3*10^(1/2)))/(-1+10^(1/2))
^(1/2)/(-18*(x-2/3-1/3*10^(1/2))^2+9*(1/3-4/3*10^(1/2))*(x-2/3-1/3*10^(1/2)
)-1+10^(1/2))^(1/2))+1/2/(-1/9+1/9*10^(1/2))/(-1+10^(1/2))^(1/2)*arctanh(9/
2*(-2/9+2/9*10^(1/2)+(1/3-4/3*10^(1/2))*(x-2/3-1/3*10^(1/2)))/(-1+10^(1/2))
^(1/2)/(-18*(x-2/3-1/3*10^(1/2))^2+9*(1/3-4/3*10^(1/2))*(x-2/3-1/3*10^(1/2)
)-1+10^(1/2))^(1/2))
```

maxima [B] time = 1.11, size = 678, normalized size = 4.08

$$\frac{1}{340}\sqrt{10}\left(\frac{124\sqrt{10}\,x}{\sqrt{10}\sqrt{-2\,x^2+3\,x+1}+\sqrt{-2\,x^2+3\,x+1}}-\frac{124\sqrt{10}\,x}{\sqrt{10}\sqrt{-2\,x^2+3\,x+1}-\sqrt{-2\,x^2+3\,x+1}}+\frac{153\sqrt{10}\,\mathrm{arc}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(-3*x^2+4*x+2)/(-2*x^2+3*x+1)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/340*sqrt(10)*(124*sqrt(10)*x/(sqrt(10)*sqrt(-2*x^2 + 3*x + 1) + sqrt(-2*x
^2 + 3*x + 1)) - 124*sqrt(10)*x/(sqrt(10)*sqrt(-2*x^2 + 3*x + 1) - sqrt(-2*
x^2 + 3*x + 1)) + 153*sqrt(10)*arcsin(8/17*sqrt(17)*sqrt(10)*x/abs(6*x + 2*
sqrt(10) - 4) + 2/17*sqrt(17)*x/abs(6*x + 2*sqrt(10) - 4) - 6/17*sqrt(17)*s
qrt(10)/abs(6*x + 2*sqrt(10) - 4) + 24/17*sqrt(17)/abs(6*x + 2*sqrt(10) - 4
))/(sqrt(10)*sqrt(sqrt(10) + 1) + sqrt(sqrt(10) + 1)) - 128*x/(sqrt(10)*sqr
t(-2*x^2 + 3*x + 1) + sqrt(-2*x^2 + 3*x + 1)) - 128*x/(sqrt(10)*sqrt(-2*x^2
 + 3*x + 1) - sqrt(-2*x^2 + 3*x + 1)) - 1224*arcsin(8/17*sqrt(17)*sqrt(10)*
x/abs(6*x + 2*sqrt(10) - 4) + 2/17*sqrt(17)*x/abs(6*x + 2*sqrt(10) - 4) - 6
/17*sqrt(17)*sqrt(10)/abs(6*x + 2*sqrt(10) - 4) + 24/17*sqrt(17)/abs(6*x +
2*sqrt(10) - 4))/(sqrt(10)*sqrt(sqrt(10) + 1) + sqrt(sqrt(10) + 1)) + 153*s
qrt(10)*log(-2/9*sqrt(10) + 2/3*sqrt(-2*x^2 + 3*x + 1)*sqrt(sqrt(10) - 1)/a
bs(6*x - 2*sqrt(10) - 4) + 2/9*sqrt(10)/abs(6*x - 2*sqrt(10) - 4) - 2/9/abs
(6*x - 2*sqrt(10) - 4) + 1/18)/(sqrt(10) - 1)^(3/2) - 42*sqrt(10)/(sqrt(10)
*sqrt(-2*x^2 + 3*x + 1) + sqrt(-2*x^2 + 3*x + 1)) + 42*sqrt(10)/(sqrt(10)*s
qrt(-2*x^2 + 3*x + 1) - sqrt(-2*x^2 + 3*x + 1)) + 1224*log(-2/9*sqrt(10) +
2/3*sqrt(-2*x^2 + 3*x + 1)*sqrt(sqrt(10) - 1)/abs(6*x - 2*sqrt(10) - 4) + 2
/9*sqrt(10)/abs(6*x - 2*sqrt(10) - 4) - 2/9/abs(6*x - 2*sqrt(10) - 4) + 1/1
8)/(sqrt(10) - 1)^(3/2) - 312/(sqrt(10)*sqrt(-2*x^2 + 3*x + 1) + sqrt(-2*x^
2 + 3*x + 1)) - 312/(sqrt(10)*sqrt(-2*x^2 + 3*x + 1) - sqrt(-2*x^2 + 3*x +
1)))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x+2}{\left(-2\,x^2+3\,x+1\right)^{3/2}\,\left(-3\,x^2+4\,x+2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 2)/((3*x - 2*x^2 + 1)^(3/2)*(4*x - 3*x^2 + 2)),x)
```

```
[Out] int((x + 2)/((3*x - 2*x^2 + 1)^(3/2)*(4*x - 3*x^2 + 2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{-6x^4\sqrt{-2x^2+3x+1}+17x^3\sqrt{-2x^2+3x+1}-5x^2\sqrt{-2x^2+3x+1}-10x\sqrt{-2x^2+3x+1}-2\sqrt{-2x^2+3x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(-3*x**2+4*x+2)/(-2*x**2+3*x+1)**(3/2),x)
```

```
[Out] -Integral(x/(-6*x**4*sqrt(-2*x**2 + 3*x + 1) + 17*x**3*sqrt(-2*x**2 + 3*x +
 1) - 5*x**2*sqrt(-2*x**2 + 3*x + 1) - 10*x*sqrt(-2*x**2 + 3*x + 1) - 2*sqr
t(-2*x**2 + 3*x + 1)), x) - Integral(2/(-6*x**4*sqrt(-2*x**2 + 3*x + 1) + 1
7*x**3*sqrt(-2*x**2 + 3*x + 1) - 5*x**2*sqrt(-2*x**2 + 3*x + 1) - 10*x*sqrt
(-2*x**2 + 3*x + 1) - 2*sqrt(-2*x**2 + 3*x + 1)), x)
```

3.27 $\int \frac{2+x}{(2+4x-3x^2)(1+3x-2x^2)^{5/2}}\,dx$

Optimal. Leaf size=193

$$-\frac{2(14x+15)}{51\left(-2x^2+3x+1\right)^{3/2}}-\frac{2(4814x+291)}{867\sqrt{-2x^2+3x+1}}+\frac{9}{2}\sqrt{\frac{1}{5}\left(17\sqrt{10}-53\right)}\tan^{-1}\left(\frac{\left(1+4\sqrt{10}\right)x+3\left(4-\sqrt{10}\right)}{2\sqrt{1+\sqrt{10}}\sqrt{-2x^2+3x+1}}\right)+\frac{9}{2}\sqrt{\frac{1}{5}}$$

```
[Out] -2/51*(15+14*x)/(-2*x^2+3*x+1)^(3/2)-2/867*(291+4814*x)/(-2*x^2+3*x+1)^(1/2
)+9/10*arctan(1/2*(12-3*10^(1/2)+x*(1+4*10^(1/2)))/(-2*x^2+3*x+1)^(1/2)/(1+
10^(1/2))^(1/2))*(-265+85*10^(1/2))^(1/2)+9/10*arctanh(1/2*(x*(1-4*10^(1/2)
)+12+3*10^(1/2))/(-2*x^2+3*x+1)^(1/2)/(-1+10^(1/2))^(1/2))*(265+85*10^(1/2)
)^(1/2)
```

Rubi [A] time = 0.27, antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1016, 1060, 1032, 724, 204, 206}

$$-\frac{2(14x+15)}{51\left(-2x^2+3x+1\right)^{3/2}}-\frac{2(4814x+291)}{867\sqrt{-2x^2+3x+1}}+\frac{9}{2}\sqrt{\frac{1}{5}\left(17\sqrt{10}-53\right)}\tan^{-1}\left(\frac{\left(1+4\sqrt{10}\right)x+3\left(4-\sqrt{10}\right)}{2\sqrt{1+\sqrt{10}}\sqrt{-2x^2+3x+1}}\right)+\frac{9}{2}\sqrt{\frac{1}{5}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x)/((2 + 4*x - 3*x^2)*(1 + 3*x - 2*x^2)^(5/2)),x]
```

```
[Out] (-2*(15 + 14*x))/(51*(1 + 3*x - 2*x^2)^(3/2)) - (2*(291 + 4814*x))/(867*Sqr
t[1 + 3*x - 2*x^2]) + (9*Sqrt[(-53 + 17*Sqrt[10])/5]*ArcTan[(3*(4 - Sqrt[10
]) + (1 + 4*Sqrt[10])*x)/(2*Sqrt[1 + Sqrt[10]]*Sqrt[1 + 3*x - 2*x^2])])/2 +
 (9*Sqrt[(53 + 17*Sqrt[10])/5]*ArcTanh[(3*(4 + Sqrt[10]) + (1 - 4*Sqrt[10])
*x)/(2*Sqrt[-1 + Sqrt[10]]*Sqrt[1 + 3*x - 2*x^2])])/2
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1016

```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e
_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((a + b*x + c*x^2)^(p + 1)
*(d + e*x + f*x^2)^(q + 1)*(g*c*(2*a*c*e - b*(c*d + a*f)) + (g*b - a*h)*(2*
c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(g*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))
 - h*(b*c*d - 2*a*c*e + a*b*f))*x))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d -
a*e)*(c*e - b*f))*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*
```

```
d - a*e)*(c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*
x^2)^q*Simp[(b*h - 2*g*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)
 + (b^2*(g*f) - b*(h*c*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) - a*(-(h*c*e
))))*(a*f*(p + 1) - c*d*(p + 2)) - e*((g*c)*(2*a*c*e - b*(c*d + a*f)) + (g*
b - a*h)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (2*f*((g*c)*(2*
a*c*e - b*(c*d + a*f)) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(
p + q + 2) - (b^2*g*f - b*(h*c*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) - a*
(-(h*c*e))))*(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(g*f) - b*(h*c
*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) + a*h*c*e))*(2*p + 2*q + 5)*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, q}, x] && NeQ[b^2 - 4*a*c, 0] &
& NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (b*d - a*e)*(c*e
 - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1])
```

Rule 1032

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1060

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((a + b*x +
 c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q + 1)*((A*c - a*C)*(2*a*c*e - b*(c*d +
a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*(2*c^2*d + b
^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*e + a*b*f) + C*(b^2*d - a*b*e -
2*a*(c*d - a*f)))*x))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*
f))*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b
*B - 2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^
2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*
(c*C*d - B*c*e - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
 + q + 2) - (2*f*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^
2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d +
 A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*
(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*
c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*
p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] &&
 NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f
)^2 - (b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !
IGtQ[q, 0]
```

Rubi steps

$$\int \frac{2+x}{\left(2+4x-3x^2\right)\left(1+3x-2x^2\right)^{5/2}}\,dx = -\frac{2(15+14x)}{51\left(1+3x-2x^2\right)^{3/2}} + \frac{2}{51}\int \frac{-56+\frac{235x}{2}+84x^2}{\left(2+4x-3x^2\right)\left(1+3x-2x^2\right)^{3/2}}\,dx$$

$$= -\frac{2(15+14x)}{51\left(1+3x-2x^2\right)^{3/2}} - \frac{2(291+4814x)}{867\sqrt{1+3x-2x^2}} + \frac{4}{867}\int \frac{\frac{7803}{2}+23}{\left(2+4x-3x^2\right)\sqrt{1}}$$

$$= -\frac{2(15+14x)}{51\left(1+3x-2x^2\right)^{3/2}} - \frac{2(291+4814x)}{867\sqrt{1+3x-2x^2}} + \frac{1}{5}\left(27\left(5-2\sqrt{10}\right)\right)\int \frac{}{(4}$$

$$= -\frac{2(15+14x)}{51\left(1+3x-2x^2\right)^{3/2}} - \frac{2(291+4814x)}{867\sqrt{1+3x-2x^2}} - \frac{1}{5}\left(54\left(5-2\sqrt{10}\right)\right)\text{Subst}$$

$$= -\frac{2(15+14x)}{51\left(1+3x-2x^2\right)^{3/2}} - \frac{2(291+4814x)}{867\sqrt{1+3x-2x^2}} + \frac{9}{2}\sqrt{\frac{1}{5}\left(-53+17\sqrt{10}\right)}\,\text{ta}$$

Mathematica [A] time = 0.60, size = 185, normalized size = 0.96

$$\frac{3}{10}\sqrt{1+\sqrt{10}}\left(7\sqrt{10}-25\right)\tan^{-1}\left(\frac{3\left(\sqrt{10}-4\right)-\left(1+4\sqrt{10}\right)x}{2\sqrt{1+\sqrt{10}}\sqrt{-2x^2+3x+1}}\right)-\frac{3}{10}\sqrt{\sqrt{10}-1}\left(25+7\sqrt{10}\right)\tanh^{-1}\left(\frac{\left(4\sqrt{10}-\right.}{2\sqrt{\sqrt{10}-}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x)/((2 + 4*x - 3*x^2)*(1 + 3*x - 2*x^2)^(5/2)),x]
```

```
[Out] (-2*(546 + 5925*x + 13860*x^2 - 9628*x^3))/(867*(1 + 3*x - 2*x^2)^(3/2)) +
(3*Sqrt[1 + Sqrt[10]]*(-25 + 7*Sqrt[10])*ArcTan[(3*(-4 + Sqrt[10]) - (1 + 4
*Sqrt[10])*x)/(2*Sqrt[1 + Sqrt[10]]*Sqrt[1 + 3*x - 2*x^2])])/10 - (3*Sqrt[-
1 + Sqrt[10]]*(25 + 7*Sqrt[10])*ArcTanh[(-3*(4 + Sqrt[10]) + (-1 + 4*Sqrt[1
0])*x)/(2*Sqrt[-1 + Sqrt[10]]*Sqrt[1 + 3*x - 2*x^2])])/10
```

fricas [B] time = 0.99, size = 439, normalized size = 2.27

$$-\frac{43680\,x^4-131040\,x^3-31212\,\sqrt{5}\left(4\,x^4-12\,x^3+5\,x^2+6\,x+1\right)\sqrt{17\,\sqrt{10}-53}\,\arctan\left(\frac{\sqrt{2}\left(\sqrt{10}\,\sqrt{5}\,x+10\,\sqrt{5}\,x\right)\sqrt{17\,\sqrt{}}}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(-3*x^2+4*x+2)/(-2*x^2+3*x+1)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/8670*(43680*x^4 - 131040*x^3 - 31212*sqrt(5)*(4*x^4 - 12*x^3 + 5*x^2 + 6
*x + 1)*sqrt(17*sqrt(10) - 53)*arctan(1/90*(sqrt(2)*(sqrt(10)*sqrt(5)*x + 1
0*sqrt(5)*x)*sqrt(17*sqrt(10) - 53)*sqrt((6*x^2 + sqrt(10)*(3*x^2 + 2*x) -
2*sqrt(-2*x^2 + 3*x + 1)*(sqrt(10)*x + 2*x + 2) + 10*x + 4)/x^2) + 2*(sqrt(
10)*sqrt(5)*(6*x + 1) - sqrt(-2*x^2 + 3*x + 1)*(sqrt(10)*sqrt(5) + 10*sqrt(
5)) + 5*sqrt(5)*(3*x + 2))*sqrt(17*sqrt(10) - 53))/x) - 7803*sqrt(5)*(4*x^4
 - 12*x^3 + 5*x^2 + 6*x + 1)*sqrt(17*sqrt(10) + 53)*log(9*(45*sqrt(10)*x +
(13*sqrt(10)*sqrt(5)*x - 40*sqrt(5)*x)*sqrt(17*sqrt(10) + 53) - 90*x + 90*s
qrt(-2*x^2 + 3*x + 1) - 90)/x) + 7803*sqrt(5)*(4*x^4 - 12*x^3 + 5*x^2 + 6*x
 + 1)*sqrt(17*sqrt(10) + 53)*log(9*(45*sqrt(10)*x - (13*sqrt(10)*sqrt(5)*x
- 40*sqrt(5)*x)*sqrt(17*sqrt(10) + 53) - 90*x + 90*sqrt(-2*x^2 + 3*x + 1) -
```

```
90)/x) + 54600*x^2 - 20*(9628*x^3 - 13860*x^2 - 5925*x - 546)*sqrt(-2*x^2
+ 3*x + 1) + 65520*x + 10920)/(4*x^4 - 12*x^3 + 5*x^2 + 6*x + 1)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(-3*x^2+4*x+2)/(-2*x^2+3*x+1)^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.02, size = 1560, normalized size = 8.08

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2+x)/(-3*x^2+4*x+2)/(-2*x^2+3*x+1)^(5/2),x)
```

```
[Out] 248/2601/(-1/9-1/9*10^(1/2))/(-2*(x-2/3+1/3*10^(1/2))^2+(1/3+4/3*10^(1/2))*
(x-2/3+1/3*10^(1/2))-1/9-1/9*10^(1/2))^(1/2)+248/2601/(-1/9+1/9*10^(1/2))/(
-2*(x-2/3-1/3*10^(1/2))^2+(1/3-4/3*10^(1/2))*(x-2/3-1/3*10^(1/2))-1/9+1/9*1
0^(1/2))^(1/2)+7/153/(-1/9+1/9*10^(1/2))/(-2*(x-2/3-1/3*10^(1/2))^2+(1/3-4/
3*10^(1/2))*(x-2/3-1/3*10^(1/2))-1/9+1/9*10^(1/2))^(3/2)+7/153/(-1/9-1/9*10
^(1/2))/(-2*(x-2/3+1/3*10^(1/2))^2+(1/3+4/3*10^(1/2))*(x-2/3+1/3*10^(1/2))-
1/9-1/9*10^(1/2))^(3/2)+7/51/(-1/9+1/9*10^(1/2))^2/(-2*(x-2/3-1/3*10^(1/2))
^2+(1/3-4/3*10^(1/2))*(x-2/3-1/3*10^(1/2))-1/9+1/9*10^(1/2))^(1/2)+7/51/(-1
/9-1/9*10^(1/2))^2/(-2*(x-2/3+1/3*10^(1/2))^2+(1/3+4/3*10^(1/2))*(x-2/3+1/3
*10^(1/2))-1/9-1/9*10^(1/2))^(1/2)-32/765/(-1/9+1/9*10^(1/2))^2/(-2*(x-2/3-
1/3*10^(1/2))^2+(1/3-4/3*10^(1/2))*(x-2/3-1/3*10^(1/2))-1/9+1/9*10^(1/2))^(
1/2)*10^(1/2)*x-32/2295/(-1/9+1/9*10^(1/2))*10^(1/2)/(-2*(x-2/3-1/3*10^(1/2
))^2+(1/3-4/3*10^(1/2))*(x-2/3-1/3*10^(1/2))-1/9+1/9*10^(1/2))^(3/2)*x+2/5*
10^(1/2)/(-1/9-1/9*10^(1/2))^2/(1+10^(1/2))^(1/2)*arctan(9/2*(-2/9-2/9*10^(
1/2)+(1/3+4/3*10^(1/2))*(x-2/3+1/3*10^(1/2)))/(1+10^(1/2))^(1/2)/(-18*(x-2/
3+1/3*10^(1/2))^2+9*(1/3+4/3*10^(1/2))*(x-2/3+1/3*10^(1/2))-1-10^(1/2))^(1/
2))+32/2295/(-1/9-1/9*10^(1/2))*10^(1/2)/(-2*(x-2/3+1/3*10^(1/2))^2+(1/3+4/
3*10^(1/2))*(x-2/3+1/3*10^(1/2))-1/9-1/9*10^(1/2))^(3/2)*x+32/765/(-1/9-1/9
*10^(1/2))^2/(-2*(x-2/3+1/3*10^(1/2))^2+(1/3+4/3*10^(1/2))*(x-2/3+1/3*10^(1
/2))-1/9-1/9*10^(1/2))^(1/2)*10^(1/2)*x+2/5*10^(1/2)/(-1/9+1/9*10^(1/2))^2/
(-1+10^(1/2))^(1/2)*arctanh(9/2*(-2/9+2/9*10^(1/2)+(1/3-4/3*10^(1/2))*(x-2/
3-1/3*10^(1/2)))/(-1+10^(1/2))^(1/2)/(-18*(x-2/3-1/3*10^(1/2))^2+9*(1/3-4/3
*10^(1/2))*(x-2/3-1/3*10^(1/2))-1+10^(1/2))^(1/2))+512/39015/(-1/9-1/9*10^(
1/2))/(-2*(x-2/3+1/3*10^(1/2))^2+(1/3+4/3*10^(1/2))*(x-2/3+1/3*10^(1/2))-1/
9-1/9*10^(1/2))^(1/2)*10^(1/2)*x-512/39015/(-1/9+1/9*10^(1/2))/(-2*(x-2/3-1
/3*10^(1/2))^2+(1/3-4/3*10^(1/2))*(x-2/3-1/3*10^(1/2))-1/9+1/9*10^(1/2))^(1
/2)*10^(1/2)*x-62/153/(-1/9-1/9*10^(1/2))^2/(-2*(x-2/3+1/3*10^(1/2))^2+(1/3
+4/3*10^(1/2))*(x-2/3+1/3*10^(1/2))-1/9-1/9*10^(1/2))^(1/2)*x+26/255/(-1/9-
1/9*10^(1/2))^2/(-2*(x-2/3+1/3*10^(1/2))^2+(1/3+4/3*10^(1/2))*(x-2/3+1/3*10
^(1/2))-1/9-1/9*10^(1/2))^(1/2)*10^(1/2)-1/2/(-1/9-1/9*10^(1/2))^2/(1+10^(1
/2))^(1/2)*arctan(9/2*(-2/9-2/9*10^(1/2)+(1/3+4/3*10^(1/2))*(x-2/3+1/3*10^(
1/2)))/(1+10^(1/2))^(1/2)/(-18*(x-2/3+1/3*10^(1/2))^2+9*(1/3+4/3*10^(1/2))*
(x-2/3+1/3*10^(1/2))-1-10^(1/2))^(1/2))-62/459/(-1/9-1/9*10^(1/2))/(-2*(x-2
/3+1/3*10^(1/2))^2+(1/3+4/3*10^(1/2))*(x-2/3+1/3*10^(1/2))-1/9-1/9*10^(1/2)
)^(3/2)*x+26/765/(-1/9-1/9*10^(1/2))*10^(1/2)/(-2*(x-2/3+1/3*10^(1/2))^2+(1
/3+4/3*10^(1/2))*(x-2/3+1/3*10^(1/2))-1/9-1/9*10^(1/2))^(3/2)-26/255/(-1/9+
1/9*10^(1/2))^2/(-2*(x-2/3-1/3*10^(1/2))^2+(1/3-4/3*10^(1/2))*(x-2/3-1/3*10
^(1/2))-1/9+1/9*10^(1/2))^(1/2)*10^(1/2)+1/2/(-1/9+1/9*10^(1/2))^2/(-1+10^(
1/2))^(1/2)*arctanh(9/2*(-2/9+2/9*10^(1/2)+(1/3-4/3*10^(1/2))*(x-2/3-1/3*10
^(1/2)))/(-1+10^(1/2))^(1/2)/(-18*(x-2/3-1/3*10^(1/2))^2+9*(1/3-4/3*10^(1/2
```

```
))*(x-2/3-1/3*10^(1/2))-1+10^(1/2))^(1/2))-26/765/(-1/9+1/9*10^(1/2))*10^(1
/2)/(-2*(x-2/3-1/3*10^(1/2))^2+(1/3-4/3*10^(1/2))*(x-2/3-1/3*10^(1/2))-1/9+
1/9*10^(1/2))^(3/2)-62/153/(-1/9+1/9*10^(1/2))^2/(-2*(x-2/3-1/3*10^(1/2))^2
+(1/3-4/3*10^(1/2))*(x-2/3-1/3*10^(1/2))-1/9+1/9*10^(1/2))^(1/2)*x-62/459/(
-1/9+1/9*10^(1/2))/(-2*(x-2/3-1/3*10^(1/2))^2+(1/3-4/3*10^(1/2))*(x-2/3-1/3
*10^(1/2))-1/9+1/9*10^(1/2))^(3/2)*x+128/13005*10^(1/2)/(-1/9+1/9*10^(1/2))
/(-2*(x-2/3-1/3*10^(1/2))^2+(1/3-4/3*10^(1/2))*(x-2/3-1/3*10^(1/2))-1/9+1/9
*10^(1/2))^(1/2)-992/7803/(-1/9+1/9*10^(1/2))/(-2*(x-2/3-1/3*10^(1/2))^2+(1
/3-4/3*10^(1/2))*(x-2/3-1/3*10^(1/2))-1/9+1/9*10^(1/2))^(1/2)*x-128/13005*1
0^(1/2)/(-1/9-1/9*10^(1/2))/(-2*(x-2/3+1/3*10^(1/2))^2+(1/3+4/3*10^(1/2))*(
x-2/3+1/3*10^(1/2))-1/9-1/9*10^(1/2))^(1/2)-992/7803/(-1/9-1/9*10^(1/2))/(-
2*(x-2/3+1/3*10^(1/2))^2+(1/3+4/3*10^(1/2))*(x-2/3+1/3*10^(1/2))-1/9-1/9*10
^(1/2))^(1/2)*x
```

maxima [B]   time = 1.26, size = 1276, normalized size = 6.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(-3*x^2+4*x+2)/(-2*x^2+3*x+1)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/17340*sqrt(10)*(2108*sqrt(10)*x/(sqrt(10)*(-2*x^2 + 3*x + 1)^(3/2) + (-2*
x^2 + 3*x + 1)^(3/2)) - 2108*sqrt(10)*x/(sqrt(10)*(-2*x^2 + 3*x + 1)^(3/2)
- (-2*x^2 + 3*x + 1)^(3/2)) - 56916*sqrt(10)*x/(2*sqrt(10)*sqrt(-2*x^2 + 3*
x + 1) + 11*sqrt(-2*x^2 + 3*x + 1)) + 56916*sqrt(10)*x/(2*sqrt(10)*sqrt(-2*
x^2 + 3*x + 1) - 11*sqrt(-2*x^2 + 3*x + 1)) + 1984*sqrt(10)*x/(sqrt(10)*sqr
t(-2*x^2 + 3*x + 1) + sqrt(-2*x^2 + 3*x + 1)) - 1984*sqrt(10)*x/(sqrt(10)*s
qrt(-2*x^2 + 3*x + 1) - sqrt(-2*x^2 + 3*x + 1)) - 70227*sqrt(10)*arcsin(8/1
7*sqrt(17)*sqrt(10)*x/abs(6*x + 2*sqrt(10) - 4) + 2/17*sqrt(17)*x/abs(6*x +
 2*sqrt(10) - 4) - 6/17*sqrt(17)*sqrt(10)/abs(6*x + 2*sqrt(10) - 4) + 24/17
*sqrt(17)/abs(6*x + 2*sqrt(10) - 4))/(2*sqrt(10)*sqrt(sqrt(10) + 1) + 11*sq
rt(sqrt(10) + 1)) - 2176*x/(sqrt(10)*(-2*x^2 + 3*x + 1)^(3/2) + (-2*x^2 + 3
*x + 1)^(3/2)) - 2176*x/(sqrt(10)*(-2*x^2 + 3*x + 1)^(3/2) - (-2*x^2 + 3*x
+ 1)^(3/2)) + 58752*x/(2*sqrt(10)*sqrt(-2*x^2 + 3*x + 1) + 11*sqrt(-2*x^2 +
 3*x + 1)) + 58752*x/(2*sqrt(10)*sqrt(-2*x^2 + 3*x + 1) - 11*sqrt(-2*x^2 +
3*x + 1)) - 2048*x/(sqrt(10)*sqrt(-2*x^2 + 3*x + 1) + sqrt(-2*x^2 + 3*x + 1
)) - 2048*x/(sqrt(10)*sqrt(-2*x^2 + 3*x + 1) - sqrt(-2*x^2 + 3*x + 1)) + 56
1816*arcsin(8/17*sqrt(17)*sqrt(10)*x/abs(6*x + 2*sqrt(10) - 4) + 2/17*sqrt(
17)*x/abs(6*x + 2*sqrt(10) - 4) - 6/17*sqrt(17)*sqrt(10)/abs(6*x + 2*sqrt(1
0) - 4) + 24/17*sqrt(17)/abs(6*x + 2*sqrt(10) - 4))/(2*sqrt(10)*sqrt(sqrt(1
0) + 1) + 11*sqrt(sqrt(10) + 1)) - 714*sqrt(10)/(sqrt(10)*(-2*x^2 + 3*x + 1
)^(3/2) + (-2*x^2 + 3*x + 1)^(3/2)) + 714*sqrt(10)/(sqrt(10)*(-2*x^2 + 3*x
+ 1)^(3/2) - (-2*x^2 + 3*x + 1)^(3/2)) + 19278*sqrt(10)/(2*sqrt(10)*sqrt(-2
*x^2 + 3*x + 1) + 11*sqrt(-2*x^2 + 3*x + 1)) - 19278*sqrt(10)/(2*sqrt(10)*s
qrt(-2*x^2 + 3*x + 1) - 11*sqrt(-2*x^2 + 3*x + 1)) - 1488*sqrt(10)/(sqrt(10
)*sqrt(-2*x^2 + 3*x + 1) + sqrt(-2*x^2 + 3*x + 1)) + 1488*sqrt(10)/(sqrt(10
)*sqrt(-2*x^2 + 3*x + 1) - sqrt(-2*x^2 + 3*x + 1)) - 5304/(sqrt(10)*(-2*x^2
 + 3*x + 1)^(3/2) + (-2*x^2 + 3*x + 1)^(3/2)) - 5304/(sqrt(10)*(-2*x^2 + 3*
x + 1)^(3/2) - (-2*x^2 + 3*x + 1)^(3/2)) + 143208/(2*sqrt(10)*sqrt(-2*x^2 +
 3*x + 1) + 11*sqrt(-2*x^2 + 3*x + 1)) + 143208/(2*sqrt(10)*sqrt(-2*x^2 + 3
*x + 1) - 11*sqrt(-2*x^2 + 3*x + 1)) + 1536/(sqrt(10)*sqrt(-2*x^2 + 3*x + 1
) + sqrt(-2*x^2 + 3*x + 1)) + 1536/(sqrt(10)*sqrt(-2*x^2 + 3*x + 1) - sqrt(
-2*x^2 + 3*x + 1)) + 70227*sqrt(10)*log(-2/9*sqrt(10) + 2/3*sqrt(-2*x^2 + 3
*x + 1)*sqrt(sqrt(10) - 1)/abs(6*x - 2*sqrt(10) - 4) + 2/9*sqrt(10)/abs(6*x
 - 2*sqrt(10) - 4) - 2/9/abs(6*x - 2*sqrt(10) - 4) + 1/18)/(sqrt(10) - 1)^(
5/2) + 561816*log(-2/9*sqrt(10) + 2/3*sqrt(-2*x^2 + 3*x + 1)*sqrt(sqrt(10)
- 1)/abs(6*x - 2*sqrt(10) - 4) + 2/9*sqrt(10)/abs(6*x - 2*sqrt(10) - 4) - 2
/9/abs(6*x - 2*sqrt(10) - 4) + 1/18)/(sqrt(10) - 1)^(5/2))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x+2}{\left(-2\,x^2+3\,x+1\right)^{5/2}\,\left(-3\,x^2+4\,x+2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 2)/((3*x - 2*x^2 + 1)^(5/2)*(4*x - 3*x^2 + 2)),x)
```

```
[Out] int((x + 2)/((3*x - 2*x^2 + 1)^(5/2)*(4*x - 3*x^2 + 2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{12x^6\sqrt{-2x^2+3x+1} - 52x^5\sqrt{-2x^2+3x+1} + 55x^4\sqrt{-2x^2+3x+1} + 22x^3\sqrt{-2x^2+3x+1} - 31x^2\sqrt{-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(-3*x**2+4*x+2)/(-2*x**2+3*x+1)**(5/2),x)
```

```
[Out] -Integral(x/(12*x**6*sqrt(-2*x**2 + 3*x + 1) - 52*x**5*sqrt(-2*x**2 + 3*x +
 1) + 55*x**4*sqrt(-2*x**2 + 3*x + 1) + 22*x**3*sqrt(-2*x**2 + 3*x + 1) - 3
1*x**2*sqrt(-2*x**2 + 3*x + 1) - 16*x*sqrt(-2*x**2 + 3*x + 1) - 2*sqrt(-2*x
**2 + 3*x + 1)), x) - Integral(2/(12*x**6*sqrt(-2*x**2 + 3*x + 1) - 52*x**5
*sqrt(-2*x**2 + 3*x + 1) + 55*x**4*sqrt(-2*x**2 + 3*x + 1) + 22*x**3*sqrt(-
2*x**2 + 3*x + 1) - 31*x**2*sqrt(-2*x**2 + 3*x + 1) - 16*x*sqrt(-2*x**2 + 3
*x + 1) - 2*sqrt(-2*x**2 + 3*x + 1)), x)
```

3.28 $\int \frac{2+x}{(2+4x-3x^2)\sqrt{1+3x+2x^2}}\,dx$

Optimal. Leaf size=151

$$\frac{1}{2}\sqrt{1-\frac{7\sqrt{\frac{2}{5}}}{5}}\tanh^{-1}\left(\frac{\left(17+4\sqrt{10}\right)x+3\left(4+\sqrt{10}\right)}{2\sqrt{55+17\sqrt{10}}\sqrt{2x^2+3x+1}}\right)-\frac{1}{2}\sqrt{1+\frac{7\sqrt{\frac{2}{5}}}{5}}\tanh^{-1}\left(\frac{\left(17-4\sqrt{10}\right)x+3\left(4-\sqrt{10}\right)}{2\sqrt{55-17\sqrt{10}}\sqrt{2x^2+3x+1}}\right)$$

```
[Out] 1/10*arctanh(1/2*(12+3*10^(1/2)+x*(17+4*10^(1/2)))/(2*x^2+3*x+1)^(1/2)/(55+
17*10^(1/2))^(1/2))*(25-7*10^(1/2))^(1/2)-1/10*arctanh(1/2*(x*(17-4*10^(1/2
))+12-3*10^(1/2))/(2*x^2+3*x+1)^(1/2)/(55-17*10^(1/2))^(1/2))*(25+7*10^(1/2
))^(1/2)
```

Rubi [A] time = 0.23, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1032, 724, 206}

$$\frac{1}{2}\sqrt{1-\frac{7\sqrt{\frac{2}{5}}}{5}}\tanh^{-1}\left(\frac{\left(17+4\sqrt{10}\right)x+3\left(4+\sqrt{10}\right)}{2\sqrt{55+17\sqrt{10}}\sqrt{2x^2+3x+1}}\right)-\frac{1}{2}\sqrt{1+\frac{7\sqrt{\frac{2}{5}}}{5}}\tanh^{-1}\left(\frac{\left(17-4\sqrt{10}\right)x+3\left(4-\sqrt{10}\right)}{2\sqrt{55-17\sqrt{10}}\sqrt{2x^2+3x+1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x)/((2 + 4*x - 3*x^2)*Sqrt[1 + 3*x + 2*x^2]),x]
```

```
[Out] -(Sqrt[1 + (7*Sqrt[2/5])/5]*ArcTanh[(3*(4 - Sqrt[10]) + (17 - 4*Sqrt[10])*x
)/(2*Sqrt[55 - 17*Sqrt[10]]*Sqrt[1 + 3*x + 2*x^2])])/2 + (Sqrt[1 - (7*Sqrt[
2/5])/5]*ArcTanh[(3*(4 + Sqrt[10]) + (17 + 4*Sqrt[10])*x)/(2*Sqrt[55 + 17*S
qrt[10]]*Sqrt[1 + 3*x + 2*x^2])])/2
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1032

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{2+x}{\left(2+4x-3x^2\right)\sqrt{1+3x+2x^2}}\,dx = \frac{1}{5}\left(5-4\sqrt{10}\right)\int \frac{1}{\left(4-2\sqrt{10}-6x\right)\sqrt{1+3x+2x^2}}\,dx + \frac{1}{5}\left(5+4\sqrt{10}\right)$$

$$= -\left(\frac{1}{5}\left(2\left(5-4\sqrt{10}\right)\right)\text{Subst}\left(\int \frac{1}{144+72\left(4-2\sqrt{10}\right)+8\left(4-2\sqrt{10}\right)^2}\right.\right.$$

$$= -\frac{1}{10}\sqrt{25+7\sqrt{10}}\,\tanh^{-1}\left(\frac{3\left(4-\sqrt{10}\right)+\left(17-4\sqrt{10}\right)x}{2\sqrt{55-17\sqrt{10}}\,\sqrt{1+3x+2x^2}}\right)+\frac{1}{10}\sqrt{25}$$

Mathematica [A] time = 0.36, size = 148, normalized size = 0.98

$$\frac{\left(5-4\sqrt{10}\right)\tanh^{-1}\left(\frac{-4\sqrt{10}x+17x-3\sqrt{10}+12}{2\sqrt{55-17\sqrt{10}}\sqrt{2x^2+3x+1}}\right)+3\sqrt{285-90\sqrt{10}}\,\tanh^{-1}\left(\frac{\left(17+4\sqrt{10}\right)x+3\left(4+\sqrt{10}\right)}{2\sqrt{55+17\sqrt{10}}\sqrt{2x^2+3x+1}}\right)}{10\sqrt{55-17\sqrt{10}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(2 + x)/((2 + 4*x - 3*x^2)*Sqrt[1 + 3*x + 2*x^2]),x]
```

```
[Out] ((5 - 4*Sqrt[10])*ArcTanh[(12 - 3*Sqrt[10] + 17*x - 4*Sqrt[10]*x)/(2*Sqrt[5
5 - 17*Sqrt[10]]*Sqrt[1 + 3*x + 2*x^2])] + 3*Sqrt[285 - 90*Sqrt[10]]*ArcTan
h[(3*(4 + Sqrt[10]) + (17 + 4*Sqrt[10])*x)/(2*Sqrt[55 + 17*Sqrt[10]]*Sqrt[1
 + 3*x + 2*x^2])])/(10*Sqrt[55 - 17*Sqrt[10]])
```

fricas [B] time = 1.23, size = 245, normalized size = 1.62

$$\frac{1}{10}\sqrt{7\sqrt{10}+25}\,\log\left(-\frac{3\sqrt{10}x+\left(\sqrt{10}x-4x\right)\sqrt{7\sqrt{10}+25}+6x-6\sqrt{2x^2+3x+1}+6}{x}\right)-\frac{1}{10}\sqrt{7\sqrt{10}+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(-3*x^2+4*x+2)/(2*x^2+3*x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/10*sqrt(7*sqrt(10) + 25)*log(-(3*sqrt(10)*x + (sqrt(10)*x - 4*x)*sqrt(7*s
qrt(10) + 25) + 6*x - 6*sqrt(2*x^2 + 3*x + 1) + 6)/x) - 1/10*sqrt(7*sqrt(10
) + 25)*log(-(3*sqrt(10)*x - (sqrt(10)*x - 4*x)*sqrt(7*sqrt(10) + 25) + 6*x
 - 6*sqrt(2*x^2 + 3*x + 1) + 6)/x) + 1/10*sqrt(-7*sqrt(10) + 25)*log((3*sqr
t(10)*x + (sqrt(10)*x + 4*x)*sqrt(-7*sqrt(10) + 25) - 6*x + 6*sqrt(2*x^2 +
3*x + 1) - 6)/x) - 1/10*sqrt(-7*sqrt(10) + 25)*log((3*sqrt(10)*x - (sqrt(10
)*x + 4*x)*sqrt(-7*sqrt(10) + 25) - 6*x + 6*sqrt(2*x^2 + 3*x + 1) - 6)/x)
```

giac [A] time = 0.48, size = 93, normalized size = 0.62

$$0.169235232112667\,\log\left(-\sqrt{2}x+\sqrt{2x^2+3x+1}+5.90976932712000\right)-0.686556214893333\,\log\left(-\sqrt{2}x+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(-3*x^2+4*x+2)/(2*x^2+3*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] 0.169235232112667*log(-sqrt(2)*x + sqrt(2*x^2 + 3*x + 1) + 5.90976932712000
) - 0.686556214893333*log(-sqrt(2)*x + sqrt(2*x^2 + 3*x + 1) - 0.1765271563
27000) + 0.686556214893333*log(-sqrt(2)*x + sqrt(2*x^2 + 3*x + 1) - 0.91927
8730509000) - 0.169235232112667*log(-sqrt(2)*x + sqrt(2*x^2 + 3*x + 1) - 1.
04272727395000)
```

maple [A] time = 0.05, size = 186, normalized size = 1.23

$$\frac{\left(-8+\sqrt{10}\right)\sqrt{10}\ \operatorname{arctanh}\left(\frac{55-17\sqrt{10}+\frac{9\left(\frac{17}{3}-\frac{4\sqrt{10}}{3}\right)\left(x-\frac{2}{3}+\frac{\sqrt{10}}{3}\right)}{2}}{\sqrt{55-17\sqrt{10}}\ \sqrt{18\left(x-\frac{2}{3}+\frac{\sqrt{10}}{3}\right)^{2}+9\left(\frac{17}{3}-\frac{4\sqrt{10}}{3}\right)\left(x-\frac{2}{3}+\frac{\sqrt{10}}{3}\right)+55-17\sqrt{10}}}\right)}{20\sqrt{55-17\sqrt{10}}}+\left(8+\sqrt{10}\right)\sqrt{10}\ \operatorname{arctanh}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2+x)/(-3*x^2+4*x+2)/(2*x^2+3*x+1)^(1/2),x)
```

```
[Out] 1/20*(-8+10^(1/2))*10^(1/2)/(55-17*10^(1/2))^(1/2)*arctanh(9/2*(110/9-34/9*
10^(1/2)+(17/3-4/3*10^(1/2))*(x-2/3+1/3*10^(1/2)))/(55-17*10^(1/2))^(1/2)/(
18*(x-2/3+1/3*10^(1/2))^2+9*(17/3-4/3*10^(1/2))*(x-2/3+1/3*10^(1/2))+55-17*
10^(1/2))^(1/2))+1/20*(8+10^(1/2))*10^(1/2)/(55+17*10^(1/2))^(1/2)*arctanh(
9/2*(110/9+34/9*10^(1/2)+(17/3+4/3*10^(1/2))*(x-2/3-1/3*10^(1/2)))/(55+17*1
0^(1/2))^(1/2)/(18*(x-2/3-1/3*10^(1/2))^2+9*(17/3+4/3*10^(1/2))*(x-2/3-1/3*
10^(1/2))+55+17*10^(1/2))^(1/2))
```

maxima [B] time = 1.07, size = 363, normalized size = 2.40

$$\frac{1}{60}\sqrt{10}\left(\frac{3\sqrt{10}\log\left(\frac{2}{9}\sqrt{10}+\frac{2\sqrt{2x^{2}+3x+1}\sqrt{17\sqrt{10}+55}}{3\left|6x-2\sqrt{10}-4\right|}+\frac{34\sqrt{10}}{9\left|6x-2\sqrt{10}-4\right|}+\frac{110}{9\left|6x-2\sqrt{10}-4\right|}+\frac{17}{18}\right)}{\sqrt{17\sqrt{10}+55}}+\frac{\sqrt{10}\log\left(-\frac{2}{9}\sqrt{10}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(-3*x^2+4*x+2)/(2*x^2+3*x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/60*sqrt(10)*(3*sqrt(10)*log(2/9*sqrt(10) + 2/3*sqrt(2*x^2 + 3*x + 1)*sqrt
(17*sqrt(10) + 55)/abs(6*x - 2*sqrt(10) - 4) + 34/9*sqrt(10)/abs(6*x - 2*sq
rt(10) - 4) + 110/9/abs(6*x - 2*sqrt(10) - 4) + 17/18)/sqrt(17*sqrt(10) + 5
5) + sqrt(10)*log(-2/9*sqrt(10) + 2*sqrt(2*x^2 + 3*x + 1)*sqrt(-17/9*sqrt(1
0) + 55/9)/abs(6*x + 2*sqrt(10) - 4) - 34/9*sqrt(10)/abs(6*x + 2*sqrt(10) -
 4) + 110/9/abs(6*x + 2*sqrt(10) - 4) + 17/18)/sqrt(-17/9*sqrt(10) + 55/9)
+ 24*log(2/9*sqrt(10) + 2/3*sqrt(2*x^2 + 3*x + 1)*sqrt(17*sqrt(10) + 55)/ab
s(6*x - 2*sqrt(10) - 4) + 34/9*sqrt(10)/abs(6*x - 2*sqrt(10) - 4) + 110/9/a
bs(6*x - 2*sqrt(10) - 4) + 17/18)/sqrt(17*sqrt(10) + 55) - 8*log(-2/9*sqrt(
10) + 2*sqrt(2*x^2 + 3*x + 1)*sqrt(-17/9*sqrt(10) + 55/9)/abs(6*x + 2*sqrt(
10) - 4) - 34/9*sqrt(10)/abs(6*x + 2*sqrt(10) - 4) + 110/9/abs(6*x + 2*sqrt
(10) - 4) + 17/18)/sqrt(-17/9*sqrt(10) + 55/9))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x+2}{\sqrt{2\,x^2+3\,x+1}\,\left(-3\,x^2+4\,x+2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 2)/((3*x + 2*x^2 + 1)^(1/2)*(4*x - 3*x^2 + 2)),x)
```

```
[Out] int((x + 2)/((3*x + 2*x^2 + 1)^(1/2)*(4*x - 3*x^2 + 2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{3x^{2}\sqrt{2x^{2}+3x+1}-4x\sqrt{2x^{2}+3x+1}-2\sqrt{2x^{2}+3x+1}}\,dx-\int \frac{2}{3x^{2}\sqrt{2x^{2}+3x+1}-4x\sqrt{2x^{2}+3x+1}-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(-3*x**2+4*x+2)/(2*x**2+3*x+1)**(1/2),x)
```

```
[Out] -Integral(x/(3*x**2*sqrt(2*x**2 + 3*x + 1) - 4*x*sqrt(2*x**2 + 3*x + 1) - 2
*sqrt(2*x**2 + 3*x + 1)), x) - Integral(2/(3*x**2*sqrt(2*x**2 + 3*x + 1) -
4*x*sqrt(2*x**2 + 3*x + 1) - 2*sqrt(2*x**2 + 3*x + 1)), x)
```

3.29 $\int \frac{2+x}{(2+4x-3x^2)(1+3x+2x^2)^{3/2}} \, dx$

Optimal. Leaf size=174

$$\frac{2(22x+21)}{5\sqrt{2x^2+3x+1}} - \frac{1}{10}\sqrt{\frac{3}{5}\left(2065+653\sqrt{10}\right)} \tanh^{-1}\left(\frac{\left(17-4\sqrt{10}\right)x+3\left(4-\sqrt{10}\right)}{2\sqrt{55-17\sqrt{10}}\sqrt{2x^2+3x+1}}\right) + \frac{1}{10}\sqrt{\frac{3}{5}\left(2065-653\sqrt{10}\right)}$$

```
[Out] 2/5*(21+22*x)/(2*x^2+3*x+1)^(1/2)+1/50*arctanh(1/2*(12+3*10^(1/2)+x*(17+4*1
0^(1/2)))/(2*x^2+3*x+1)^(1/2)/(55+17*10^(1/2))^(1/2))*(30975-9795*10^(1/2))
^(1/2)-1/50*arctanh(1/2*(x*(17-4*10^(1/2))+12-3*10^(1/2))/(2*x^2+3*x+1)^(1/
2)/(55-17*10^(1/2))^(1/2))*(30975+9795*10^(1/2))^(1/2)
```

Rubi [A] time = 0.25, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1016, 1032, 724, 206}

$$\frac{2(22x+21)}{5\sqrt{2x^2+3x+1}} - \frac{1}{10}\sqrt{\frac{3}{5}\left(2065+653\sqrt{10}\right)} \tanh^{-1}\left(\frac{\left(17-4\sqrt{10}\right)x+3\left(4-\sqrt{10}\right)}{2\sqrt{55-17\sqrt{10}}\sqrt{2x^2+3x+1}}\right) + \frac{1}{10}\sqrt{\frac{3}{5}\left(2065-653\sqrt{10}\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x)/((2 + 4*x - 3*x^2)*(1 + 3*x + 2*x^2)^(3/2)),x]
```

```
[Out] (2*(21 + 22*x))/(5*Sqrt[1 + 3*x + 2*x^2]) - (Sqrt[(3*(2065 + 653*Sqrt[10]))
/5]*ArcTanh[(3*(4 - Sqrt[10]) + (17 - 4*Sqrt[10])*x)/(2*Sqrt[55 - 17*Sqrt[1
0]]*Sqrt[1 + 3*x + 2*x^2])])/10 + (Sqrt[(3*(2065 - 653*Sqrt[10]))/5]*ArcTan
h[(3*(4 + Sqrt[10]) + (17 + 4*Sqrt[10])*x)/(2*Sqrt[55 + 17*Sqrt[10]]*Sqrt[1
 + 3*x + 2*x^2])])/10
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1016

```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e
_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((a + b*x + c*x^2)^(p + 1)
*(d + e*x + f*x^2)^(q + 1)*(g*c*(2*a*c*e - b*(c*d + a*f)) + (g*b - a*h)*(2*
c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(g*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))
 - h*(b*c*d - 2*a*c*e + a*b*f))*x))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d -
a*e)*(c*e - b*f))*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*
d - a*e)*(c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*
x^2)^q*Simp[(b*h - 2*g*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)
 + (b^2*(g*f) - b*(h*c*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) - a*(-(h*c*e
))))*(a*f*(p + 1) - c*d*(p + 2)) - e*((g*c)*(2*a*c*e - b*(c*d + a*f)) + (g*
b - a*h)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (2*f*((g*c)*(2*
a*c*e - b*(c*d + a*f)) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(
p + q + 2) - (b^2*g*f - b*(h*c*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) - a*
```

```
(-(h*c*e))))*(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(g*f) - b*(h*c
*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) + a*h*c*e))*(2*p + 2*q + 5)*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, q}, x] && NeQ[b^2 - 4*a*c, 0] &
& NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (b*d - a*e)*(c*e
 - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1])
```

Rule 1032

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{2+x}{\left(2+4x-3x^2\right)\left(1+3x+2x^2\right)^{3/2}}\,dx = \frac{2(21+22x)}{5\sqrt{1+3x+2x^2}} - \frac{2}{15}\int \frac{-72+\frac{81x}{2}}{\left(2+4x-3x^2\right)\sqrt{1+3x+2x^2}}\,dx$$

$$= \frac{2(21+22x)}{5\sqrt{1+3x+2x^2}} - \frac{1}{5}\left(9\left(3-\sqrt{10}\right)\right)\int \frac{1}{\left(4+2\sqrt{10}-6x\right)\sqrt{1+3x+2}}$$

$$= \frac{2(21+22x)}{5\sqrt{1+3x+2x^2}} + \frac{1}{5}\left(18\left(3-\sqrt{10}\right)\right)\text{Subst}\left(\int \frac{}{144+72\left(4+2\sqrt{10}\right.}\right.$$

$$= \frac{2(21+22x)}{5\sqrt{1+3x+2x^2}} - \frac{1}{10}\sqrt{\frac{3}{5}\left(2065+653\sqrt{10}\right)}\tanh^{-1}\left(\frac{3\left(4-\sqrt{10}\right)}{2\sqrt{55-17\sqrt{1}}}\right.$$

Mathematica [A] time = 0.57, size = 172, normalized size = 0.99

$$\frac{1}{50}\left(\frac{\sqrt{30975-9795\sqrt{10}}\sqrt{2x^2+3x+1}\tanh^{-1}\left(\frac{4\sqrt{10}x+17x+3\sqrt{10}+12}{2\sqrt{55+17\sqrt{10}}\sqrt{2x^2+3x+1}}\right)+440x+420}{\sqrt{2x^2+3x+1}} - \sqrt{30975+9795\sqrt{10}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x)/((2 + 4*x - 3*x^2)*(1 + 3*x + 2*x^2)^(3/2)),x]
```

```
[Out] (-(Sqrt[30975 + 9795*Sqrt[10]]*ArcTanh[(12 - 3*Sqrt[10] + 17*x - 4*Sqrt[10]
*x)/(2*Sqrt[55 - 17*Sqrt[10]]*Sqrt[1 + 3*x + 2*x^2])]) + (420 + 440*x + Sqr
t[30975 - 9795*Sqrt[10]]*Sqrt[1 + 3*x + 2*x^2]*ArcTanh[(12 + 3*Sqrt[10] + 1
7*x + 4*Sqrt[10]*x)/(2*Sqrt[55 + 17*Sqrt[10]]*Sqrt[1 + 3*x + 2*x^2])])/Sqrt
[1 + 3*x + 2*x^2])/50
```

fricas [B] time = 0.97, size = 365, normalized size = 2.10

$$\sqrt{5}\left(2x^2+3x+1\right)\sqrt{1959\sqrt{10}+6195}\,\log\left(-\frac{45\sqrt{10}x+\left(41\sqrt{10}\sqrt{5}x-130\sqrt{5}x\right)\sqrt{1959\sqrt{10}+6195}+90x-90\sqrt{2x^2+3x+1}+90}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(-3*x^2+4*x+2)/(2*x^2+3*x+1)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/50*(sqrt(5)*(2*x^2 + 3*x + 1)*sqrt(1959*sqrt(10) + 6195)*log(-(45*sqrt(10
)*x + (41*sqrt(10)*sqrt(5)*x - 130*sqrt(5)*x)*sqrt(1959*sqrt(10) + 6195) +
90*x - 90*sqrt(2*x^2 + 3*x + 1) + 90)/x) - sqrt(5)*(2*x^2 + 3*x + 1)*sqrt(1
959*sqrt(10) + 6195)*log(-(45*sqrt(10)*x - (41*sqrt(10)*sqrt(5)*x - 130*sqr
t(5)*x)*sqrt(1959*sqrt(10) + 6195) + 90*x - 90*sqrt(2*x^2 + 3*x + 1) + 90)/
x) + sqrt(5)*(2*x^2 + 3*x + 1)*sqrt(-1959*sqrt(10) + 6195)*log((45*sqrt(10)
*x + (41*sqrt(10)*sqrt(5)*x + 130*sqrt(5)*x)*sqrt(-1959*sqrt(10) + 6195) -
90*x + 90*sqrt(2*x^2 + 3*x + 1) - 90)/x) - sqrt(5)*(2*x^2 + 3*x + 1)*sqrt(-
1959*sqrt(10) + 6195)*log((45*sqrt(10)*x - (41*sqrt(10)*sqrt(5)*x + 130*sqr
t(5)*x)*sqrt(-1959*sqrt(10) + 6195) - 90*x + 90*sqrt(2*x^2 + 3*x + 1) - 90)
/x) + 840*x^2 + 20*sqrt(2*x^2 + 3*x + 1)*(22*x + 21) + 1260*x + 420)/(2*x^2
 + 3*x + 1)
```

giac [**A**] time = 0.50, size = 112, normalized size = 0.64

$$\frac{2\,(22\,x+21)}{5\,\sqrt{2\,x^2+3\,x+1}}+0.0140045514133333\,\log\left(-\sqrt{2}\,x+\sqrt{2\,x^2+3\,x+1}+5.90976932712000\right)-4.977931686200$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(-3*x^2+4*x+2)/(2*x^2+3*x+1)^(3/2),x, algorithm="giac")
```

```
[Out] 2/5*(22*x + 21)/sqrt(2*x^2 + 3*x + 1) + 0.0140045514133333*log(-sqrt(2)*x +
 sqrt(2*x^2 + 3*x + 1) + 5.90976932712000) - 4.97793168620000*log(-sqrt(2)*
x + sqrt(2*x^2 + 3*x + 1) - 0.176527156327000) + 4.97793168620000*log(-sqrt
(2)*x + sqrt(2*x^2 + 3*x + 1) - 0.919278730509000) - 0.0140045514125333*log
(-sqrt(2)*x + sqrt(2*x^2 + 3*x + 1) - 1.04272727395000)
```

maple [**B**] time = 0.02, size = 466, normalized size = 2.68

$$-\frac{\left(-8+\sqrt{10}\right)\sqrt{10}\left(-\frac{\operatorname{arctanh}\left(\frac{55-17\sqrt{10}+\frac{9\left(\frac{17}{3}-\frac{4\sqrt{10}}{3}\right)\left(x-\frac{2}{3}+\frac{\sqrt{10}}{3}\right)}{2}}{\sqrt{55-17\sqrt{10}}\,\sqrt{18\left(x-\frac{2}{3}+\frac{\sqrt{10}}{3}\right)^2+9\left(\frac{17}{3}-\frac{4\sqrt{10}}{3}\right)\left(x-\frac{2}{3}+\frac{\sqrt{10}}{3}\right)+55-17\sqrt{10}}}\right)}{\left(\frac{55}{9}-\frac{17\sqrt{10}}{9}\right)\sqrt{55-17\sqrt{10}}}+\frac{1}{3\left(\frac{55}{9}-\frac{17\sqrt{10}}{9}\right)\sqrt{2\left(x-\frac{2}{3}+\frac{\sqrt{10}}{3}\right)^2+\left(\frac{17}{3}-\right.}}\right.}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2+x)/(-3*x^2+4*x+2)/(2*x^2+3*x+1)^(3/2),x)
```

```
[Out] -1/20*(-8+10^(1/2))*10^(1/2)*(1/3/(55/9-17/9*10^(1/2))/(2*(x-2/3+1/3*10^(1/
2))^2+(17/3-4/3*10^(1/2))*(x-2/3+1/3*10^(1/2))+55/9-17/9*10^(1/2))^(1/2)-1/
3*(17/3-4/3*10^(1/2))/(55/9-17/9*10^(1/2))*(3+4*x)/(440/9-136/9*10^(1/2)-(1
7/3-4/3*10^(1/2))^2)/(2*(x-2/3+1/3*10^(1/2))^2+(17/3-4/3*10^(1/2))*(x-2/3+1
/3*10^(1/2))+55/9-17/9*10^(1/2))^(1/2)-1/(55/9-17/9*10^(1/2))/(55-17*10^(1/
2))^(1/2)*arctanh(9/2*(110/9-34/9*10^(1/2)+(17/3-4/3*10^(1/2))*(x-2/3+1/3*1
0^(1/2)))/(55-17*10^(1/2))^(1/2)/(18*(x-2/3+1/3*10^(1/2))^2+9*(17/3-4/3*10^
(1/2))*(x-2/3+1/3*10^(1/2))+55-17*10^(1/2))^(1/2)))-1/20*(8+10^(1/2))*10^(1
/2)*(1/3/(55/9+17/9*10^(1/2))/(2*(x-2/3-1/3*10^(1/2))^2+(17/3+4/3*10^(1/2))
*(x-2/3-1/3*10^(1/2))+55/9+17/9*10^(1/2))^(1/2)-1/3*(17/3+4/3*10^(1/2))/(55
/9+17/9*10^(1/2))*(3+4*x)/(440/9+136/9*10^(1/2)-(17/3+4/3*10^(1/2))^2)/(2*(
x-2/3-1/3*10^(1/2))^2+(17/3+4/3*10^(1/2))*(x-2/3-1/3*10^(1/2))+55/9+17/9*10
^(1/2))^(1/2)-1/(55/9+17/9*10^(1/2))/(55+17*10^(1/2))^(1/2)*arctanh(9/2*(11
0/9+34/9*10^(1/2)+(17/3+4/3*10^(1/2))*(x-2/3-1/3*10^(1/2)))/(55+17*10^(1/2)
)^(1/2)/(18*(x-2/3-1/3*10^(1/2))^2+9*(17/3+4/3*10^(1/2))*(x-2/3-1/3*10^(1/2
))+55+17*10^(1/2))^(1/2)))
```

maxima [B] time = 1.10, size = 668, normalized size = 3.84

$$-\frac{1}{60}\sqrt{10}\left(\frac{588\sqrt{10}\,x}{17\sqrt{10}\sqrt{2x^2+3x+1}+55\sqrt{2x^2+3x+1}}-\frac{588\sqrt{10}\,x}{17\sqrt{10}\sqrt{2x^2+3x+1}-55\sqrt{2x^2+3x+1}}+\frac{}{17\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(-3*x^2+4*x+2)/(2*x^2+3*x+1)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/60*sqrt(10)*(588*sqrt(10)*x/(17*sqrt(10)*sqrt(2*x^2 + 3*x + 1) + 55*sqrt
(2*x^2 + 3*x + 1)) - 588*sqrt(10)*x/(17*sqrt(10)*sqrt(2*x^2 + 3*x + 1) - 55
*sqrt(2*x^2 + 3*x + 1)) + 2112*x/(17*sqrt(10)*sqrt(2*x^2 + 3*x + 1) + 55*sq
rt(2*x^2 + 3*x + 1)) + 2112*x/(17*sqrt(10)*sqrt(2*x^2 + 3*x + 1) - 55*sqrt(
2*x^2 + 3*x + 1)) - 27*sqrt(10)*log(2/9*sqrt(10) + 2/3*sqrt(2*x^2 + 3*x + 1
)*sqrt(17*sqrt(10) + 55)/abs(6*x - 2*sqrt(10) - 4) + 34/9*sqrt(10)/abs(6*x
- 2*sqrt(10) - 4) + 110/9/abs(6*x - 2*sqrt(10) - 4) + 17/18)/(17*sqrt(10) +
 55)^(3/2) - sqrt(10)*log(-2/9*sqrt(10) + 2*sqrt(2*x^2 + 3*x + 1)*sqrt(-17/
9*sqrt(10) + 55/9)/abs(6*x + 2*sqrt(10) - 4) - 34/9*sqrt(10)/abs(6*x + 2*sq
rt(10) - 4) + 110/9/abs(6*x + 2*sqrt(10) - 4) + 17/18)/(-17/9*sqrt(10) + 55
/9)^(3/2) + 450*sqrt(10)/(17*sqrt(10)*sqrt(2*x^2 + 3*x + 1) + 55*sqrt(2*x^2
 + 3*x + 1)) - 450*sqrt(10)/(17*sqrt(10)*sqrt(2*x^2 + 3*x + 1) - 55*sqrt(2*
x^2 + 3*x + 1)) - 216*log(2/9*sqrt(10) + 2/3*sqrt(2*x^2 + 3*x + 1)*sqrt(17*
sqrt(10) + 55)/abs(6*x - 2*sqrt(10) - 4) + 34/9*sqrt(10)/abs(6*x - 2*sqrt(1
0) - 4) + 110/9/abs(6*x - 2*sqrt(10) - 4) + 17/18)/(17*sqrt(10) + 55)^(3/2)
 + 8*log(-2/9*sqrt(10) + 2*sqrt(2*x^2 + 3*x + 1)*sqrt(-17/9*sqrt(10) + 55/9
)/abs(6*x + 2*sqrt(10) - 4) - 34/9*sqrt(10)/abs(6*x + 2*sqrt(10) - 4) + 110
/9/abs(6*x + 2*sqrt(10) - 4) + 17/18)/(-17/9*sqrt(10) + 55/9)^(3/2) + 1656/
(17*sqrt(10)*sqrt(2*x^2 + 3*x + 1) + 55*sqrt(2*x^2 + 3*x + 1)) + 1656/(17*s
qrt(10)*sqrt(2*x^2 + 3*x + 1) - 55*sqrt(2*x^2 + 3*x + 1)))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x+2}{\left(2x^2+3x+1\right)^{3/2}\left(-3x^2+4x+2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 2)/((3*x + 2*x^2 + 1)^(3/2)*(4*x - 3*x^2 + 2)),x)
```

```
[Out] int((x + 2)/((3*x + 2*x^2 + 1)^(3/2)*(4*x - 3*x^2 + 2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{6x^4\sqrt{2x^2+3x+1}+x^3\sqrt{2x^2+3x+1}-13x^2\sqrt{2x^2+3x+1}-10x\sqrt{2x^2+3x+1}-2\sqrt{2x^2+3x+1}}\,dx-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(-3*x**2+4*x+2)/(2*x**2+3*x+1)**(3/2),x)
```

```
[Out] -Integral(x/(6*x**4*sqrt(2*x**2 + 3*x + 1) + x**3*sqrt(2*x**2 + 3*x + 1) -
13*x**2*sqrt(2*x**2 + 3*x + 1) - 10*x*sqrt(2*x**2 + 3*x + 1) - 2*sqrt(2*x**
2 + 3*x + 1)), x) - Integral(2/(6*x**4*sqrt(2*x**2 + 3*x + 1) + x**3*sqrt(2
*x**2 + 3*x + 1) - 13*x**2*sqrt(2*x**2 + 3*x + 1) - 10*x*sqrt(2*x**2 + 3*x
+ 1) - 2*sqrt(2*x**2 + 3*x + 1)), x)
```

3.30 $\int \frac{2+x}{\left(2+4x-3x^2\right)\left(1+3x+2x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=197

$$\frac{2(22x+21)}{15\left(2x^2+3x+1\right)^{3/2}}+\frac{2(230x+273)}{15\sqrt{2x^2+3x+1}}-\frac{1}{50}\sqrt{\frac{1}{3}\left(4885115+1544809\sqrt{10}\right)}\tanh^{-1}\left(\frac{\left(17-4\sqrt{10}\right)x+3\left(4-\sqrt{1}\right.}{2\sqrt{55-17\sqrt{10}}\sqrt{2x^2+3x}}\right.$$

```
[Out] 2/15*(21+22*x)/(2*x^2+3*x+1)^(3/2)+2/15*(273+230*x)/(2*x^2+3*x+1)^(1/2)+1/1
50*arctanh(1/2*(12+3*10^(1/2)+x*(17+4*10^(1/2)))/(2*x^2+3*x+1)^(1/2)/(55+17
*10^(1/2))^(1/2))*(14655345-4634427*10^(1/2))^(1/2)-1/150*arctanh(1/2*(x*(1
7-4*10^(1/2))+12-3*10^(1/2))/(2*x^2+3*x+1)^(1/2)/(55-17*10^(1/2))^(1/2))*(1
4655345+4634427*10^(1/2))^(1/2)
```

Rubi [A] time = 0.30, antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1016, 1060, 1032, 724, 206}

$$\frac{2(22x+21)}{15\left(2x^2+3x+1\right)^{3/2}}+\frac{2(230x+273)}{15\sqrt{2x^2+3x+1}}-\frac{1}{50}\sqrt{\frac{1}{3}\left(4885115+1544809\sqrt{10}\right)}\tanh^{-1}\left(\frac{\left(17-4\sqrt{10}\right)x+3\left(4-\sqrt{1}\right.}{2\sqrt{55-17\sqrt{10}}\sqrt{2x^2+3x}}\right.$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x)/((2 + 4*x - 3*x^2)*(1 + 3*x + 2*x^2)^(5/2)),x]
```

```
[Out] (2*(21 + 22*x))/(15*(1 + 3*x + 2*x^2)^(3/2)) + (2*(273 + 230*x))/(15*Sqrt[1
 + 3*x + 2*x^2]) - (Sqrt[(4885115 + 1544809*Sqrt[10])/3]*ArcTanh[(3*(4 - Sq
rt[10]) + (17 - 4*Sqrt[10])*x)/(2*Sqrt[55 - 17*Sqrt[10]]*Sqrt[1 + 3*x + 2*x
^2])]])/50 + (Sqrt[(4885115 - 1544809*Sqrt[10])/3]*ArcTanh[(3*(4 + Sqrt[10])
 + (17 + 4*Sqrt[10])*x)/(2*Sqrt[55 + 17*Sqrt[10]]*Sqrt[1 + 3*x + 2*x^2])]])/
50
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1016

```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e
_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((a + b*x + c*x^2)^(p + 1)
*(d + e*x + f*x^2)^(q + 1)*(g*c*(2*a*c*e - b*(c*d + a*f)) + (g*b - a*h)*(2*
c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(g*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))
 - h*(b*c*d - 2*a*c*e + a*b*f))*x))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d -
a*e)*(c*e - b*f))*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*
d - a*e)*(c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*
x^2)^q*Simp[(b*h - 2*g*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)
 + (b^2*(g*f) - b*(h*c*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) - a*(-(h*c*e
))))*(a*f*(p + 1) - c*d*(p + 2)) - e*((g*c)*(2*a*c*e - b*(c*d + a*f)) + (g*
b - a*h)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (2*f*((g*c)*(2*
```

```
a*c*e - b*(c*d + a*f)) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(
p + q + 2) - (b^2*g*f - b*(h*c*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) - a*
(-(h*c*e))))*(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(g*f) - b*(h*c
*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) + a*h*c*e))*(2*p + 2*q + 5)*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, q}, x] && NeQ[b^2 - 4*a*c, 0] &
& NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (b*d - a*e)*(c*e
 - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1])
```

Rule 1032

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1060

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((a + b*x +
 c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q + 1)*((A*c - a*C)*(2*a*c*e - b*(c*d +
a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*(2*c^2*d + b
^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*e + a*b*f) + C*(b^2*d - a*b*e -
2*a*(c*d - a*f)))*x))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*
f))*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b
*B - 2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^
2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*
(c*C*d - B*c*e - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
 + q + 2) - (2*f*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^
2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d +
 A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*
(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*
c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*
p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] &&
 NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f
)^2 - (b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !
IGtQ[q, 0]
```

Rubi steps

$$\int \frac{2+x}{(2+4x-3x^2)(1+3x+2x^2)^{5/2}}\,dx = \frac{2(21+22x)}{15(1+3x+2x^2)^{3/2}} - \frac{2}{45}\int \frac{-480-\frac{813x}{2}+396x^2}{(2+4x-3x^2)(1+3x+2x^2)^{3/2}}\,dx$$

$$= \frac{2(21+22x)}{15(1+3x+2x^2)^{3/2}} + \frac{2(273+230x)}{15\sqrt{1+3x+2x^2}} + \frac{4}{675}\int \frac{\frac{23355}{2}-\frac{2713}{4}}{(2+4x-3x^2)\sqrt{1+}}$$

$$= \frac{2(21+22x)}{15(1+3x+2x^2)^{3/2}} + \frac{2(273+230x)}{15\sqrt{1+3x+2x^2}} - \frac{1}{25}\left(3\left(335-106\sqrt{10}\right)\right)\int$$

$$= \frac{2(21+22x)}{15(1+3x+2x^2)^{3/2}} + \frac{2(273+230x)}{15\sqrt{1+3x+2x^2}} + \frac{1}{25}\left(6\left(335-106\sqrt{10}\right)\right)\text{Sub}$$

$$= \frac{2(21+22x)}{15(1+3x+2x^2)^{3/2}} + \frac{2(273+230x)}{15\sqrt{1+3x+2x^2}} - \frac{1}{50}\sqrt{\frac{1}{3}\left(4885115+154480\right.}$$

Mathematica [A] time = 0.71, size = 190, normalized size = 0.96

$$\frac{1}{450}\left(\sqrt{55-17\sqrt{10}}\left(7289+2305\sqrt{10}\right)\tanh^{-1}\left(\frac{\left(4\sqrt{10}-17\right)x+3\left(\sqrt{10}-4\right)}{2\sqrt{55-17\sqrt{10}}\sqrt{2x^2+3x+1}}\right)-\sqrt{55+17\sqrt{10}}\left(2305\sqrt{10}-\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x)/((2 + 4*x - 3*x^2)*(1 + 3*x + 2*x^2)^(5/2)),x]
```

```
[Out] ((60*(294 + 1071*x + 1236*x^2 + 460*x^3))/(1 + 3*x + 2*x^2)^(3/2) + Sqrt[55
 - 17*Sqrt[10]]*(7289 + 2305*Sqrt[10])*ArcTanh[(3*(-4 + Sqrt[10]) + (-17 +
4*Sqrt[10])*x)/(2*Sqrt[55 - 17*Sqrt[10]]*Sqrt[1 + 3*x + 2*x^2])] - Sqrt[55
+ 17*Sqrt[10]]*(-7289 + 2305*Sqrt[10])*ArcTanh[(-3*(4 + Sqrt[10]) - (17 + 4
*Sqrt[10])*x)/(2*Sqrt[55 + 17*Sqrt[10]]*Sqrt[1 + 3*x + 2*x^2])])/450
```

fricas [B] time = 1.02, size = 435, normalized size = 2.21

$$\frac{23520\,x^4+70560\,x^3+\sqrt{3}\left(4\,x^4+12\,x^3+13\,x^2+6\,x+1\right)\sqrt{1544809\,\sqrt{10}+4885115}\,\log\left(-\frac{243\,\sqrt{10}\,x+\left(893\,\sqrt{10}\,\sqrt{3}\,x\right.}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(-3*x^2+4*x+2)/(2*x^2+3*x+1)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/150*(23520*x^4 + 70560*x^3 + sqrt(3)*(4*x^4 + 12*x^3 + 13*x^2 + 6*x + 1)*
sqrt(1544809*sqrt(10) + 4885115)*log(-(243*sqrt(10)*x + (893*sqrt(10)*sqrt(
3)*x - 2824*sqrt(3)*x)*sqrt(1544809*sqrt(10) + 4885115) + 486*x - 486*sqrt(
2*x^2 + 3*x + 1) + 486)/x) - sqrt(3)*(4*x^4 + 12*x^3 + 13*x^2 + 6*x + 1)*sq
rt(1544809*sqrt(10) + 4885115)*log(-(243*sqrt(10)*x - (893*sqrt(10)*sqrt(3)
*x - 2824*sqrt(3)*x)*sqrt(1544809*sqrt(10) + 4885115) + 486*x - 486*sqrt(2*
x^2 + 3*x + 1) + 486)/x) + sqrt(3)*(4*x^4 + 12*x^3 + 13*x^2 + 6*x + 1)*sqrt
(-1544809*sqrt(10) + 4885115)*log((243*sqrt(10)*x + (893*sqrt(10)*sqrt(3)*x
 + 2824*sqrt(3)*x)*sqrt(-1544809*sqrt(10) + 4885115) - 486*x + 486*sqrt(2*x
^2 + 3*x + 1) - 486)/x) - sqrt(3)*(4*x^4 + 12*x^3 + 13*x^2 + 6*x + 1)*sqrt(
-1544809*sqrt(10) + 4885115)*log((243*sqrt(10)*x - (893*sqrt(10)*sqrt(3)*x
+ 2824*sqrt(3)*x)*sqrt(-1544809*sqrt(10) + 4885115) - 486*x + 486*sqrt(2*x^
2 + 3*x + 1) - 486)/x) + 76440*x^2 + 20*(460*x^3 + 1236*x^2 + 1071*x + 294)
```

```
*sqrt(2*x^2 + 3*x + 1) + 35280*x + 5880)/(4*x^4 + 12*x^3 + 13*x^2 + 6*x + 1
)
```

giac [A] time = 0.54, size = 121, normalized size = 0.61

$$\frac{2\left(\left(4\left(115\,x+309\right)x+1071\right)x+294\right)}{15\left(2\,x^2+3\,x+1\right)^{\frac{3}{2}}}+0.00115890443050800\,\log\left(-\sqrt{2}\,x+\sqrt{2\,x^2+3\,x+1}+5.90976932712\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(-3*x^2+4*x+2)/(2*x^2+3*x+1)^(5/2),x, algorithm="giac")
```

```
[Out] 2/15*((4*(115*x + 309)*x + 1071)*x + 294)/(2*x^2 + 3*x + 1)^(3/2) + 0.00115
890443050800*log(-sqrt(2)*x + sqrt(2*x^2 + 3*x + 1) + 5.90976932712000) - 3
6.0928986365333*log(-sqrt(2)*x + sqrt(2*x^2 + 3*x + 1) - 0.176527156327000)
 + 36.0928986365333*log(-sqrt(2)*x + sqrt(2*x^2 + 3*x + 1) - 0.919278730509
000) - 0.00115890442528267*log(-sqrt(2)*x + sqrt(2*x^2 + 3*x + 1) - 1.04272
727395000)
```

maple [B] time = 0.02, size = 878, normalized size = 4.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2+x)/(-3*x^2+4*x+2)/(2*x^2+3*x+1)^(5/2),x)
```

```
[Out] -1/20*(-8+10^(1/2))*10^(1/2)*(1/9/(55/9-17/9*10^(1/2))/(2*(x-2/3+1/3*10^(1/
2))^2+(17/3-4/3*10^(1/2))*(x-2/3+1/3*10^(1/2))+55/9-17/9*10^(1/2))^(3/2)-1/
6*(17/3-4/3*10^(1/2))/(55/9-17/9*10^(1/2))*(2/3*(4*x+3)/(440/9-136/9*10^(1/
2)-(17/3-4/3*10^(1/2))^2)/(2*(x-2/3+1/3*10^(1/2))^2+(17/3-4/3*10^(1/2))*(x-
2/3+1/3*10^(1/2))+55/9-17/9*10^(1/2))^(3/2)+32/3/(440/9-136/9*10^(1/2)-(17/
3-4/3*10^(1/2))^2)^2*(4*x+3)/(2*(x-2/3+1/3*10^(1/2))^2+(17/3-4/3*10^(1/2))*
(x-2/3+1/3*10^(1/2))+55/9-17/9*10^(1/2))^(1/2))+1/3/(55/9-17/9*10^(1/2))*(1
/(55/9-17/9*10^(1/2))/(2*(x-2/3+1/3*10^(1/2))^2+(17/3-4/3*10^(1/2))*(x-2/3+
1/3*10^(1/2))+55/9-17/9*10^(1/2))^(1/2)-(17/3-4/3*10^(1/2))/(55/9-17/9*10^(
1/2))*(4*x+3)/(440/9-136/9*10^(1/2)-(17/3-4/3*10^(1/2))^2)/(2*(x-2/3+1/3*10
^(1/2))^2+(17/3-4/3*10^(1/2))*(x-2/3+1/3*10^(1/2))+55/9-17/9*10^(1/2))^(1/2
)-3/(55/9-17/9*10^(1/2))/(55-17*10^(1/2))^(1/2)*arctanh(9/2*(110/9-34/9*10^
(1/2)+(17/3-4/3*10^(1/2))*(x-2/3+1/3*10^(1/2)))/(55-17*10^(1/2))^(1/2)/(18*
(x-2/3+1/3*10^(1/2))^2+9*(17/3-4/3*10^(1/2))*(x-2/3+1/3*10^(1/2))+55-17*10^
(1/2))^(1/2))))-1/20*(8+10^(1/2))*10^(1/2)*(1/9/(55/9+17/9*10^(1/2))/(2*(x-
2/3-1/3*10^(1/2))^2+(17/3+4/3*10^(1/2))*(x-2/3-1/3*10^(1/2))+55/9+17/9*10^(
1/2))^(3/2)-1/6*(17/3+4/3*10^(1/2))/(55/9+17/9*10^(1/2))*(2/3*(4*x+3)/(440/
9+136/9*10^(1/2)-(17/3+4/3*10^(1/2))^2)/(2*(x-2/3-1/3*10^(1/2))^2+(17/3+4/3
*10^(1/2))*(x-2/3-1/3*10^(1/2))+55/9+17/9*10^(1/2))^(3/2)+32/3/(440/9+136/9
*10^(1/2)-(17/3+4/3*10^(1/2))^2)^2*(4*x+3)/(2*(x-2/3-1/3*10^(1/2))^2+(17/3+
4/3*10^(1/2))*(x-2/3-1/3*10^(1/2))+55/9+17/9*10^(1/2))^(1/2))+1/3/(55/9+17/
9*10^(1/2))*(1/(55/9+17/9*10^(1/2))/(2*(x-2/3-1/3*10^(1/2))^2+(17/3+4/3*10^
(1/2))*(x-2/3-1/3*10^(1/2))+55/9+17/9*10^(1/2))^(1/2)-(17/3+4/3*10^(1/2))/(
55/9+17/9*10^(1/2))*(4*x+3)/(440/9+136/9*10^(1/2)-(17/3+4/3*10^(1/2))^2)/(2
*(x-2/3-1/3*10^(1/2))^2+(17/3+4/3*10^(1/2))*(x-2/3-1/3*10^(1/2))+55/9+17/9*
10^(1/2))^(1/2)-3/(55/9+17/9*10^(1/2))/(55+17*10^(1/2))^(1/2)*arctanh(9/2*(
110/9+34/9*10^(1/2)+(17/3+4/3*10^(1/2))*(x-2/3-1/3*10^(1/2)))/(55+17*10^(1/
2))^(1/2)/(18*(x-2/3-1/3*10^(1/2))^2+9*(17/3+4/3*10^(1/2))*(x-2/3-1/3*10^(1
/2))+55+17*10^(1/2))^(1/2))))
```

maxima [B] time = 1.24, size = 1276, normalized size = 6.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(-3*x^2+4*x+2)/(2*x^2+3*x+1)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/300*sqrt(10)*(980*sqrt(10)*x/(17*sqrt(10)*(2*x^2 + 3*x + 1)^(3/2) + 55*(
2*x^2 + 3*x + 1)^(3/2)) - 980*sqrt(10)*x/(17*sqrt(10)*(2*x^2 + 3*x + 1)^(3/
2) - 55*(2*x^2 + 3*x + 1)^(3/2)) + 5292*sqrt(10)*x/(374*sqrt(10)*sqrt(2*x^2
 + 3*x + 1) + 1183*sqrt(2*x^2 + 3*x + 1)) - 5292*sqrt(10)*x/(374*sqrt(10)*s
qrt(2*x^2 + 3*x + 1) - 1183*sqrt(2*x^2 + 3*x + 1)) - 15680*sqrt(10)*x/(17*s
qrt(10)*sqrt(2*x^2 + 3*x + 1) + 55*sqrt(2*x^2 + 3*x + 1)) + 15680*sqrt(10)*
x/(17*sqrt(10)*sqrt(2*x^2 + 3*x + 1) - 55*sqrt(2*x^2 + 3*x + 1)) + 3520*x/(
17*sqrt(10)*(2*x^2 + 3*x + 1)^(3/2) + 55*(2*x^2 + 3*x + 1)^(3/2)) + 3520*x/
(17*sqrt(10)*(2*x^2 + 3*x + 1)^(3/2) - 55*(2*x^2 + 3*x + 1)^(3/2)) + 19008*
x/(374*sqrt(10)*sqrt(2*x^2 + 3*x + 1) + 1183*sqrt(2*x^2 + 3*x + 1)) + 19008
*x/(374*sqrt(10)*sqrt(2*x^2 + 3*x + 1) - 1183*sqrt(2*x^2 + 3*x + 1)) - 5632
0*x/(17*sqrt(10)*sqrt(2*x^2 + 3*x + 1) + 55*sqrt(2*x^2 + 3*x + 1)) - 56320*
x/(17*sqrt(10)*sqrt(2*x^2 + 3*x + 1) - 55*sqrt(2*x^2 + 3*x + 1)) + 750*sqrt
(10)/(17*sqrt(10)*(2*x^2 + 3*x + 1)^(3/2) + 55*(2*x^2 + 3*x + 1)^(3/2)) - 7
50*sqrt(10)/(17*sqrt(10)*(2*x^2 + 3*x + 1)^(3/2) - 55*(2*x^2 + 3*x + 1)^(3/
2)) + 4050*sqrt(10)/(374*sqrt(10)*sqrt(2*x^2 + 3*x + 1) + 1183*sqrt(2*x^2 +
 3*x + 1)) - 4050*sqrt(10)/(374*sqrt(10)*sqrt(2*x^2 + 3*x + 1) - 1183*sqrt(
2*x^2 + 3*x + 1)) - 11760*sqrt(10)/(17*sqrt(10)*sqrt(2*x^2 + 3*x + 1) + 55*
sqrt(2*x^2 + 3*x + 1)) + 11760*sqrt(10)/(17*sqrt(10)*sqrt(2*x^2 + 3*x + 1)
 - 55*sqrt(2*x^2 + 3*x + 1)) + 2760/(17*sqrt(10)*(2*x^2 + 3*x + 1)^(3/2) + 5
5*(2*x^2 + 3*x + 1)^(3/2)) + 2760/(17*sqrt(10)*(2*x^2 + 3*x + 1)^(3/2) - 55
*(2*x^2 + 3*x + 1)^(3/2)) + 14904/(374*sqrt(10)*sqrt(2*x^2 + 3*x + 1) + 118
3*sqrt(2*x^2 + 3*x + 1)) + 14904/(374*sqrt(10)*sqrt(2*x^2 + 3*x + 1) - 1183
*sqrt(2*x^2 + 3*x + 1)) - 42240/(17*sqrt(10)*sqrt(2*x^2 + 3*x + 1) + 55*sqr
t(2*x^2 + 3*x + 1)) - 42240/(17*sqrt(10)*sqrt(2*x^2 + 3*x + 1) - 55*sqrt(2*
x^2 + 3*x + 1)) - 1215*sqrt(10)*log(2/9*sqrt(10) + 2/3*sqrt(2*x^2 + 3*x + 1
)*sqrt(17*sqrt(10) + 55)/abs(6*x - 2*sqrt(10) - 4) + 34/9*sqrt(10)/abs(6*x
 - 2*sqrt(10) - 4) + 110/9/abs(6*x - 2*sqrt(10) - 4) + 17/18)/(17*sqrt(10) +
 55)^(5/2) - 5*sqrt(10)*log(-2/9*sqrt(10) + 2*sqrt(2*x^2 + 3*x + 1)*sqrt(-1
7/9*sqrt(10) + 55/9)/abs(6*x + 2*sqrt(10) - 4) - 34/9*sqrt(10)/abs(6*x + 2*
sqrt(10) - 4) + 110/9/abs(6*x + 2*sqrt(10) - 4) + 17/18)/(-17/9*sqrt(10) +
55/9)^(5/2) - 9720*log(2/9*sqrt(10) + 2/3*sqrt(2*x^2 + 3*x + 1)*sqrt(17*sqr
t(10) + 55)/abs(6*x - 2*sqrt(10) - 4) + 34/9*sqrt(10)/abs(6*x - 2*sqrt(10)
 - 4) + 110/9/abs(6*x - 2*sqrt(10) - 4) + 17/18)/(17*sqrt(10) + 55)^(5/2) +
40*log(-2/9*sqrt(10) + 2*sqrt(2*x^2 + 3*x + 1)*sqrt(-17/9*sqrt(10) + 55/9)/
abs(6*x + 2*sqrt(10) - 4) - 34/9*sqrt(10)/abs(6*x + 2*sqrt(10) - 4) + 110/9
/abs(6*x + 2*sqrt(10) - 4) + 17/18)/(-17/9*sqrt(10) + 55/9)^(5/2))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x+2}{\left(2\,x^2+3\,x+1\right)^{5/2}\,\left(-3\,x^2+4\,x+2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 2)/((3*x + 2*x^2 + 1)^(5/2)*(4*x - 3*x^2 + 2)),x)
```

```
[Out] int((x + 2)/((3*x + 2*x^2 + 1)^(5/2)*(4*x - 3*x^2 + 2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{12x^6\sqrt{2x^2+3x+1}+20x^5\sqrt{2x^2+3x+1}-17x^4\sqrt{2x^2+3x+1}-58x^3\sqrt{2x^2+3x+1}-47x^2\sqrt{2x^2+3x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(-3*x**2+4*x+2)/(2*x**2+3*x+1)**(5/2),x)
```

```
[Out] -Integral(x/(12*x**6*sqrt(2*x**2 + 3*x + 1) + 20*x**5*sqrt(2*x**2 + 3*x + 1
) - 17*x**4*sqrt(2*x**2 + 3*x + 1) - 58*x**3*sqrt(2*x**2 + 3*x + 1) - 47*x**
```

```
*2*sqrt(2*x**2 + 3*x + 1) - 16*x*sqrt(2*x**2 + 3*x + 1) - 2*sqrt(2*x**2 + 3
*x + 1)), x) - Integral(2/(12*x**6*sqrt(2*x**2 + 3*x + 1) + 20*x**5*sqrt(2*
x**2 + 3*x + 1) - 17*x**4*sqrt(2*x**2 + 3*x + 1) - 58*x**3*sqrt(2*x**2 + 3*
x + 1) - 47*x**2*sqrt(2*x**2 + 3*x + 1) - 16*x*sqrt(2*x**2 + 3*x + 1) - 2*s
qrt(2*x**2 + 3*x + 1)), x)
```

3.31 $\int \frac{1+x}{(4+2x+x^2)\sqrt{5+2x+x^2}}\,dx$

Optimal. Leaf size=15

$$-\tanh^{-1}\left(\sqrt{x^2+2x+5}\right)$$

```
[Out] -arctanh((x^2+2*x+5)^(1/2))
```

Rubi [A] time = 0.02, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1024, 206}

$$-\tanh^{-1}\left(\sqrt{x^2+2x+5}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)/((4 + 2*x + x^2)*Sqrt[5 + 2*x + x^2]),x]
```

```
[Out] -ArcTanh[Sqrt[5 + 2*x + x^2]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1024

```
Int[((g_) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e
_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[-2*g, Subst[Int[1/(b*d - a*e -
 b*x^2), x], x, Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h
}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e - b*f, 0] &&
 EqQ[h*e - 2*g*f, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1+x}{(4+2x+x^2)\sqrt{5+2x+x^2}}\,dx &= -\left(2\,\text{Subst}\left(\int \frac{1}{2-2x^2}\,dx, x, \sqrt{5+2x+x^2}\right)\right)\\ &= -\tanh^{-1}\left(\sqrt{5+2x+x^2}\right)\end{aligned}$$

Mathematica [C] time = 0.04, size = 79, normalized size = 5.27

$$\frac{1}{2}\left(-\tanh^{-1}\left(\frac{-i\sqrt{3}\,x-i\sqrt{3}+4}{\sqrt{x^2+2x+5}}\right)-\tanh^{-1}\left(\frac{i\sqrt{3}\,x+i\sqrt{3}+4}{\sqrt{x^2+2x+5}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)/((4 + 2*x + x^2)*Sqrt[5 + 2*x + x^2]),x]
```

```
[Out] (-ArcTanh[(4 - I*Sqrt[3] - I*Sqrt[3]*x)/Sqrt[5 + 2*x + x^2]] - ArcTanh[(4 +
 I*Sqrt[3] + I*Sqrt[3]*x)/Sqrt[5 + 2*x + x^2]])/2
```

fricas [B] time = 0.87, size = 49, normalized size = 3.27

$$\frac{1}{2}\log\left(x^2-\sqrt{x^2+2\,x+5}\,(x+2)+3\,x+6\right)-\frac{1}{2}\log\left(x^2-\sqrt{x^2+2\,x+5}\,x+x+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(x^2+2*x+4)/(x^2+2*x+5)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*log(x^2 - sqrt(x^2 + 2*x + 5)*(x + 2) + 3*x + 6) - 1/2*log(x^2 - sqrt(x
^2 + 2*x + 5)*x + x + 4)
```

giac [B] time = 0.35, size = 31, normalized size = 2.07

$$-\frac{1}{2}\log\left(\sqrt{x^2+2x+5}+1\right)+\frac{1}{2}\log\left(\sqrt{x^2+2x+5}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(x^2+2*x+4)/(x^2+2*x+5)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*log(sqrt(x^2 + 2*x + 5) + 1) + 1/2*log(sqrt(x^2 + 2*x + 5) - 1)
```

maple [A] time = 0.02, size = 14, normalized size = 0.93

$$-\operatorname{arctanh}\left(\sqrt{x^2+2x+5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+1)/(x^2+2*x+4)/(x^2+2*x+5)^(1/2),x)
```

```
[Out] -arctanh((x^2+2*x+5)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x+1}{\sqrt{x^2+2x+5}\left(x^2+2x+4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(x^2+2*x+4)/(x^2+2*x+5)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x + 1)/(sqrt(x^2 + 2*x + 5)*(x^2 + 2*x + 4)), x)
```

mupad [B] time = 3.76, size = 13, normalized size = 0.87

$$-\operatorname{atanh}\left(\sqrt{x^2+2x+5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)/((2*x + x^2 + 4)*(2*x + x^2 + 5)^(1/2)),x)
```

```
[Out] -atanh((2*x + x^2 + 5)^(1/2))
```

sympy [B] time = 7.00, size = 36, normalized size = 2.40

$$\frac{\log\left(-1+\frac{1}{\sqrt{x^2+2x+5}}\right)}{2}-\frac{\log\left(1+\frac{1}{\sqrt{x^2+2x+5}}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(x**2+2*x+4)/(x**2+2*x+5)**(1/2),x)
```

```
[Out] log(-1 + 1/sqrt(x**2 + 2*x + 5))/2 - log(1 + 1/sqrt(x**2 + 2*x + 5))/2
```

3.32 $\int \frac{4+x}{\left(4+2x+x^2\right)\sqrt{5+2x+x^2}}\,dx$

Optimal. Leaf size=44

$$\sqrt{3}\tan^{-1}\left(\frac{x+1}{\sqrt{3}\sqrt{x^2+2x+5}}\right)-\tanh^{-1}\left(\sqrt{x^2+2x+5}\right)$$

```
[Out] -arctanh((x^2+2*x+5)^(1/2))+arctan(1/3*(1+x)*3^(1/2)/(x^2+2*x+5)^(1/2))*3^(
1/2)
```

Rubi [A] time = 0.05, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {1025, 982, 204, 1024, 206}

$$\sqrt{3}\tan^{-1}\left(\frac{x+1}{\sqrt{3}\sqrt{x^2+2x+5}}\right)-\tanh^{-1}\left(\sqrt{x^2+2x+5}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(4 + x)/((4 + 2*x + x^2)*Sqrt[5 + 2*x + x^2]),x]
```

```
[Out] Sqrt[3]*ArcTan[(1 + x)/(Sqrt[3]*Sqrt[5 + 2*x + x^2])] - ArcTanh[Sqrt[5 + 2*
x + x^2]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 982

```
Int[1/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(
x_)^2]), x_Symbol] :> Dist[-2*e, Subst[Int[1/(e*(b*e - 4*a*f) - (b*d - a*e)
*x^2), x], x, (e + 2*f*x)/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e - b*f, 0
]
```

Rule 1024

```
Int[((g_) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e
_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[-2*g, Subst[Int[1/(b*d - a*e -
 b*x^2), x], x, Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h
}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e - b*f, 0] &&
 EqQ[h*e - 2*g*f, 0]
```

Rule 1025

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> -Dist[(h*e - 2*g*f)/(2*f), Int[1/(
(a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/(2*f), Int[(e + 2
*f*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c,
d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e
```

```
- b*f, 0] && NeQ[h*e - 2*g*f, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{4+x}{(4+2x+x^2)\sqrt{5+2x+x^2}}\,dx &= \frac{1}{2}\int \frac{2+2x}{(4+2x+x^2)\sqrt{5+2x+x^2}}\,dx+3\int \frac{1}{(4+2x+x^2)\sqrt{5+2x+x^2}}\,d\\ &= -\left(2\,\mathrm{Subst}\left(\int \frac{1}{2-2x^2}\,dx,x,\sqrt{5+2x+x^2}\right)\right)-12\,\mathrm{Subst}\left(\int \frac{1}{-24-2x^2}\,d\right.\\ &= \sqrt{3}\,\tan^{-1}\left(\frac{1+x}{\sqrt{3}\sqrt{5+2x+x^2}}\right)-\tanh^{-1}\left(\sqrt{5+2x+x^2}\right)\end{aligned}$$

Mathematica [C] time = 0.05, size = 101, normalized size = 2.30

$$-\frac{1}{2}\left(1+i\sqrt{3}\right)\tanh^{-1}\left(\frac{-i\sqrt{3}x-i\sqrt{3}+4}{\sqrt{x^2+2x+5}}\right)-\frac{1}{2}\left(1-i\sqrt{3}\right)\tanh^{-1}\left(\frac{i\sqrt{3}x+i\sqrt{3}+4}{\sqrt{x^2+2x+5}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 + x)/((4 + 2*x + x^2)*Sqrt[5 + 2*x + x^2]),x]
```

```
[Out] -1/2*((1 + I*Sqrt[3])*ArcTanh[(4 - I*Sqrt[3] - I*Sqrt[3]*x)/Sqrt[5 + 2*x +
x^2]]) - ((1 - I*Sqrt[3])*ArcTanh[(4 + I*Sqrt[3] + I*Sqrt[3]*x)/Sqrt[5 + 2*
x + x^2]])/2
```

fricas [B] time = 0.99, size = 106, normalized size = 2.41

$$-\sqrt{3}\arctan\left(-\frac{1}{3}\sqrt{3}(x+2)+\frac{1}{3}\sqrt{3}\sqrt{x^2+2x+5}\right)+\sqrt{3}\arctan\left(-\frac{1}{3}\sqrt{3}x+\frac{1}{3}\sqrt{3}\sqrt{x^2+2x+5}\right)+\frac{1}{2}\log\left(x^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4+x)/(x^2+2*x+4)/(x^2+2*x+5)^(1/2),x, algorithm="fricas")
```

```
[Out] -sqrt(3)*arctan(-1/3*sqrt(3)*(x + 2) + 1/3*sqrt(3)*sqrt(x^2 + 2*x + 5)) + s
qrt(3)*arctan(-1/3*sqrt(3)*x + 1/3*sqrt(3)*sqrt(x^2 + 2*x + 5)) + 1/2*log(x
^2 - sqrt(x^2 + 2*x + 5)*(x + 2) + 3*x + 6) - 1/2*log(x^2 - sqrt(x^2 + 2*x
+ 5)*x + x + 4)
```

giac [B] time = 0.37, size = 108, normalized size = 2.45

$$-\sqrt{3}\arctan\left(-\frac{1}{3}\sqrt{3}\left(x-\sqrt{x^2+2x+5}+2\right)\right)+\sqrt{3}\arctan\left(-\frac{1}{3}\sqrt{3}\left(x-\sqrt{x^2+2x+5}\right)\right)+\frac{1}{2}\log\left(\left(x-\sqrt{x^2+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4+x)/(x^2+2*x+4)/(x^2+2*x+5)^(1/2),x, algorithm="giac")
```

```
[Out] -sqrt(3)*arctan(-1/3*sqrt(3)*(x - sqrt(x^2 + 2*x + 5) + 2)) + sqrt(3)*arcta
n(-1/3*sqrt(3)*(x - sqrt(x^2 + 2*x + 5))) + 1/2*log((x - sqrt(x^2 + 2*x + 5
))^2 + 4*x - 4*sqrt(x^2 + 2*x + 5) + 7) - 1/2*log((x - sqrt(x^2 + 2*x + 5))
^2 + 3)
```

maple [A] time = 0.01, size = 40, normalized size = 0.91

$$-\operatorname{arctanh}\left(\sqrt{x^2+2x+5}\right)+\sqrt{3}\arctan\left(\frac{\sqrt{3}\,(2x+2)}{6\sqrt{x^2+2x+5}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4+x)/(x^2+2*x+4)/(x^2+2*x+5)^(1/2),x)
```

```
[Out] -arctanh((x^2+2*x+5)^(1/2))+3^(1/2)*arctan(1/6*3^(1/2)/(x^2+2*x+5)^(1/2)*(2
*x+2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+4}{\sqrt{x^2+2\,x+5}\left(x^2+2\,x+4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4+x)/(x^2+2*x+4)/(x^2+2*x+5)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x + 4)/(sqrt(x^2 + 2*x + 5)*(x^2 + 2*x + 4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x+4}{\left(x^2+2\,x+4\right)\,\sqrt{x^2+2\,x+5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 4)/((2*x + x^2 + 4)*(2*x + x^2 + 5)^(1/2)),x)
```

```
[Out] int((x + 4)/((2*x + x^2 + 4)*(2*x + x^2 + 5)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+4}{\left(x^2+2x+4\right)\sqrt{x^2+2x+5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4+x)/(x**2+2*x+4)/(x**2+2*x+5)**(1/2),x)
```

```
[Out] Integral((x + 4)/((x**2 + 2*x + 4)*sqrt(x**2 + 2*x + 5)), x)
```

3.33 $\int \frac{1+2x}{(3+x+x^2)\sqrt{5+x+x^2}} \, dx$

Optimal. Leaf size=24

$$-\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{x^2+x+5}}{\sqrt{2}}\right)$$

```
[Out] -arctanh(1/2*(x^2+x+5)^(1/2)*2^(1/2))*2^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1024, 206}

$$-\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{x^2+x+5}}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x)/((3 + x + x^2)*Sqrt[5 + x + x^2]),x]
```

```
[Out] -(Sqrt[2]*ArcTanh[Sqrt[5 + x + x^2]/Sqrt[2]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1024

```
Int[((g_) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e
_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[-2*g, Subst[Int[1/(b*d - a*e -
 b*x^2), x], x, Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h
}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e - b*f, 0] &&
 EqQ[h*e - 2*g*f, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1+2x}{(3+x+x^2)\sqrt{5+x+x^2}} \, dx &= -\left(2\operatorname{Subst}\left(\int \frac{1}{2-x^2} \, dx, x, \sqrt{5+x+x^2}\right)\right)\\ &= -\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{5+x+x^2}}{\sqrt{2}}\right)\end{aligned}$$

Mathematica [C] time = 0.06, size = 90, normalized size = 3.75

$$-\frac{\tanh^{-1}\left(\frac{-2i\sqrt{11}x-i\sqrt{11}+19}{4\sqrt{2}\sqrt{x^2+x+5}}\right)+\tanh^{-1}\left(\frac{2i\sqrt{11}x+i\sqrt{11}+19}{4\sqrt{2}\sqrt{x^2+x+5}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x)/((3 + x + x^2)*Sqrt[5 + x + x^2]),x]
```

```
[Out] -((ArcTanh[(19 - I*Sqrt[11] - (2*I)*Sqrt[11]*x)/(4*Sqrt[2]*Sqrt[5 + x + x^2
])] + ArcTanh[(19 + I*Sqrt[11] + (2*I)*Sqrt[11]*x)/(4*Sqrt[2]*Sqrt[5 + x +
x^2])])/Sqrt[2])
```

fricas [A] time = 1.10, size = 34, normalized size = 1.42

$$\frac{1}{2}\sqrt{2}\log\left(\frac{x^2 - 2\sqrt{2}\sqrt{x^2 + x + 5} + x + 7}{x^2 + x + 3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)/(x^2+x+3)/(x^2+x+5)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(2)*log((x^2 - 2*sqrt(2)*sqrt(x^2 + x + 5) + x + 7)/(x^2 + x + 3))
```

giac [B] time = 0.36, size = 39, normalized size = 1.62

$$-\frac{1}{2}\sqrt{2}\log\left(\sqrt{2} + \sqrt{x^2 + x + 5}\right) + \frac{1}{2}\sqrt{2}\log\left(-\sqrt{2} + \sqrt{x^2 + x + 5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)/(x^2+x+3)/(x^2+x+5)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*log(sqrt(2) + sqrt(x^2 + x + 5)) + 1/2*sqrt(2)*log(-sqrt(2) +
sqrt(x^2 + x + 5))
```

maple [A] time = 0.02, size = 20, normalized size = 0.83

$$-\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{x^2 + x + 5}\,\sqrt{2}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+2*x)/(x^2+x+3)/(x^2+x+5)^(1/2),x)
```

```
[Out] -arctanh(1/2*(x^2+x+5)^(1/2)*2^(1/2))*2^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2x + 1}{\sqrt{x^2 + x + 5}\left(x^2 + x + 3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)/(x^2+x+3)/(x^2+x+5)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((2*x + 1)/(sqrt(x^2 + x + 5)*(x^2 + x + 3)), x)
```

mupad [B] time = 3.78, size = 19, normalized size = 0.79

$$-\sqrt{2}\operatorname{atanh}\left(\frac{\sqrt{2}\,\sqrt{x^2 + x + 5}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x + 1)/((x + x^2 + 3)*(x + x^2 + 5)^(1/2)),x)
```

```
[Out] -2^(1/2)*atanh((2^(1/2)*(x + x^2 + 5)^(1/2))/2)
```

sympy [A] time = 6.81, size = 68, normalized size = 2.83

$$2\left(\begin{cases} -\frac{\sqrt{2}\operatorname{acoth}\left(\frac{\sqrt{2}}{\sqrt{x^2+x+5}}\right)}{2} & \text{for}\ \frac{1}{x^2+x+5} > \frac{1}{2} \\ -\frac{\sqrt{2}\operatorname{atanh}\left(\frac{\sqrt{2}}{\sqrt{x^2+x+5}}\right)}{2} & \text{for}\ \frac{1}{x^2+x+5} < \frac{1}{2} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)/(x**2+x+3)/(x**2+x+5)**(1/2),x)
```

```
[Out] 2*Piecewise((-sqrt(2)*acoth(sqrt(2)/sqrt(x**2 + x + 5))/2, 1/(x**2 + x + 5)
 > 1/2), (-sqrt(2)*atanh(sqrt(2)/sqrt(x**2 + x + 5))/2, 1/(x**2 + x + 5) <
1/2))
```

3.34 $\int \frac{x}{(3+x+x^2)\sqrt{5+x+x^2}}\,dx$

Optimal. Leaf size=56

$$-\frac{\tan^{-1}\left(\frac{\sqrt{\frac{2}{11}}(2x+1)}{\sqrt{x^2+x+5}}\right)}{\sqrt{22}}-\frac{\tanh^{-1}\left(\frac{\sqrt{x^2+x+5}}{\sqrt{2}}\right)}{\sqrt{2}}$$

[Out] -1/2*arctanh(1/2*(x^2+x+5)^(1/2)*2^(1/2))*2^(1/2)-1/22*arctan(1/11*(1+2*x)*22^(1/2)/(x^2+x+5)^(1/2))*22^(1/2)

Rubi [A] time = 0.05, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1025, 982, 204, 1024, 206}

$$-\frac{\tan^{-1}\left(\frac{\sqrt{\frac{2}{11}}(2x+1)}{\sqrt{x^2+x+5}}\right)}{\sqrt{22}}-\frac{\tanh^{-1}\left(\frac{\sqrt{x^2+x+5}}{\sqrt{2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[x/((3 + x + x^2)*Sqrt[5 + x + x^2]),x]

[Out] -(ArcTan[(Sqrt[2/11]*(1 + 2*x))/Sqrt[5 + x + x^2]]/Sqrt[22]) - ArcTanh[Sqrt[5 + x + x^2]/Sqrt[2]]/Sqrt[2]

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rule 982

```
Int[1/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[-2*e, Subst[Int[1/(e*(b*e - 4*a*f) - (b*d - a*e)*x^2), x], x, (e + 2*f*x)/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e - b*f, 0]
```

Rule 1024

```
Int[((g_) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[-2*g, Subst[Int[1/(b*d - a*e - b*x^2), x], x, Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e - b*f, 0] && EqQ[h*e - 2*g*f, 0]
```

Rule 1025

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> -Dist[(h*e - 2*g*f)/(2*f), Int[1/(
```

```
(a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/(2*f), Int[(e + 2
*f*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c,
d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e
 - b*f, 0] && NeQ[h*e - 2*g*f, 0]
```

Rubi steps

$$\int \frac{x}{\left(3+x+x^2\right)\sqrt{5+x+x^2}}\,dx = -\left(\frac{1}{2}\int \frac{1}{\left(3+x+x^2\right)\sqrt{5+x+x^2}}\,dx\right)+\frac{1}{2}\int \frac{1+2x}{\left(3+x+x^2\right)\sqrt{5+x+x^2}}\,dx$$

$$= \text{Subst}\left(\int \frac{1}{-11-2x^2}\,dx, x, \frac{1+2x}{\sqrt{5+x+x^2}}\right)-\text{Subst}\left(\int \frac{1}{2-x^2}\,dx, x, \sqrt{5+x}\right.$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{\frac{2}{11}}(1+2x)}{\sqrt{5+x+x^2}}\right)}{\sqrt{22}}-\frac{\tanh^{-1}\left(\frac{\sqrt{5+x+x^2}}{\sqrt{2}}\right)}{\sqrt{2}}$$

Mathematica [C] time = 0.06, size = 114, normalized size = 2.04

$$\frac{-\left(\left(\sqrt{11}-i\right)\tanh^{-1}\left(\frac{-2i\sqrt{11}x-i\sqrt{11}+19}{4\sqrt{2}\sqrt{x^2+x+5}}\right)\right)-\left(\sqrt{11}+i\right)\tanh^{-1}\left(\frac{2i\sqrt{11}x+i\sqrt{11}+19}{4\sqrt{2}\sqrt{x^2+x+5}}\right)}{2\sqrt{22}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((3 + x + x^2)*Sqrt[5 + x + x^2]),x]
```

```
[Out] (-((-I + Sqrt[11])*ArcTanh[(19 - I*Sqrt[11] - (2*I)*Sqrt[11]*x)/(4*Sqrt[2]*
Sqrt[5 + x + x^2])]) - (I + Sqrt[11])*ArcTanh[(19 + I*Sqrt[11] + (2*I)*Sqrt
[11]*x)/(4*Sqrt[2]*Sqrt[5 + x + x^2])])/(2*Sqrt[22])
```

fricas [B] time = 1.10, size = 307, normalized size = 5.48

$$-\frac{1}{33}\sqrt{11}\sqrt{6}\sqrt{3}\arctan\left(\frac{2}{33}\sqrt{11}\sqrt{3}\sqrt{\sqrt{6}\sqrt{3}(2x+1)+6x^2-\sqrt{x^2+x+5}\left(2\sqrt{6}\sqrt{3}+6x+3\right)+6x+30}\right.+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2+x+3)/(x^2+x+5)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/33*sqrt(11)*sqrt(6)*sqrt(3)*arctan(2/33*sqrt(11)*sqrt(3)*sqrt(sqrt(6)*sq
rt(3)*(2*x + 1) + 6*x^2 - sqrt(x^2 + x + 5)*(2*sqrt(6)*sqrt(3) + 6*x + 3) +
 6*x + 30) + 1/33*sqrt(11)*(2*sqrt(6)*sqrt(3) + 6*x + 3) - 2/11*sqrt(11)*sq
rt(x^2 + x + 5)) + 1/33*sqrt(11)*sqrt(6)*sqrt(3)*arctan(-1/33*sqrt(11)*(2*s
qrt(6)*sqrt(3) - 6*x - 3) + 1/33*sqrt(11)*sqrt(-12*sqrt(6)*sqrt(3)*(2*x + 1
) + 72*x^2 + 12*sqrt(x^2 + x + 5)*(2*sqrt(6)*sqrt(3) - 6*x - 3) + 72*x + 36
0) - 2/11*sqrt(11)*sqrt(x^2 + x + 5)) + 1/12*sqrt(6)*sqrt(3)*log(12*sqrt(6)
*sqrt(3)*(2*x + 1) + 72*x^2 - 12*sqrt(x^2 + x + 5)*(2*sqrt(6)*sqrt(3) + 6*x
 + 3) + 72*x + 360) - 1/12*sqrt(6)*sqrt(3)*log(-12*sqrt(6)*sqrt(3)*(2*x + 1
) + 72*x^2 + 12*sqrt(x^2 + x + 5)*(2*sqrt(6)*sqrt(3) - 6*x - 3) + 72*x + 36
0)
```

giac [B] time = 0.32, size = 133, normalized size = 2.38

$$\frac{1}{22}\sqrt{11}\sqrt{2}\arctan\left(-\frac{1}{11}\sqrt{11}\left(2x+2\sqrt{2}-2\sqrt{x^2+x+5}+1\right)\right)-\frac{1}{22}\sqrt{11}\sqrt{2}\arctan\left(-\frac{1}{11}\sqrt{11}\left(2x-2\sqrt{2}-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2+x+3)/(x^2+x+5)^(1/2),x, algorithm="giac")
```

```
[Out] 1/22*sqrt(11)*sqrt(2)*arctan(-1/11*sqrt(11)*(2*x + 2*sqrt(2) - 2*sqrt(x^2 +
 x + 5) + 1)) - 1/22*sqrt(11)*sqrt(2)*arctan(-1/11*sqrt(11)*(2*x - 2*sqrt(2
) - 2*sqrt(x^2 + x + 5) + 1)) + 1/4*sqrt(2)*log(324*(2*x + 2*sqrt(2) - 2*sq
rt(x^2 + x + 5) + 1)^2 + 3564) - 1/4*sqrt(2)*log(324*(2*x - 2*sqrt(2) - 2*s
qrt(x^2 + x + 5) + 1)^2 + 3564)
```

maple [A] time = 0.01, size = 45, normalized size = 0.80

$$-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{x^2+x+5}\,\sqrt{2}}{2}\right)}{2}-\frac{\sqrt{22}\,\arctan\left(\frac{(2x+1)\sqrt{22}}{11\sqrt{x^2+x+5}}\right)}{22}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^2+x+3)/(x^2+x+5)^(1/2),x)
```

```
[Out] -1/2*arctanh(1/2*(x^2+x+5)^(1/2)*2^(1/2))*2^(1/2)-1/22*arctan(1/11*(1+2*x)*
22^(1/2)/(x^2+x+5)^(1/2))*22^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x}{\sqrt{x^2+x+5}\left(x^2+x+3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2+x+3)/(x^2+x+5)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/(sqrt(x^2 + x + 5)*(x^2 + x + 3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{x}{\left(x^2+x+3\right)\,\sqrt{x^2+x+5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((x + x^2 + 3)*(x + x^2 + 5)^(1/2)),x)
```

```
[Out] int(x/((x + x^2 + 3)*(x + x^2 + 5)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x}{\left(x^2+x+3\right)\sqrt{x^2+x+5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**2+x+3)/(x**2+x+5)**(1/2),x)
```

```
[Out] Integral(x/((x**2 + x + 3)*sqrt(x**2 + x + 5)), x)
```

3.35 $\int \frac{A+Bx}{\sqrt{d+ex+fx^2}\left(ae+bex+bfx^2\right)^2}\,dx$

Optimal. Leaf size=249

$$\frac{(Be-2Af)\left(8aef-b\left(4df+e^2\right)\right)\tanh^{-1}\left(\frac{(e+2fx)\sqrt{bd-ae}}{\sqrt{e}\sqrt{be-4af}\sqrt{d+ex+fx^2}}\right)}{2e^{3/2}f(bd-ae)^{3/2}(be-4af)^{3/2}}-\frac{\sqrt{d+ex+fx^2}\,(e(Ab-2aB)-bx(Be-2Af))}{e(bd-ae)(be-4af)\left(ae+bex+bfx^2\right)}$$

```
[Out] 1/2*(-2*A*f+B*e)*(8*a*e*f-b*(4*d*f+e^2))*arctanh((2*f*x+e)*(-a*e+b*d)^(1/2)
/e^(1/2)/(-4*a*f+b*e)^(1/2)/(f*x^2+e*x+d)^(1/2))/e^(3/2)/(-a*e+b*d)^(3/2)/f
/(-4*a*f+b*e)^(3/2)+1/2*B*arctanh(b^(1/2)*(f*x^2+e*x+d)^(1/2)/(-a*e+b*d)^(1
/2))/(-a*e+b*d)^(3/2)/f/b^(1/2)-((A*b-2*B*a)*e-b*(-2*A*f+B*e)*x)*(f*x^2+e*x
+d)^(1/2)/e/(-a*e+b*d)/(-4*a*f+b*e)/(b*f*x^2+b*e*x+a*e)
```

Rubi [A] time = 0.91, antiderivative size = 249, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.139, Rules used = {1016, 1025, 982, 208, 1024}

$$\frac{(Be-2Af)\left(8aef-b\left(4df+e^2\right)\right)\tanh^{-1}\left(\frac{(e+2fx)\sqrt{bd-ae}}{\sqrt{e}\sqrt{be-4af}\sqrt{d+ex+fx^2}}\right)}{2e^{3/2}f(bd-ae)^{3/2}(be-4af)^{3/2}}-\frac{\sqrt{d+ex+fx^2}\,(e(Ab-2aB)-bx(Be-2Af))}{e(bd-ae)(be-4af)\left(ae+bex+bfx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x)/(Sqrt[d + e*x + f*x^2]*(a*e + b*e*x + b*f*x^2)^2),x]
```

```
[Out] -((((A*b - 2*a*B)*e - b*(B*e - 2*A*f)*x)*Sqrt[d + e*x + f*x^2])/(e*(b*d - a
*e)*(b*e - 4*a*f)*(a*e + b*e*x + b*f*x^2))) + ((B*e - 2*A*f)*(8*a*e*f - b*(
e^2 + 4*d*f))*ArcTanh[(Sqrt[b*d - a*e]*(e + 2*f*x))/(Sqrt[e]*Sqrt[b*e - 4*a
*f]*Sqrt[d + e*x + f*x^2])])/(2*e^(3/2)*(b*d - a*e)^(3/2)*f*(b*e - 4*a*f)^(
3/2)) + (B*ArcTanh[(Sqrt[b]*Sqrt[d + e*x + f*x^2])/Sqrt[b*d - a*e]])/(2*Sqr
t[b]*(b*d - a*e)^(3/2)*f)
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 982

```
Int[1/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(
x_)^2]), x_Symbol] :> Dist[-2*e, Subst[Int[1/(e*(b*e - 4*a*f) - (b*d - a*e)
*x^2), x], x, (e + 2*f*x)/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e - b*f, 0
]
```

Rule 1016

```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e
_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((a + b*x + c*x^2)^(p + 1)
*(d + e*x + f*x^2)^(q + 1)*(g*c*(2*a*c*e - b*(c*d + a*f)) + (g*b - a*h)*(2*
c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(g*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))
 - h*(b*c*d - 2*a*c*e + a*b*f))*x))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d -
a*e)*(c*e - b*f))*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*
d - a*e)*(c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*
x^2)^q*Simp[(b*h - 2*g*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)
 + (b^2*(g*f) - b*(h*c*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) - a*(-(h*c*e
))))*(a*f*(p + 1) - c*d*(p + 2)) - e*((g*c)*(2*a*c*e - b*(c*d + a*f)) + (g*
```

```
b - a*h)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (2*f*((g*c)*(2*
a*c*e - b*(c*d + a*f)) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(
p + q + 2) - (b^2*g*f - b*(h*c*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) - a*
(-(h*c*e))))*(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(g*f) - b*(h*c
*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) + a*h*c*e))*(2*p + 2*q + 5)*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, q}, x] && NeQ[b^2 - 4*a*c, 0] &
& NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (b*d - a*e)*(c*e
 - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1])
```

Rule 1024

```
Int[((g_) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e
_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[-2*g, Subst[Int[1/(b*d - a*e -
 b*x^2), x], x, Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h
}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e - b*f, 0] &&
 EqQ[h*e - 2*g*f, 0]
```

Rule 1025

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> -Dist[(h*e - 2*g*f)/(2*f), Int[1/(
(a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/(2*f), Int[(e + 2
*f*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c,
d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e
 - b*f, 0] && NeQ[h*e - 2*g*f, 0]
```

Rubi steps

$$\int \frac{A+Bx}{\sqrt{d+ex+fx^2}\left(ae+bex+bfx^2\right)^2}\,dx = -\frac{((Ab-2aB)e-b(Be-2Af)x)\sqrt{d+ex+fx^2}}{e(bd-ae)(be-4af)\left(ae+bex+bfx^2\right)} - \frac{\int \frac{-\frac{1}{2}b(bd-ae)f^2\left(2bBde-\right.}{}}{}$$

$$= -\frac{((Ab-2aB)e-b(Be-2Af)x)\sqrt{d+ex+fx^2}}{e(bd-ae)(be-4af)\left(ae+bex+bfx^2\right)} - \frac{B\int \frac{e+2fx}{\sqrt{d+ex+fx^2}\left(ae+be\right.}}{4(bd-ae)}$$

$$= -\frac{((Ab-2aB)e-b(Be-2Af)x)\sqrt{d+ex+fx^2}}{e(bd-ae)(be-4af)\left(ae+bex+bfx^2\right)} + \frac{(Be)\,\mathrm{Subst}\left(\int \frac{}{bde-a}\right.}{2}$$

$$= -\frac{((Ab-2aB)e-b(Be-2Af)x)\sqrt{d+ex+fx^2}}{e(bd-ae)(be-4af)\left(ae+bex+bfx^2\right)} + \frac{(Be-2Af)\left(8aef\right.}{2e}$$

Mathematica [B] time = 1.49, size = 767, normalized size = 3.08

$$-\frac{-(ae+bx(e+fx))\log\left(b(e+2fx)-\sqrt{b}\sqrt{e}\sqrt{be-4af}\right)\left(-8abef(Be-2Af)-b^{3/2}Be^{5/2}\sqrt{be-4af}+4a\sqrt{b}Be^{3/2}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x)/(Sqrt[d + e*x + f*x^2]*(a*e + b*e*x + b*f*x^2)^2),x]
```

```
[Out] -1/4*(4*b*Sqrt[e]*Sqrt[b*d - a*e]*f*Sqrt[b*e - 4*a*f]*Sqrt[d + x*(e + f*x)]
*(-(B*e*(2*a + b*x)) + A*b*(e + 2*f*x)) - (-(b^(3/2)*B*e^(5/2)*Sqrt[b*e - 4
*a*f]) + 4*a*Sqrt[b]*B*e^(3/2)*f*Sqrt[b*e - 4*a*f] - 8*a*b*e*f*(B*e - 2*A*f
) + b^2*(B*e - 2*A*f)*(e^2 + 4*d*f))*(a*e + b*x*(e + f*x))*Log[-(Sqrt[b]*Sq
```

```
rt[e]*Sqrt[b*e - 4*a*f]) + b*(e + 2*f*x)] + (b^(3/2)*B*e^(5/2)*Sqrt[b*e - 4
*a*f] - 4*a*Sqrt[b]*B*e^(3/2)*f*Sqrt[b*e - 4*a*f] - 8*a*b*e*f*(B*e - 2*A*f)
 + b^2*(B*e - 2*A*f)*(e^2 + 4*d*f))*(a*e + b*x*(e + f*x))*Log[Sqrt[b]*Sqrt[
e]*Sqrt[b*e - 4*a*f] + b*(e + 2*f*x)] - (b^(3/2)*B*e^(5/2)*Sqrt[b*e - 4*a*f
] - 4*a*Sqrt[b]*B*e^(3/2)*f*Sqrt[b*e - 4*a*f] - 8*a*b*e*f*(B*e - 2*A*f) + b
^2*(B*e - 2*A*f)*(e^2 + 4*d*f))*(a*e + b*x*(e + f*x))*Log[Sqrt[b]*(e^(3/2)*
Sqrt[b*e - 4*a*f] + Sqrt[b]*(e^2 - 4*d*f) + 2*Sqrt[e]*f*Sqrt[b*e - 4*a*f]*x
 - 4*Sqrt[b*d - a*e]*f*Sqrt[d + x*(e + f*x)])] + (-(b^(3/2)*B*e^(5/2)*Sqrt[
b*e - 4*a*f]) + 4*a*Sqrt[b]*B*e^(3/2)*f*Sqrt[b*e - 4*a*f] - 8*a*b*e*f*(B*e
- 2*A*f) + b^2*(B*e - 2*A*f)*(e^2 + 4*d*f))*(a*e + b*x*(e + f*x))*Log[Sqrt[
b]*(e^(3/2)*Sqrt[b*e - 4*a*f] - Sqrt[b]*(e^2 - 4*d*f) + 2*Sqrt[e]*f*Sqrt[b*
e - 4*a*f]*x + 4*Sqrt[b*d - a*e]*f*Sqrt[d + x*(e + f*x)])])/(b*e^(3/2)*(b*d
 - a*e)^(3/2)*f*(b*e - 4*a*f)^(3/2)*(a*e + b*x*(e + f*x)))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(b*f*x^2+b*e*x+a*e)^2/(f*x^2+e*x+d)^(1/2),x, algorithm="f
ricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(b*f*x^2+b*e*x+a*e)^2/(f*x^2+e*x+d)^(1/2),x, algorithm="g
iac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to divide, perhaps due to rounding er
ror%%%{%%%{1,[2]%%%},[8,2,0,0,0]%%%}+%%%{%%{[%%%{-4,[1]%%%},0]:[1,0,%%%{-1,
[1]%%%}]%%},[7,2,1,0,0]%%%}+%%%{%%%{6,[1]%%%},[6,2,2,0,0]%%%}+%%%{%%%{-4,[2
]%%%},[6,2,0,0,1]%%%}+%%%{%%%{8,[2]%%%},[6,1,1,1,0]%%%}+%%%{%%{[-4,0]:[1,0,
%%%{-1,[1]%%%}]%%},[5,2,3,0,0]%%%}+%%%{%%{[%%%{12,[1]%%%},0]:[1,0,%%%{-1,[1
]%%%}]%%},[5,2,1,0,1]%%%}+%%%{%%{[%%%{-24,[1]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%
},[5,1,2,1,0]%%%}+%%%{1,[4,2,4,0,0]%%%}+%%%{%%%{-14,[1]%%%},[4,2,2,0,1]%%%}
+%%%{%%%{6,[2]%%%},[4,2,0,0,2]%%%}+%%%{%%%{26,[1]%%%},[4,1,3,1,0]%%%}+%%%{%
%%{-16,[2]%%%},[4,1,1,1,1]%%%}+%%%{%%%{16,[2]%%%},[4,0,2,2,0]%%%}+%%%{%%{[8
,0]:[1,0,%%%{-1,[1]%%%}]%%},[3,2,3,0,1]%%%}+%%%{%%{[%%%{-12,[1]%%%},0]:[1,0
,%%%{-1,[1]%%%}]%%},[3,2,1,0,2]%%%}+%%%{%%{[-12,0]:[1,0,%%%{-1,[1]%%%}]%%},
[3,1,4,1,0]%%%}+%%%{%%{[%%%{32,[1]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[3,1,2,1,
1]%%%}+%%%{%%{[%%%{-32,[1]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[3,0,3,2,0]%%%}+%
%%{-2,[2,2,4,0,1]%%%}+%%%{%%%{10,[1]%%%},[2,2,2,0,2]%%%}+%%%{%%%{-4,[2]%%%}
,[2,2,0,0,3]%%%}+%%%{2,[2,1,5,1,0]%%%}+%%%{%%%{-28,[1]%%%},[2,1,3,1,1]%%%}+
%%%{%%%{8,[2]%%%},[2,1,1,1,2]%%%}+%%%{%%%{24,[1]%%%},[2,0,4,2,0]%%%}+%%%{%%
{[-4,0]:[1,0,%%%{-1,[1]%%%}]%%},[1,2,3,0,2]%%%}+%%%{%%{[%%%{4,[1]%%%},0]:[1
,0,%%%{-1,[1]%%%}]%%},[1,2,1,0,3]%%%}+%%%{%%{[12,0]:[1,0,%%%{-1,[1]%%%}]%%}
,[1,1,4,1,1]%%%}+%%%{%%{[%%%{-8,[1]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[1,1,2,1
,2]%%%}+%%%{%%{[-8,0]:[1,0,%%%{-1,[1]%%%}]%%},[1,0,5,2,0]%%%}+%%%{1,[0,2,4,
0,2]%%%}+%%%{%%%{-2,[1]%%%},[0,2,2,0,3]%%%}+%%%{%%%{1,[2]%%%},[0,2,0,0,4]%%
%}+%%%{-2,[0,1,5,1,1]%%%}+%%%{%%%{2,[1]%%%},[0,1,3,1,2]%%%}+%%%{1,[0,0,6,2,
0]%%%} / %%%{%%%{1,[3]%%%},[8,2,0,0,0]%%%}+%%%{%%{poly1[%%%{-4,[2]%%%},0]:[
1,0,%%%{-1,[1]%%%}]%%},[7,2,1,0,0]%%%}+%%%{%%%{6,[2]%%%},[6,2,2,0,0]%%%}+%%
%{%%%{-4,[3]%%%},[6,2,0,0,1]%%%}+%%%{%%%{8,[3]%%%},[6,1,1,1,0]%%%}+%%%{%%{p
oly1[%%%{-4,[1]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[5,2,3,0,0]%%%}+%%%{%%{poly1
[%%%{12,[2]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[5,2,1,0,1]%%%}+%%%{%%{poly1[%%%
```

```
{-24,[2]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[5,1,2,1,0]%%%}+%%%{%%%{1,[1]%%%},[
4,2,4,0,0]%%%}+%%%{%%%{-14,[2]%%%},[4,2,2,0,1]%%%}+%%%{%%%{6,[3]%%%},[4,2,0
,0,2]%%%}+%%%{%%%{26,[2]%%%},[4,1,3,1,0]%%%}+%%%{%%%{-16,[3]%%%},[4,1,1,1,1
]%%%}+%%%{%%%{16,[3]%%%},[4,0,2,2,0]%%%}+%%%{%%{poly1[%%%{8,[1]%%%},0]:[1,0
,%%%{-1,[1]%%%}]%%},[3,2,3,0,1]%%%}+%%%{%%{poly1[%%%{-12,[2]%%%},0]:[1,0,%%
%{-1,[1]%%%}]%%},[3,2,1,0,2]%%%}+%%%{%%{poly1[%%%{-12,[1]%%%},0]:[1,0,%%%{-
1,[1]%%%}]%%},[3,1,4,1,0]%%%}+%%%{%%{poly1[%%%{32,[2]%%%},0]:[1,0,%%%{-1,[1
]%%%}]%%},[3,1,2,1,1]%%%}+%%%{%%{poly1[%%%{-32,[2]%%%},0]:[1,0,%%%{-1,[1]%%
%}]%%},[3,0,3,2,0]%%%}+%%%{%%%{-2,[1]%%%},[2,2,4,0,1]%%%}+%%%{%%%{10,[2]%%%
},[2,2,2,0,2]%%%}+%%%{%%%{-4,[3]%%%},[2,2,0,0,3]%%%}+%%%{%%%{2,[1]%%%},[2,1
,5,1,0]%%%}+%%%{%%%{-28,[2]%%%},[2,1,3,1,1]%%%}+%%%{%%%{8,[3]%%%},[2,1,1,1,
2]%%%}+%%%{%%%{24,[2]%%%},[2,0,4,2,0]%%%}+%%%{%%{poly1[%%%{-4,[1]%%%},0]:[1
,0,%%%{-1,[1]%%%}]%%},[1,2,3,0,2]%%%}+%%%{%%{poly1[%%%{4,[2]%%%},0]:[1,0,%%
%{-1,[1]%%%}]%%},[1,2,1,0,3]%%%}+%%%{%%{poly1[%%%{12,[1]%%%},0]:[1,0,%%%{-1
,[1]%%%}]%%},[1,1,4,1,1]%%%}+%%%{%%{poly1[%%%{-8,[2]%%%},0]:[1,0,%%%{-1,[1]
%%%}]%%},[1,1,2,1,2]%%%}+%%%{%%{poly1[%%%{-8,[1]%%%},0]:[1,0,%%%{-1,[1]%%%}
]%%},[1,0,5,2,0]%%%}+%%%{%%%{1,[1]%%%},[0,2,4,0,2]%%%}+%%%{%%%{-2,[2]%%%},[
0,2,2,0,3]%%%}+%%%{%%%{1,[3]%%%},[0,2,0,0,4]%%%}+%%%{%%%{-2,[1]%%%},[0,1,5,
1,1]%%%}+%%%{%%%{2,[2]%%%},[0,1,3,1,2]%%%}+%%%{%%%{1,[1]%%%},[0,0,6,2,0]%%%
} Error: Bad Argument Value
```

maple [B] time = 0.05, size = 3606, normalized size = 14.48

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x+A)/(b*f*x^2+b*e*x+a*e)^2/(f*x^2+e*x+d)^(1/2),x)
```

```
[Out] -1/e/(4*a*f-b*e)/(a*e-b*d)/(x+1/2/f*e-1/2/b/f*(-b*e*(4*a*f-b*e))^(1/2))*((x
-1/2*(-b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/f)^2*f+(-b*e*(4*a*f-b*e))^(1/2)/b*(x
-1/2*(-b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/f)-1/b*(a*e-b*d))^(1/2)*A+1/2/f/(4*a
*f-b*e)/(a*e-b*d)/(x+1/2/f*e-1/2/b/f*(-b*e*(4*a*f-b*e))^(1/2))*((x-1/2*(-b*
e+(-b*e*(4*a*f-b*e))^(1/2))/b/f)^2*f+(-b*e*(4*a*f-b*e))^(1/2)/b*(x-1/2*(-b*
e+(-b*e*(4*a*f-b*e))^(1/2))/b/f)-1/b*(a*e-b*d))^(1/2)*B-1/2/f/e/(4*a*f-b*e)
/b/(a*e-b*d)/(x+1/2/f*e-1/2/b/f*(-b*e*(4*a*f-b*e))^(1/2))*((x-1/2*(-b*e+(-b
*e*(4*a*f-b*e))^(1/2))/b/f)^2*f+(-b*e*(4*a*f-b*e))^(1/2)/b*(x-1/2*(-b*e+(-b
*e*(4*a*f-b*e))^(1/2))/b/f)-1/b*(a*e-b*d))^(1/2)*B*(-b*e*(4*a*f-b*e))^(1/2)
+1/2/e/(4*a*f-b*e)/b*(-b*e*(4*a*f-b*e))^(1/2)/(a*e-b*d)/(-1/b*(a*e-b*d))^(1
/2)*ln((-2/b*(a*e-b*d)+(-b*e*(4*a*f-b*e))^(1/2)/b*(x-1/2*(-b*e+(-b*e*(4*a*f
-b*e))^(1/2))/b/f)+2*(-1/b*(a*e-b*d))^(1/2)*((x-1/2*(-b*e+(-b*e*(4*a*f-b*e)
)^(1/2))/b/f)^2*f+(-b*e*(4*a*f-b*e))^(1/2)/b*(x-1/2*(-b*e+(-b*e*(4*a*f-b*e)
)^(1/2))/b/f)-1/b*(a*e-b*d))^(1/2))/(x-1/2*(-b*e+(-b*e*(4*a*f-b*e))^(1/2))/
b/f))*A-1/4/f/(4*a*f-b*e)/b*(-b*e*(4*a*f-b*e))^(1/2)/(a*e-b*d)/(-1/b*(a*e-b
*d))^(1/2)*ln((-2/b*(a*e-b*d)+(-b*e*(4*a*f-b*e))^(1/2)/b*(x-1/2*(-b*e+(-b*e
*(4*a*f-b*e))^(1/2))/b/f)+2*(-1/b*(a*e-b*d))^(1/2)*((x-1/2*(-b*e+(-b*e*(4*a
*f-b*e))^(1/2))/b/f)^2*f+(-b*e*(4*a*f-b*e))^(1/2)/b*(x-1/2*(-b*e+(-b*e*(4*a
*f-b*e))^(1/2))/b/f)-1/b*(a*e-b*d))^(1/2))/(x-1/2*(-b*e+(-b*e*(4*a*f-b*e))^
(1/2))/b/f))*B-1/4/f/b/(a*e-b*d)/(-1/b*(a*e-b*d))^(1/2)*ln((-2/b*(a*e-b*d)+
(-b*e*(4*a*f-b*e))^(1/2)/b*(x-1/2*(-b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/f)+2*(-
1/b*(a*e-b*d))^(1/2)*((x-1/2*(-b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/f)^2*f+(-b*e
*(4*a*f-b*e))^(1/2)/b*(x-1/2*(-b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/f)-1/b*(a*e-
b*d))^(1/2))/(x-1/2*(-b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/f))*B-2/e/(4*a*f-b*e)
/(-b*e*(4*a*f-b*e))^(1/2)/(-1/b*(a*e-b*d))^(1/2)*ln((-2/b*(a*e-b*d)+(-b*e*(
4*a*f-b*e))^(1/2)/b*(x-1/2*(-b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/f)+2*(-1/b*(a*
e-b*d))^(1/2)*((x-1/2*(-b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/f)^2*f+(-b*e*(4*a*f
-b*e))^(1/2)/b*(x-1/2*(-b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/f)-1/b*(a*e-b*d))^(
1/2))/(x-1/2*(-b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/f))*A*f+1/(4*a*f-b*e)/(-b*e*
(4*a*f-b*e))^(1/2)/(-1/b*(a*e-b*d))^(1/2)*ln((-2/b*(a*e-b*d)+(-b*e*(4*a*f-b
*e))^(1/2)/b*(x-1/2*(-b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/f)+2*(-1/b*(a*e-b*d))
^(1/2)*((x-1/2*(-b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/f)^2*f+(-b*e*(4*a*f-b*e))^
```

```
(1/2)/b*(x-1/2*(-b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/f)-1/b*(a*e-b*d))^(1/2))/(
x-1/2*(-b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/f))*B+2/e/(4*a*f-b*e)/(-b*e*(4*a*f-
b*e))^(1/2)/(-1/b*(a*e-b*d))^(1/2)*ln((-2/b*(a*e-b*d)-(-b*e*(4*a*f-b*e))^(1
/2)/b*(x+1/2*(b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/f)+2*(-1/b*(a*e-b*d))^(1/2)*(
(x+1/2*(b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/f)^2*f-(-b*e*(4*a*f-b*e))^(1/2)/b*(
x+1/2*(b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/f)-1/b*(a*e-b*d))^(1/2))/(x+1/2*(b*e
+(-b*e*(4*a*f-b*e))^(1/2))/b/f))*A*f-1/(4*a*f-b*e)/(-b*e*(4*a*f-b*e))^(1/2)
/(-1/b*(a*e-b*d))^(1/2)*ln((-2/b*(a*e-b*d)-(-b*e*(4*a*f-b*e))^(1/2)/b*(x+1/
2*(b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/f)+2*(-1/b*(a*e-b*d))^(1/2)*((x+1/2*(b*e
+(-b*e*(4*a*f-b*e))^(1/2))/b/f)^2*f-(-b*e*(4*a*f-b*e))^(1/2)/b*(x+1/2*(b*e+
(-b*e*(4*a*f-b*e))^(1/2))/b/f)-1/b*(a*e-b*d))^(1/2))/(x+1/2*(b*e+(-b*e*(4*a
*f-b*e))^(1/2))/b/f))*B-1/e/(4*a*f-b*e)/(a*e-b*d)/(x+1/2/f*e+1/2/b/f*(-b*e*
(4*a*f-b*e))^(1/2))*((x+1/2*(b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/f)^2*f-(-b*e*(
4*a*f-b*e))^(1/2)/b*(x+1/2*(b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/f)-1/b*(a*e-b*d
))^(1/2)*A+1/2/f/(4*a*f-b*e)/(a*e-b*d)/(x+1/2/f*e+1/2/b/f*(-b*e*(4*a*f-b*e)
)^(1/2))*((x+1/2*(b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/f)^2*f-(-b*e*(4*a*f-b*e))
^(1/2)/b*(x+1/2*(b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/f)-1/b*(a*e-b*d))^(1/2)*B+
1/2/f/e/(4*a*f-b*e)/b/(a*e-b*d)/(x+1/2/f*e+1/2/b/f*(-b*e*(4*a*f-b*e))^(1/2)
)*((x+1/2*(b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/f)^2*f-(-b*e*(4*a*f-b*e))^(1/2)/
b*(x+1/2*(b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/f)-1/b*(a*e-b*d))^(1/2)*B*(-b*e*(
4*a*f-b*e))^(1/2)-1/2/e/(4*a*f-b*e)/b*(-b*e*(4*a*f-b*e))^(1/2)/(a*e-b*d)/(-
1/b*(a*e-b*d))^(1/2)*ln((-2/b*(a*e-b*d)-(-b*e*(4*a*f-b*e))^(1/2)/b*(x+1/2*(
b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/f)+2*(-1/b*(a*e-b*d))^(1/2)*((x+1/2*(b*e+(-
b*e*(4*a*f-b*e))^(1/2))/b/f)^2*f-(-b*e*(4*a*f-b*e))^(1/2)/b*(x+1/2*(b*e+(-b
*e*(4*a*f-b*e))^(1/2))/b/f)-1/b*(a*e-b*d))^(1/2))/(x+1/2*(b*e+(-b*e*(4*a*f-
b*e))^(1/2))/b/f))*A+1/4/f/(4*a*f-b*e)/b*(-b*e*(4*a*f-b*e))^(1/2)/(a*e-b*d)
/(-1/b*(a*e-b*d))^(1/2)*ln((-2/b*(a*e-b*d)-(-b*e*(4*a*f-b*e))^(1/2)/b*(x+1/
2*(b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/f)+2*(-1/b*(a*e-b*d))^(1/2)*((x+1/2*(b*e
+(-b*e*(4*a*f-b*e))^(1/2))/b/f)^2*f-(-b*e*(4*a*f-b*e))^(1/2)/b*(x+1/2*(b*e+
(-b*e*(4*a*f-b*e))^(1/2))/b/f)-1/b*(a*e-b*d))^(1/2))/(x+1/2*(b*e+(-b*e*(4*a
*f-b*e))^(1/2))/b/f))*B-1/4/f/b/(a*e-b*d)/(-1/b*(a*e-b*d))^(1/2)*ln((-2/b*(
a*e-b*d)-(-b*e*(4*a*f-b*e))^(1/2)/b*(x+1/2*(b*e+(-b*e*(4*a*f-b*e))^(1/2))/b
/f)+2*(-1/b*(a*e-b*d))^(1/2)*((x+1/2*(b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/f)^2*
f-(-b*e*(4*a*f-b*e))^(1/2)/b*(x+1/2*(b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/f)-1/b
*(a*e-b*d))^(1/2))/(x+1/2*(b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/f))*B
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Bx + A}{\left(bfx^2 + bex + ae\right)^2 \sqrt{fx^2 + ex + d}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(b*f*x^2+b*e*x+a*e)^2/(f*x^2+e*x+d)^(1/2),x, algorithm="m
axima")
```

```
[Out] integrate((B*x + A)/((b*f*x^2 + b*e*x + a*e)^2*sqrt(f*x^2 + e*x + d)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + B\,x}{\left(b\,f\,x^2 + b\,e\,x + a\,e\right)^2 \sqrt{f\,x^2 + e\,x + d}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x)/((a*e + b*e*x + b*f*x^2)^2*(d + e*x + f*x^2)^(1/2)),x)
```

```
[Out] int((A + B*x)/((a*e + b*e*x + b*f*x^2)^2*(d + e*x + f*x^2)^(1/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(b*f*x**2+b*e*x+a*e)**2/(f*x**2+e*x+d)**(1/2),x)
```

```
[Out] Timed out
```

3.36 $\int \frac{(g+hx)\sqrt{a+bx+cx^2}}{\left(ad+bdx+cdx^2\right)^2}\,dx$

Optimal. Leaf size=48

$$-\frac{2(-2ah+x(2cg-bh)+bg)}{d^2\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}$$

```
[Out] -2*(b*g-2*a*h+(-b*h+2*c*g)*x)/(-4*a*c+b^2)/d^2/(c*x^2+b*x+a)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {998, 636}

$$-\frac{2(-2ah+x(2cg-bh)+bg)}{d^2\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((g + h*x)*Sqrt[a + b*x + c*x^2])/(a*d + b*d*x + c*d*x^2)^2,x]
```

```
[Out] (-2*(b*g - 2*a*h + (2*c*g - b*h)*x))/((b^2 - 4*a*c)*d^2*Sqrt[a + b*x + c*x^
2])
```

Rule 636

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[(-2*(b*d - 2*a*e + (2*c*d - b*e)*x))/((b^2 - 4*a*c)*Sqrt[a + b*x
 + c*x^2]), x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b
^2 - 4*a*c, 0]
```

Rule 998

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_
) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Dist[(c/f)^p, Int[(g + h*
x)^m*(d + e*x + f*x^2)^(p + q), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, p,
 q}, x] && EqQ[c*d - a*f, 0] && EqQ[b*d - a*e, 0] && (IntegerQ[p] || GtQ[c/
f, 0]) && ( !IntegerQ[q] || LeafCount[d + e*x + f*x^2] <= LeafCount[a + b*x
 + c*x^2])
```

Rubi steps

$$\begin{aligned}\int \frac{(g+hx)\sqrt{a+bx+cx^2}}{\left(ad+bdx+cdx^2\right)^2}\,dx &= \frac{\int \frac{g+hx}{\left(a+bx+cx^2\right)^{3/2}}\,dx}{d^2}\\ &= -\frac{2(bg-2ah+(2cg-bh)x)}{\left(b^2-4ac\right)d^2\sqrt{a+bx+cx^2}}\end{aligned}$$

Mathematica [A] time = 0.19, size = 46, normalized size = 0.96

$$\frac{4ah-2bg+2bhx-4cgx}{d^2\left(b^2-4ac\right)\sqrt{a+x(b+cx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g + h*x)*Sqrt[a + b*x + c*x^2])/(a*d + b*d*x + c*d*x^2)^2,x]
```

```
[Out] (-2*b*g + 4*a*h - 4*c*g*x + 2*b*h*x)/((b^2 - 4*a*c)*d^2*Sqrt[a + x*(b + c*x
)])
```

fricas [A] time = 1.97, size = 85, normalized size = 1.77

$$-\frac{2\sqrt{cx^2+bx+a}\left(bg-2\,ah+\left(2\,cg-bh\right)x\right)}{\left(b^2c-4\,ac^2\right)d^2x^2+\left(b^3-4\,abc\right)d^2x+\left(ab^2-4\,a^2c\right)d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(c*x^2+b*x+a)^(1/2)/(c*d*x^2+b*d*x+a*d)^2,x, algorithm="f
ricas")
```

```
[Out] -2*sqrt(c*x^2 + b*x + a)*(b*g - 2*a*h + (2*c*g - b*h)*x)/((b^2*c - 4*a*c^2)
*d^2*x^2 + (b^3 - 4*a*b*c)*d^2*x + (a*b^2 - 4*a^2*c)*d^2)
```

giac [A] time = 0.27, size = 81, normalized size = 1.69

$$-\frac{2\left(\frac{\left(2\,cd^2g-bd^2h\right)x}{b^2d^4-4\,acd^4}+\frac{bd^2g-2\,ad^2h}{b^2d^4-4\,acd^4}\right)}{\sqrt{cx^2+bx+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(c*x^2+b*x+a)^(1/2)/(c*d*x^2+b*d*x+a*d)^2,x, algorithm="g
iac")
```

```
[Out] -2*((2*c*d^2*g - b*d^2*h)*x/(b^2*d^4 - 4*a*c*d^4) + (b*d^2*g - 2*a*d^2*h)/(
b^2*d^4 - 4*a*c*d^4))/sqrt(c*x^2 + b*x + a)
```

maple [A] time = 0.00, size = 48, normalized size = 1.00

$$-\frac{2\left(bhx-2cgx+2ah-bg\right)}{\sqrt{c\,x^2+bx+a}\,\left(4ac-b^2\right)d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)*(c*x^2+b*x+a)^(1/2)/(c*d*x^2+b*d*x+a*d)^2,x)
```

```
[Out] -2/(c*x^2+b*x+a)^(1/2)*(b*h*x-2*c*g*x+2*a*h-b*g)/d^2/(4*a*c-b^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{cx^2+bx+a}\left(hx+g\right)}{\left(cdx^2+bdx+ad\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(c*x^2+b*x+a)^(1/2)/(c*d*x^2+b*d*x+a*d)^2,x, algorithm="m
axima")
```

```
[Out] integrate(sqrt(c*x^2 + b*x + a)*(h*x + g)/(c*d*x^2 + b*d*x + a*d)^2, x)
```

mupad [B] time = 3.75, size = 49, normalized size = 1.02

$$\frac{4\,a\,h-2\,b\,g+2\,b\,h\,x-4\,c\,g\,x}{\left(b^2\,d^2-4\,a\,c\,d^2\right)\,\sqrt{c\,x^2+b\,x+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g + h*x)*(a + b*x + c*x^2)^(1/2))/(a*d + b*d*x + c*d*x^2)^2,x)
```

```
[Out] (4*a*h - 2*b*g + 2*b*h*x - 4*c*g*x)/((b^2*d^2 - 4*a*c*d^2)*(a + b*x + c*x^2
)^(1/2))
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{g}{a\sqrt{a+bx+cx^2}+bx\sqrt{a+bx+cx^2}+cx^2\sqrt{a+bx+cx^2}}\,dx+\int \frac{hx}{a\sqrt{a+bx+cx^2}+bx\sqrt{a+bx+cx^2}+cx^2\sqrt{a+bx+cx^2}}\,dx}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(c*x**2+b*x+a)**(1/2)/(c*d*x**2+b*d*x+a*d)**2,x)
```

```
[Out] (Integral(g/(a*sqrt(a + b*x + c*x**2) + b*x*sqrt(a + b*x + c*x**2) + c*x**2
*sqrt(a + b*x + c*x**2)), x) + Integral(h*x/(a*sqrt(a + b*x + c*x**2) + b*x
*sqrt(a + b*x + c*x**2) + c*x**2*sqrt(a + b*x + c*x**2)), x))/d**2
```

3.37 $\int \frac{3+2x}{\sqrt{-3-4x-x^2}\left(3+4x+2x^2\right)} \, dx$

Optimal. Leaf size=17

$$\tanh^{-1}\left(\frac{x}{\sqrt{-x^2-4x-3}}\right)$$

```
[Out] arctanh(x/(-x^2-4*x-3)^(1/2))
```

Rubi [A] time = 0.02, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1027, 206}

$$\tanh^{-1}\left(\frac{x}{\sqrt{-x^2-4x-3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(3 + 2*x)/(Sqrt[-3 - 4*x - x^2]*(3 + 4*x + 2*x^2)),x]
```

```
[Out] ArcTanh[x/Sqrt[-3 - 4*x - x^2]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1027

```
Int[((g_) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (e_
.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[g, Subst[Int[1/(a + (c*d - a*f)
*x^2), x], x, x/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h
}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[b*d - a*e, 0] &&
 EqQ[2*h*d - g*e, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{3+2x}{\sqrt{-3-4x-x^2}\left(3+4x+2x^2\right)} \, dx &= 3\,\text{Subst}\left(\int \frac{1}{3-3x^2} \, dx, x, \frac{x}{\sqrt{-3-4x-x^2}}\right)\\ &= \tanh^{-1}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right)\end{aligned}$$

Mathematica [C] time = 0.29, size = 165, normalized size = 9.71

$$\frac{1}{6}\left(\sqrt{1-2i\sqrt{2}}\left(\sqrt{2}+i\right)\tanh^{-1}\left(\frac{\left(2-i\sqrt{2}\right)x-2i\sqrt{2}+2}{\sqrt{2+4i\sqrt{2}}\sqrt{-x^2-4x-3}}\right)+\sqrt{1+2i\sqrt{2}}\left(\sqrt{2}-i\right)\tanh^{-1}\left(\frac{\left(2+i\sqrt{2}\right)x+2}{\sqrt{2-4i\sqrt{2}}\sqrt{-x^2}}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + 2*x)/(Sqrt[-3 - 4*x - x^2]*(3 + 4*x + 2*x^2)),x]
```

```
[Out] (Sqrt[1 - (2*I)*Sqrt[2]]*(I + Sqrt[2])*ArcTanh[(2 - (2*I)*Sqrt[2] + (2 - I*
Sqrt[2])*x)/(Sqrt[2 + (4*I)*Sqrt[2]]*Sqrt[-3 - 4*x - x^2])] + Sqrt[1 + (2*I
)*Sqrt[2]]*(-I + Sqrt[2])*ArcTanh[(2 + (2*I)*Sqrt[2] + (2 + I*Sqrt[2])*x)/(
Sqrt[2 - (4*I)*Sqrt[2]]*Sqrt[-3 - 4*x - x^2])])/6
```

fricas [B] time = 1.34, size = 56, normalized size = 3.29

$$-\frac{1}{4}\log\left(-\frac{2\sqrt{-x^2-4x-3}x+4x+3}{x^2}\right)+\frac{1}{4}\log\left(\frac{2\sqrt{-x^2-4x-3}x-4x-3}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+2*x)/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*log(-(2*sqrt(-x^2 - 4*x - 3)*x + 4*x + 3)/x^2) + 1/4*log((2*sqrt(-x^2
- 4*x - 3)*x - 4*x - 3)/x^2)
```

giac [B] time = 0.23, size = 98, normalized size = 5.76

$$\frac{1}{2}\log\left(\frac{2\left(\sqrt{-x^2-4x-3}-1\right)}{x+2}+\frac{3\left(\sqrt{-x^2-4x-3}-1\right)^2}{(x+2)^2}+1\right)-\frac{1}{2}\log\left(\frac{2\left(\sqrt{-x^2-4x-3}-1\right)}{x+2}+\frac{\left(\sqrt{-x^2-4x\cdots}\right.}{(x+\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+2*x)/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*log(2*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 3*(sqrt(-x^2 - 4*x - 3) - 1)
^2/(x + 2)^2 + 1) - 1/2*log(2*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + (sqrt(-x
^2 - 4*x - 3) - 1)^2/(x + 2)^2 + 3)
```

maple [B] time = 0.02, size = 94, normalized size = 5.53

$$-\frac{\sqrt{3}\,\sqrt{4}\,\sqrt{\frac{3x^2}{\left(-x-\frac{3}{2}\right)^2}-12}\;\operatorname{arctanh}\left(\frac{3x}{\left(-x-\frac{3}{2}\right)\sqrt{\frac{3x^2}{\left(-x-\frac{3}{2}\right)^2}-12}}\right)}{6\sqrt{\frac{\frac{x^2}{\left(-x-\frac{3}{2}\right)^2}-4}{\left(\frac{x}{-x-\frac{3}{2}}+1\right)^2}}\left(\frac{x}{-x-\frac{3}{2}}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3+2*x)/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x)
```

```
[Out] -1/6*3^(1/2)*4^(1/2)/((x^2/(-3/2-x)^2-4)/(x/(-3/2-x)+1)^2)^(1/2)/(x/(-3/2-x
)+1)*(3*x^2/(-3/2-x)^2-12)^(1/2)*arctanh(3*x/(-3/2-x)/(3*x^2/(-3/2-x)^2-12)
^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{2x+3}{\left(2x^2+4x+3\right)\sqrt{-x^2-4x-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+2*x)/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((2*x + 3)/((2*x^2 + 4*x + 3)*sqrt(-x^2 - 4*x - 3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.06

$$\int\frac{2x+3}{\sqrt{-x^2-4x-3}\,\left(2x^2+4x+3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x + 3)/((- 4*x - x^2 - 3)^(1/2)*(4*x + 2*x^2 + 3)),x)
[Out] int((2*x + 3)/((- 4*x - x^2 - 3)^(1/2)*(4*x + 2*x^2 + 3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2x+3}{\sqrt{-(x+1)(x+3)}\left(2x^2+4x+3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+2*x)/(2*x**2+4*x+3)/(-x**2-4*x-3)**(1/2),x)
[Out] Integral((2*x + 3)/(sqrt(-(x + 1)*(x + 3))*(2*x**2 + 4*x + 3)), x)
```

3.38 $\int \frac{3+4x}{\sqrt{-3-4x-x^2}\left(3+4x+2x^2\right)} \, dx$

Optimal. Leaf size=86

$$\sqrt{2}\tan^{-1}\left(\frac{1-\frac{x+3}{\sqrt{-x^2-4x-3}}}{\sqrt{2}}\right)-\sqrt{2}\tan^{-1}\left(\frac{\frac{x+3}{\sqrt{-x^2-4x-3}}+1}{\sqrt{2}}\right)+\tanh^{-1}\left(\frac{x}{\sqrt{-x^2-4x-3}}\right)$$

```
[Out] arctanh(x/(-x^2-4*x-3)^(1/2))+arctan(1/2*(1+(-3-x)/(-x^2-4*x-3)^(1/2))*2^(1
/2))*2^(1/2)-arctan(1/2*(1+(3+x)/(-x^2-4*x-3)^(1/2))*2^(1/2))*2^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.281, Rules used = {1028, 986, 12, 1026, 1161, 618, 204, 1027, 206}

$$\sqrt{2}\tan^{-1}\left(\frac{1-\frac{x+3}{\sqrt{-x^2-4x-3}}}{\sqrt{2}}\right)-\sqrt{2}\tan^{-1}\left(\frac{\frac{x+3}{\sqrt{-x^2-4x-3}}+1}{\sqrt{2}}\right)+\tanh^{-1}\left(\frac{x}{\sqrt{-x^2-4x-3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(3 + 4*x)/(Sqrt[-3 - 4*x - x^2]*(3 + 4*x + 2*x^2)),x]
```

```
[Out] Sqrt[2]*ArcTan[(1 - (3 + x)/Sqrt[-3 - 4*x - x^2])/Sqrt[2]] - Sqrt[2]*ArcTan
[(1 + (3 + x)/Sqrt[-3 - 4*x - x^2])/Sqrt[2]] + ArcTanh[x/Sqrt[-3 - 4*x - x^
2]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 986

```
Int[1/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*
(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f)
, 2]}, Dist[1/(2*q), Int[(c*d - a*f + q + (c*e - b*f)*x)/((a + b*x + c*x^2)
*Sqrt[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[(c*d - a*f - q + (c*e -
 b*f)*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b,
 c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[c*e -
 b*f, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1026

```
Int[(x_)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (e_.)*(x_) + (f_.)
*(x_)^2]), x_Symbol] :> Dist[-2*e, Subst[Int[(1 - d*x^2)/(c*e - b*f - e*(2*
c*d - b*e + 2*a*f)*x^2 + d^2*(c*e - b*f)*x^4), x], x, (1 + ((e + Sqrt[e^2 -
 4*d*f])*x)/(2*d))/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f},
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[b*d - a*e, 0]
```

Rule 1027

```
Int[((g_) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (e_
.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[g, Subst[Int[1/(a + (c*d - a*f)
*x^2), x], x, x/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h
}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[b*d - a*e, 0] &&
 EqQ[2*h*d - g*e, 0]
```

Rule 1028

```
Int[((g_) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (e_
.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> -Dist[(2*h*d - g*e)/e, Int[1/((a + b
*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/e, Int[(2*d + e*x)/((a
+ b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, g
, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[b*d - a*e, 0]
 && NeQ[2*h*d - g*e, 0]
```

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rubi steps

$$\begin{aligned}
\int \frac{3+4x}{\sqrt{-3-4x-x^2}\left(3+4x+2x^2\right)}\,dx &= -\left(3\int \frac{1}{\sqrt{-3-4x-x^2}\left(3+4x+2x^2\right)}\,dx\right)-\int \frac{-6-4x}{\sqrt{-3-4x-x^2}\left(3+4x+\right.} \\
&= \frac{1}{2}\int \frac{-6-4x}{\sqrt{-3-4x-x^2}\left(3+4x+2x^2\right)}\,dx-\frac{1}{2}\int -\frac{4x}{\sqrt{-3-4x-x^2}\left(3+4x+\right.} \\
&= 2\tanh^{-1}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right)+2\int \frac{x}{\sqrt{-3-4x-x^2}\left(3+4x+2x^2\right)}\,dx-3\,\mathrm{S} \\
&= \tanh^{-1}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right)+16\,\mathrm{Subst}\left(\int \frac{1+3x^2}{-4-8x^2-36x^4}\,dx,x,\frac{1+}{\sqrt{-3-4}}\right. \\
&= \tanh^{-1}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right)-\frac{2}{3}\,\mathrm{Subst}\left(\int \frac{1}{\frac{1}{3}-\frac{2x}{3}+x^2}\,dx,x,\frac{1+\frac{x}{3}}{\sqrt{-3-4x-x^2}}\right. \\
&= \tanh^{-1}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right)+\frac{4}{3}\,\mathrm{Subst}\left(\int \frac{1}{-\frac{8}{9}-x^2}\,dx,x,\frac{2}{3}\left(-1+\frac{3+}{\sqrt{-3-4x}}\right.\right. \\
&= \sqrt{2}\,\tan^{-1}\left(\frac{1-\frac{3+x}{\sqrt{-3-4x-x^2}}}{\sqrt{2}}\right)-\sqrt{2}\,\tan^{-1}\left(\frac{1+\frac{3+x}{\sqrt{-3-4x-x^2}}}{\sqrt{2}}\right)+\tanh^{-1}\left(\frac{}{\sqrt{-3-}}\right.
\end{aligned}$$

Mathematica [C] time = 0.11, size = 150, normalized size = 1.74

$$-\frac{1}{2}i\left(\sqrt{1+2i\sqrt{2}}\ \tanh^{-1}\left(\frac{\left(2-i\sqrt{2}\right)x-2i\sqrt{2}+2}{\sqrt{2+4i\sqrt{2}}\ \sqrt{-x^2-4x-3}}\right)-\sqrt{1-2i\sqrt{2}}\ \tanh^{-1}\left(\frac{\left(2+i\sqrt{2}\right)x+2i\sqrt{2}+2}{\sqrt{2-4i\sqrt{2}}\ \sqrt{-x^2-4x-3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + 4*x)/(Sqrt[-3 - 4*x - x^2]*(3 + 4*x + 2*x^2)),x]
```

```
[Out] (-1/2*I)*(Sqrt[1 + (2*I)*Sqrt[2]]*ArcTanh[(2 - (2*I)*Sqrt[2] + (2 - I*Sqrt[
2])*x)/(Sqrt[2 + (4*I)*Sqrt[2]]*Sqrt[-3 - 4*x - x^2])] - Sqrt[1 - (2*I)*Sqr
t[2]]*ArcTanh[(2 + (2*I)*Sqrt[2] + (2 + I*Sqrt[2])*x)/(Sqrt[2 - (4*I)*Sqrt[
2]]*Sqrt[-3 - 4*x - x^2])])
```

fricas [A] time = 1.02, size = 132, normalized size = 1.53

$$\frac{1}{2}\sqrt{2}\arctan\left(\frac{\sqrt{2}x+3\sqrt{2}\sqrt{-x^2-4x-3}}{2(2x+3)}\right)+\frac{1}{2}\sqrt{2}\arctan\left(-\frac{\sqrt{2}x-3\sqrt{2}\sqrt{-x^2-4x-3}}{2(2x+3)}\right)-\frac{1}{4}\log\left(-\frac{2\sqrt{-x^2}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+4*x)/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(2)*arctan(1/2*(sqrt(2)*x + 3*sqrt(2)*sqrt(-x^2 - 4*x - 3))/(2*x +
3)) + 1/2*sqrt(2)*arctan(-1/2*(sqrt(2)*x - 3*sqrt(2)*sqrt(-x^2 - 4*x - 3))/
(2*x + 3)) - 1/4*log(-(2*sqrt(-x^2 - 4*x - 3)*x + 4*x + 3)/x^2) + 1/4*log((
2*sqrt(-x^2 - 4*x - 3)*x - 4*x - 3)/x^2)
```

giac [B] time = 0.27, size = 163, normalized size = 1.90

$$\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\frac{3\left(\sqrt{-x^2-4x-3}-1\right)}{x+2}+1\right)\right)+\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\frac{\sqrt{-x^2-4x-3}-1}{x+2}+1\right)\right)+\frac{1}{2}\log\left(\frac{2\left(\sqrt{-}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+4*x)/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="giac")
```

```
[Out] sqrt(2)*arctan(1/2*sqrt(2)*(3*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 1)) + sq
rt(2)*arctan(1/2*sqrt(2)*((sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 1)) + 1/2*lo
g(2*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 3*(sqrt(-x^2 - 4*x - 3) - 1)^2/(x
+ 2)^2 + 1) - 1/2*log(2*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + (sqrt(-x^2 - 4
*x - 3) - 1)^2/(x + 2)^2 + 3)
```

maple [A] time = 0.01, size = 123, normalized size = 1.43

$$\frac{\sqrt{3}\ \sqrt{4}\ \sqrt{\frac{3x^2}{\left(-x-\frac{3}{2}\right)^2}-12}\ \left(-\operatorname{arctanh}\left(\frac{3x}{\left(-x-\frac{3}{2}\right)\sqrt{\frac{3x^2}{\left(-x-\frac{3}{2}\right)^2}-12}}\right)+\sqrt{2}\ \arctan\left(\frac{\sqrt{\frac{3x^2}{\left(-x-\frac{3}{2}\right)^2}-12}\ \sqrt{2}}{6}\right)\right)}{6\sqrt{\frac{\frac{x^2}{\left(-x-\frac{3}{2}\right)^2}-4}{\left(\frac{x}{-x-\frac{3}{2}}+1\right)^2}}\ \left(\frac{x}{-x-\frac{3}{2}}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x+3)/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x)
```

```
[Out] 1/6*3^(1/2)*4^(1/2)*(3/(-x-3/2)^2*x^2-12)^(1/2)*(2^(1/2)*arctan(1/6*(3/(-x-
3/2)^2*x^2-12)^(1/2)*2^(1/2))-arctanh(3/(-x-3/2)/(3/(-x-3/2)^2*x^2-12)^(1/2
)*x))/((1/(-x-3/2)^2*x^2-4)/(1/(-x-3/2)*x+1)^2)^(1/2)/(1/(-x-3/2)*x+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{4\,x+3}{\left(2\,x^2+4\,x+3\right)\sqrt{-x^2-4\,x-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+4*x)/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="maxima")
[Out] integrate((4*x + 3)/((2*x^2 + 4*x + 3)*sqrt(-x^2 - 4*x - 3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{4\,x+3}{\sqrt{-x^2-4\,x-3}\,\left(2\,x^2+4\,x+3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x + 3)/((- 4*x - x^2 - 3)^(1/2)*(4*x + 2*x^2 + 3)),x)
[Out] int((4*x + 3)/((- 4*x - x^2 - 3)^(1/2)*(4*x + 2*x^2 + 3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{4x+3}{\sqrt{-(x+1)(x+3)}\,\left(2x^2+4x+3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+4*x)/(2*x**2+4*x+3)/(-x**2-4*x-3)**(1/2),x)
[Out] Integral((4*x + 3)/(sqrt(-(x + 1)*(x + 3))*(2*x**2 + 4*x + 3)), x)
```

3.39 $\int \frac{(g+hx)\sqrt{a+bx+cx^2}}{\left(ad+bdx+cdx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=136

$$\frac{h\sqrt{a+bx+cx^2}\log\left(a+bx+cx^2\right)}{2cd\sqrt{ad+bdx+cdx^2}}-\frac{\sqrt{a+bx+cx^2}\,(2cg-bh)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{cd\sqrt{b^2-4ac}\,\sqrt{ad+bdx+cdx^2}}$$

```
[Out] 1/2*h*ln(c*x^2+b*x+a)*(c*x^2+b*x+a)^(1/2)/c/d/(c*d*x^2+b*d*x+a*d)^(1/2)-(-b
*h+2*c*g)*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))*(c*x^2+b*x+a)^(1/2)/c/d/(-4
*a*c+b^2)^(1/2)/(c*d*x^2+b*d*x+a*d)^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.132, Rules used = {999, 634, 618, 206, 628}

$$\frac{h\sqrt{a+bx+cx^2}\log\left(a+bx+cx^2\right)}{2cd\sqrt{ad+bdx+cdx^2}}-\frac{\sqrt{a+bx+cx^2}\,(2cg-bh)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{cd\sqrt{b^2-4ac}\,\sqrt{ad+bdx+cdx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((g + h*x)*Sqrt[a + b*x + c*x^2])/(a*d + b*d*x + c*d*x^2)^(3/2),x]
```

```
[Out] -(((2*c*g - b*h)*Sqrt[a + b*x + c*x^2]*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c
]])/(c*Sqrt[b^2 - 4*a*c]*d*Sqrt[a*d + b*d*x + c*d*x^2])) + (h*Sqrt[a + b*x
+ c*x^2]*Log[a + b*x + c*x^2])/(2*c*d*Sqrt[a*d + b*d*x + c*d*x^2])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 999

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_
) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b
*x + c*x^2)^FracPart[p])/(d^IntPart[p]*(d + e*x + f*x^2)^FracPart[p]), Int[
(g + h*x)^m*(d + e*x + f*x^2)^(p + q), x], x] /; FreeQ[{a, b, c, d, e, f, g
```

```
, h, p, q}, x] && EqQ[c*d - a*f, 0] && EqQ[b*d - a*e, 0] &&  !IntegerQ[p] &
&  !IntegerQ[q] &&  !GtQ[c/f, 0]
```

Rubi steps

$$\int \frac{(g+hx)\sqrt{a+bx+cx^2}}{\left(ad+bdx+cdx^2\right)^{3/2}}\,dx = \frac{\sqrt{a+bx+cx^2}\int \frac{g+hx}{ad+bdx+cdx^2}\,dx}{\sqrt{ad+bdx+cdx^2}}$$

$$= \frac{\left(h\sqrt{a+bx+cx^2}\right)\int \frac{bd+2cdx}{ad+bdx+cdx^2}\,dx}{2cd\sqrt{ad+bdx+cdx^2}} + \frac{\left((2cdg-bdh)\sqrt{a+bx+cx^2}\right)\int \frac{1}{ad+bdx+cdx^2}$$

$$= \frac{h\sqrt{a+bx+cx^2}\,\log\left(a+bx+cx^2\right)}{2cd\sqrt{ad+bdx+cdx^2}} - \frac{\left((2cdg-bdh)\sqrt{a+bx+cx^2}\right)\text{Subst}\left(\int \frac{}{(b^2}\right.}{cd\sqrt{ad+bdx+cd}}$$

$$= -\frac{(2cg-bh)\sqrt{a+bx+cx^2}\,\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}\,d\sqrt{ad+bdx+cdx^2}} + \frac{h\sqrt{a+bx+cx^2}\,\log\left(a+bx+cx\right.}{2cd\sqrt{ad+bdx+cdx^2}}$$

Mathematica [A] time = 0.09, size = 108, normalized size = 0.79

$$\frac{(a+x(b+cx))^{3/2}\left((4cg-2bh)\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)+h\sqrt{4ac-b^2}\,\log(a+x(b+cx))\right)}{2c\sqrt{4ac-b^2}\,(d(a+x(b+cx)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g + h*x)*Sqrt[a + b*x + c*x^2])/(a*d + b*d*x + c*d*x^2)^(3/2),x
]
```

```
[Out] ((a + x*(b + c*x))^(3/2)*((4*c*g - 2*b*h)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*
a*c]] + Sqrt[-b^2 + 4*a*c]*h*Log[a + x*(b + c*x)]))/(2*c*Sqrt[-b^2 + 4*a*c]
*(d*(a + x*(b + c*x)))^(3/2))
```

fricas [F] time = 119.13, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{cdx^2+bdx+ad}\,\sqrt{cx^2+bx+a}\,\left(hx+g\right)}{c^2d^2x^4+2\,bcd^2x^3+2\,abd^2x+\left(b^2+2\,ac\right)d^2x^2+a^2d^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(c*x^2+b*x+a)^(1/2)/(c*d*x^2+b*d*x+a*d)^(3/2),x, algorith
m="fricas")
```

```
[Out] integral(sqrt(c*d*x^2 + b*d*x + a*d)*sqrt(c*x^2 + b*x + a)*(h*x + g)/(c^2*d
^2*x^4 + 2*b*c*d^2*x^3 + 2*a*b*d^2*x + (b^2 + 2*a*c)*d^2*x^2 + a^2*d^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^2+bx+a}\,\left(hx+g\right)}{\left(cdx^2+bdx+ad\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(c*x^2+b*x+a)^(1/2)/(c*d*x^2+b*d*x+a*d)^(3/2),x, algorith
m="giac")
```

```
[Out] integrate(sqrt(c*x^2 + b*x + a)*(h*x + g)/(c*d*x^2 + b*d*x + a*d)^(3/2), x)
```

maple [A] time = 0.03, size = 121, normalized size = 0.89

$$\frac{\sqrt{\left(c\,x^{2}+bx+a\right)d}\,\left(-2bh\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^{2}}}\right)+4cg\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^{2}}}\right)+\sqrt{4ac-b^{2}}\,h\ln\left(c\,x^{2}+bx+a\right)\right)}{2\sqrt{c\,x^{2}+bx+a}\,\sqrt{4ac-b^{2}}\,c\,d^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)*(c*x^2+b*x+a)^(1/2)/(c*d*x^2+b*d*x+a*d)^(3/2),x)
```

```
[Out] 1/2/(c*x^2+b*x+a)^(1/2)*(d*(c*x^2+b*x+a))^(1/2)*(h*ln(c*x^2+b*x+a)*(4*a*c-b
^2)^(1/2)-2*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b*h+4*arctan((2*c*x+b)/(4*a
*c-b^2)^(1/2))*c*g)/d^2/c/(4*a*c-b^2)^(1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(c*x^2+b*x+a)^(1/2)/(c*d*x^2+b*d*x+a*d)^(3/2),x, algorith
m="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(g+h\,x\right)\,\sqrt{c\,x^2+b\,x+a}}{{\left(c\,d\,x^2+b\,d\,x+a\,d\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g + h*x)*(a + b*x + c*x^2)^(1/2))/(a*d + b*d*x + c*d*x^2)^(3/2),x)
```

```
[Out] int(((g + h*x)*(a + b*x + c*x^2)^(1/2))/(a*d + b*d*x + c*d*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(g + hx\right) \sqrt{a + bx + cx^{2}}}{\left(d \left(a + bx + cx^{2}\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(c*x**2+b*x+a)**(1/2)/(c*d*x**2+b*d*x+a*d)**(3/2),x)
```

```
[Out] Integral((g + h*x)*sqrt(a + b*x + c*x**2)/(d*(a + b*x + c*x**2))**(3/2), x)
```

3.40 $\int x^2\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}\,dx$

Optimal. Leaf size=212

$$-\frac{ac^2\sqrt{a^2+2abx+b^2x^2}\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{8d^{3/2}(a+bx)}-\frac{\sqrt{a^2+2abx+b^2x^2}\left(c+dx^2\right)^{3/2}(8bc-15adx)}{60d^2(a+bx)}-\frac{acx\sqrt{a^2+2abx+b^2x^2}}{8d(a+bx)}$$

```
[Out] 1/5*b*x^2*(d*x^2+c)^(3/2)*((b*x+a)^2)^(1/2)/d/(b*x+a)-1/60*(-15*a*d*x+8*b*c
)*(d*x^2+c)^(3/2)*((b*x+a)^2)^(1/2)/d^2/(b*x+a)-1/8*a*c^2*arctanh(x*d^(1/2)
/(d*x^2+c)^(1/2))*((b*x+a)^2)^(1/2)/d^(3/2)/(b*x+a)-1/8*a*c*x*((b*x+a)^2)^(
1/2)*(d*x^2+c)^(1/2)/d/(b*x+a)
```

Rubi [A] time = 0.12, antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {1001, 833, 780, 195, 217, 206}

$$-\frac{ac^2\sqrt{a^2+2abx+b^2x^2}\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{8d^{3/2}(a+bx)}-\frac{\sqrt{a^2+2abx+b^2x^2}\left(c+dx^2\right)^{3/2}(8bc-15adx)}{60d^2(a+bx)}-\frac{acx\sqrt{a^2+2abx+b^2x^2}}{8d(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + d*x^2],x]
```

```
[Out] -(a*c*x*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + d*x^2])/(8*d*(a + b*x)) + (b
*x^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*(c + d*x^2)^(3/2))/(5*d*(a + b*x)) - ((8
*b*c - 15*a*d*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*(c + d*x^2)^(3/2))/(60*d^2*(
a + b*x)) - (a*c^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*ArcTanh[(Sqrt[d]*x)/Sqrt[c
 + d*x^2]])/(8*d^(3/2)*(a + b*x))
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1001

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_
) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((4
*c)^IntPart[p]*(b + 2*c*x)^(2*FracPart[p])), Int[(g + h*x)^m*(b + 2*c*x)^(2
*p)*(d + f*x^2)^q, x], x] /; FreeQ[{a, b, c, d, f, g, h, m, p, q}, x] && Eq
Q[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int x^2\sqrt{a^2+2abx+b^2x^2}\,\sqrt{c+dx^2}\,dx = \frac{\sqrt{a^2+2abx+b^2x^2}\int x^2\left(2ab+2b^2x\right)\sqrt{c+dx^2}\,dx}{2ab+2b^2x}$$

$$= \frac{bx^2\sqrt{a^2+2abx+b^2x^2}\left(c+dx^2\right)^{3/2}}{5d(a+bx)} + \frac{\sqrt{a^2+2abx+b^2x^2}\int x\left(-4b^2c\cdots\right.}{5d\left(2ab+2b\cdots\right.}$$

$$= \frac{bx^2\sqrt{a^2+2abx+b^2x^2}\left(c+dx^2\right)^{3/2}}{5d(a+bx)} - \frac{(8bc-15adx)\sqrt{a^2+2abx+b^2x\cdots}}{60d^2(a+bx)}$$

$$= -\frac{acx\sqrt{a^2+2abx+b^2x^2}\,\sqrt{c+dx^2}}{8d(a+bx)} + \frac{bx^2\sqrt{a^2+2abx+b^2x^2}\left(c+dx^2\right)\cdots}{5d(a+bx)}$$

$$= -\frac{acx\sqrt{a^2+2abx+b^2x^2}\,\sqrt{c+dx^2}}{8d(a+bx)} + \frac{bx^2\sqrt{a^2+2abx+b^2x^2}\left(c+dx^2\right)\cdots}{5d(a+bx)}$$

$$= -\frac{acx\sqrt{a^2+2abx+b^2x^2}\,\sqrt{c+dx^2}}{8d(a+bx)} + \frac{bx^2\sqrt{a^2+2abx+b^2x^2}\left(c+dx^2\right)\cdots}{5d(a+bx)}$$

Mathematica [A] time = 0.15, size = 129, normalized size = 0.61

$$\frac{\sqrt{(a+bx)^2}\,\sqrt{c+dx^2}\left(\sqrt{\frac{dx^2}{c}+1}\left(15adx\left(c+2dx^2\right)+8b\left(-2c^2+cdx^2+3d^2x^4\right)\right)-15ac^{3/2}\sqrt{d}\,\sinh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)\right)}{120d^2(a+bx)\sqrt{\frac{dx^2}{c}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + d*x^2],x]
```

```
[Out] (Sqrt[(a + b*x)^2]*Sqrt[c + d*x^2]*(Sqrt[1 + (d*x^2)/c]*(15*a*d*x*(c + 2*d*
x^2) + 8*b*(-2*c^2 + c*d*x^2 + 3*d^2*x^4)) - 15*a*c^(3/2)*Sqrt[d]*ArcSinh[(
Sqrt[d]*x)/Sqrt[c]]))/(120*d^2*(a + b*x)*Sqrt[1 + (d*x^2)/c])
```

fricas [A] time = 0.94, size = 175, normalized size = 0.83

$$\left[\frac{15\,ac^2\sqrt{d}\,\log\left(-2\,dx^2+2\,\sqrt{dx^2+c}\,\sqrt{d}\,x-c\right)+2\left(24\,bd^2x^4+30\,ad^2x^3+8\,bcdx^2+15\,acdx-16\,bc^2\right)\sqrt{dx^2+\cdots}}{240\,d^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/240*(15*a*c^2*sqrt(d)*log(-2*d*x^2 + 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) +
2*(24*b*d^2*x^4 + 30*a*d^2*x^3 + 8*b*c*d*x^2 + 15*a*c*d*x - 16*b*c^2)*sqrt(
d*x^2 + c))/d^2, 1/120*(15*a*c^2*sqrt(-d)*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)
) + (24*b*d^2*x^4 + 30*a*d^2*x^3 + 8*b*c*d*x^2 + 15*a*c*d*x - 16*b*c^2)*sqr
t(d*x^2 + c))/d^2]
```

giac [A] time = 0.25, size = 117, normalized size = 0.55

$$\frac{ac^2\log\left(\left|-\sqrt{d}\,x+\sqrt{dx^2+c}\right|\right)\operatorname{sgn}(bx+a)}{8\,d^{\frac{3}{2}}}+\frac{1}{120}\sqrt{dx^2+c}\left(\left(2\left(3\left(4\,bx\operatorname{sgn}(bx+a)+5\,a\operatorname{sgn}(bx+a)\right)x+\frac{4\,bc\operatorname{sgn}}{}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*a*c^2*log(abs(-sqrt(d)*x + sqrt(d*x^2 + c)))*sgn(b*x + a)/d^(3/2) + 1/1
20*sqrt(d*x^2 + c)*((2*(3*(4*b*x*sgn(b*x + a) + 5*a*sgn(b*x + a))*x + 4*b*c
*sgn(b*x + a)/d)*x + 15*a*c*sgn(b*x + a)/d)*x - 16*b*c^2*sgn(b*x + a)/d^2)
```

maple [C] time = 0.05, size = 103, normalized size = 0.49

$$-\frac{\left(15a\,c^2d\ln\left(\sqrt{d}\;x+\sqrt{d\,x^2+c}\right)+15\sqrt{d\,x^2+c}\;ac\,d^{\frac{3}{2}}x-24\left(d\,x^2+c\right)^{\frac{3}{2}}b\,d^{\frac{3}{2}}x^2-30\left(d\,x^2+c\right)^{\frac{3}{2}}a\,d^{\frac{3}{2}}x+16\left(d\,x^2\right.\right.}{120d^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2),x)
```

```
[Out] -1/120*csgn(b*x+a)*(-24*d^(3/2)*(d*x^2+c)^(3/2)*x^2*b-30*d^(3/2)*(d*x^2+c)^
(3/2)*x*a+16*d^(1/2)*(d*x^2+c)^(3/2)*b*c+15*d^(3/2)*(d*x^2+c)^(1/2)*x*a*c+1
5*ln(x*d^(1/2)+(d*x^2+c)^(1/2))*a*c^2*d)/d^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{dx^2+c}\,\sqrt{(bx+a)^2}\,x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^2 + c)*sqrt((b*x + a)^2)*x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,\sqrt{(a+b\,x)^2}\,\sqrt{d\,x^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*((a + b*x)^2)^(1/2)*(c + d*x^2)^(1/2),x)
```

```
[Out] int(x^2*((a + b*x)^2)^(1/2)*(c + d*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\sqrt{c+dx^2}\,\sqrt{(a+bx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*((b*x+a)**2)**(1/2)*(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(c + d*x**2)*sqrt((a + b*x)**2), x)
```

3.41 $\int x\sqrt{a^2+2abx+b^2x^2}\,\sqrt{c+dx^2}\,dx$

Optimal. Leaf size=161

$$-\frac{bc^2\sqrt{a^2+2abx+b^2x^2}\,\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{8d^{3/2}(a+bx)}-\frac{bcx\sqrt{a^2+2abx+b^2x^2}\,\sqrt{c+dx^2}}{8d(a+bx)}+\frac{(4a+3bx)\sqrt{a^2+2abx+b^2x^2}\,(c+}{12d(a+bx)}$$

```
[Out] 1/12*(3*b*x+4*a)*(d*x^2+c)^(3/2)*((b*x+a)^2)^(1/2)/d/(b*x+a)-1/8*b*c^2*arct
anh(x*d^(1/2)/(d*x^2+c)^(1/2))*((b*x+a)^2)^(1/2)/d^(3/2)/(b*x+a)-1/8*b*c*x*
((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2)/d/(b*x+a)
```

Rubi [A] time = 0.07, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {1001, 780, 195, 217, 206}

$$-\frac{bc^2\sqrt{a^2+2abx+b^2x^2}\,\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{8d^{3/2}(a+bx)}-\frac{bcx\sqrt{a^2+2abx+b^2x^2}\,\sqrt{c+dx^2}}{8d(a+bx)}+\frac{(4a+3bx)\sqrt{a^2+2abx+b^2x^2}\,(c+}{12d(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + d*x^2],x]
```

```
[Out] -(b*c*x*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + d*x^2])/(8*d*(a + b*x)) + ((
4*a + 3*b*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*(c + d*x^2)^(3/2))/(12*d*(a + b*
x)) - (b*c^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x
^2]])/(8*d^(3/2)*(a + b*x))
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 1001

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_
) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((4
*c)^IntPart[p]*(b + 2*c*x)^(2*FracPart[p])), Int[(g + h*x)^m*(b + 2*c*x)^(2
```

```
*p)*(d + f*x^2)^q, x], x] /; FreeQ[{a, b, c, d, f, g, h, m, p, q}, x] && Eq
Q[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int x\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}\,dx = \frac{\sqrt{a^2+2abx+b^2x^2}\int x\left(2ab+2b^2x\right)\sqrt{c+dx^2}\,dx}{2ab+2b^2x}$$

$$= \frac{(4a+3bx)\sqrt{a^2+2abx+b^2x^2}\left(c+dx^2\right)^{3/2}}{12d(a+bx)} - \frac{\left(b^2c\sqrt{a^2+2abx+b^2x^2}\right)}{2d\left(2ab+2b^2\right.}$$

$$= -\frac{bcx\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}}{8d(a+bx)} + \frac{(4a+3bx)\sqrt{a^2+2abx+b^2x^2}\left(c\right.}{12d(a+bx)}$$

$$= -\frac{bcx\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}}{8d(a+bx)} + \frac{(4a+3bx)\sqrt{a^2+2abx+b^2x^2}\left(c\right.}{12d(a+bx)}$$

$$= -\frac{bcx\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}}{8d(a+bx)} + \frac{(4a+3bx)\sqrt{a^2+2abx+b^2x^2}\left(c\right.}{12d(a+bx)}$$

Mathematica [A] time = 0.12, size = 117, normalized size = 0.73

$$\frac{\sqrt{(a+bx)^2}\sqrt{c+dx^2}\left(\sqrt{d}\sqrt{\frac{dx^2}{c}+1}\left(8a\left(c+dx^2\right)+3bx\left(c+2dx^2\right)\right)-3bc^{3/2}\sinh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\right)}{24d^{3/2}(a+bx)\sqrt{\frac{dx^2}{c}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + d*x^2],x]
```

```
[Out] (Sqrt[(a + b*x)^2]*Sqrt[c + d*x^2]*(Sqrt[d]*Sqrt[1 + (d*x^2)/c]*(8*a*(c + d
*x^2) + 3*b*x*(c + 2*d*x^2)) - 3*b*c^(3/2)*ArcSinh[(Sqrt[d]*x)/Sqrt[c]]))/(
24*d^(3/2)*(a + b*x)*Sqrt[1 + (d*x^2)/c])
```

fricas [A] time = 1.00, size = 157, normalized size = 0.98

$$\left[\frac{3\,bc^2\sqrt{d}\,\log\left(-2\,dx^2+2\,\sqrt{dx^2+c}\,\sqrt{d}\,x-c\right)+2\left(6\,bd^2x^3+8\,ad^2x^2+3\,bcdx+8\,acd\right)\sqrt{dx^2+c}}{48\,d^2}, \frac{3\,bc^2\sqrt{-d}\,\text{ar}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/48*(3*b*c^2*sqrt(d)*log(-2*d*x^2 + 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) + 2*
(6*b*d^2*x^3 + 8*a*d^2*x^2 + 3*b*c*d*x + 8*a*c*d)*sqrt(d*x^2 + c))/d^2, 1/2
4*(3*b*c^2*sqrt(-d)*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) + (6*b*d^2*x^3 + 8*a
*d^2*x^2 + 3*b*c*d*x + 8*a*c*d)*sqrt(d*x^2 + c))/d^2]
```

giac [A] time = 0.21, size = 98, normalized size = 0.61

$$\frac{bc^2\log\left(\left|-\sqrt{d}\,x+\sqrt{dx^2+c}\right|\right)\text{sgn}\,(bx+a)}{8\,d^{\frac{3}{2}}}+\frac{1}{24}\sqrt{dx^2+c}\left(\left(2\left(3\,bx\text{sgn}\,(bx+a)+4\,a\text{sgn}\,(bx+a)\right)x+\frac{3\,bc\text{sgn}}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*b*c^2*log(abs(-sqrt(d)*x + sqrt(d*x^2 + c)))*sgn(b*x + a)/d^(3/2) + 1/2
4*sqrt(d*x^2 + c)*((2*(3*b*x*sgn(b*x + a) + 4*a*sgn(b*x + a))*x + 3*b*c*sgn
(b*x + a)/d)*x + 8*a*c*sgn(b*x + a)/d)
```

maple [C] time = 0.01, size = 83, normalized size = 0.52

$$\frac{\left(-3b\,c^{2}\ln\left(\sqrt{d}\,x+\sqrt{d\,x^{2}+c}\right)-3\sqrt{d\,x^{2}+c}\,bc\sqrt{d}\,x+6\left(d\,x^{2}+c\right)^{\frac{3}{2}}b\sqrt{d}\,x+8\left(d\,x^{2}+c\right)^{\frac{3}{2}}a\sqrt{d}\right)\mathrm{csgn}\left(bx+a\right)}{24d^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2),x)
```

```
[Out] 1/24*csgn(b*x+a)*(6*d^(1/2)*(d*x^2+c)^(3/2)*x*b+8*a*(d*x^2+c)^(3/2)*d^(1/2)
-3*d^(1/2)*(d*x^2+c)^(1/2)*x*b*c-3*ln(d^(1/2)*x+(d*x^2+c)^(1/2))*b*c^2)/d^(
3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{dx^{2}+c}\,\sqrt{(bx+a)^{2}}\,x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^2 + c)*sqrt((b*x + a)^2)*x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\sqrt{(a+b\,x)^{2}}\,\sqrt{d\,x^{2}+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*((a + b*x)^2)^(1/2)*(c + d*x^2)^(1/2),x)
```

```
[Out] int(x*((a + b*x)^2)^(1/2)*(c + d*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\sqrt{c+dx^{2}}\,\sqrt{(a+bx)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((b*x+a)**2)**(1/2)*(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(x*sqrt(c + d*x**2)*sqrt((a + b*x)**2), x)
```

3.42 $\int \sqrt{a^2+2abx+b^2x^2}\,\sqrt{c+dx^2}\,dx$

Optimal. Leaf size=148

$$\frac{b\sqrt{a^2+2abx+b^2x^2}\left(c+dx^2\right)^{3/2}}{3d(a+bx)}+\frac{ax\sqrt{a^2+2abx+b^2x^2}\,\sqrt{c+dx^2}}{2(a+bx)}+\frac{ac\sqrt{a^2+2abx+b^2x^2}\,\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{2\sqrt{d}\,(a+bx)}$$

```
[Out] 1/3*b*(d*x^2+c)^(3/2)*((b*x+a)^2)^(1/2)/d/(b*x+a)+1/2*a*c*arctanh(x*d^(1/2)
/(d*x^2+c)^(1/2))*((b*x+a)^2)^(1/2)/(b*x+a)/d^(1/2)+1/2*a*x*((b*x+a)^2)^(1/
2)*(d*x^2+c)^(1/2)/(b*x+a)
```

Rubi [A] time = 0.06, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {970, 641, 195, 217, 206}

$$\frac{b\sqrt{a^2+2abx+b^2x^2}\left(c+dx^2\right)^{3/2}}{3d(a+bx)}+\frac{ax\sqrt{a^2+2abx+b^2x^2}\,\sqrt{c+dx^2}}{2(a+bx)}+\frac{ac\sqrt{a^2+2abx+b^2x^2}\,\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{2\sqrt{d}\,(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + d*x^2],x]
```

```
[Out] (a*x*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + d*x^2])/(2*(a + b*x)) + (b*Sqrt
[a^2 + 2*a*b*x + b^2*x^2]*(c + d*x^2)^(3/2))/(3*d*(a + b*x)) + (a*c*Sqrt[a^
2 + 2*a*b*x + b^2*x^2]*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/(2*Sqrt[d]*(a
+ b*x))
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 970

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (f_.)*(x_)^2)^(q_.), x_
Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((4*c)^IntPart[p]*(b + 2*c*x)
^(2*FracPart[p])), Int[(b + 2*c*x)^(2*p)*(d + f*x^2)^q, x], x] /; FreeQ[{a,
 b, c, d, f, p, q}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}\,dx = \frac{\sqrt{a^2+2abx+b^2x^2}\int \left(2ab+2b^2x\right)\sqrt{c+dx^2}\,dx}{2ab+2b^2x}$$

$$= \frac{b\sqrt{a^2+2abx+b^2x^2}\left(c+dx^2\right)^{3/2}}{3d(a+bx)} + \frac{\left(2ab\sqrt{a^2+2abx+b^2x^2}\right)\int \sqrt{c+dx^2}\,d}{2ab+2b^2x}$$

$$= \frac{ax\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}}{2(a+bx)} + \frac{b\sqrt{a^2+2abx+b^2x^2}\left(c+dx^2\right)^{3/2}}{3d(a+bx)} + \frac{\left(abc\right.}{}$$

$$= \frac{ax\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}}{2(a+bx)} + \frac{b\sqrt{a^2+2abx+b^2x^2}\left(c+dx^2\right)^{3/2}}{3d(a+bx)} + \frac{\left(abc\right.}{}$$

$$= \frac{ax\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}}{2(a+bx)} + \frac{b\sqrt{a^2+2abx+b^2x^2}\left(c+dx^2\right)^{3/2}}{3d(a+bx)} + \frac{ac\sqrt{}}{}$$

Mathematica [A] time = 0.06, size = 85, normalized size = 0.57

$$\frac{\sqrt{(a+bx)^2}\left(\sqrt{c+dx^2}\left(3adx+2b\left(c+dx^2\right)\right)+3ac\sqrt{d}\log\left(\sqrt{d}\sqrt{c+dx^2}+dx\right)\right)}{6d(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + d*x^2],x]
```

```
[Out] (Sqrt[(a + b*x)^2]*(Sqrt[c + d*x^2]*(3*a*d*x + 2*b*(c + d*x^2)) + 3*a*c*Sqr
t[d]*Log[d*x + Sqrt[d]*Sqrt[c + d*x^2]]))/(6*d*(a + b*x))
```

fricas [A] time = 0.79, size = 128, normalized size = 0.86

$$\left[\frac{3\,ac\sqrt{d}\,\log\left(-2\,dx^2-2\,\sqrt{dx^2+c}\,\sqrt{d}\,x-c\right)+2\left(2\,bdx^2+3\,adx+2\,bc\right)\sqrt{dx^2+c}}{12\,d}, -\frac{3\,ac\sqrt{-d}\,\arctan\left(\frac{\sqrt{-d}\,x}{\sqrt{dx^2+c}}\right)-}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*a*c*sqrt(d)*log(-2*d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) + 2*(2
*b*d*x^2 + 3*a*d*x + 2*b*c)*sqrt(d*x^2 + c))/d, -1/6*(3*a*c*sqrt(-d)*arctan
(sqrt(-d)*x/sqrt(d*x^2 + c)) - (2*b*d*x^2 + 3*a*d*x + 2*b*c)*sqrt(d*x^2 + c
))/d]
```

giac [A] time = 0.27, size = 79, normalized size = 0.53

$$-\frac{ac\log\left(\left|-\sqrt{d}\,x+\sqrt{dx^2+c}\right|\right)\mathrm{sgn}\left(bx+a\right)}{2\sqrt{d}}+\frac{1}{6}\sqrt{dx^2+c}\left(\left(2\,bx\mathrm{sgn}\left(bx+a\right)+3\,a\mathrm{sgn}\left(bx+a\right)\right)x+\frac{2\,bc\mathrm{sgn}\left(bx+\right.}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*a*c*log(abs(-sqrt(d)*x + sqrt(d*x^2 + c)))*sgn(b*x + a)/sqrt(d) + 1/6*
sqrt(d*x^2 + c)*((2*b*x*sgn(b*x + a) + 3*a*sgn(b*x + a))*x + 2*b*c*sgn(b*x
+ a)/d)
```

maple [C] time = 0.01, size = 65, normalized size = 0.44

$$\frac{\left(3acd\ln\left(\sqrt{d}\,x+\sqrt{d\,x^2+c}\right)+3\sqrt{d\,x^2+c}\,a\,d^{\frac{3}{2}}x+2\left(d\,x^2+c\right)^{\frac{3}{2}}b\sqrt{d}\right)\operatorname{csgn}\left(bx+a\right)}{6d^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2),x)
```

```
[Out] 1/6*csgn(b*x+a)*(2*b*(d*x^2+c)^(3/2)*d^(1/2)+3*d^(3/2)*(d*x^2+c)^(1/2)*x*a+
3*ln(d^(1/2)*x+(d*x^2+c)^(1/2))*a*c*d)/d^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{dx^2+c}\,\sqrt{(bx+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^2 + c)*sqrt((b*x + a)^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\sqrt{(a+b\,x)^2}\,\sqrt{d\,x^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x)^2)^(1/2)*(c + d*x^2)^(1/2),x)
```

```
[Out] int(((a + b*x)^2)^(1/2)*(c + d*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{c+dx^2}\,\sqrt{(a+bx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)**2)**(1/2)*(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(sqrt(c + d*x**2)*sqrt((a + b*x)**2), x)
```

3.43 $\int \frac{\sqrt{a^2+2abx+b^2x^2}\,\sqrt{c+dx^2}}{x}\,dx$

Optimal. Leaf size=160

$$\frac{\sqrt{a^2+2abx+b^2x^2}\,(2a+bx)\sqrt{c+dx^2}}{2(a+bx)}+\frac{bc\sqrt{a^2+2abx+b^2x^2}\,\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{2\sqrt{d}\,(a+bx)}-\frac{a\sqrt{c}\,\sqrt{a^2+2abx+b^2x^2}\,\tanh^{-1}}{a+bx}$$

```
[Out] -a*arctanh((d*x^2+c)^(1/2)/c^(1/2))*c^(1/2)*((b*x+a)^2)^(1/2)/(b*x+a)+1/2*b
*c*arctanh(x*d^(1/2)/(d*x^2+c)^(1/2))*((b*x+a)^2)^(1/2)/(b*x+a)/d^(1/2)+1/2
*(b*x+2*a)*((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2)/(b*x+a)
```

Rubi [A] time = 0.12, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {1001, 815, 844, 217, 206, 266, 63, 208}

$$\frac{\sqrt{a^2+2abx+b^2x^2}\,(2a+bx)\sqrt{c+dx^2}}{2(a+bx)}+\frac{bc\sqrt{a^2+2abx+b^2x^2}\,\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{2\sqrt{d}\,(a+bx)}-\frac{a\sqrt{c}\,\sqrt{a^2+2abx+b^2x^2}\,\tanh^{-1}}{a+bx}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + d*x^2])/x,x]
```

```
[Out] ((2*a + b*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + d*x^2])/(2*(a + b*x)) +
 (b*c*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/(
2*Sqrt[d]*(a + b*x)) - (a*Sqrt[c]*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*ArcTanh[Sqr
t[c + d*x^2]/Sqrt[c]])/(a + b*x)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 815

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
 + 1) + g*c*e*(m + 2*p + 1)*x)*(a + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p
 + 2)), x] + Dist[(2*p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1001

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_
) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((4
*c)^IntPart[p]*(b + 2*c*x)^(2*FracPart[p])), Int[(g + h*x)^m*(b + 2*c*x)^(2
*p)*(d + f*x^2)^q, x], x] /; FreeQ[{a, b, c, d, f, g, h, m, p, q}, x] && Eq
Q[b^2 - 4*a*c, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{a^2+2abx+b^2x^2}\,\sqrt{c+dx^2}}{x}\,dx &= \frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{\left(2ab+2b^2x\right)\sqrt{c+dx^2}}{x}\,dx}{2ab+2b^2x}\\
&= \frac{(2a+bx)\sqrt{a^2+2abx+b^2x^2}\,\sqrt{c+dx^2}}{2(a+bx)} + \frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{4abcd+2b^2\cdots}{x\sqrt{c+dx\cdots}}}{2d\left(2ab+2b^2x\right)}\\
&= \frac{(2a+bx)\sqrt{a^2+2abx+b^2x^2}\,\sqrt{c+dx^2}}{2(a+bx)} + \frac{\left(2abc\sqrt{a^2+2abx+b^2x^2}\right)\int \cdots}{2ab+2b^2x}\\
&= \frac{(2a+bx)\sqrt{a^2+2abx+b^2x^2}\,\sqrt{c+dx^2}}{2(a+bx)} + \frac{\left(abc\sqrt{a^2+2abx+b^2x^2}\right)\mathrm{Subs}\cdots}{2ab+2b^2\cdots}\\
&= \frac{(2a+bx)\sqrt{a^2+2abx+b^2x^2}\,\sqrt{c+dx^2}}{2(a+bx)} + \frac{bc\sqrt{a^2+2abx+b^2x^2}\,\tanh^{-1}(\cdots}{2\sqrt{d}\,(a+bx)}\\
&= \frac{(2a+bx)\sqrt{a^2+2abx+b^2x^2}\,\sqrt{c+dx^2}}{2(a+bx)} + \frac{bc\sqrt{a^2+2abx+b^2x^2}\,\tanh^{-1}(\cdots}{2\sqrt{d}\,(a+bx)}
\end{aligned}
$$

Mathematica [A] time = 0.14, size = 139, normalized size = 0.87

$$
\frac{\sqrt{(a+bx)^2}\left(\sqrt{d}\sqrt{\frac{dx^2}{c}+1}\left((2a+bx)\sqrt{c+dx^2}-2a\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)\right)+b\sqrt{c}\sqrt{c+dx^2}\sinh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)\right)}{2\sqrt{d}\,(a+bx)\sqrt{\frac{dx^2}{c}+1}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + d*x^2])/x,x]
```

```
[Out] (Sqrt[(a + b*x)^2]*(b*Sqrt[c]*Sqrt[c + d*x^2]*ArcSinh[(Sqrt[d]*x)/Sqrt[c]]
+ Sqrt[d]*Sqrt[1 + (d*x^2)/c]*((2*a + b*x)*Sqrt[c + d*x^2] - 2*a*Sqrt[c]*Ar
cTanh[Sqrt[c + d*x^2]/Sqrt[c]])))/(2*Sqrt[d]*(a + b*x)*Sqrt[1 + (d*x^2)/c])
```

fricas [A] time = 0.99, size = 341, normalized size = 2.13

$$\left[\frac{bc\sqrt{d}\log\left(-2\,dx^2 - 2\,\sqrt{dx^2+c}\,\sqrt{d}\,x - c\right) + 2\,a\sqrt{c}\,d\log\left(-\frac{dx^2 - 2\,\sqrt{dx^2+c}\,\sqrt{c} + 2\,c}{x^2}\right) + 2\,(bdx + 2\,ad)\sqrt{dx^2+c}}{4\,d}, -\frac{bc\sqrt{-}\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [1/4*(b*c*sqrt(d)*log(-2*d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) + 2*a*sqr
t(c)*d*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) + 2*(b*d*x + 2*a
*d)*sqrt(d*x^2 + c))/d, -1/2*(b*c*sqrt(-d)*arctan(sqrt(-d)*x/sqrt(d*x^2 + c
)) - a*sqrt(c)*d*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) - (b*d
*x + 2*a*d)*sqrt(d*x^2 + c))/d, 1/4*(4*a*sqrt(-c)*d*arctan(sqrt(-c)/sqrt(d*
x^2 + c)) + b*c*sqrt(d)*log(-2*d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) + 2
*(b*d*x + 2*a*d)*sqrt(d*x^2 + c))/d, -1/2*(b*c*sqrt(-d)*arctan(sqrt(-d)*x/s
qrt(d*x^2 + c)) - 2*a*sqrt(-c)*d*arctan(sqrt(-c)/sqrt(d*x^2 + c)) - (b*d*x
+ 2*a*d)*sqrt(d*x^2 + c))/d]
```

giac [A] time = 0.25, size = 102, normalized size = 0.64

$$\frac{2\,ac\arctan\left(-\frac{\sqrt{d}\,x-\sqrt{dx^2+c}}{\sqrt{-c}}\right)\mathrm{sgn}\left(bx+a\right)}{\sqrt{-c}} - \frac{bc\log\left(\left|-\sqrt{d}\,x + \sqrt{dx^2+c}\right|\right)\mathrm{sgn}\left(bx+a\right)}{2\,\sqrt{d}} + \frac{1}{2}\,\sqrt{dx^2+c}\left(bx\mathrm{sgn}\left(bx+a\right)\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2)/x,x, algorithm="giac")
```

```
[Out] 2*a*c*arctan(-(sqrt(d)*x - sqrt(d*x^2 + c))/sqrt(-c))*sgn(b*x + a)/sqrt(-c)
 - 1/2*b*c*log(abs(-sqrt(d)*x + sqrt(d*x^2 + c)))*sgn(b*x + a)/sqrt(d) + 1/
2*sqrt(d*x^2 + c)*(b*x*sgn(b*x + a) + 2*a*sgn(b*x + a))
```

maple [C] time = 0.01, size = 94, normalized size = 0.59

$$-\frac{\left(2a\sqrt{c}\,\sqrt{d}\,\ln\left(\frac{2c+2\sqrt{d\,x^2+c}\,\sqrt{c}}{x}\right) - bc\ln\left(\sqrt{d}\,x + \sqrt{d\,x^2+c}\right) - \sqrt{d\,x^2+c}\,b\sqrt{d}\,x - 2\sqrt{d\,x^2+c}\,a\sqrt{d}\right)\mathrm{csgn}\left(bx+a\cdots\right.}{2\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2)/x,x)
```

```
[Out] -1/2*csgn(b*x+a)*(2*c^(1/2)*d^(1/2)*ln(2*(c^(1/2)*(d*x^2+c)^(1/2)+c)/x)*a-d
^(1/2)*(d*x^2+c)^(1/2)*x*b-2*d^(1/2)*(d*x^2+c)^(1/2)*a-ln(d^(1/2)*x+(d*x^2+
c)^(1/2))*b*c)/d^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx^2+c}\sqrt{(bx+a)^2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^2 + c)*sqrt((b*x + a)^2)/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{(a+b\,x)^2}\,\sqrt{d\,x^2+c}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((((a + b*x)^2)^(1/2)*(c + d*x^2)^(1/2))/x,x)
```

```
[Out] int((((a + b*x)^2)^(1/2)*(c + d*x^2)^(1/2))/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c+dx^2}\,\sqrt{(a+bx)^2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)**2)**(1/2)*(d*x**2+c)**(1/2)/x,x)
```

```
[Out] Integral(sqrt(c + d*x**2)*sqrt((a + b*x)**2)/x, x)
```

3.44 $\int \frac{\sqrt{a^2+2abx+b^2x^2}\,\sqrt{c+dx^2}}{x^2}\,dx$

Optimal. Leaf size=156

$$-\frac{\sqrt{a^2+2abx+b^2x^2}\,(a-bx)\sqrt{c+dx^2}}{x(a+bx)}+\frac{a\sqrt{d}\,\sqrt{a^2+2abx+b^2x^2}\,\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{a+bx}-\frac{b\sqrt{c}\,\sqrt{a^2+2abx+b^2x^2}\,\tanh}{a+bx}$$

```
[Out] -b*arctanh((d*x^2+c)^(1/2)/c^(1/2))*c^(1/2)*((b*x+a)^2)^(1/2)/(b*x+a)+a*arc
tanh(x*d^(1/2)/(d*x^2+c)^(1/2))*d^(1/2)*((b*x+a)^2)^(1/2)/(b*x+a)-(-b*x+a)*
((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2)/x/(b*x+a)
```

Rubi [A] time = 0.11, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {1001, 813, 844, 217, 206, 266, 63, 208}

$$-\frac{\sqrt{a^2+2abx+b^2x^2}\,(a-bx)\sqrt{c+dx^2}}{x(a+bx)}+\frac{a\sqrt{d}\,\sqrt{a^2+2abx+b^2x^2}\,\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{a+bx}-\frac{b\sqrt{c}\,\sqrt{a^2+2abx+b^2x^2}\,\tanh}{a+bx}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + d*x^2])/x^2,x]
```

```
[Out] -(((a - b*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + d*x^2])/(x*(a + b*x)))
+ (a*Sqrt[d]*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x
^2]])/(a + b*x) - (b*Sqrt[c]*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*ArcTanh[Sqrt[c +
 d*x^2]/Sqrt[c]])/(a + b*x)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 813

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1
) + e*g*(m + 1)*x)*(a + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1001

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_
) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((4
*c)^IntPart[p]*(b + 2*c*x)^(2*FracPart[p])), Int[(g + h*x)^m*(b + 2*c*x)^(2
*p)*(d + f*x^2)^q, x], x] /; FreeQ[{a, b, c, d, f, g, h, m, p, q}, x] && Eq
Q[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{\sqrt{a^2+2abx+b^2x^2}\,\sqrt{c+dx^2}}{x^2}\,dx = \frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{\left(2ab+2b^2x\right)\sqrt{c+dx^2}}{x^2}\,dx}{2ab+2b^2x}$$

$$= -\frac{(a-bx)\sqrt{a^2+2abx+b^2x^2}\,\sqrt{c+dx^2}}{x(a+bx)} - \frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{-4b^2c-4ab}{x\sqrt{c+dx^2}}}{2\left(2ab+2b^2x\right)}$$

$$= -\frac{(a-bx)\sqrt{a^2+2abx+b^2x^2}\,\sqrt{c+dx^2}}{x(a+bx)} + \frac{\left(2b^2c\sqrt{a^2+2abx+b^2x^2}\right)\int -}{2ab+2b^2x}$$

$$= -\frac{(a-bx)\sqrt{a^2+2abx+b^2x^2}\,\sqrt{c+dx^2}}{x(a+bx)} + \frac{\left(b^2c\sqrt{a^2+2abx+b^2x^2}\right)\text{Subs}}{2ab+2b^2}$$

$$= -\frac{(a-bx)\sqrt{a^2+2abx+b^2x^2}\,\sqrt{c+dx^2}}{x(a+bx)} + \frac{a\sqrt{d}\,\sqrt{a^2+2abx+b^2x^2}\,\tanh}{a+bx}$$

$$= -\frac{(a-bx)\sqrt{a^2+2abx+b^2x^2}\,\sqrt{c+dx^2}}{x(a+bx)} + \frac{a\sqrt{d}\,\sqrt{a^2+2abx+b^2x^2}\,\tanh}{a+bx}$$

Mathematica [A] time = 0.28, size = 118, normalized size = 0.76

$$\frac{\sqrt{(a+bx)^2}\left(\frac{(bx-a)\sqrt{c+dx^2}}{x}+\frac{a\sqrt{c}\,\sqrt{d}\,\sqrt{\frac{dx^2}{c}+1}\,\sinh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{\sqrt{c+dx^2}}-b\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)\right)}{a+bx}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + d*x^2])/x^2,x]
```

```
[Out] (Sqrt[(a + b*x)^2]*(((-a + b*x)*Sqrt[c + d*x^2])/x + (a*Sqrt[c]*Sqrt[d]*Sqr
t[1 + (d*x^2)/c]*ArcSinh[(Sqrt[d]*x)/Sqrt[c]])/Sqrt[c + d*x^2] - b*Sqrt[c]*
ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]]))/(a + b*x)
```

fricas [A] time = 1.05, size = 333, normalized size = 2.13

$$\left[\frac{a\sqrt{d}\,x\log\left(-2\,dx^2-2\,\sqrt{dx^2+c}\,\sqrt{d}\,x-c\right)+b\sqrt{c}\,x\log\left(-\frac{dx^2-2\,\sqrt{dx^2+c}\,\sqrt{c}+2\,c}{x^2}\right)+2\,\sqrt{dx^2+c}\,(bx-a)}{2\,x},-\frac{2\,a\sqrt{-d}\,x\,a}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] [1/2*(a*sqrt(d)*x*log(-2*d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) + b*sqrt(
c)*x*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) + 2*sqrt(d*x^2 + c
)*(b*x - a))/x, -1/2*(2*a*sqrt(-d)*x*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) - b
*sqrt(c)*x*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) - 2*sqrt(d*x
^2 + c)*(b*x - a))/x, 1/2*(2*b*sqrt(-c)*x*arctan(sqrt(-c)/sqrt(d*x^2 + c))
+ a*sqrt(d)*x*log(-2*d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) + 2*sqrt(d*x^
2 + c)*(b*x - a))/x, -(a*sqrt(-d)*x*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) - b*
sqrt(-c)*x*arctan(sqrt(-c)/sqrt(d*x^2 + c)) - sqrt(d*x^2 + c)*(b*x - a))/x]
```

giac [A] time = 0.26, size = 126, normalized size = 0.81

$$\frac{2\,bc\arctan\left(-\frac{\sqrt{d}\,x-\sqrt{dx^2+c}}{\sqrt{-c}}\right)\mathrm{sgn}\left(bx+a\right)}{\sqrt{-c}}-a\sqrt{d}\,\log\left(\left|-\sqrt{d}\,x+\sqrt{dx^2+c}\right|\right)\mathrm{sgn}\left(bx+a\right)+\sqrt{dx^2+c}\,b\mathrm{sgn}\left(bx+a\right)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] 2*b*c*arctan(-(sqrt(d)*x - sqrt(d*x^2 + c))/sqrt(-c))*sgn(b*x + a)/sqrt(-c)
 - a*sqrt(d)*log(abs(-sqrt(d)*x + sqrt(d*x^2 + c)))*sgn(b*x + a) + sqrt(d*x
^2 + c)*b*sgn(b*x + a) + 2*a*c*sqrt(d)*sgn(b*x + a)/((sqrt(d)*x - sqrt(d*x^
2 + c))^2 - c)
```

maple [C] time = 0.01, size = 118, normalized size = 0.76

$$\frac{\left(acdx\ln\left(\sqrt{d}\;x+\sqrt{d\,x^2+c}\right)-b\,c^{\frac{3}{2}}\sqrt{d}\;x\ln\left(\frac{2c+2\sqrt{d\,x^2+c}\;\sqrt{c}}{x}\right)+\sqrt{d\,x^2+c}\;a\,d^{\frac{3}{2}}x^2+\sqrt{d\,x^2+c}\;bc\sqrt{d}\;x-\left(d\,x^2+c\right)\right.}{c\sqrt{d}\;x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2)/x^2,x)
```

```
[Out] csgn(b*x+a)*(d^(3/2)*(d*x^2+c)^(1/2)*x^2*a-d^(1/2)*c^(3/2)*ln(2*(c+(d*x^2+c
)^(1/2)*c^(1/2))/x)*x*b-(d*x^2+c)^(3/2)*a*d^(1/2)+(d*x^2+c)^(1/2)*b*c*d^(1/
2)*x+ln(d^(1/2)*x+(d*x^2+c)^(1/2))*x*a*c*d)/c/x/d^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{dx^2+c}\,\sqrt{(bx+a)^2}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2)/x^2,x, algorithm="maxima")
[Out] integrate(sqrt(d*x^2 + c)*sqrt((b*x + a)^2)/x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{(a+b\,x)^2}\,\sqrt{d\,x^2+c}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((((a + b*x)^2)^(1/2)*(c + d*x^2)^(1/2))/x^2,x)
[Out] int((((a + b*x)^2)^(1/2)*(c + d*x^2)^(1/2))/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c+dx^2}\,\sqrt{(a+bx)^2}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)**2)**(1/2)*(d*x**2+c)**(1/2)/x**2,x)
[Out] Integral(sqrt(c + d*x**2)*sqrt((a + b*x)**2)/x**2, x)
```

3.45 $\int \frac{\sqrt{a^2+2abx+b^2x^2}\,\sqrt{c+dx^2}}{x^3}\,dx$

Optimal. Leaf size=161

$$-\frac{\sqrt{a^2+2abx+b^2x^2}\,(a+2bx)\sqrt{c+dx^2}}{2x^2(a+bx)}+\frac{b\sqrt{d}\,\sqrt{a^2+2abx+b^2x^2}\,\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{a+bx}-\frac{ad\sqrt{a^2+2abx+b^2x^2}\,\tanh}{2\sqrt{c}\,(a+bx)}$$

```
[Out] -1/2*a*d*arctanh((d*x^2+c)^(1/2)/c^(1/2))*((b*x+a)^2)^(1/2)/(b*x+a)/c^(1/2)
+b*arctanh(x*d^(1/2)/(d*x^2+c)^(1/2))*d^(1/2)*((b*x+a)^2)^(1/2)/(b*x+a)-1/2
*(2*b*x+a)*((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2)/x^2/(b*x+a)
```

Rubi [A] time = 0.12, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.229$, Rules used = {1001, 811, 844, 217, 206, 266, 63, 208}

$$-\frac{\sqrt{a^2+2abx+b^2x^2}\,(a+2bx)\sqrt{c+dx^2}}{2x^2(a+bx)}+\frac{b\sqrt{d}\,\sqrt{a^2+2abx+b^2x^2}\,\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{a+bx}-\frac{ad\sqrt{a^2+2abx+b^2x^2}\,\tanh}{2\sqrt{c}\,(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + d*x^2])/x^3,x]
```

```
[Out] -((a + 2*b*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + d*x^2])/(2*x^2*(a + b*
x)) + (b*Sqrt[d]*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*ArcTanh[(Sqrt[d]*x)/Sqrt[c +
 d*x^2]])/(a + b*x) - (a*d*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*ArcTanh[Sqrt[c + d
*x^2]/Sqrt[c]])/(2*Sqrt[c]*(a + b*x))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 811

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(a + c*x^2)^p*((d*g - e*f*(m + 2
))*(c*d^2 + a*e^2) - 2*c*d^2*p*(e*f - d*g) - e*(g*(m + 1)*(c*d^2 + a*e^2) +
 2*c*d*p*(e*f - d*g))*x))/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), x] - Dist[
p/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 2)*(a + c*x^2)^
(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) - c*(2*c*d*(d*g*(2*p + 1) - e*f*(m
 + 2*p + 2)) - 2*a*e^2*g*(m + 1))*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0]
 &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1001

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_
) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((4
*c)^IntPart[p]*(b + 2*c*x)^(2*FracPart[p])), Int[(g + h*x)^m*(b + 2*c*x)^(2
*p)*(d + f*x^2)^q, x], x] /; FreeQ[{a, b, c, d, f, g, h, m, p, q}, x] && Eq
Q[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{\sqrt{a^2+2abx+b^2x^2}\,\sqrt{c+dx^2}}{x^3}\,dx = \frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{\left(2ab+2b^2x\right)\sqrt{c+dx^2}}{x^3}\,dx}{2ab+2b^2x}$$

$$= -\frac{(a+2bx)\sqrt{a^2+2abx+b^2x^2}\,\sqrt{c+dx^2}}{2x^2(a+bx)} - \frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{-4abcd-}{x\sqrt{c+}}}{4c\left(2ab+2b^2x\right)}$$

$$= -\frac{(a+2bx)\sqrt{a^2+2abx+b^2x^2}\,\sqrt{c+dx^2}}{2x^2(a+bx)} + \frac{\left(abd\sqrt{a^2+2abx+b^2x^2}\right)\int}{2ab+2b^2x}$$

$$= -\frac{(a+2bx)\sqrt{a^2+2abx+b^2x^2}\,\sqrt{c+dx^2}}{2x^2(a+bx)} + \frac{\left(abd\sqrt{a^2+2abx+b^2x^2}\right)\mathrm{Su}}{2\left(2ab+2\right.}$$

$$= -\frac{(a+2bx)\sqrt{a^2+2abx+b^2x^2}\,\sqrt{c+dx^2}}{2x^2(a+bx)} + \frac{b\sqrt{d}\,\sqrt{a^2+2abx+b^2x^2}\,\tan}{a+bx}$$

$$= -\frac{(a+2bx)\sqrt{a^2+2abx+b^2x^2}\,\sqrt{c+dx^2}}{2x^2(a+bx)} + \frac{b\sqrt{d}\,\sqrt{a^2+2abx+b^2x^2}\,\tan}{a+bx}$$

Mathematica [A] time = 0.12, size = 126, normalized size = 0.78

$$-\frac{\sqrt{(a+bx)^2}\,\sqrt{c+dx^2}\left(c(a+2bx)\sqrt{\frac{dx^2}{c}+1}+adx^2\tanh^{-1}\left(\sqrt{\frac{dx^2}{c}+1}\right)-2b\sqrt{c}\,\sqrt{d}\,x^2\sinh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)\right)}{2cx^2(a+bx)\sqrt{\frac{dx^2}{c}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + d*x^2])/x^3,x]
```

```
[Out] -1/2*(Sqrt[(a + b*x)^2]*Sqrt[c + d*x^2]*(c*(a + 2*b*x)*Sqrt[1 + (d*x^2)/c]
- 2*b*Sqrt[c]*Sqrt[d]*x^2*ArcSinh[(Sqrt[d]*x)/Sqrt[c]] + a*d*x^2*ArcTanh[Sq
rt[1 + (d*x^2)/c]]))/(c*x^2*(a + b*x)*Sqrt[1 + (d*x^2)/c])
```

fricas [A] time = 0.76, size = 377, normalized size = 2.34

$$\left[\frac{2\,bc\sqrt{d}\,x^2\log\left(-2\,dx^2-2\,\sqrt{dx^2+c}\,\sqrt{d}\,x-c\right)+a\sqrt{c}\,dx^2\log\left(-\frac{dx^2-2\,\sqrt{dx^2+c}\,\sqrt{c}+2\,c}{x^2}\right)-2\,(2\,bcx+ac)\sqrt{dx^2+c}}{4\,cx^2},-\frac{4\ldots}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] [1/4*(2*b*c*sqrt(d)*x^2*log(-2*d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) + a
*sqrt(c)*d*x^2*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) - 2*(2*b
*c*x + a*c)*sqrt(d*x^2 + c))/(c*x^2), -1/4*(4*b*c*sqrt(-d)*x^2*arctan(sqrt(
-d)*x/sqrt(d*x^2 + c)) - a*sqrt(c)*d*x^2*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sq
rt(c) + 2*c)/x^2) + 2*(2*b*c*x + a*c)*sqrt(d*x^2 + c))/(c*x^2), 1/2*(a*sqrt
(-c)*d*x^2*arctan(sqrt(-c)/sqrt(d*x^2 + c)) + b*c*sqrt(d)*x^2*log(-2*d*x^2
- 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) - (2*b*c*x + a*c)*sqrt(d*x^2 + c))/(c*x^
2), -1/2*(2*b*c*sqrt(-d)*x^2*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) - a*sqrt(-c
)*d*x^2*arctan(sqrt(-c)/sqrt(d*x^2 + c)) + (2*b*c*x + a*c)*sqrt(d*x^2 + c))
/(c*x^2)]
```

giac [A] time = 0.28, size = 199, normalized size = 1.24

$$\frac{ad\arctan\left(-\frac{\sqrt{d}\,x-\sqrt{dx^2+c}}{\sqrt{-c}}\right)\mathrm{sgn}\left(bx+a\right)}{\sqrt{-c}}-b\sqrt{d}\,\log\left(\left|-\sqrt{d}\,x+\sqrt{dx^2+c}\right|\right)\mathrm{sgn}\left(bx+a\right)+\frac{\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^3 ad\mathrm{sgn}\ldots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] a*d*arctan(-(sqrt(d)*x - sqrt(d*x^2 + c))/sqrt(-c))*sgn(b*x + a)/sqrt(-c) -
 b*sqrt(d)*log(abs(-sqrt(d)*x + sqrt(d*x^2 + c)))*sgn(b*x + a) + ((sqrt(d)*
x - sqrt(d*x^2 + c))^3*a*d*sgn(b*x + a) + 2*(sqrt(d)*x - sqrt(d*x^2 + c))^2
*b*c*sqrt(d)*sgn(b*x + a) + (sqrt(d)*x - sqrt(d*x^2 + c))*a*c*d*sgn(b*x + a
) - 2*b*c^2*sqrt(d)*sgn(b*x + a))/((sqrt(d)*x - sqrt(d*x^2 + c))^2 - c)^2
```

maple [C] time = 0.02, size = 141, normalized size = 0.88

$$-\frac{\left(a\sqrt{c}\;d^{\frac{3}{2}}x^2\ln\left(\frac{2c+2\sqrt{d\,x^2+c}\;\sqrt{c}}{x}\right)-2bcd\,x^2\ln\left(\sqrt{d}\;x+\sqrt{d\,x^2+c}\right)-2\sqrt{d\,x^2+c}\;b\,d^{\frac{3}{2}}x^3-\sqrt{d\,x^2+c}\;a\,d^{\frac{3}{2}}x^2+2\left(d\,x\ldots\right.\right.}{2c\sqrt{d}\;x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2)/x^3,x)
```

```
[Out] -1/2*csgn(b*x+a)*(c^(1/2)*ln(2*(c+(d*x^2+c)^(1/2)*c^(1/2))/x)*d^(3/2)*x^2*a
-2*d^(3/2)*(d*x^2+c)^(1/2)*x^3*b+2*(d*x^2+c)^(3/2)*b*d^(1/2)*x-(d*x^2+c)^(1
/2)*a*d^(3/2)*x^2-2*ln(d^(1/2)*x+(d*x^2+c)^(1/2))*x^2*b*c*d+(d*x^2+c)^(3/2)
*a*d^(1/2))/x^2/c/d^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx^2+c}\,\sqrt{(bx+a)^2}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2)/x^3,x, algorithm="maxima")
[Out] integrate(sqrt(d*x^2 + c)*sqrt((b*x + a)^2)/x^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{(a+b\,x)^2}\,\sqrt{d\,x^2+c}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((((a + b*x)^2)^(1/2)*(c + d*x^2)^(1/2))/x^3,x)
[Out] int((((a + b*x)^2)^(1/2)*(c + d*x^2)^(1/2))/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c+dx^2}\,\sqrt{(a+bx)^2}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)**2)**(1/2)*(d*x**2+c)**(1/2)/x**3,x)
[Out] Integral(sqrt(c + d*x**2)*sqrt((a + b*x)**2)/x**3, x)
```

3.46 $\int x^2\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}\,dx$

Optimal. Leaf size=317

$$-\frac{\sqrt{a^2+2abx+b^2x^2}\,(2dx+e)\sqrt{c+dx^2+ex}\,\left(2ad\left(4cd-5e^2\right)-b\left(12cde-7e^3\right)\right)}{128d^4(a+bx)}-\frac{\sqrt{a^2+2abx+b^2x^2}\,\left(c+dx^2+\right.}{}$$

[Out] 1/5*b*x^2*(d*x^2+e*x+c)^(3/2)*((b*x+a)^2)^(1/2)/d/(b*x+a)-1/240*(32*b*c*d+50*a*d*e-35*b*e^2-6*d*(10*a*d-7*b*e)*x)*(d*x^2+e*x+c)^(3/2)*((b*x+a)^2)^(1/2)/d^3/(b*x+a)-1/256*(4*c*d-e^2)*(8*a*c*d^2-10*a*d*e^2-12*b*c*d*e+7*b*e^3)*arctanh(1/2*(2*d*x+e)/d^(1/2)/(d*x^2+e*x+c)^(1/2))*((b*x+a)^2)^(1/2)/d^(9/2)/(b*x+a)-1/128*(2*a*d*(4*c*d-5*e^2)-b*(12*c*d*e-7*e^3))*(2*d*x+e)*((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2)/d^4/(b*x+a)

Rubi [A] time = 0.33, antiderivative size = 317, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1000, 832, 779, 612, 621, 206}

$$-\frac{\sqrt{a^2+2abx+b^2x^2}\,\left(c+dx^2+ex\right)^{3/2}\left(-6dx(10ad-7be)+50ade+32bcd-35be^2\right)}{240d^3(a+bx)}-\frac{\sqrt{a^2+2abx+b^2x^2}\,(2dx+}{}$$

Antiderivative was successfully verified.

[In] Int[x^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + e*x + d*x^2],x]

[Out] -((2*a*d*(4*c*d - 5*e^2) - b*(12*c*d*e - 7*e^3))*(e + 2*d*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + e*x + d*x^2])/(128*d^4*(a + b*x)) + (b*x^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*(c + e*x + d*x^2)^(3/2))/(5*d*(a + b*x)) - ((32*b*c*d + 50*a*d*e - 35*b*e^2 - 6*d*(10*a*d - 7*b*e)*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*(c + e*x + d*x^2)^(3/2))/(240*d^3*(a + b*x)) - ((4*c*d - e^2)*(8*a*c*d^2 - 12*b*c*d*e - 10*a*d*e^2 + 7*b*e^3)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*ArcTanh[(e + 2*d*x)/(2*Sqrt[d]*Sqrt[c + e*x + d*x^2])])/(256*d^(9/2)*(a + b*x))

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) - 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p + 3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
```

```
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && !LeQ[p, -1]
```

Rule 832

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p +
1))/(c*(m + 2*p + 2)), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) && !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1000

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_
) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Dist[(a + b*x + c*x^2)^Fr
acPart[p]/((4*c)^IntPart[p]*(b + 2*c*x)^(2*FracPart[p])), Int[(g + h*x)^m*(
b + 2*c*x)^(2*p)*(d + e*x + f*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 h, m, p, q}, x] && EqQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int x^2\sqrt{a^2+2abx+b^2x^2}\,\sqrt{c+ex+dx^2}\,dx = \frac{\sqrt{a^2+2abx+b^2x^2}\int x^2\left(2ab+2b^2x\right)\sqrt{c+ex+dx^2}\,dx}{2ab+2b^2x}$$

$$= \frac{bx^2\sqrt{a^2+2abx+b^2x^2}\left(c+ex+dx^2\right)^{3/2}}{5d(a+bx)} + \frac{\sqrt{a^2+2abx+b^2x^2}\int \cdots}{\cdots}$$

$$= \frac{bx^2\sqrt{a^2+2abx+b^2x^2}\left(c+ex+dx^2\right)^{3/2}}{5d(a+bx)} - \frac{\left(32bcd+50ade-35b\cdots\right.}{\cdots}$$

$$= -\frac{\left(8acd^2-12bcde-10ade^2+7be^3\right)(e+2dx)\sqrt{a^2+2abx+b^2x^2}\cdots}{128d^4(a+bx)}$$

$$= -\frac{\left(8acd^2-12bcde-10ade^2+7be^3\right)(e+2dx)\sqrt{a^2+2abx+b^2x^2}\cdots}{128d^4(a+bx)}$$

$$= -\frac{\left(8acd^2-12bcde-10ade^2+7be^3\right)(e+2dx)\sqrt{a^2+2abx+b^2x^2}\cdots}{128d^4(a+bx)}$$

Mathematica [A] time = 0.27, size = 198, normalized size = 0.62

$$\frac{\sqrt{(a+bx)^2}\left(-\frac{5\left(2ad\left(4cd-5e^2\right)+b\left(7e^3-12cde\right)\right)\left(\left(4cd-e^2\right)\tanh^{-1}\left(\frac{2dx+e}{2\sqrt{d}\sqrt{c+x(dx+e)}}\right)+2\sqrt{d}(2dx+e)\sqrt{c+x(dx+e)}\right)}{256d^{7/2}}+\frac{(c+x(dx+e))^{3/2}(10ad(6dx-5\cdots}{48d\cdots}\cdots\right.}{5d(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + e*x + d*x^2],x]
```

```
[Out] (Sqrt[(a + b*x)^2]*(b*x^2*(c + x*(e + d*x))^(3/2) + ((c + x*(e + d*x))^(3/2
)*(-32*b*c*d + 7*b*e*(5*e - 6*d*x) + 10*a*d*(-5*e + 6*d*x)))/(48*d^2) - (5*
(2*a*d*(4*c*d - 5*e^2) + b*(-12*c*d*e + 7*e^3))*(2*Sqrt[d]*(e + 2*d*x)*Sqrt
```

```
[c + x*(e + d*x)] + (4*c*d - e^2)*ArcTanh[(e + 2*d*x)/(2*Sqrt[d]*Sqrt[c + x
*(e + d*x)])]))/(256*d^(7/2))))/(5*d*(a + b*x))
```

fricas [A] time = 1.03, size = 517, normalized size = 1.63

$$\left[-\frac{15\left(32\,ac^2d^3-48\,bc^2d^2e-48\,acd^2e^2+40\,bcde^3+10\,ade^4-7\,be^5\right)\sqrt{d}\,\log\left(8\,d^2x^2+8\,dex+4\,\sqrt{dx^2+ex+c}\,(2\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/7680*(15*(32*a*c^2*d^3 - 48*b*c^2*d^2*e - 48*a*c*d^2*e^2 + 40*b*c*d*e^3
 + 10*a*d*e^4 - 7*b*e^5)*sqrt(d)*log(8*d^2*x^2 + 8*d*e*x + 4*sqrt(d*x^2 + e
*x + c)*(2*d*x + e)*sqrt(d) + 4*c*d + e^2) - 4*(384*b*d^5*x^4 - 256*b*c^2*d
^3 - 520*a*c*d^3*e + 460*b*c*d^2*e^2 + 150*a*d^2*e^3 - 105*b*d*e^4 + 48*(10
*a*d^5 + b*d^4*e)*x^3 + 8*(16*b*c*d^4 + 10*a*d^4*e - 7*b*d^3*e^2)*x^2 + 2*(
120*a*c*d^4 - 116*b*c*d^3*e - 50*a*d^3*e^2 + 35*b*d^2*e^3)*x)*sqrt(d*x^2 +
e*x + c))/d^5, 1/3840*(15*(32*a*c^2*d^3 - 48*b*c^2*d^2*e - 48*a*c*d^2*e^2 +
 40*b*c*d*e^3 + 10*a*d*e^4 - 7*b*e^5)*sqrt(-d)*arctan(1/2*sqrt(d*x^2 + e*x
+ c)*(2*d*x + e)*sqrt(-d)/(d^2*x^2 + d*e*x + c*d)) + 2*(384*b*d^5*x^4 - 256
*b*c^2*d^3 - 520*a*c*d^3*e + 460*b*c*d^2*e^2 + 150*a*d^2*e^3 - 105*b*d*e^4
+ 48*(10*a*d^5 + b*d^4*e)*x^3 + 8*(16*b*c*d^4 + 10*a*d^4*e - 7*b*d^3*e^2)*x
^2 + 2*(120*a*c*d^4 - 116*b*c*d^3*e - 50*a*d^3*e^2 + 35*b*d^2*e^3)*x)*sqrt(
d*x^2 + e*x + c))/d^5]
```

giac [A] time = 0.31, size = 368, normalized size = 1.16

$$\frac{1}{1920}\,\sqrt{dx^2+xe+c}\left(2\left(4\left(6\left(8\,bx\mathrm{sgn}\left(bx+a\right)+\frac{10\,ad^4\mathrm{sgn}\left(bx+a\right)+bd^3e\mathrm{sgn}\left(bx+a\right)}{d^4}\right)x+\frac{16\,bcd^3\mathrm{sgn}\left(bx+a\right)}{}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/1920*sqrt(d*x^2 + x*e + c)*(2*(4*(6*(8*b*x*sgn(b*x + a) + (10*a*d^4*sgn(b
*x + a) + b*d^3*e*sgn(b*x + a))/d^4)*x + (16*b*c*d^3*sgn(b*x + a) + 10*a*d^
3*e*sgn(b*x + a) - 7*b*d^2*e^2*sgn(b*x + a))/d^4)*x + (120*a*c*d^3*sgn(b*x
+ a) - 116*b*c*d^2*e*sgn(b*x + a) - 50*a*d^2*e^2*sgn(b*x + a) + 35*b*d*e^3*
sgn(b*x + a))/d^4)*x - (256*b*c^2*d^2*sgn(b*x + a) + 520*a*c*d^2*e*sgn(b*x
+ a) - 460*b*c*d*e^2*sgn(b*x + a) - 150*a*d*e^3*sgn(b*x + a) + 105*b*e^4*sg
n(b*x + a))/d^4) + 1/256*(32*a*c^2*d^3*sgn(b*x + a) - 48*b*c^2*d^2*e*sgn(b*
x + a) - 48*a*c*d^2*e^2*sgn(b*x + a) + 40*b*c*d*e^3*sgn(b*x + a) + 10*a*d*e
^4*sgn(b*x + a) - 7*b*e^5*sgn(b*x + a))*log(abs(-2*(sqrt(d)*x - sqrt(d*x^2
+ x*e + c))*sqrt(d) - e))/d^(9/2)
```

maple [C] time = 0.02, size = 530, normalized size = 1.67

$$\frac{\left(-480a\,c^2d^4\ln\left(\frac{2dx+e+2\sqrt{d\,x^2+ex+c}\,\sqrt{d}}{2\sqrt{d}}\right)+720ac\,d^3e^2\ln\left(\frac{2dx+e+2\sqrt{d\,x^2+ex+c}\,\sqrt{d}}{2\sqrt{d}}\right)-150a\,d^2e^4\ln\left(\frac{2dx+e+2\sqrt{d\,x^2+ex+c}\,\sqrt{d}}{2\sqrt{d}}\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2),x)
```

```
[Out] 1/3840*csgn(b*x+a)*(768*(d*x^2+e*x+c)^(3/2)*d^(9/2)*x^2*b+960*(d*x^2+e*x+c)
^(3/2)*d^(9/2)*x*a-672*(d*x^2+e*x+c)^(3/2)*d^(7/2)*x*b*e-800*(d*x^2+e*x+c)^
```

```
(3/2)*d^(7/2)*a*e-512*(d*x^2+e*x+c)^(3/2)*d^(7/2)*b*c+560*(d*x^2+e*x+c)^(3/
2)*d^(5/2)*b*e^2-480*(d*x^2+e*x+c)^(1/2)*d^(9/2)*x*a*c+600*(d*x^2+e*x+c)^(1
/2)*d^(7/2)*x*a*e^2+720*(d*x^2+e*x+c)^(1/2)*d^(7/2)*x*b*c*e-420*(d*x^2+e*x+
c)^(1/2)*d^(5/2)*x*b*e^3-240*(d*x^2+e*x+c)^(1/2)*d^(7/2)*a*c*e+300*(d*x^2+e
*x+c)^(1/2)*d^(5/2)*a*e^3+360*(d*x^2+e*x+c)^(1/2)*d^(5/2)*b*c*e^2-210*(d*x^
2+e*x+c)^(1/2)*d^(3/2)*b*e^4-480*ln(1/2*(2*(d*x^2+e*x+c)^(1/2)*d^(1/2)+2*d*
x+e)/d^(1/2))*a*c^2*d^4+720*ln(1/2*(2*(d*x^2+e*x+c)^(1/2)*d^(1/2)+2*d*x+e)/
d^(1/2))*a*c*d^3*e^2-150*ln(1/2*(2*(d*x^2+e*x+c)^(1/2)*d^(1/2)+2*d*x+e)/d^(
1/2))*a*d^2*e^4+720*ln(1/2*(2*(d*x^2+e*x+c)^(1/2)*d^(1/2)+2*d*x+e)/d^(1/2))
*b*c^2*d^3*e-600*ln(1/2*(2*(d*x^2+e*x+c)^(1/2)*d^(1/2)+2*d*x+e)/d^(1/2))*b*
c*d^2*e^3+105*ln(1/2*(2*(d*x^2+e*x+c)^(1/2)*d^(1/2)+2*d*x+e)/d^(1/2))*b*d*e
^5)/d^(11/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{dx^2 + ex + c}\,\sqrt{(bx + a)^2}\,x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^2 + e*x + c)*sqrt((b*x + a)^2)*x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,\sqrt{(a + b\,x)^2}\,\sqrt{d\,x^2 + e\,x + c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*((a + b*x)^2)^(1/2)*(c + e*x + d*x^2)^(1/2),x)
```

```
[Out] int(x^2*((a + b*x)^2)^(1/2)*(c + e*x + d*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\sqrt{c + dx^2 + ex}\,\sqrt{(a + bx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*((b*x+a)**2)**(1/2)*(d*x**2+e*x+c)**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(c + d*x**2 + e*x)*sqrt((a + b*x)**2), x)
```

3.47 $\int x\sqrt{a^2+2abx+b^2x^2}\,\sqrt{c+ex+dx^2}\,dx$

Optimal. Leaf size=227

$$-\frac{\sqrt{a^2+2abx+b^2x^2}\,(4cd-e^2)\,(8ade+4bcd-5be^2)\tanh^{-1}\left(\frac{2dx+e}{2\sqrt{d}\,\sqrt{c+dx^2+ex}}\right)}{128d^{7/2}(a+bx)}-\frac{\sqrt{a^2+2abx+b^2x^2}\,(2dx+e)\sqrt{c+\ldots}}{64d^3(a+\ldots}$$

```
[Out] 1/24*(6*b*d*x+8*a*d-5*b*e)*(d*x^2+e*x+c)^(3/2)*((b*x+a)^2)^(1/2)/d^2/(b*x+a
)-1/128*(4*c*d-e^2)*(8*a*d*e+4*b*c*d-5*b*e^2)*arctanh(1/2*(2*d*x+e)/d^(1/2)
/(d*x^2+e*x+c)^(1/2))*((b*x+a)^2)^(1/2)/d^(7/2)/(b*x+a)-1/64*(8*a*d*e+4*b*c
*d-5*b*e^2)*(2*d*x+e)*((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2)/d^3/(b*x+a)
```

Rubi [A] time = 0.13, antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.139, Rules used = {1000, 779, 612, 621, 206}

$$-\frac{\sqrt{a^2+2abx+b^2x^2}\,(2dx+e)\sqrt{c+dx^2+ex}\,(8ade+4bcd-5be^2)}{64d^3(a+bx)}-\frac{\sqrt{a^2+2abx+b^2x^2}\,(4cd-e^2)\,(8ade+4bcd\ldots}{128d^{7/2}(a+bx\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + e*x + d*x^2],x]
```

```
[Out] -((4*b*c*d + 8*a*d*e - 5*b*e^2)*(e + 2*d*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*S
qrt[c + e*x + d*x^2])/(64*d^3*(a + b*x)) + ((8*a*d - 5*b*e + 6*b*d*x)*Sqrt[
a^2 + 2*a*b*x + b^2*x^2]*(c + e*x + d*x^2)^(3/2))/(24*d^2*(a + b*x)) - ((4*
c*d - e^2)*(4*b*c*d + 8*a*d*e - 5*b*e^2)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*ArcT
anh[(e + 2*d*x)/(2*Sqrt[d]*Sqrt[c + e*x + d*x^2])])/(128*d^(7/2)*(a + b*x))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 1000

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_
) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Dist[(a + b*x + c*x^2)^Fr
acPart[p]/((4*c)^IntPart[p]*(b + 2*c*x)^(2*FracPart[p])), Int[(g + h*x)^m*(
b + 2*c*x)^(2*p)*(d + e*x + f*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 h, m, p, q}, x] && EqQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int x\sqrt{a^2+2abx+b^2x^2}\,\sqrt{c+ex+dx^2}\,dx = \frac{\sqrt{a^2+2abx+b^2x^2}\,\int x\left(2ab+2b^2x\right)\sqrt{c+ex+dx^2}\,dx}{2ab+2b^2x}$$

$$= \frac{(8ad-5be+6bdx)\sqrt{a^2+2abx+b^2x^2}\,\left(c+ex+dx^2\right)^{3/2}}{24d^2(a+bx)} - \frac{\left(b\left(4bc\right.\right.}{}$$

$$= -\frac{\left(4bcd+8ade-5be^2\right)(e+2dx)\sqrt{a^2+2abx+b^2x^2}\,\sqrt{c+ex+dx^2}}{64d^3(a+bx)}$$

$$= -\frac{\left(4bcd+8ade-5be^2\right)(e+2dx)\sqrt{a^2+2abx+b^2x^2}\,\sqrt{c+ex+dx^2}}{64d^3(a+bx)}$$

$$= -\frac{\left(4bcd+8ade-5be^2\right)(e+2dx)\sqrt{a^2+2abx+b^2x^2}\,\sqrt{c+ex+dx^2}}{64d^3(a+bx)}$$

Mathematica [A] time = 0.13, size = 147, normalized size = 0.65

$$\frac{\sqrt{(a+bx)^2}\left((c+x(dx+e))^{3/2}(8ad+6bdx-5be) - \frac{3\left(8ade+4bcd-5be^2\right)\left(\left(4cd-e^2\right)\tanh^{-1}\left(\frac{2dx+e}{2\sqrt{d}\,\sqrt{c+x(dx+e)}}\right)+2\sqrt{d}\,(2dx+e)\sqrt{c+x(d}\right.}{16d^{3/2}}\right.}{24d^2(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + e*x + d*x^2],x]
```

```
[Out] (Sqrt[(a + b*x)^2]*((8*a*d - 5*b*e + 6*b*d*x)*(c + x*(e + d*x))^(3/2) - (3*
(4*b*c*d + 8*a*d*e - 5*b*e^2)*(2*Sqrt[d]*(e + 2*d*x)*Sqrt[c + x*(e + d*x)]
+ (4*c*d - e^2)*ArcTanh[(e + 2*d*x)/(2*Sqrt[d]*Sqrt[c + x*(e + d*x)])]))/(1
6*d^(3/2))))/(24*d^2*(a + b*x))
```

fricas [A] time = 0.90, size = 391, normalized size = 1.72

$$\left[\frac{3\left(16\,bc^2d^2+32\,acd^2e-24\,bcde^2-8\,ade^3+5\,be^4\right)\sqrt{d}\,\log\left(8\,d^2x^2+8\,dex-4\,\sqrt{dx^2+ex+c}\,(2\,dx+e)\sqrt{d}\,+\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/768*(3*(16*b*c^2*d^2 + 32*a*c*d^2*e - 24*b*c*d*e^2 - 8*a*d*e^3 + 5*b*e^4
)*sqrt(d)*log(8*d^2*x^2 + 8*d*e*x - 4*sqrt(d*x^2 + e*x + c)*(2*d*x + e)*sqr
t(d) + 4*c*d + e^2) + 4*(48*b*d^4*x^3 + 64*a*c*d^3 - 52*b*c*d^2*e - 24*a*d^
2*e^2 + 15*b*d*e^3 + 8*(8*a*d^4 + b*d^3*e)*x^2 + 2*(12*b*c*d^3 + 8*a*d^3*e
- 5*b*d^2*e^2)*x)*sqrt(d*x^2 + e*x + c))/d^4, 1/384*(3*(16*b*c^2*d^2 + 32*a
*c*d^2*e - 24*b*c*d*e^2 - 8*a*d*e^3 + 5*b*e^4)*sqrt(-d)*arctan(1/2*sqrt(d*x
```

```
^2 + e*x + c)*(2*d*x + e)*sqrt(-d)/(d^2*x^2 + d*e*x + c*d)) + 2*(48*b*d^4*x
^3 + 64*a*c*d^3 - 52*b*c*d^2*e - 24*a*d^2*e^2 + 15*b*d*e^3 + 8*(8*a*d^4 + b
*d^3*e)*x^2 + 2*(12*b*c*d^3 + 8*a*d^3*e - 5*b*d^2*e^2)*x)*sqrt(d*x^2 + e*x
+ c))/d^4]
```

giac [A] time = 0.28, size = 268, normalized size = 1.18

$$\frac{1}{192}\sqrt{dx^2+xe+c}\left(2\left(4\left(6\,bx\mathrm{sgn}\left(bx+a\right)+\frac{8\,ad^3\mathrm{sgn}\left(bx+a\right)+bd^2e\mathrm{sgn}\left(bx+a\right)}{d^3}\right)x+\frac{12\,bcd^2\mathrm{sgn}\left(bx+a\right)+8\,a\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/192*sqrt(d*x^2 + x*e + c)*(2*(4*(6*b*x*sgn(b*x + a) + (8*a*d^3*sgn(b*x +
a) + b*d^2*e*sgn(b*x + a))/d^3)*x + (12*b*c*d^2*sgn(b*x + a) + 8*a*d^2*e*sg
n(b*x + a) - 5*b*d*e^2*sgn(b*x + a))/d^3)*x + (64*a*c*d^2*sgn(b*x + a) - 52
*b*c*d*e*sgn(b*x + a) - 24*a*d*e^2*sgn(b*x + a) + 15*b*e^3*sgn(b*x + a))/d^
3) + 1/128*(16*b*c^2*d^2*sgn(b*x + a) + 32*a*c*d^2*e*sgn(b*x + a) - 24*b*c*
d*e^2*sgn(b*x + a) - 8*a*d*e^3*sgn(b*x + a) + 5*b*e^4*sgn(b*x + a))*log(abs
(-2*(sqrt(d)*x - sqrt(d*x^2 + x*e + c))*sqrt(d) - e))/d^(7/2)
```

maple [C] time = 0.01, size = 381, normalized size = 1.68

$$\frac{\left(-96ac\,d^3e\ln\left(\frac{2dx+e+2\sqrt{d\,x^2+ex+c}\,\sqrt{d}}{2\sqrt{d}}\right)+24a\,d^2e^3\ln\left(\frac{2dx+e+2\sqrt{d\,x^2+ex+c}\,\sqrt{d}}{2\sqrt{d}}\right)-48b\,c^2d^3\ln\left(\frac{2dx+e+2\sqrt{d\,x^2+ex+c}\,\sqrt{d}}{2\sqrt{d}}\right)+7\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2),x)
```

```
[Out] 1/384*csgn(b*x+a)*(96*d^(7/2)*(d*x^2+e*x+c)^(3/2)*x*b+128*d^(7/2)*(d*x^2+e*
x+c)^(3/2)*a-80*d^(5/2)*(d*x^2+e*x+c)^(3/2)*b*e-96*d^(7/2)*(d*x^2+e*x+c)^(1
/2)*x*a*e-48*d^(7/2)*(d*x^2+e*x+c)^(1/2)*x*b*c+60*d^(5/2)*(d*x^2+e*x+c)^(1/
2)*x*b*e^2-48*d^(5/2)*(d*x^2+e*x+c)^(1/2)*a*e^2-24*d^(5/2)*(d*x^2+e*x+c)^(1
/2)*b*c*e+30*d^(3/2)*(d*x^2+e*x+c)^(1/2)*b*e^3-96*ln(1/2*(2*d*x+e+2*(d*x^2+
e*x+c)^(1/2)*d^(1/2))/d^(1/2))*a*c*d^3*e+24*ln(1/2*(2*d*x+e+2*(d*x^2+e*x+c)
^(1/2)*d^(1/2))/d^(1/2))*a*d^2*e^3-48*ln(1/2*(2*d*x+e+2*(d*x^2+e*x+c)^(1/2)
*d^(1/2))/d^(1/2))*b*c^2*d^3+72*ln(1/2*(2*d*x+e+2*(d*x^2+e*x+c)^(1/2)*d^(1/
2))/d^(1/2))*b*c*d^2*e^2-15*ln(1/2*(2*d*x+e+2*(d*x^2+e*x+c)^(1/2)*d^(1/2))/
d^(1/2))*b*d*e^4)/d^(9/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{dx^2+ex+c}\,\sqrt{(bx+a)^2}\,x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^2 + e*x + c)*sqrt((b*x + a)^2)*x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x\,\sqrt{{\left(a+b\,x\right)}^2}\,\sqrt{d\,x^2+e\,x+c}\,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*((a + b*x)^2)^(1/2)*(c + e*x + d*x^2)^(1/2),x)
```

```
[Out] int(x*((a + b*x)^2)^(1/2)*(c + e*x + d*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\sqrt{c + dx^2 + ex}\,\sqrt{(a + bx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((b*x+a)**2)**(1/2)*(d*x**2+e*x+c)**(1/2),x)
```

```
[Out] Integral(x*sqrt(c + d*x**2 + e*x)*sqrt((a + b*x)**2), x)
```

3.48 $\int \sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}\,dx$

Optimal. Leaf size=198

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(4cd-e^2\right)(2ad-be)\tanh^{-1}\left(\frac{2dx+e}{2\sqrt{d}\sqrt{c+dx^2+ex}}\right)}{16d^{5/2}(a+bx)}+\frac{\sqrt{a^2+2abx+b^2x^2}\,(2dx+e)(2ad-be)\sqrt{c+dx}}{8d^2(a+bx)}$$

```
[Out] 1/3*b*(d*x^2+e*x+c)^(3/2)*((b*x+a)^2)^(1/2)/d/(b*x+a)+1/16*(2*a*d-b*e)*(4*c
*d-e^2)*arctanh(1/2*(2*d*x+e)/d^(1/2)/(d*x^2+e*x+c)^(1/2))*((b*x+a)^2)^(1/2
)/d^(5/2)/(b*x+a)+1/8*(2*a*d-b*e)*(2*d*x+e)*((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)
^(1/2)/d^2/(b*x+a)
```

Rubi [A] time = 0.10, antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {969, 640, 612, 621, 206}

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(4cd-e^2\right)(2ad-be)\tanh^{-1}\left(\frac{2dx+e}{2\sqrt{d}\sqrt{c+dx^2+ex}}\right)}{16d^{5/2}(a+bx)}+\frac{\sqrt{a^2+2abx+b^2x^2}\,(2dx+e)(2ad-be)\sqrt{c+dx}}{8d^2(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + e*x + d*x^2],x]
```

```
[Out] ((2*a*d - b*e)*(e + 2*d*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + e*x + d*x
^2])/(8*d^2*(a + b*x)) + (b*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*(c + e*x + d*x^2)
^(3/2))/(3*d*(a + b*x)) + ((2*a*d - b*e)*(4*c*d - e^2)*Sqrt[a^2 + 2*a*b*x +
 b^2*x^2]*ArcTanh[(e + 2*d*x)/(2*Sqrt[d]*Sqrt[c + e*x + d*x^2])])/(16*d^(5/
2)*(a + b*x))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 969

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f_.)*(x_)
^2)^(q_.), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((4*c)^IntPart[p
```

```
]*(b + 2*c*x)^(2*FracPart[p])), Int[(b + 2*c*x)^(2*p)*(d + e*x + f*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, f, p, q}, x] && EqQ[b^2 - 4*a*c, 0] && !In
tegerQ[p]
```

Rubi steps

$$\int \sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}\,dx = \frac{\sqrt{a^2+2abx+b^2x^2}\int \left(2ab+2b^2x\right)\sqrt{c+ex+dx^2}\,dx}{2ab+2b^2x}$$

$$= \frac{b\sqrt{a^2+2abx+b^2x^2}\left(c+ex+dx^2\right)^{3/2}}{3d(a+bx)} + \frac{\left(b(2ad-be)\sqrt{a^2+2abx+\right.}{d\left(2ab\right.}$$

$$= \frac{(2ad-be)(e+2dx)\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}}{8d^2(a+bx)} + \frac{b\sqrt{a^2+2a}}{}$$

$$= \frac{(2ad-be)(e+2dx)\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}}{8d^2(a+bx)} + \frac{b\sqrt{a^2+2a}}{}$$

$$= \frac{(2ad-be)(e+2dx)\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}}{8d^2(a+bx)} + \frac{b\sqrt{a^2+2a}}{}$$

Mathematica [A] time = 0.13, size = 134, normalized size = 0.68

$$\frac{\sqrt{(a+bx)^2}\left(2\sqrt{d}\sqrt{c+x(dx+e)}\left(6ad(2dx+e)+b\left(8cd+8d^2x^2+2dex-3e^2\right)\right)+3\left(4cd-e^2\right)(2ad-be)\tanh\right.}{48d^{5/2}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + e*x + d*x^2],x]
```

```
[Out] (Sqrt[(a + b*x)^2]*(2*Sqrt[d]*Sqrt[c + x*(e + d*x)]*(6*a*d*(e + 2*d*x) + b*
(8*c*d - 3*e^2 + 2*d*e*x + 8*d^2*x^2)) + 3*(2*a*d - b*e)*(4*c*d - e^2)*ArcT
anh[(e + 2*d*x)/(2*Sqrt[d]*Sqrt[c + x*(e + d*x)])]))/(48*d^(5/2)*(a + b*x))
```

fricas [A] time = 0.93, size = 287, normalized size = 1.45

$$\left[\frac{3\left(8\,acd^2-4\,bcde-2\,ade^2+be^3\right)\sqrt{d}\,\log\left(8\,d^2x^2+8\,dex+4\,\sqrt{dx^2+ex+c}\,(2\,dx+e)\sqrt{d}\,+4\,cd+e^2\right)+4\left(8\,b\right.}{96\,d^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/96*(3*(8*a*c*d^2 - 4*b*c*d*e - 2*a*d*e^2 + b*e^3)*sqrt(d)*log(8*d^2*x^2
+ 8*d*e*x + 4*sqrt(d*x^2 + e*x + c)*(2*d*x + e)*sqrt(d) + 4*c*d + e^2) + 4*
(8*b*d^3*x^2 + 8*b*c*d^2 + 6*a*d^2*e - 3*b*d*e^2 + 2*(6*a*d^3 + b*d^2*e)*x)
*sqrt(d*x^2 + e*x + c))/d^3, -1/48*(3*(8*a*c*d^2 - 4*b*c*d*e - 2*a*d*e^2 +
b*e^3)*sqrt(-d)*arctan(1/2*sqrt(d*x^2 + e*x + c)*(2*d*x + e)*sqrt(-d)/(d^2*
x^2 + d*e*x + c*d)) - 2*(8*b*d^3*x^2 + 8*b*c*d^2 + 6*a*d^2*e - 3*b*d*e^2 +
2*(6*a*d^3 + b*d^2*e)*x)*sqrt(d*x^2 + e*x + c))/d^3]
```

giac [A] time = 0.30, size = 185, normalized size = 0.93

$$\frac{1}{24}\sqrt{dx^2+xe+c}\left(2\left(4\,bx\mathrm{sgn}\left(bx+a\right)+\frac{6\,ad^2\mathrm{sgn}\left(bx+a\right)+bde\mathrm{sgn}\left(bx+a\right)}{d^2}\right)x+\frac{8\,bcd\mathrm{sgn}\left(bx+a\right)+6\,ade\mathrm{s}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/24*sqrt(d*x^2 + x*e + c)*(2*(4*b*x*sgn(b*x + a) + (6*a*d^2*sgn(b*x + a) +
 b*d*e*sgn(b*x + a))/d^2)*x + (8*b*c*d*sgn(b*x + a) + 6*a*d*e*sgn(b*x + a)
- 3*b*e^2*sgn(b*x + a))/d^2) - 1/16*(8*a*c*d^2*sgn(b*x + a) - 4*b*c*d*e*sgn
(b*x + a) - 2*a*d*e^2*sgn(b*x + a) + b*e^3*sgn(b*x + a))*log(abs(-2*(sqrt(d
)*x - sqrt(d*x^2 + x*e + c))*sqrt(d) - e))/d^(5/2)
```

maple [C] time = 0.01, size = 257, normalized size = 1.30

$$\frac{\left(24ac\,d^{3}\ln\left(\frac{2dx+e+2\sqrt{d\,x^{2}+ex+c}\,\sqrt{d}}{2\sqrt{d}}\right)-6a\,d^{2}e^{2}\ln\left(\frac{2dx+e+2\sqrt{d\,x^{2}+ex+c}\,\sqrt{d}}{2\sqrt{d}}\right)-12bc\,d^{2}e\ln\left(\frac{2dx+e+2\sqrt{d\,x^{2}+ex+c}\,\sqrt{d}}{2\sqrt{d}}\right)+3bd\,e^{3}\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2),x)
```

```
[Out] 1/48*csgn(b*x+a)*(16*d^(5/2)*(d*x^2+e*x+c)^(3/2)*b+24*d^(7/2)*(d*x^2+e*x+c)
^(1/2)*x*a-12*d^(5/2)*(d*x^2+e*x+c)^(1/2)*x*b*e+12*d^(5/2)*(d*x^2+e*x+c)^(1
/2)*a*e-6*d^(3/2)*(d*x^2+e*x+c)^(1/2)*b*e^2+24*ln(1/2*(2*d*x+e+2*(d*x^2+e*x
+c)^(1/2)*d^(1/2))/d^(1/2))*a*c*d^3-6*ln(1/2*(2*d*x+e+2*(d*x^2+e*x+c)^(1/2)
*d^(1/2))/d^(1/2))*a*d^2*e^2-12*ln(1/2*(2*d*x+e+2*(d*x^2+e*x+c)^(1/2)*d^(1/
2))/d^(1/2))*b*c*d^2*e+3*ln(1/2*(2*d*x+e+2*(d*x^2+e*x+c)^(1/2)*d^(1/2))/d^(
1/2))*b*d*e^3)/d^(7/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{dx^2 + ex + c}\,\sqrt{(bx + a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^2 + e*x + c)*sqrt((b*x + a)^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{(a + b\,x)^2}\,\sqrt{d\,x^2 + e\,x + c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x)^2)^(1/2)*(c + e*x + d*x^2)^(1/2),x)
```

```
[Out] int(((a + b*x)^2)^(1/2)*(c + e*x + d*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{c + dx^2 + ex}\,\sqrt{(a + bx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)**2)**(1/2)*(d*x**2+e*x+c)**(1/2),x)
```

```
[Out] Integral(sqrt(c + d*x**2 + e*x)*sqrt((a + b*x)**2), x)
```

3.49 $\int \frac{\sqrt{a^2+2abx+b^2x^2}\,\sqrt{c+ex+dx^2}}{x}\,dx$

Optimal. Leaf size=211

$$\frac{\sqrt{a^2+2abx+b^2x^2}\,(4ade+4bcd-be^2)\tanh^{-1}\left(\frac{2dx+e}{2\sqrt{d}\,\sqrt{c+dx^2+ex}}\right)}{8d^{3/2}(a+bx)}+\frac{\sqrt{a^2+2abx+b^2x^2}\,\sqrt{c+dx^2+ex}\,(4ad+2bc$$

```
[Out] 1/8*(4*a*d*e+4*b*c*d-b*e^2)*arctanh(1/2*(2*d*x+e)/d^(1/2)/(d*x^2+e*x+c)^(1/
2))*((b*x+a)^2)^(1/2)/d^(3/2)/(b*x+a)-a*arctanh(1/2*(e*x+2*c)/c^(1/2)/(d*x^
2+e*x+c)^(1/2))*c^(1/2)*((b*x+a)^2)^(1/2)/(b*x+a)+1/4*(2*b*d*x+4*a*d+b*e)*(
(b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2)/d/(b*x+a)
```

Rubi [A] time = 0.22, antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1000, 814, 843, 621, 206, 724}

$$\frac{\sqrt{a^2+2abx+b^2x^2}\,(4ade+4bcd-be^2)\tanh^{-1}\left(\frac{2dx+e}{2\sqrt{d}\,\sqrt{c+dx^2+ex}}\right)}{8d^{3/2}(a+bx)}+\frac{\sqrt{a^2+2abx+b^2x^2}\,\sqrt{c+dx^2+ex}\,(4ad+2bc$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + e*x + d*x^2])/x,x]
```

```
[Out] ((4*a*d + b*e + 2*b*d*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + e*x + d*x^2
])/(4*d*(a + b*x)) + ((4*b*c*d + 4*a*d*e - b*e^2)*Sqrt[a^2 + 2*a*b*x + b^2*
x^2]*ArcTanh[(e + 2*d*x)/(2*Sqrt[d]*Sqrt[c + e*x + d*x^2])])/(8*d^(3/2)*(a
+ b*x)) - (a*Sqrt[c]*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*ArcTanh[(2*c + e*x)/(2*S
qrt[c]*Sqrt[c + e*x + d*x^2])])/(a + b*x)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
```

```
/; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1000

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_
) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Dist[(a + b*x + c*x^2)^Fr
acPart[p]/((4*c)^IntPart[p]*(b + 2*c*x)^(2*FracPart[p])), Int[(g + h*x)^m*(
b + 2*c*x)^(2*p)*(d + e*x + f*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 h, m, p, q}, x] && EqQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{\sqrt{a^2+2abx+b^2x^2}\,\sqrt{c+ex+dx^2}}{x}\,dx = \frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{\left(2ab+2b^2x\right)\sqrt{c+ex+dx^2}}{x}\,dx}{2ab+2b^2x}$$

$$= \frac{(4ad+be+2bdx)\sqrt{a^2+2abx+b^2x^2}\,\sqrt{c+ex+dx^2}}{4d(a+bx)} - \frac{\sqrt{a^2+2abx+ \cdots}}{\cdots}$$

$$= \frac{(4ad+be+2bdx)\sqrt{a^2+2abx+b^2x^2}\,\sqrt{c+ex+dx^2}}{4d(a+bx)} + \frac{\left(2abc\sqrt{a^2+2 \cdots}\right.}{\cdots}$$

$$= \frac{(4ad+be+2bdx)\sqrt{a^2+2abx+b^2x^2}\,\sqrt{c+ex+dx^2}}{4d(a+bx)} - \frac{\left(4abc\sqrt{a^2+2 \cdots}\right.}{\cdots}$$

$$= \frac{(4ad+be+2bdx)\sqrt{a^2+2abx+b^2x^2}\,\sqrt{c+ex+dx^2}}{4d(a+bx)} + \frac{\left(4bcd+4ade \cdots\right.}{\cdots}$$

Mathematica [A] time = 0.20, size = 149, normalized size = 0.71

$$\frac{\sqrt{(a+bx)^2}\left(\left(4ade+4bcd-be^2\right)\tanh^{-1}\left(\frac{2dx+e}{2\sqrt{d}\,\sqrt{c+x(dx+e)}}\right)+2\sqrt{d}\left(\sqrt{c+x(dx+e)}\,(4ad+b(2dx+e))-4a\sqrt{c}\,d\tan \cdots\right.\right.}{8d^{3/2}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + e*x + d*x^2])/x,x]
```

```
[Out] (Sqrt[(a + b*x)^2]*((4*b*c*d + 4*a*d*e - b*e^2)*ArcTanh[(e + 2*d*x)/(2*Sqrt
[d]*Sqrt[c + x*(e + d*x)])] + 2*Sqrt[d]*(Sqrt[c + x*(e + d*x)]*(4*a*d + b*(
e + 2*d*x)) - 4*a*Sqrt[c]*d*ArcTanh[(2*c + e*x)/(2*Sqrt[c]*Sqrt[c + x*(e +
d*x)])])))/(8*d^(3/2)*(a + b*x))
```

fricas [A] time = 2.54, size = 651, normalized size = 3.09

$$\left[\frac{8\,a\sqrt{c}\,d^2\log\left(\frac{8\,cex+\left(4\,cd+e^2\right)x^2-4\,\sqrt{dx^2+ex+c}\,(ex+2\,c)\sqrt{c}+8\,c^2}{x^2}\right)-\left(4\,bcd+4\,ade-be^2\right)\sqrt{d}\,\log\left(8\,d^2x^2+8\,dex-4\,\sqrt{dx^2}\right.}{16\,d^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [1/16*(8*a*sqrt(c)*d^2*log((8*c*e*x + (4*c*d + e^2)*x^2 - 4*sqrt(d*x^2 + e*
x + c)*(e*x + 2*c)*sqrt(c) + 8*c^2)/x^2) - (4*b*c*d + 4*a*d*e - b*e^2)*sqrt
(d)*log(8*d^2*x^2 + 8*d*e*x - 4*sqrt(d*x^2 + e*x + c)*(2*d*x + e)*sqrt(d) +
 4*c*d + e^2) + 4*(2*b*d^2*x + 4*a*d^2 + b*d*e)*sqrt(d*x^2 + e*x + c))/d^2,
 1/8*(4*a*sqrt(c)*d^2*log((8*c*e*x + (4*c*d + e^2)*x^2 - 4*sqrt(d*x^2 + e*x
 + c)*(e*x + 2*c)*sqrt(c) + 8*c^2)/x^2) - (4*b*c*d + 4*a*d*e - b*e^2)*sqrt(
-d)*arctan(1/2*sqrt(d*x^2 + e*x + c)*(2*d*x + e)*sqrt(-d)/(d^2*x^2 + d*e*x
+ c*d)) + 2*(2*b*d^2*x + 4*a*d^2 + b*d*e)*sqrt(d*x^2 + e*x + c))/d^2, 1/16*
(16*a*sqrt(-c)*d^2*arctan(1/2*sqrt(d*x^2 + e*x + c)*(e*x + 2*c)*sqrt(-c)/(c
*d*x^2 + c*e*x + c^2)) - (4*b*c*d + 4*a*d*e - b*e^2)*sqrt(d)*log(8*d^2*x^2
+ 8*d*e*x - 4*sqrt(d*x^2 + e*x + c)*(2*d*x + e)*sqrt(d) + 4*c*d + e^2) + 4*
(2*b*d^2*x + 4*a*d^2 + b*d*e)*sqrt(d*x^2 + e*x + c))/d^2, 1/8*(8*a*sqrt(-c)
*d^2*arctan(1/2*sqrt(d*x^2 + e*x + c)*(e*x + 2*c)*sqrt(-c)/(c*d*x^2 + c*e*x
 + c^2)) - (4*b*c*d + 4*a*d*e - b*e^2)*sqrt(-d)*arctan(1/2*sqrt(d*x^2 + e*x
 + c)*(2*d*x + e)*sqrt(-d)/(d^2*x^2 + d*e*x + c*d)) + 2*(2*b*d^2*x + 4*a*d^
2 + b*d*e)*sqrt(d*x^2 + e*x + c))/d^2]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2)/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(b*x
+a)]Warning, replacing 0 by ` u`, a substitution variable should perhaps be
 purged.Warning, replacing 0 by ` u`, a substitution variable should perhap
s be purged.Warning, replacing 0 by ` u`, a substitution variable should pe
rhaps be purged.index.cc index_m operator + Error: Bad Argument Value
```

maple [C] time = 0.01, size = 214, normalized size = 1.01

$$-\frac{\left(8a\sqrt{c}\,d^{\frac{5}{2}}\ln\left(\frac{ex+2c+2\sqrt{d\,x^2+ex+c}\,\sqrt{c}}{x}\right)-4a\,d^2e\ln\left(\frac{2dx+e+2\sqrt{d\,x^2+ex+c}\,\sqrt{d}}{2\sqrt{d}}\right)-4bc\,d^2\ln\left(\frac{2dx+e+2\sqrt{d\,x^2+ex+c}\,\sqrt{d}}{2\sqrt{d}}\right)+bd\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2)/x,x)
```

```
[Out] -1/8*csgn(b*x+a)*(8*c^(1/2)*d^(5/2)*ln((2*c+e*x+2*c^(1/2)*(d*x^2+e*x+c)^(1/
2))/x)*a-4*d^(5/2)*(d*x^2+e*x+c)^(1/2)*x*b-8*d^(5/2)*(d*x^2+e*x+c)^(1/2)*a-
2*d^(3/2)*(d*x^2+e*x+c)^(1/2)*b*e-4*d^2*ln(1/2*(2*d*x+e+2*(d*x^2+e*x+c)^(1/
2)*d^(1/2))/d^(1/2))*a*e-4*ln(1/2*(2*d*x+e+2*(d*x^2+e*x+c)^(1/2)*d^(1/2))/d
^(1/2))*b*c*d^2+ln(1/2*(2*d*x+e+2*(d*x^2+e*x+c)^(1/2)*d^(1/2))/d^(1/2))*b*d
*e^2)/d^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx^2 + ex + c}\,\sqrt{(bx + a)^2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2)/x,x, algorithm="maxima")
[Out] integrate(sqrt(d*x^2 + e*x + c)*sqrt((b*x + a)^2)/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{(a + b\,x)^2}\,\sqrt{d\,x^2 + e\,x + c}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((((a + b*x)^2)^(1/2)*(c + e*x + d*x^2)^(1/2))/x,x)
[Out] int((((a + b*x)^2)^(1/2)*(c + e*x + d*x^2)^(1/2))/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c + dx^2 + ex}\,\sqrt{(a + bx)^2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)**2)**(1/2)*(d*x**2+e*x+c)**(1/2)/x,x)
[Out] Integral(sqrt(c + d*x**2 + e*x)*sqrt((a + b*x)**2)/x, x)
```

3.50 $\int \frac{\sqrt{a^2+2abx+b^2x^2}\,\sqrt{c+ex+dx^2}}{x^2}\,dx$

Optimal. Leaf size=202

$$-\frac{\sqrt{a^2+2abx+b^2x^2}\,(a-bx)\sqrt{c+dx^2+ex}}{x(a+bx)}+\frac{\sqrt{a^2+2abx+b^2x^2}\,(2ad+be)\tanh^{-1}\left(\frac{2dx+e}{2\sqrt{d}\,\sqrt{c+dx^2+ex}}\right)}{2\sqrt{d}\,(a+bx)}-\frac{\sqrt{a^2+2a}}{}$$

```
[Out] -1/2*(a*e+2*b*c)*arctanh(1/2*(e*x+2*c)/c^(1/2)/(d*x^2+e*x+c)^(1/2))*((b*x+a
)^2)^(1/2)/(b*x+a)/c^(1/2)+1/2*(2*a*d+b*e)*arctanh(1/2*(2*d*x+e)/d^(1/2)/(d
*x^2+e*x+c)^(1/2))*((b*x+a)^2)^(1/2)/(b*x+a)/d^(1/2)-(-b*x+a)*((b*x+a)^2)^(
1/2)*(d*x^2+e*x+c)^(1/2)/x/(b*x+a)
```

Rubi [A] time = 0.19, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {1000, 812, 843, 621, 206, 724}

$$-\frac{\sqrt{a^2+2abx+b^2x^2}\,(a-bx)\sqrt{c+dx^2+ex}}{x(a+bx)}+\frac{\sqrt{a^2+2abx+b^2x^2}\,(2ad+be)\tanh^{-1}\left(\frac{2dx+e}{2\sqrt{d}\,\sqrt{c+dx^2+ex}}\right)}{2\sqrt{d}\,(a+bx)}-\frac{\sqrt{a^2+2a}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + e*x + d*x^2])/x^2,x]
```

```
[Out] -(((a - b*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + e*x + d*x^2])/(x*(a + b
*x))) + ((2*a*d + b*e)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*ArcTanh[(e + 2*d*x)/(2
*Sqrt[d]*Sqrt[c + e*x + d*x^2])])/(2*Sqrt[d]*(a + b*x)) - ((2*b*c + a*e)*Sq
rt[a^2 + 2*a*b*x + b^2*x^2]*ArcTanh[(2*c + e*x)/(2*Sqrt[c]*Sqrt[c + e*x + d
*x^2])])/(2*Sqrt[c]*(a + b*x))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 812

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2)
- d*g*(2*p + 1) + e*g*(m + 1)*x)*(a + b*x + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p
 + 2)), x] + Dist[p/(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m +
 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
```

```
NeQ[c*d^2 - b*d*e + a*e^2, 0] && RationalQ[p] && p > 0 && (LtQ[m, -1] || Eq
Q[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p
 + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1000

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_
) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Dist[(a + b*x + c*x^2)^Fr
acPart[p]/((4*c)^IntPart[p]*(b + 2*c*x)^(2*FracPart[p])), Int[(g + h*x)^m*(
b + 2*c*x)^(2*p)*(d + e*x + f*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 h, m, p, q}, x] && EqQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}}{x^2}\,dx = \frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{\left(2ab+2b^2x\right)\sqrt{c+ex+dx^2}}{x^2}\,dx}{2ab+2b^2x}$$

$$= -\frac{(a-bx)\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}}{x(a+bx)} - \frac{\sqrt{a^2+2abx+b^2x^2}\int \cdots}{2\left(2ab+\cdots\right.}$$

$$= -\frac{(a-bx)\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}}{x(a+bx)} + \frac{\left(b(2bc+ae)\sqrt{a^2+2ab\cdots\right.}{2ab\cdots}$$

$$= -\frac{(a-bx)\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}}{x(a+bx)} - \frac{\left(2b(2bc+ae)\sqrt{a^2+2a\cdots\right.}{\cdots}$$

$$= -\frac{(a-bx)\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}}{x(a+bx)} + \frac{(2ad+be)\sqrt{a^2+2abx\cdots}}{2\sqrt{\cdots}}$$

Mathematica [A] time = 0.19, size = 155, normalized size = 0.77

$$\frac{\sqrt{(a+bx)^2}\left(\sqrt{c}\,x(2ad+be)\tanh^{-1}\left(\frac{2dx+e}{2\sqrt{d}\sqrt{c+x(dx+e)}}\right)+\sqrt{d}\left(2\sqrt{c}\,(bx-a)\sqrt{c+x(dx+e)}-x(ae+2bc)\tanh^{-1}\left(\frac{\cdots}{2\sqrt{\cdots}}\right.\right.\right.}{2\sqrt{c}\sqrt{d}\,x(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + e*x + d*x^2])/x^2,x]
```

```
[Out] (Sqrt[(a + b*x)^2]*(Sqrt[c]*(2*a*d + b*e)*x*ArcTanh[(e + 2*d*x)/(2*Sqrt[d]*
Sqrt[c + x*(e + d*x)])] + Sqrt[d]*(2*Sqrt[c]*(-a + b*x)*Sqrt[c + x*(e + d*x
)] - (2*b*c + a*e)*x*ArcTanh[(2*c + e*x)/(2*Sqrt[c]*Sqrt[c + x*(e + d*x)])]
)))/(2*Sqrt[c]*Sqrt[d]*x*(a + b*x))
```

fricas [A] time = 1.40, size = 647, normalized size = 3.20

$$\left[\frac{(2\,acd + bce)\sqrt{d}\,x\log\left(8\,d^2x^2 + 8\,dex + 4\,\sqrt{dx^2 + ex + c}\,(2\,dx + e)\sqrt{d} + 4\,cd + e^2\right) + (2\,bcd + ade)\sqrt{c}\,x\log\left(\frac{8}{-}\right.}{4\,cdx}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] [1/4*((2*a*c*d + b*c*e)*sqrt(d)*x*log(8*d^2*x^2 + 8*d*e*x + 4*sqrt(d*x^2 +
e*x + c)*(2*d*x + e)*sqrt(d) + 4*c*d + e^2) + (2*b*c*d + a*d*e)*sqrt(c)*x*l
og((8*c*e*x + (4*c*d + e^2)*x^2 - 4*sqrt(d*x^2 + e*x + c)*(e*x + 2*c)*sqrt(
c) + 8*c^2)/x^2) + 4*(b*c*d*x - a*c*d)*sqrt(d*x^2 + e*x + c))/(c*d*x), -1/4
*(2*(2*a*c*d + b*c*e)*sqrt(-d)*x*arctan(1/2*sqrt(d*x^2 + e*x + c)*(2*d*x +
e)*sqrt(-d)/(d^2*x^2 + d*e*x + c*d)) - (2*b*c*d + a*d*e)*sqrt(c)*x*log((8*c
*e*x + (4*c*d + e^2)*x^2 - 4*sqrt(d*x^2 + e*x + c)*(e*x + 2*c)*sqrt(c) + 8*
c^2)/x^2) - 4*(b*c*d*x - a*c*d)*sqrt(d*x^2 + e*x + c))/(c*d*x), 1/4*(2*(2*b
*c*d + a*d*e)*sqrt(-c)*x*arctan(1/2*sqrt(d*x^2 + e*x + c)*(e*x + 2*c)*sqrt(
-c)/(c*d*x^2 + c*e*x + c^2)) + (2*a*c*d + b*c*e)*sqrt(d)*x*log(8*d^2*x^2 +
8*d*e*x + 4*sqrt(d*x^2 + e*x + c)*(2*d*x + e)*sqrt(d) + 4*c*d + e^2) + 4*(b
*c*d*x - a*c*d)*sqrt(d*x^2 + e*x + c))/(c*d*x), 1/2*((2*b*c*d + a*d*e)*sqrt
(-c)*x*arctan(1/2*sqrt(d*x^2 + e*x + c)*(e*x + 2*c)*sqrt(-c)/(c*d*x^2 + c*e
*x + c^2)) - (2*a*c*d + b*c*e)*sqrt(-d)*x*arctan(1/2*sqrt(d*x^2 + e*x + c)*
(2*d*x + e)*sqrt(-d)/(d^2*x^2 + d*e*x + c*d)) + 2*(b*c*d*x - a*c*d)*sqrt(d*
x^2 + e*x + c))/(c*d*x)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(b*x
+a)]Unable to divide, perhaps due to rounding error%%%{2,[4,4,0]%%%}+%%%{%%
%{-16,[1]%%%},[4,2,1]%%%}+%%%{%%%{32,[2]%%%},[4,0,2]%%%}+%%%{-4,[2,4,1]%%%}
+%%%{%%%{32,[1]%%%},[2,2,2]%%%}+%%%{%%%{-64,[2]%%%},[2,0,3]%%%}+%%%{2,[0,4,
2]%%%}+%%%{%%%{-16,[1]%%%},[0,2,3]%%%}+%%%{%%%{32,[2]%%%},[0,0,4]%%%} / %%%
{4,[4,0,0]%%%}+%%%{-8,[2,0,1]%%%}+%%%{4,[0,0,2]%%%} Error: Bad Argument Val
ue
```

maple [C] time = 0.01, size = 249, normalized size = 1.23

$$\frac{\left(2ac\,d^2x\ln\left(\frac{2dx+e+2\sqrt{d\,x^2+ex+c}\,\sqrt{d}}{2\sqrt{d}}\right) - a\sqrt{c}\,d^{\frac{3}{2}}ex\ln\left(\frac{ex+2c+2\sqrt{d\,x^2+ex+c}\,\sqrt{c}}{x}\right) - 2b\,c^{\frac{3}{2}}d^{\frac{3}{2}}x\ln\left(\frac{ex+2c+2\sqrt{d\,x^2+ex+c}\,\sqrt{c}}{x}\right) + \right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2)/x^2,x)
```

```
[Out] 1/2*csgn(b*x+a)*(2*d^(5/2)*(d*x^2+e*x+c)^(1/2)*x^2*a-2*d^(3/2)*c^(3/2)*ln((
e*x+2*c+2*(d*x^2+e*x+c)^(1/2)*c^(1/2))/x)*x*b-d^(3/2)*c^(1/2)*ln((e*x+2*c+2
*(d*x^2+e*x+c)^(1/2)*c^(1/2))/x)*x*a*e-2*d^(3/2)*(d*x^2+e*x+c)^(3/2)*a+2*d^
(3/2)*(d*x^2+e*x+c)^(1/2)*x*a*e+2*d^(3/2)*(d*x^2+e*x+c)^(1/2)*x*b*c+2*ln(1/
```

```
2*(2*d*x+e+2*(d*x^2+e*x+c)^(1/2)*d^(1/2))/d^(1/2))*x*a*c*d^2+ln(1/2*(2*d*x+
e+2*(d*x^2+e*x+c)^(1/2)*d^(1/2))/d^(1/2))*d*x*b*c*e)/c/x/d^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx^2 + ex + c}\,\sqrt{(bx + a)^2}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^2 + e*x + c)*sqrt((b*x + a)^2)/x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{(a + b\,x)^2}\,\sqrt{d\,x^2 + e\,x + c}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((((a + b*x)^2)^(1/2)*(c + e*x + d*x^2)^(1/2))/x^2,x)
```

```
[Out] int((((a + b*x)^2)^(1/2)*(c + e*x + d*x^2)^(1/2))/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c + dx^2 + ex}\,\sqrt{(a + bx)^2}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)**2)**(1/2)*(d*x**2+e*x+c)**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(c + d*x**2 + e*x)*sqrt((a + b*x)**2)/x**2, x)
```

3.51 $\int \frac{\sqrt{a^2+2abx+b^2x^2}\,\sqrt{c+ex+dx^2}}{x^3}\,dx$

Optimal. Leaf size=215

$$-\frac{\sqrt{a^2+2abx+b^2x^2}\,\left(4acd-ae^2+4bce\right)\tanh^{-1}\left(\frac{2c+ex}{2\sqrt{c}\,\sqrt{c+dx^2+ex}}\right)}{8c^{3/2}(a+bx)}-\frac{\sqrt{a^2+2abx+b^2x^2}\,\sqrt{c+dx^2+ex}\,(x(ae+4}{4cx^2(a+bx)}$$

```
[Out] -1/8*(4*a*c*d-a*e^2+4*b*c*e)*arctanh(1/2*(e*x+2*c)/c^(1/2)/(d*x^2+e*x+c)^(1
/2))*((b*x+a)^2)^(1/2)/c^(3/2)/(b*x+a)+b*arctanh(1/2*(2*d*x+e)/d^(1/2)/(d*x
^2+e*x+c)^(1/2))*d^(1/2)*((b*x+a)^2)^(1/2)/(b*x+a)-1/4*(2*a*c+(a*e+4*b*c)*x
)*((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2)/c/x^2/(b*x+a)
```

Rubi [A] time = 0.18, antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1000, 810, 843, 621, 206, 724}

$$-\frac{\sqrt{a^2+2abx+b^2x^2}\,\left(4acd-ae^2+4bce\right)\tanh^{-1}\left(\frac{2c+ex}{2\sqrt{c}\,\sqrt{c+dx^2+ex}}\right)}{8c^{3/2}(a+bx)}-\frac{\sqrt{a^2+2abx+b^2x^2}\,\sqrt{c+dx^2+ex}\,(x(ae+4}{4cx^2(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + e*x + d*x^2])/x^3,x]
```

```
[Out] -((2*a*c + (4*b*c + a*e)*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + e*x + d*
x^2])/(4*c*x^2*(a + b*x)) + (b*Sqrt[d]*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*ArcTan
h[(e + 2*d*x)/(2*Sqrt[d]*Sqrt[c + e*x + d*x^2])])/(a + b*x) - ((4*a*c*d + 4
*b*c*e - a*e^2)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*ArcTanh[(2*c + e*x)/(2*Sqrt[c
]*Sqrt[c + e*x + d*x^2])])/(8*c^(3/2)*(a + b*x))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 810

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^
p*((d*g - e*f*(m + 2))*(c*d^2 - b*d*e + a*e^2) - d*p*(2*c*d - b*e)*(e*f - d
*g) - e*(g*(m + 1)*(c*d^2 - b*d*e + a*e^2) + p*(2*c*d - b*e)*(e*f - d*g))*x
))/(e^2*(m + 1)*(m + 2)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[p/(e^2*(m + 1)*
(m + 2)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(
p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) + b^2*e*(d*g*(p + 1) - e*f*(m + p +
 2)) + b*(a*e^2*g*(m + 1) - c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2))) - c*(2
```

```
*c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2)) - e*(2*a*e*g*(m + 1) - b*(d*g*(m -
 2*p) + e*f*(m + 2*p + 2))))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g},
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] &&
LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1000

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_
) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Dist[(a + b*x + c*x^2)^Fr
acPart[p]/((4*c)^IntPart[p]*(b + 2*c*x)^(2*FracPart[p])), Int[(g + h*x)^m*(
b + 2*c*x)^(2*p)*(d + e*x + f*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 h, m, p, q}, x] && EqQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}}{x^3}\,dx = \frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{(2ab+2b^2x)\sqrt{c+ex+dx^2}}{x^3}\,dx}{2ab+2b^2x}$$

$$= -\frac{(2ac+(4bc+ae)x)\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}}{4cx^2(a+bx)} - \frac{\sqrt{a^2+2abx \cdots}}{\cdots}$$

$$= -\frac{(2ac+(4bc+ae)x)\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}}{4cx^2(a+bx)} + \frac{\left(2b^2d\sqrt{a^2+\cdots}\right.}{\cdots}$$

$$= -\frac{(2ac+(4bc+ae)x)\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}}{4cx^2(a+bx)} + \frac{\left(4b^2d\sqrt{a^2+\cdots}\right.}{\cdots}$$

$$= -\frac{(2ac+(4bc+ae)x)\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}}{4cx^2(a+bx)} + \frac{b\sqrt{d}\sqrt{a^2+\cdots}}{\cdots}$$

Mathematica [A] time = 0.22, size = 161, normalized size = 0.75

$$-\frac{\sqrt{(a+bx)^2}\left(x^2\left(4acd-ae^2+4bce\right)\tanh^{-1}\left(\frac{2c+ex}{2\sqrt{c}\sqrt{c+x(dx+e)}}\right)+2\sqrt{c}\sqrt{c+x(dx+e)}(2ac+aex+4bcx)-8bc^{3/2}\sqrt{\cdots}\right.}{8c^{3/2}x^2(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + e*x + d*x^2])/x^3,x]
```

```
[Out] -1/8*(Sqrt[(a + b*x)^2]*(2*Sqrt[c]*(2*a*c + 4*b*c*x + a*e*x)*Sqrt[c + x*(e
+ d*x)] - 8*b*c^(3/2)*Sqrt[d]*x^2*ArcTanh[(e + 2*d*x)/(2*Sqrt[d]*Sqrt[c + x
*(e + d*x)])] + (4*a*c*d + 4*b*c*e - a*e^2)*x^2*ArcTanh[(2*c + e*x)/(2*Sqrt
[c]*Sqrt[c + x*(e + d*x)])]))/(c^(3/2)*x^2*(a + b*x))
```

fricas [A] time = 1.81, size = 693, normalized size = 3.22

$$\left[\frac{8\,bc^2\sqrt{d}\,x^2\log\left(8\,d^2x^2+8\,dex+4\,\sqrt{dx^2+ex+c}\,(2\,dx+e)\sqrt{d}\;+4\,cd+e^2\right)-\left(4\,acd+4\,bce-ae^2\right)\sqrt{c}\,x^2\log\left(\cdots\right.}{16\,c^2x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] [1/16*(8*b*c^2*sqrt(d)*x^2*log(8*d^2*x^2 + 8*d*e*x + 4*sqrt(d*x^2 + e*x + c
)*(2*d*x + e)*sqrt(d) + 4*c*d + e^2) - (4*a*c*d + 4*b*c*e - a*e^2)*sqrt(c)*
x^2*log((8*c*e*x + (4*c*d + e^2)*x^2 + 4*sqrt(d*x^2 + e*x + c)*(e*x + 2*c)*
sqrt(c) + 8*c^2)/x^2) - 4*(2*a*c^2 + (4*b*c^2 + a*c*e)*x)*sqrt(d*x^2 + e*x
+ c))/(c^2*x^2), -1/16*(16*b*c^2*sqrt(-d)*x^2*arctan(1/2*sqrt(d*x^2 + e*x +
 c)*(2*d*x + e)*sqrt(-d)/(d^2*x^2 + d*e*x + c*d)) + (4*a*c*d + 4*b*c*e - a*
e^2)*sqrt(c)*x^2*log((8*c*e*x + (4*c*d + e^2)*x^2 + 4*sqrt(d*x^2 + e*x + c)
*(e*x + 2*c)*sqrt(c) + 8*c^2)/x^2) + 4*(2*a*c^2 + (4*b*c^2 + a*c*e)*x)*sqrt
(d*x^2 + e*x + c))/(c^2*x^2), 1/8*(4*b*c^2*sqrt(d)*x^2*log(8*d^2*x^2 + 8*d*
e*x + 4*sqrt(d*x^2 + e*x + c)*(2*d*x + e)*sqrt(d) + 4*c*d + e^2) + (4*a*c*d
 + 4*b*c*e - a*e^2)*sqrt(-c)*x^2*arctan(1/2*sqrt(d*x^2 + e*x + c)*(e*x + 2*
c)*sqrt(-c)/(c*d*x^2 + c*e*x + c^2)) - 2*(2*a*c^2 + (4*b*c^2 + a*c*e)*x)*sq
rt(d*x^2 + e*x + c))/(c^2*x^2), -1/8*(8*b*c^2*sqrt(-d)*x^2*arctan(1/2*sqrt(
d*x^2 + e*x + c)*(2*d*x + e)*sqrt(-d)/(d^2*x^2 + d*e*x + c*d)) - (4*a*c*d +
 4*b*c*e - a*e^2)*sqrt(-c)*x^2*arctan(1/2*sqrt(d*x^2 + e*x + c)*(e*x + 2*c)
*sqrt(-c)/(c*d*x^2 + c*e*x + c^2)) + 2*(2*a*c^2 + (4*b*c^2 + a*c*e)*x)*sqrt
(d*x^2 + e*x + c))/(c^2*x^2)]
```

giac [B] time = 0.41, size = 450, normalized size = 2.09

$$-b\sqrt{d}\,\log\left(\left|-2\left(\sqrt{d}\,x-\sqrt{dx^2+xe+c}\right)d-\sqrt{d}\,e\right|\right)\mathrm{sgn}\left(bx+a\right)+\frac{\left(4\,acd\mathrm{sgn}\left(bx+a\right)+4\,bce\mathrm{sgn}\left(bx+a\right)-ae^2\mathrm{s}\cdots\right.}{4\,\sqrt{-c}\,c\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] -b*sqrt(d)*log(abs(-2*(sqrt(d)*x - sqrt(d*x^2 + x*e + c))*d - sqrt(d)*e))*s
gn(b*x + a) + 1/4*(4*a*c*d*sgn(b*x + a) + 4*b*c*e*sgn(b*x + a) - a*e^2*sgn(
b*x + a))*arctan(-(sqrt(d)*x - sqrt(d*x^2 + x*e + c))/sqrt(-c))/(sqrt(-c)*c
) + 1/4*(4*(sqrt(d)*x - sqrt(d*x^2 + x*e + c))^3*a*c*d*sgn(b*x + a) + 4*(sq
rt(d)*x - sqrt(d*x^2 + x*e + c))^3*b*c*e*sgn(b*x + a) + 8*(sqrt(d)*x - sqrt
(d*x^2 + x*e + c))^2*b*c^2*sqrt(d)*sgn(b*x + a) + 8*(sqrt(d)*x - sqrt(d*x^2
 + x*e + c))^2*a*c*sqrt(d)*e*sgn(b*x + a) + 4*(sqrt(d)*x - sqrt(d*x^2 + x*e
 + c))*a*c^2*d*sgn(b*x + a) + (sqrt(d)*x - sqrt(d*x^2 + x*e + c))^3*a*e^2*s
gn(b*x + a) - 4*(sqrt(d)*x - sqrt(d*x^2 + x*e + c))*b*c^2*e*sgn(b*x + a) -
8*b*c^3*sqrt(d)*sgn(b*x + a) + (sqrt(d)*x - sqrt(d*x^2 + x*e + c))*a*c*e^2*
sgn(b*x + a))/(((sqrt(d)*x - sqrt(d*x^2 + x*e + c))^2 - c)^2*c)
```

maple [C] time = 0.02, size = 358, normalized size = 1.67

$$\frac{\left(-4a\,c^{\frac{3}{2}}d^{\frac{5}{2}}x^2\ln\left(\frac{ex+2c+2\sqrt{d\,x^2+ex+c}\;\sqrt{c}}{x}\right)+a\sqrt{c}\;d^{\frac{3}{2}}e^2x^2\ln\left(\frac{ex+2c+2\sqrt{d\,x^2+ex+c}\;\sqrt{c}}{x}\right)+8b\,c^2d^2x^2\ln\left(\frac{2dx+e+2\sqrt{d\,x^2+ex+c}\cdots}{2\sqrt{d}}\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2)/x^3,x)
```

```
[Out] 1/8*csgn(b*x+a)*(-4*d^(5/2)*c^(3/2)*ln((e*x+2*c+2*(d*x^2+e*x+c)^(1/2)*c^(1/
2))/x)*x^2*a-2*d^(5/2)*(d*x^2+e*x+c)^(1/2)*x^3*a*e+8*d^(5/2)*(d*x^2+e*x+c)^
(1/2)*x^3*b*c-4*d^(3/2)*c^(3/2)*ln((e*x+2*c+2*(d*x^2+e*x+c)^(1/2)*c^(1/2))/
x)*x^2*b*e+4*d^(5/2)*(d*x^2+e*x+c)^(1/2)*x^2*a*c+d^(3/2)*c^(1/2)*ln((e*x+2*
c+2*(d*x^2+e*x+c)^(1/2)*c^(1/2))/x)*x^2*a*e^2+2*d^(3/2)*(d*x^2+e*x+c)^(3/2)
*x*a*e-8*d^(3/2)*(d*x^2+e*x+c)^(3/2)*x*b*c-2*d^(3/2)*(d*x^2+e*x+c)^(1/2)*x^
2*a*e^2+8*d^(3/2)*(d*x^2+e*x+c)^(1/2)*x^2*b*c*e+8*ln(1/2*(2*d*x+e+2*(d*x^2+
e*x+c)^(1/2)*d^(1/2))/d^(1/2))*x^2*b*c^2*d^2-4*d^(3/2)*(d*x^2+e*x+c)^(3/2)*
a*c)/x^2/c^2/d^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx^2+ex+c}\,\sqrt{(bx+a)^2}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^2 + e*x + c)*sqrt((b*x + a)^2)/x^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{(a+b\,x)^2}\,\sqrt{d\,x^2+e\,x+c}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((((a + b*x)^2)^(1/2)*(c + e*x + d*x^2)^(1/2))/x^3,x)
```

```
[Out] int((((a + b*x)^2)^(1/2)*(c + e*x + d*x^2)^(1/2))/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c+dx^2+ex}\,\sqrt{(a+bx)^2}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)**2)**(1/2)*(d*x**2+e*x+c)**(1/2)/x**3,x)
```

```
[Out] Integral(sqrt(c + d*x**2 + e*x)*sqrt((a + b*x)**2)/x**3, x)
```

3.52 $\int \frac{x^2 \sqrt{a+cx^2}}{d+ex+fx^2} \, dx$

Optimal. Leaf size=452

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)\left(af^2+2c\left(e^2-df\right)\right)}{2\sqrt{c}f^3}-\frac{\left(e\left(e-\sqrt{e^2-4df}\right)\left(af^2+c\left(e^2-2df\right)\right)-2df\left(af^2+c\left(e^2-df\right)\right)\right)\tan}{\sqrt{2}f^3\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}\right.}}$$

```
[Out] 1/2*(a*f^2+2*c*(-d*f+e^2))*arctanh(x*c^(1/2)/(c*x^2+a)^(1/2))/f^3/c^(1/2)-1
/2*(-f*x+2*e)*(c*x^2+a)^(1/2)/f^2-1/2*arctanh(1/2*(2*a*f-c*x*(e-(-4*d*f+e^2
)^(1/2)))*2^(1/2)/(c*x^2+a)^(1/2)/(2*a*f^2+(-(-4*d*f+e^2)^(1/2)*e-2*d*f+e^2
)*c)^(1/2))*(-2*d*f*(a*f^2+c*(-d*f+e^2))+e*(a*f^2+c*(-2*d*f+e^2))*(e-(-4*d*
f+e^2)^(1/2)))/f^3*2^(1/2)/(-4*d*f+e^2)^(1/2)/(2*a*f^2+(-(-4*d*f+e^2)^(1/2)
*e-2*d*f+e^2)*c)^(1/2)+1/2*arctanh(1/2*(2*a*f-c*x*(e+(-4*d*f+e^2)^(1/2)))*2
^(1/2)/(c*x^2+a)^(1/2)/(2*a*f^2+((-4*d*f+e^2)^(1/2)*e-2*d*f+e^2)*c)^(1/2))*
(-2*d*f*(a*f^2+c*(-d*f+e^2))+e*(a*f^2+c*(-2*d*f+e^2))*(e+(-4*d*f+e^2)^(1/2)
))/f^3*2^(1/2)/(-4*d*f+e^2)^(1/2)/(2*a*f^2+((-4*d*f+e^2)^(1/2)*e-2*d*f+e^2)
*c)^(1/2)
```

Rubi [A] time = 1.97, antiderivative size = 452, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1069, 1080, 217, 206, 1034, 725}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)\left(af^2+2c\left(e^2-df\right)\right)}{2\sqrt{c}f^3}-\frac{\left(e\left(e-\sqrt{e^2-4df}\right)\left(af^2+c\left(e^2-2df\right)\right)-2df\left(af^2+c\left(e^2-df\right)\right)\right)\tan}{\sqrt{2}f^3\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}\right.}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sqrt[a + c*x^2])/(d + e*x + f*x^2),x]
```

```
[Out] -((2*e - f*x)*Sqrt[a + c*x^2])/(2*f^2) + ((a*f^2 + 2*c*(e^2 - d*f))*ArcTanh
[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(2*Sqrt[c]*f^3) - ((e*(e - Sqrt[e^2 - 4*d*f]
)*(a*f^2 + c*(e^2 - 2*d*f)) - 2*d*f*(a*f^2 + c*(e^2 - d*f)))*ArcTanh[(2*a*f
 - c*(e - Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*
Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[2]*f^3*Sqrt[e^2 - 4*d*f]*Sqrt[
2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]) + ((e*(e + Sqrt[e^2 - 4*d
*f])*(a*f^2 + c*(e^2 - 2*d*f)) - 2*d*f*(a*f^2 + c*(e^2 - d*f)))*ArcTanh[(2*
a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f +
 e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[2]*f^3*Sqrt[e^2 - 4*d*f]*Sq
rt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
```

```
[{a, c, d, e}, x]
```

Rule 1034

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f
_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(
b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Dist[(2*c*g -
h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a,
 b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1069

```
Int[((a_) + (c_.)*(x_)^2)^(p_)*((A_.) + (C_.)*(x_)^2)*((d_) + (e_.)*(x_) +
(f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((C*(-(c*e*(2*p + q + 2))) + 2*c*C*f*
(p + q + 1)*x)*(a + c*x^2)^p*(d + e*x + f*x^2)^(q + 1))/(2*c*f^2*(p + q + 1
)*(2*p + 2*q + 3)), x] - Dist[1/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3)), Int[
(a + c*x^2)^(p - 1)*(d + e*x + f*x^2)^q*Simp[p*(-(a*e))*(C*(c*e)*(q + 1) -
c*(C*e)*(2*p + 2*q + 3)) + (p + q + 1)*(a*c*(C*(2*d*f - e^2*(2*p + q + 2))
+ f*(-2*A*f)*(2*p + 2*q + 3))) + (2*p*(c*d - a*f)*(C*(c*e)*(q + 1) - c*(C*e
)*(2*p + 2*q + 3)) + (p + q + 1)*(C*e*f*p*(-4*a*c)))*x + (p*(c*e)*(C*(c*e)*
(q + 1) - c*(C*e)*(2*p + 2*q + 3)) + (p + q + 1)*(C*f^2*p*(-4*a*c) - c^2*(C
*(e^2 - 4*d*f)*(2*p + q + 2) + f*(2*C*d + 2*A*f)*(2*p + 2*q + 3))))*x^2, x]
, x], x] /; FreeQ[{a, c, d, e, f, A, C, q}, x] && NeQ[e^2 - 4*d*f, 0] && Gt
Q[p, 0] && NeQ[p + q + 1, 0] && NeQ[2*p + 2*q + 3, 0] &&  !IGtQ[p, 0] &&  !
IGtQ[q, 0]
```

Rule 1080

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*Sqrt[(d_.) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqrt[d + f*x^2],
 x], x] + Dist[1/c, Int[(A*c - a*C + (B*c - b*C)*x)/((a + b*x + c*x^2)*Sqrt
[d + f*x^2]), x], x] /; FreeQ[{a, b, c, d, f, A, B, C}, x] && NeQ[b^2 - 4*a
*c, 0]
```

Rubi steps

$$\int \frac{x^2\sqrt{a+cx^2}}{d+ex+fx^2}\,dx = -\frac{(2e-fx)\sqrt{a+cx^2}}{2f^2} - \frac{\int \frac{acdf-ce(2cd-af)x-c\left(af^2+2c\left(e^2-df\right)\right)x^2}{\sqrt{a+cx^2}\left(d+ex+fx^2\right)}\,dx}{2cf^2}$$

$$= -\frac{(2e-fx)\sqrt{a+cx^2}}{2f^2} - \frac{\int \frac{acdf^2+cd\left(af^2+2c\left(e^2-df\right)\right)+\left(-cef(2cd-af)+ce\left(af^2+2c\left(e^2-df\right)\right)\right)x}{\sqrt{a+cx^2}\left(d+ex+fx^2\right)}\,dx}{2cf^3} + \frac{\left(af^2+2\cdots\right.}{\cdots}$$

$$= -\frac{(2e-fx)\sqrt{a+cx^2}}{2f^2} + \frac{\left(af^2+2c\left(e^2-df\right)\right)\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x}{\sqrt{a+cx^2}}\right)}{2f^3} + \frac{\left(e\left(e-\sqrt{e^2-4\cdots}\right.\right.}{\cdots}$$

$$= -\frac{(2e-fx)\sqrt{a+cx^2}}{2f^2} + \frac{\left(af^2+2c\left(e^2-df\right)\right)\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{2\sqrt{c}\,f^3} - \frac{\left(e\left(e-\sqrt{e^2-4df}\right)\left(af^2+\cdots\right.\right.}{\cdots}$$

$$= -\frac{(2e-fx)\sqrt{a+cx^2}}{2f^2} + \frac{\left(af^2+2c\left(e^2-df\right)\right)\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{2\sqrt{c}\,f^3} - \frac{\left(e\left(e-\sqrt{e^2-4df}\right)\left(af^2+\cdots\right.\right.}{\cdots}$$

Mathematica [A]   time = 2.50, size = 516, normalized size = 1.14

$$\frac{\frac{2f\left(\frac{a^{3/2}\sqrt{\frac{cx^2}{a}+1}\sinh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{\sqrt{c}}+ax+cx^3\right)}{\sqrt{a+cx^2}}+\frac{\left(\frac{2df-e^2}{\sqrt{e^2-4df}}+e\right)\left(\sqrt{4af^2-2c\left(e\sqrt{e^2-4df}+2df-e^2\right)}\tanh^{-1}\left(\frac{2af+cx\left(\sqrt{e^2-4df}-e\right)}{\sqrt{a+cx^2}\sqrt{4af^2-2c\left(e\sqrt{e^2-4df}+2df-e^2\right)}}\right)-\sqrt{c}\left(\sqrt{e\cdots}\right.\right.}{f}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sqrt[a + c*x^2])/(d + e*x + f*x^2),x]
```

```
[Out] (-2*(e + (e^2 - 2*d*f)/Sqrt[e^2 - 4*d*f])*Sqrt[a + c*x^2] - 2*(e + (-e^2 +
2*d*f)/Sqrt[e^2 - 4*d*f])*Sqrt[a + c*x^2] + (2*f*(a*x + c*x^3 + (a^(3/2)*Sq
rt[1 + (c*x^2)/a]*ArcSinh[(Sqrt[c]*x)/Sqrt[a]])/Sqrt[c]))/Sqrt[a + c*x^2] +
 ((e + (-e^2 + 2*d*f)/Sqrt[e^2 - 4*d*f])*(-(Sqrt[c]*(-e + Sqrt[e^2 - 4*d*f]
)*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]]) + Sqrt[4*a*f^2 - 2*c*(-e^2 + 2*d*f
+ e*Sqrt[e^2 - 4*d*f])]*ArcTanh[(2*a*f + c*(-e + Sqrt[e^2 - 4*d*f])*x)/(Sqr
t[4*a*f^2 - 2*c*(-e^2 + 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])]))/f
 + ((e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])*(Sqrt[c]*(e + Sqrt[e^2 - 4*d*f])*Ar
cTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]] + Sqrt[4*a*f^2 + 2*c*(e^2 - 2*d*f + e*Sq
rt[e^2 - 4*d*f])]*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[4*a*f
^2 + 2*c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])]))/(f*Sqrt[e
^2 - 4*d*f]))/(4*f^2)
```

fricas [F(-1)]   time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^2+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-1)]   time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^2+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B]   time = 0.04, size = 7739, normalized size = 17.12

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*x^2+a)^(1/2)/(f*x^2+e*x+d),x)
```

```
[Out] result too large to display
```

maxima [F(-2)]   time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^2+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for mo
re details)Is 4*d*f-e^2 positive, negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\,\sqrt{c\,x^2+a}}{f\,x^2+e\,x+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + c*x^2)^(1/2))/(d + e*x + f*x^2),x)
```

```
[Out] int((x^2*(a + c*x^2)^(1/2))/(d + e*x + f*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\sqrt{a+cx^2}}{d+ex+fx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c*x**2+a)**(1/2)/(f*x**2+e*x+d),x)
```

```
[Out] Integral(x**2*sqrt(a + c*x**2)/(d + e*x + f*x**2), x)
```

3.53 $\int \frac{x\sqrt{a+cx^2}}{d+ex+fx^2}\,dx$

Optimal. Leaf size=395

$$-\frac{\left(2cdef-\left(e-\sqrt{e^2-4df}\right)\left(af^2+c\left(e^2-df\right)\right)\right)\tanh^{-1}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}f^2\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}+\frac{\left(2cdef-\left(\sqrt{e^2\right.\right.}{}$$

```
[Out] -e*arctanh(x*c^(1/2)/(c*x^2+a)^(1/2))*c^(1/2)/f^2+(c*x^2+a)^(1/2)/f-1/2*arc
tanh(1/2*(2*a*f-c*x*(e-(-4*d*f+e^2)^(1/2)))*2^(1/2)/(c*x^2+a)^(1/2)/(2*a*f^
2+(-(-4*d*f+e^2)^(1/2)*e-2*d*f+e^2)*c)^(1/2))*(2*c*d*e*f-(a*f^2+c*(-d*f+e^2
))*(e-(-4*d*f+e^2)^(1/2)))/f^2*2^(1/2)/(-4*d*f+e^2)^(1/2)/(2*a*f^2+(-(-4*d*
f+e^2)^(1/2)*e-2*d*f+e^2)*c)^(1/2)+1/2*arctanh(1/2*(2*a*f-c*x*(e+(-4*d*f+e^
2)^(1/2)))*2^(1/2)/(c*x^2+a)^(1/2)/(2*a*f^2+((-4*d*f+e^2)^(1/2)*e-2*d*f+e^2
)*c)^(1/2))*(2*c*d*e*f-(a*f^2+c*(-d*f+e^2))*(e+(-4*d*f+e^2)^(1/2)))/f^2*2^(
1/2)/(-4*d*f+e^2)^(1/2)/(2*a*f^2+((-4*d*f+e^2)^(1/2)*e-2*d*f+e^2)*c)^(1/2)
```

Rubi [A] time = 0.93, antiderivative size = 395, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {1020, 1080, 217, 206, 1034, 725}

$$-\frac{\left(2cdef-\left(e-\sqrt{e^2-4df}\right)\left(af^2+c\left(e^2-df\right)\right)\right)\tanh^{-1}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}f^2\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}+\frac{\left(2cdef-\left(\sqrt{e^2\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sqrt[a + c*x^2])/(d + e*x + f*x^2),x]
```

```
[Out] Sqrt[a + c*x^2]/f - (Sqrt[c]*e*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/f^2 -
((2*c*d*e*f - (e - Sqrt[e^2 - 4*d*f])*(a*f^2 + c*(e^2 - d*f)))*ArcTanh[(2*a
*f - c*(e - Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f -
e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[2]*f^2*Sqrt[e^2 - 4*d*f]*Sqr
t[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]) + ((2*c*d*e*f - (e + Sq
rt[e^2 - 4*d*f])*(a*f^2 + c*(e^2 - d*f)))*ArcTanh[(2*a*f - c*(e + Sqrt[e^2
- 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]
*Sqrt[a + c*x^2])])/(Sqrt[2]*f^2*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 -
2*d*f + e*Sqrt[e^2 - 4*d*f])])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 1020

```
Int[((g_.) + (h_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f
_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(h*(a + c*x^2)^p*(d + e*x + f*x^2)^(q +
 1))/(2*f*(p + q + 1)), x] + Dist[1/(2*f*(p + q + 1)), Int[(a + c*x^2)^(p -
 1)*(d + e*x + f*x^2)^q*Simp[a*h*e*p - a*(h*e - 2*g*f)*(p + q + 1) - 2*h*p*
(c*d - a*f)*x - (h*c*e*p + c*(h*e - 2*g*f)*(p + q + 1))*x^2, x], x], x] /;
FreeQ[{a, c, d, e, f, g, h, q}, x] && NeQ[e^2 - 4*d*f, 0] && GtQ[p, 0] && N
eQ[p + q + 1, 0]
```

Rule 1034

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f
_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(
b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Dist[(2*c*g -
h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a,
 b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1080

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*Sqrt[(d_.) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqrt[d + f*x^2],
 x], x] + Dist[1/c, Int[(A*c - a*C + (B*c - b*C)*x)/((a + b*x + c*x^2)*Sqrt
[d + f*x^2]), x], x] /; FreeQ[{a, b, c, d, f, A, B, C}, x] && NeQ[b^2 - 4*a
*c, 0]
```

Rubi steps

$$\int \frac{x\sqrt{a+cx^2}}{d+ex+fx^2}\,dx = \frac{\sqrt{a+cx^2}}{f} + \frac{\int \frac{-(cd-af)x-cex^2}{\sqrt{a+cx^2}\left(d+ex+fx^2\right)}\,dx}{f}$$

$$= \frac{\sqrt{a+cx^2}}{f} + \frac{\int \frac{cde+\left(ce^2+f(-cd+af)\right)x}{\sqrt{a+cx^2}\left(d+ex+fx^2\right)}\,dx}{f^2} - \frac{(ce)\int \frac{1}{\sqrt{a+cx^2}}\,dx}{f^2}$$

$$= \frac{\sqrt{a+cx^2}}{f} - \frac{(ce)\,\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x}{\sqrt{a+cx^2}}\right)}{f^2} + \frac{\left(2cdef-\left(e-\sqrt{e^2-4df}\right)\left(af^2+c\left(e^2-d\right.\right.\right.}{f^2\sqrt{e^2-4d}}$$

$$= \frac{\sqrt{a+cx^2}}{f} - \frac{\sqrt{c}\,e\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{f^2} - \frac{\left(2cdef-\left(e-\sqrt{e^2-4df}\right)\left(af^2+c\left(e^2-df\right)\right)\right)\text{Subst}\left(\right.}{f^2\sqrt{e^2-4}}$$

$$= \frac{\sqrt{a+cx^2}}{f} - \frac{\sqrt{c}\,e\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{f^2} - \frac{\left(2cdef-\left(e-\sqrt{e^2-4df}\right)\left(af^2+c\left(e^2-df\right)\right)\right)\tanh^{-1}}{\sqrt{2}\,f^2\sqrt{e^2-4df}\,\sqrt{2af^2+c\left(e^2-2\right.}}$$

Mathematica [A] time = 1.52, size = 422, normalized size = 1.07

$$-\frac{\frac{\left(\sqrt{e^2-4df}-e\right)\sqrt{4af^2-2c\left(e\sqrt{e^2-4df}+2df-e^2\right)}\tanh^{-1}\left(\frac{2af+cx\left(\sqrt{e^2-4df}-e\right)}{\sqrt{a+cx^2}\sqrt{4af^2-2c\left(e\sqrt{e^2-4df}+2df-e^2\right)}}\right)}{\sqrt{e^2-4df}} + \frac{e\sqrt{4af^2+2c\left(e\sqrt{e^2-4df}-2df+e^2\right)}\tanh^{-1}\left(\frac{}{\sqrt{a+cx^2}\ \cdots}\right)}{\sqrt{e^2-4df}}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sqrt[a + c*x^2])/(d + e*x + f*x^2),x]
```

```
[Out] -1/4*(-4*f*Sqrt[a + c*x^2] + 4*Sqrt[c]*e*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2
]] + ((-e + Sqrt[e^2 - 4*d*f])*Sqrt[4*a*f^2 - 2*c*(-e^2 + 2*d*f + e*Sqrt[e^
2 - 4*d*f])]*ArcTanh[(2*a*f + c*(-e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[4*a*f^2 -
 2*c*(-e^2 + 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/Sqrt[e^2 - 4*
d*f] + Sqrt[4*a*f^2 + 2*c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*ArcTanh[(2*a
*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[4*a*f^2 + 2*c*(e^2 - 2*d*f + e*Sqrt
[e^2 - 4*d*f])]*Sqrt[a + c*x^2])] + (e*Sqrt[4*a*f^2 + 2*c*(e^2 - 2*d*f + e*
Sqrt[e^2 - 4*d*f])]*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[4*a
*f^2 + 2*c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/Sqrt[e^2
 - 4*d*f])/f^2
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.02, size = 5581, normalized size = 14.13

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^2+a)^(1/2)/(f*x^2+e*x+d),x)
```

```
[Out] result too large to display
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for mo
re details)Is 4*d*f-e^2 positive, negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\,\sqrt{c\,x^2+a}}{f\,x^2+e\,x+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + c*x^2)^(1/2))/(d + e*x + f*x^2),x)
```

```
[Out] int((x*(a + c*x^2)^(1/2))/(d + e*x + f*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt{a + cx^2}}{d + ex + fx^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x**2+a)**(1/2)/(f*x**2+e*x+d),x)
```

```
[Out] Integral(x*sqrt(a + c*x**2)/(d + e*x + f*x**2), x)
```

3.54 $\int \frac{\sqrt{a+cx^2}}{d+ex+fx^2}\,dx$

Optimal. Leaf size=298

$$-\frac{\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}\tanh^{-1}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}f\sqrt{e^2-4df}}+\frac{\sqrt{2af^2+c\left(e\sqrt{e^2-4df}\right.}}{}$$

```
[Out] arctanh(x*c^(1/2)/(c*x^2+a)^(1/2))*c^(1/2)/f-1/2*arctanh(1/2*(2*a*f-c*x*(e-
(-4*d*f+e^2)^(1/2)))*2^(1/2)/(c*x^2+a)^(1/2)/(2*a*f^2+(-(-4*d*f+e^2)^(1/2)*
e-2*d*f+e^2)*c)^(1/2))*(2*a*f^2+(-(-4*d*f+e^2)^(1/2)*e-2*d*f+e^2)*c)^(1/2)/
f*2^(1/2)/(-4*d*f+e^2)^(1/2)+1/2*arctanh(1/2*(2*a*f-c*x*(e+(-4*d*f+e^2)^(1/
2)))*2^(1/2)/(c*x^2+a)^(1/2)/(2*a*f^2+((-4*d*f+e^2)^(1/2)*e-2*d*f+e^2)*c)^(
1/2))*(2*a*f^2+((-4*d*f+e^2)^(1/2)*e-2*d*f+e^2)*c)^(1/2)/f*2^(1/2)/(-4*d*f+
e^2)^(1/2)
```

Rubi [A] time = 0.38, antiderivative size = 298, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {991, 217, 206, 1034, 725}

$$-\frac{\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}\tanh^{-1}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}f\sqrt{e^2-4df}}+\frac{\sqrt{2af^2+c\left(e\sqrt{e^2-4df}\right.}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + c*x^2]/(d + e*x + f*x^2),x]
```

```
[Out] (Sqrt[c]*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/f - (Sqrt[2*a*f^2 + c*(e^2 -
 2*d*f - e*Sqrt[e^2 - 4*d*f])]*ArcTanh[(2*a*f - c*(e - Sqrt[e^2 - 4*d*f])*x
)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c
*x^2])])/(Sqrt[2]*f*Sqrt[e^2 - 4*d*f]) + (Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e
*Sqrt[e^2 - 4*d*f])]*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]
*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(
Sqrt[2]*f*Sqrt[e^2 - 4*d*f])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 991

```
Int[Sqrt[(a_) + (c_.)*(x_)^2]/((d_) + (e_.)*(x_) + (f_.)*(x_)^2), x_Symbol]
 :> Dist[c/f, Int[1/Sqrt[a + c*x^2], x], x] - Dist[1/f, Int[(c*d - a*f + c*
e*x)/(Sqrt[a + c*x^2]*(d + e*x + f*x^2)), x], x] /; FreeQ[{a, c, d, e, f},
```

```
x] && NeQ[e^2 - 4*d*f, 0]
```

Rule 1034

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f
_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(
b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Dist[(2*c*g -
h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a,
 b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{\sqrt{a+cx^2}}{d+ex+fx^2}\,dx = -\frac{\int \frac{cd-af+cex}{\sqrt{a+cx^2}\left(d+ex+fx^2\right)}\,dx}{f}+\frac{c\int \frac{1}{\sqrt{a+cx^2}}\,dx}{f}$$

$$= \frac{c\,\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x}{\sqrt{a+cx^2}}\right)}{f}+\frac{\left(2f(cd-af)-ce\left(e+\sqrt{e^2-4df}\right)\right)\int \frac{1}{\left(e+\sqrt{e^2-4df}+2fx\right)\sqrt{a+c}}}{f\sqrt{e^2-4df}}$$

$$= \frac{\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{f}-\frac{\left(2f(cd-af)-ce\left(e+\sqrt{e^2-4df}\right)\right)\text{Subst}\left(\int \frac{1}{4af^2+c\left(e+\sqrt{e^2-4df}\right)^2-x^2}\,d\right.}{f\sqrt{e^2-4df}}$$

$$= \frac{\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{f}-\frac{\sqrt{2af^2+c\left(e^2-2df-e\sqrt{e^2-4df}\right)}\tanh^{-1}\left(\frac{2af-c\left(e-\sqrt{e^2-4d}\right.}{\sqrt{2}\sqrt{2af^2+c\left(e^2-2df-e\sqrt{e^2}\right.}}\right.}{\sqrt{2}f\sqrt{e^2-4df}}$$

Mathematica [A] time = 0.36, size = 282, normalized size = 0.95

$$\frac{-\sqrt{4af^2-2c\left(e\sqrt{e^2-4df}+2df-e^2\right)}\tanh^{-1}\left(\frac{2af+cx\left(\sqrt{e^2-4df}-e\right)}{\sqrt{a+cx^2}\sqrt{4af^2-2c\left(e\sqrt{e^2-4df}+2df-e^2\right)}}\right)+\sqrt{4af^2+2c\left(e\sqrt{e^2-4df}-2\right.}}{2f\sqrt{e^2-4df}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + c*x^2]/(d + e*x + f*x^2),x]
```

```
[Out] (2*Sqrt[c]*Sqrt[e^2 - 4*d*f]*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]] - Sqrt[4*
a*f^2 - 2*c*(-e^2 + 2*d*f + e*Sqrt[e^2 - 4*d*f])]*ArcTanh[(2*a*f + c*(-e +
Sqrt[e^2 - 4*d*f])*x)/(Sqrt[4*a*f^2 - 2*c*(-e^2 + 2*d*f + e*Sqrt[e^2 - 4*d*
f])]*Sqrt[a + c*x^2])] + Sqrt[4*a*f^2 + 2*c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d
*f])]*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[4*a*f^2 + 2*c*(e^
2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(2*f*Sqrt[e^2 - 4*d*f]
)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.01, size = 3249, normalized size = 10.90

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^(1/2)/(f*x^2+e*x+d),x)
```

```
[Out] -1/2/(-4*d*f+e^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e+(-4*
d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*((-4*d*f+e^2)^(1/2)*c*
e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)+1/2/(-4*d*f+e^2)^(1/2)*c^(1/2)/f*ln((-1
/2*c*(e+(-4*d*f+e^2)^(1/2))/f+c*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))/c^(1/2)+(
(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(
-4*d*f+e^2)^(1/2))/f)+1/2*((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^
2)^(1/2))*e+1/2*c^(1/2)/f*ln((-1/2*c*(e+(-4*d*f+e^2)^(1/2))/f+c*(x+1/2*(e+(
-4*d*f+e^2)^(1/2))/f))/c^(1/2)+((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-c*(e+(
-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*((-4*d*f+e^2)^(1/
2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))+1/2/f^2*2^(1/2)/(((-4*d*f+e^2)^(1
/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*ln((((-4*d*f+e^2)^(1/2)*c*e+2*a*f
^2-2*c*d*f+c*e^2)/f^2-c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/
2))/f)+1/2*2^(1/2)*(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/
2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e+(-4*d*f+e^2)^(1/2))/f*(x+
1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e
^2)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))*c*e+1/(-4*d*f+e^2)^(1/2)*
2^(1/2)/(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*ln((((-4
*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c*(e+(-4*d*f+e^2)^(1/2))/f*(
x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^
2-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e+
(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*((-4*d*f+e^2)^(1/2
)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))*
a-1/(-4*d*f+e^2)^(1/2)/f*2^(1/2)/(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c
*e^2)/f^2)^(1/2)*ln((((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c*(
e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*(((-4*
d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e
^2)^(1/2))/f)^2*c-4*c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2)
)/f)+2*((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))/(x+1/2*(e
+(-4*d*f+e^2)^(1/2))/f))*c*d+1/2/(-4*d*f+e^2)^(1/2)/f^2*2^(1/2)/(((-4*d*f+e
^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*ln((((-4*d*f+e^2)^(1/2)*c*e
+2*a*f^2-2*c*d*f+c*e^2)/f^2-c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^
2)^(1/2))/f)+1/2*2^(1/2)*(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^
2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e+(-4*d*f+e^2)^(1/2))
/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d
*f+c*e^2)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))*c*e^2+1/2/(-4*d*f+e
^2)^(1/2)*(4*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e-(-4*d*f+e^2)^(1/2
))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+2*(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2
*c*d*f+c*e^2)/f^2)^(1/2)+1/2*c^(1/2)/f*ln((-1/2*c*(e-(-4*d*f+e^2)^(1/2))/f+
c*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f))/c^(1/2)+((x-1/2*(-e+(-4*d*f+e^2)^(1/2)
)/f)^2*c-c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+1/2*(
-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))-1/2/(-4*d*f+e^2)
^(1/2)*c^(1/2)/f*ln((-1/2*c*(e-(-4*d*f+e^2)^(1/2))/f+c*(x-1/2*(-e+(-4*d*f+e
^2)^(1/2))/f))/c^(1/2)+((x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-c*(e-(-4*d*f+
e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+1/2*(-(-4*d*f+e^2)^(1/2)*c*
e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))*e+1/2/f^2*2^(1/2)/((-(-4*d*f+e^2)^(1/2
)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*ln(((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^
2-2*c*d*f+c*e^2)/f^2-c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/
```

```
2))/f)+1/2*2^(1/2)*((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1
/2)*(4*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e-(-4*d*f+e^2)^(1/2))/f*(
x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+2*(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f
+c*e^2)/f^2)^(1/2))/(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f))*c*e-1/(-4*d*f+e^2)^(
1/2)*2^(1/2)/((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*ln
(((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c*(e-(-4*d*f+e^2)^(1/
2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*((-(-4*d*f+e^2)^(1/2)*c
*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2
*c-4*c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+2*(-(-4*d
*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))/(x-1/2*(-e+(-4*d*f+e^2
)^(1/2))/f))*a+1/(-4*d*f+e^2)^(1/2)/f*2^(1/2)/((-(-4*d*f+e^2)^(1/2)*c*e+2*a
*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*ln(((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f
+c*e^2)/f^2-c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+1/
2*2^(1/2)*((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x
-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e
+(-4*d*f+e^2)^(1/2))/f)+2*(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f
^2)^(1/2))/(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f))*c*d-1/2/(-4*d*f+e^2)^(1/2)/f^
2*2^(1/2)/((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*ln(((
-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c*(e-(-4*d*f+e^2)^(1/2))
/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*((-(-4*d*f+e^2)^(1/2)*c*e+
2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-
4*c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+2*(-(-4*d*f+
e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))/(x-1/2*(-e+(-4*d*f+e^2)^(
1/2))/f))*c*e^2
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for mo
re details)Is 4*d*f-e^2 positive, negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c\,x^2+a}}{f\,x^2+e\,x+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^2)^(1/2)/(d + e*x + f*x^2),x)
```

```
[Out] int((a + c*x^2)^(1/2)/(d + e*x + f*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + cx^2}}{d + ex + fx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**(1/2)/(f*x**2+e*x+d),x)
```

```
[Out] Integral(sqrt(a + c*x**2)/(d + e*x + f*x**2), x)
```

3.55 $\int \frac{\sqrt{a+cx^2}}{x\left(d+ex+fx^2\right)}\,dx$

Optimal. Leaf size=358

$$\frac{\left(\left(e-\sqrt{e^2-4df}\right)(cd-af)+2aef\right)\tanh^{-1}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}\,d\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}-\frac{\left(\left(\sqrt{e^2-4df}+e\right)(cd-af)\right.}{\sqrt{2}\,d\sqrt{e^2-4d}}$$

```
[Out] -arctanh((c*x^2+a)^(1/2)/a^(1/2))*a^(1/2)/d+1/2*arctanh(1/2*(2*a*f-c*x*(e-(
-4*d*f+e^2)^(1/2)))*2^(1/2)/(c*x^2+a)^(1/2)/(2*a*f^2+(-(-4*d*f+e^2)^(1/2)*e
-2*d*f+e^2)*c)^(1/2))*(2*a*e*f+(-a*f+c*d)*(e-(-4*d*f+e^2)^(1/2)))/d*2^(1/2)
/(-4*d*f+e^2)^(1/2)/(2*a*f^2+(-(-4*d*f+e^2)^(1/2)*e-2*d*f+e^2)*c)^(1/2)-1/2
*arctanh(1/2*(2*a*f-c*x*(e+(-4*d*f+e^2)^(1/2)))*2^(1/2)/(c*x^2+a)^(1/2)/(2*
a*f^2+((-4*d*f+e^2)^(1/2)*e-2*d*f+e^2)*c)^(1/2))*(2*a*e*f+(-a*f+c*d)*(e+(-4
*d*f+e^2)^(1/2)))/d*2^(1/2)/(-4*d*f+e^2)^(1/2)/(2*a*f^2+((-4*d*f+e^2)^(1/2)
*e-2*d*f+e^2)*c)^(1/2)
```

Rubi [A] time = 1.31, antiderivative size = 358, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6728, 266, 50, 63, 208, 1020, 1034, 725, 206}

$$\frac{\left(\left(e-\sqrt{e^2-4df}\right)(cd-af)+2aef\right)\tanh^{-1}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}\,d\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}-\frac{\left(\left(\sqrt{e^2-4df}+e\right)(cd-af)\right.}{\sqrt{2}\,d\sqrt{e^2-4d}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + c*x^2]/(x*(d + e*x + f*x^2)),x]
```

```
[Out] ((2*a*e*f + (c*d - a*f)*(e - Sqrt[e^2 - 4*d*f]))*ArcTanh[(2*a*f - c*(e - Sq
rt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4
*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[2]*d*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e
^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]) - ((2*a*e*f + (c*d - a*f)*(e + Sqrt[e^2
 - 4*d*f]))*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a
*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[2]*d
*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]) -
 (Sqrt[a]*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])/d
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 1020

```
Int[((g_.) + (h_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f
_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(h*(a + c*x^2)^p*(d + e*x + f*x^2)^(q +
 1))/(2*f*(p + q + 1)), x] + Dist[1/(2*f*(p + q + 1)), Int[(a + c*x^2)^(p -
 1)*(d + e*x + f*x^2)^q*Simp[a*h*e*p - a*(h*e - 2*g*f)*(p + q + 1) - 2*h*p*
(c*d - a*f)*x - (h*c*e*p + c*(h*e - 2*g*f)*(p + q + 1))*x^2, x], x], x] /;
FreeQ[{a, c, d, e, f, g, h, q}, x] && NeQ[e^2 - 4*d*f, 0] && GtQ[p, 0] && N
eQ[p + q + 1, 0]
```

Rule 1034

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f
_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(
b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Dist[(2*c*g -
h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a,
 b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+cx^2}}{x\left(d+ex+fx^2\right)}\,dx = \int\left(\frac{\sqrt{a+cx^2}}{dx}+\frac{(-e-fx)\sqrt{a+cx^2}}{d\left(d+ex+fx^2\right)}\right)dx$$
$$= \frac{\int \frac{\sqrt{a+cx^2}}{x}\,dx}{d}+\frac{\int \frac{(-e-fx)\sqrt{a+cx^2}}{d+ex+fx^2}\,dx}{d}$$
$$= -\frac{\sqrt{a+cx^2}}{d}+\frac{\text{Subst}\left(\int \frac{\sqrt{a+cx}}{x}\,dx,x,x^2\right)}{2d}+\frac{\int \frac{-aef+f(cd-af)x}{\sqrt{a+cx^2}\left(d+ex+fx^2\right)}\,dx}{df}$$
$$= \frac{a\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+cx}}\,dx,x,x^2\right)}{2d}-\frac{\left(2aef+(cd-af)\left(e-\sqrt{e^2-4df}\right)\right)\int \frac{1}{\left(e-\sqrt{e^2-4df}+2fx\right)}}{d\sqrt{e^2-4df}}$$
$$= \frac{a\,\text{Subst}\left(\int \frac{1}{-\frac{a}{c}+\frac{x^2}{c}}\,dx,x,\sqrt{a+cx^2}\right)}{cd}+\frac{\left(2aef+(cd-af)\left(e-\sqrt{e^2-4df}\right)\right)\text{Subst}\left(\int \right.}{d\sqrt{e^2-}}$$
$$= \frac{\left(2aef+(cd-af)\left(e-\sqrt{e^2-4df}\right)\right)\tanh^{-1}\left(\frac{2af-c\left(e-\sqrt{e^2-4df}\right)x}{\sqrt{2}\sqrt{2af^2+c\left(e^2-2df-e\sqrt{e^2-4df}\right)}\sqrt{a+cx^2}}\right)}{\sqrt{2}\,d\sqrt{e^2-4df}\sqrt{2af^2+c\left(e^2-2df-e\sqrt{e^2-4df}\right)}}-\frac{(2}{}$$

Mathematica [A] time = 0.65, size = 314, normalized size = 0.88

$$\frac{\left(\sqrt{e^2-4df}+e\right)\sqrt{4af^2-2c\left(e\sqrt{e^2-4df}+2df-e^2\right)}\tanh^{-1}\left(\frac{2af+cx\left(\sqrt{e^2-4df}-e\right)}{\sqrt{a+cx^2}\sqrt{4af^2-2c\left(e\sqrt{e^2-4df}+2df-e^2\right)}}\right)+\left(\sqrt{e^2-4d\right.}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + c*x^2]/(x*(d + e*x + f*x^2)),x]
```

```
[Out] ((e + Sqrt[e^2 - 4*d*f])*Sqrt[4*a*f^2 - 2*c*(-e^2 + 2*d*f + e*Sqrt[e^2 - 4*
d*f])]*ArcTanh[(2*a*f + c*(-e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[4*a*f^2 - 2*c*(
-e^2 + 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])] + (-e + Sqrt[e^2 - 4
*d*f])*Sqrt[4*a*f^2 + 2*c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*ArcTanh[(2*a
*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[4*a*f^2 + 2*c*(e^2 - 2*d*f + e*Sqrt
[e^2 - 4*d*f])]*Sqrt[a + c*x^2])] - 4*Sqrt[a]*f*Sqrt[e^2 - 4*d*f]*ArcTanh[S
qrt[a + c*x^2]/Sqrt[a]])/(4*d*f*Sqrt[e^2 - 4*d*f])
```

fricas [B] time = 83.15, size = 2266, normalized size = 6.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2)/x/(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] [-1/4*(sqrt(2)*d*sqrt((2*c*d^2 + a*e^2 - 2*a*d*f + (d^2*e^2 - 4*d^3*f)*sqrt
(a^2*e^2/(d^4*e^2 - 4*d^5*f)))/(d^2*e^2 - 4*d^3*f))*log((2*a*c*d*e*x - a^2*
e^2 + sqrt(2)*(d^3*e^2 - 4*d^4*f)*sqrt(a^2*e^2/(d^4*e^2 - 4*d^5*f))*sqrt(c*
x^2 + a)*sqrt((2*c*d^2 + a*e^2 - 2*a*d*f + (d^2*e^2 - 4*d^3*f)*sqrt(a^2*e^2
/(d^4*e^2 - 4*d^5*f)))/(d^2*e^2 - 4*d^3*f)) - (a*d^2*e^2 - 4*a*d^3*f)*sqrt(
a^2*e^2/(d^4*e^2 - 4*d^5*f)))/x) - sqrt(2)*d*sqrt((2*c*d^2 + a*e^2 - 2*a*d*
f + (d^2*e^2 - 4*d^3*f)*sqrt(a^2*e^2/(d^4*e^2 - 4*d^5*f)))/(d^2*e^2 - 4*d^3
*f))*log((2*a*c*d*e*x - a^2*e^2 - sqrt(2)*(d^3*e^2 - 4*d^4*f)*sqrt(a^2*e^2/
(d^4*e^2 - 4*d^5*f))*sqrt(c*x^2 + a)*sqrt((2*c*d^2 + a*e^2 - 2*a*d*f + (d^2
```

```
*e^2 - 4*d^3*f)*sqrt(a^2*e^2/(d^4*e^2 - 4*d^5*f)))/(d^2*e^2 - 4*d^3*f)) - (
a*d^2*e^2 - 4*a*d^3*f)*sqrt(a^2*e^2/(d^4*e^2 - 4*d^5*f)))/x) - sqrt(2)*d*sq
rt((2*c*d^2 + a*e^2 - 2*a*d*f - (d^2*e^2 - 4*d^3*f)*sqrt(a^2*e^2/(d^4*e^2 -
 4*d^5*f)))/(d^2*e^2 - 4*d^3*f))*log((2*a*c*d*e*x - a^2*e^2 + sqrt(2)*(d^3*
e^2 - 4*d^4*f)*sqrt(a^2*e^2/(d^4*e^2 - 4*d^5*f))*sqrt(c*x^2 + a)*sqrt((2*c*
d^2 + a*e^2 - 2*a*d*f - (d^2*e^2 - 4*d^3*f)*sqrt(a^2*e^2/(d^4*e^2 - 4*d^5*f
)))/(d^2*e^2 - 4*d^3*f)) + (a*d^2*e^2 - 4*a*d^3*f)*sqrt(a^2*e^2/(d^4*e^2 -
4*d^5*f)))/x) + sqrt(2)*d*sqrt((2*c*d^2 + a*e^2 - 2*a*d*f - (d^2*e^2 - 4*d^
3*f)*sqrt(a^2*e^2/(d^4*e^2 - 4*d^5*f)))/(d^2*e^2 - 4*d^3*f))*log((2*a*c*d*e
*x - a^2*e^2 - sqrt(2)*(d^3*e^2 - 4*d^4*f)*sqrt(a^2*e^2/(d^4*e^2 - 4*d^5*f)
)*sqrt(c*x^2 + a)*sqrt((2*c*d^2 + a*e^2 - 2*a*d*f - (d^2*e^2 - 4*d^3*f)*sqr
t(a^2*e^2/(d^4*e^2 - 4*d^5*f)))/(d^2*e^2 - 4*d^3*f)) + (a*d^2*e^2 - 4*a*d^3
*f)*sqrt(a^2*e^2/(d^4*e^2 - 4*d^5*f)))/x) - 2*sqrt(a)*log(-(c*x^2 - 2*sqrt(
c*x^2 + a)*sqrt(a) + 2*a)/x^2))/d, -1/4*(sqrt(2)*d*sqrt((2*c*d^2 + a*e^2 -
2*a*d*f + (d^2*e^2 - 4*d^3*f)*sqrt(a^2*e^2/(d^4*e^2 - 4*d^5*f)))/(d^2*e^2 -
 4*d^3*f))*log((2*a*c*d*e*x - a^2*e^2 + sqrt(2)*(d^3*e^2 - 4*d^4*f)*sqrt(a^
2*e^2/(d^4*e^2 - 4*d^5*f))*sqrt(c*x^2 + a)*sqrt((2*c*d^2 + a*e^2 - 2*a*d*f
+ (d^2*e^2 - 4*d^3*f)*sqrt(a^2*e^2/(d^4*e^2 - 4*d^5*f)))/(d^2*e^2 - 4*d^3*f
)) - (a*d^2*e^2 - 4*a*d^3*f)*sqrt(a^2*e^2/(d^4*e^2 - 4*d^5*f)))/x) - sqrt(2
)*d*sqrt((2*c*d^2 + a*e^2 - 2*a*d*f + (d^2*e^2 - 4*d^3*f)*sqrt(a^2*e^2/(d^4
*e^2 - 4*d^5*f)))/(d^2*e^2 - 4*d^3*f))*log((2*a*c*d*e*x - a^2*e^2 - sqrt(2)
*(d^3*e^2 - 4*d^4*f)*sqrt(a^2*e^2/(d^4*e^2 - 4*d^5*f))*sqrt(c*x^2 + a)*sqrt
((2*c*d^2 + a*e^2 - 2*a*d*f + (d^2*e^2 - 4*d^3*f)*sqrt(a^2*e^2/(d^4*e^2 - 4
*d^5*f)))/(d^2*e^2 - 4*d^3*f)) - (a*d^2*e^2 - 4*a*d^3*f)*sqrt(a^2*e^2/(d^4*
e^2 - 4*d^5*f)))/x) - sqrt(2)*d*sqrt((2*c*d^2 + a*e^2 - 2*a*d*f - (d^2*e^2
- 4*d^3*f)*sqrt(a^2*e^2/(d^4*e^2 - 4*d^5*f)))/(d^2*e^2 - 4*d^3*f))*log((2*a
*c*d*e*x - a^2*e^2 + sqrt(2)*(d^3*e^2 - 4*d^4*f)*sqrt(a^2*e^2/(d^4*e^2 - 4*
d^5*f))*sqrt(c*x^2 + a)*sqrt((2*c*d^2 + a*e^2 - 2*a*d*f - (d^2*e^2 - 4*d^3*
f)*sqrt(a^2*e^2/(d^4*e^2 - 4*d^5*f)))/(d^2*e^2 - 4*d^3*f)) + (a*d^2*e^2 - 4
*a*d^3*f)*sqrt(a^2*e^2/(d^4*e^2 - 4*d^5*f)))/x) + sqrt(2)*d*sqrt((2*c*d^2 +
 a*e^2 - 2*a*d*f - (d^2*e^2 - 4*d^3*f)*sqrt(a^2*e^2/(d^4*e^2 - 4*d^5*f)))/(
d^2*e^2 - 4*d^3*f))*log((2*a*c*d*e*x - a^2*e^2 - sqrt(2)*(d^3*e^2 - 4*d^4*f
)*sqrt(a^2*e^2/(d^4*e^2 - 4*d^5*f))*sqrt(c*x^2 + a)*sqrt((2*c*d^2 + a*e^2 -
 2*a*d*f - (d^2*e^2 - 4*d^3*f)*sqrt(a^2*e^2/(d^4*e^2 - 4*d^5*f)))/(d^2*e^2
- 4*d^3*f)) + (a*d^2*e^2 - 4*a*d^3*f)*sqrt(a^2*e^2/(d^4*e^2 - 4*d^5*f)))/x)
 - 4*sqrt(-a)*arctan(sqrt(-a)/sqrt(c*x^2 + a)))/d]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2)/x/(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 2Error index.cc index_gcd E
rror: Bad Argument Value
```

maple [B] time = 0.02, size = 3544, normalized size = 9.90

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^(1/2)/x/(f*x^2+e*x+d),x)
```

```
[Out] f/(e+(-4*d*f+e^2)^(1/2))/(-4*d*f+e^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2)
)/f)^2*c-4*c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*((
-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)-1/(e+(-4*d*f+e^2)^(
1/2))*c^(1/2)*ln((-1/2*c*(e+(-4*d*f+e^2)^(1/2))/f+c*(x+1/2*(e+(-4*d*f+e^2)^
```

```
(1/2))/f))/c^(1/2)+((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-c*(e+(-4*d*f+e^2)^
(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*((-4*d*f+e^2)^(1/2)*c*e+2*a*f
^2-2*c*d*f+c*e^2)/f^2)^(1/2))-1/(e+(-4*d*f+e^2)^(1/2))/(-4*d*f+e^2)^(1/2)*c
^(1/2)*ln((-1/2*c*(e+(-4*d*f+e^2)^(1/2))/f+c*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/
f))/c^(1/2)+((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-c*(e+(-4*d*f+e^2)^(1/2))/
f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*
d*f+c*e^2)/f^2)^(1/2))*e-1/f/(e+(-4*d*f+e^2)^(1/2))*2^(1/2)/(((-4*d*f+e^2)^
(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*ln((((-4*d*f+e^2)^(1/2)*c*e+2*a
*f^2-2*c*d*f+c*e^2)/f^2-c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(
1/2))/f)+1/2*2^(1/2)*(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(
1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e+(-4*d*f+e^2)^(1/2))/f*(
x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c
*e^2)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))*c*e-2*f/(e+(-4*d*f+e^2)
^(1/2))/(-4*d*f+e^2)^(1/2)*2^(1/2)/(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f
+c*e^2)/f^2)^(1/2)*ln((((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c
*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*(((-
4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f
+e^2)^(1/2))/f)^2*c-4*c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/
2))/f)+2*((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))/(x+1/2*
(e+(-4*d*f+e^2)^(1/2))/f))*a+2/(e+(-4*d*f+e^2)^(1/2))/(-4*d*f+e^2)^(1/2)*2^
(1/2)/(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*ln((((-4*d
*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c*(e+(-4*d*f+e^2)^(1/2))/f*(x+
1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-
2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e+(-
4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*((-4*d*f+e^2)^(1/2)*
c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))*c*
d-1/f/(e+(-4*d*f+e^2)^(1/2))/(-4*d*f+e^2)^(1/2)*2^(1/2)/(((-4*d*f+e^2)^(1/2
)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*ln((((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2
-2*c*d*f+c*e^2)/f^2-c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2)
)/f)+1/2*2^(1/2)*(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)
*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/
2*(e+(-4*d*f+e^2)^(1/2))/f)+2*((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2
)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))*c*e^2+f/(-e+(-4*d*f+e^2)^(1
/2))/(-4*d*f+e^2)^(1/2)*(4*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e-(-4
*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+2*(-(-4*d*f+e^2)^(1/2)
*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)+1/(-e+(-4*d*f+e^2)^(1/2))*c^(1/2)*ln
((-1/2*c*(e-(-4*d*f+e^2)^(1/2))/f+c*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f))/c^(1
/2)+((x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/
2*(-e+(-4*d*f+e^2)^(1/2))/f)+1/2*(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c
*e^2)/f^2)^(1/2))-1/(-e+(-4*d*f+e^2)^(1/2))/(-4*d*f+e^2)^(1/2)*c^(1/2)*ln((
-1/2*c*(e-(-4*d*f+e^2)^(1/2))/f+c*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f))/c^(1/2
)+((x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*
(-e+(-4*d*f+e^2)^(1/2))/f)+1/2*(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e
^2)/f^2)^(1/2))*e+1/f/(-e+(-4*d*f+e^2)^(1/2))*2^(1/2)/((-(-4*d*f+e^2)^(1/2)
*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*ln(((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2
-2*c*d*f+c*e^2)/f^2-c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2
))/f)+1/2*2^(1/2)*((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/
2)*(4*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e-(-4*d*f+e^2)^(1/2))/f*(x
-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+2*(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+
c*e^2)/f^2)^(1/2))/(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f))*c*e-2*f/(-e+(-4*d*f+e
^2)^(1/2))/(-4*d*f+e^2)^(1/2)*2^(1/2)/((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c
*d*f+c*e^2)/f^2)^(1/2)*ln(((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/
f^2-c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2
)*((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x-1/2*(-e
+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f
+e^2)^(1/2))/f)+2*(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2
))/(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f))*a+2/(-e+(-4*d*f+e^2)^(1/2))/(-4*d*f+e
^2)^(1/2)*2^(1/2)/((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/
2)*ln(((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c*(e-(-4*d*f+e^2
```

```
)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*((-(-4*d*f+e^2)^(1
/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))
/f)^2*c-4*c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+2*(-
(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))/(x-1/2*(-e+(-4*d*
f+e^2)^(1/2))/f))*c*d-1/f/(-e+(-4*d*f+e^2)^(1/2))/(-4*d*f+e^2)^(1/2)*2^(1/2
)/((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*ln(((-(-4*d*f
+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/
2*(-e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-
2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e-(
-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+2*(-(-4*d*f+e^2)^(1/
2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))/(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)
)*c*e^2+4*f/(-e+(-4*d*f+e^2)^(1/2))/(e+(-4*d*f+e^2)^(1/2))*a^(1/2)*ln((2*a+
2*a^(1/2)*(c*x^2+a)^(1/2))/x)-4*f/(-e+(-4*d*f+e^2)^(1/2))/(e+(-4*d*f+e^2)^(
1/2))*(c*x^2+a)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^2 + a}}{\left(fx^2 + ex + d\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2)/x/(f*x^2+e*x+d),x, algorithm="maxima")
[Out] integrate(sqrt(c*x^2 + a)/((f*x^2 + e*x + d)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c\,x^2 + a}}{x\,\left(f\,x^2 + e\,x + d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^2)^(1/2)/(x*(d + e*x + f*x^2)),x)
[Out] int((a + c*x^2)^(1/2)/(x*(d + e*x + f*x^2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + cx^2}}{x\left(d + ex + fx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**(1/2)/x/(f*x**2+e*x+d),x)
[Out] Integral(sqrt(a + c*x**2)/(x*(d + e*x + f*x**2)), x)
```

3.56 $\int \frac{\sqrt{a+cx^2}}{x^2\left(d+ex+fx^2\right)}\,dx$

Optimal. Leaf size=382

$$-\frac{f\left(a\left(e\sqrt{e^2-4df}-2df+e^2\right)+2cd^2\right)\tanh^{-1}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}\,d^2\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}+\frac{f\left(a\left(-e\sqrt{e^2-4df}-2d\right.\right.}{\sqrt{2}\,d^2\sqrt{e^2}}$$

```
[Out] e*arctanh((c*x^2+a)^(1/2)/a^(1/2))*a^(1/2)/d^2-(c*x^2+a)^(1/2)/d/x-1/2*f*ar
ctanh(1/2*(2*a*f-c*x*(e-(-4*d*f+e^2)^(1/2)))*2^(1/2)/(c*x^2+a)^(1/2)/(2*a*f
^2+(-(-4*d*f+e^2)^(1/2)*e-2*d*f+e^2)*c)^(1/2))*(2*c*d^2+a*((-4*d*f+e^2)^(1/
2)*e-2*d*f+e^2))/d^2*2^(1/2)/(-4*d*f+e^2)^(1/2)/(2*a*f^2+(-(-4*d*f+e^2)^(1/
2)*e-2*d*f+e^2)*c)^(1/2)+1/2*f*arctanh(1/2*(2*a*f-c*x*(e+(-4*d*f+e^2)^(1/2)
))*2^(1/2)/(c*x^2+a)^(1/2)/(2*a*f^2+((-4*d*f+e^2)^(1/2)*e-2*d*f+e^2)*c)^(1/
2))*(2*c*d^2+a*(-(-4*d*f+e^2)^(1/2)*e-2*d*f+e^2))/d^2*2^(1/2)/(-4*d*f+e^2)^
(1/2)/(2*a*f^2+((-4*d*f+e^2)^(1/2)*e-2*d*f+e^2)*c)^(1/2)
```

Rubi [A] time = 1.42, antiderivative size = 382, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 12, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {6728, 277, 217, 206, 266, 50, 63, 208, 1020, 1080, 1034, 725}

$$-\frac{f\left(a\left(e\sqrt{e^2-4df}-2df+e^2\right)+2cd^2\right)\tanh^{-1}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}\,d^2\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}+\frac{f\left(a\left(-e\sqrt{e^2-4df}-2d\right.\right.}{\sqrt{2}\,d^2\sqrt{e^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + c*x^2]/(x^2*(d + e*x + f*x^2)),x]
```

```
[Out] -(Sqrt[a + c*x^2]/(d*x)) - (f*(2*c*d^2 + a*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*
f]))*ArcTanh[(2*a*f - c*(e - Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 +
c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[2]*d^2*Sqrt
[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]) + (f*(
2*c*d^2 + a*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]))*ArcTanh[(2*a*f - c*(e + Sq
rt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4
*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[2]*d^2*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*
(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]) + (Sqrt[a]*e*ArcTanh[Sqrt[a + c*x^2]/
Sqrt[a]])/d^2
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 1020

```
Int[((g_.) + (h_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f
_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(h*(a + c*x^2)^p*(d + e*x + f*x^2)^(q +
 1))/(2*f*(p + q + 1)), x] + Dist[1/(2*f*(p + q + 1)), Int[(a + c*x^2)^(p -
 1)*(d + e*x + f*x^2)^q*Simp[a*h*e*p - a*(h*e - 2*g*f)*(p + q + 1) - 2*h*p*
(c*d - a*f)*x - (h*c*e*p + c*(h*e - 2*g*f)*(p + q + 1))*x^2, x], x], x] /;
FreeQ[{a, c, d, e, f, g, h, q}, x] && NeQ[e^2 - 4*d*f, 0] && GtQ[p, 0] && N
eQ[p + q + 1, 0]
```

Rule 1034

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f
_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(
b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Dist[(2*c*g -
h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a,
 b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1080

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*Sqrt[(d_.) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqrt[d + f*x^2],
 x], x] + Dist[1/c, Int[(A*c - a*C + (B*c - b*C)*x)/((a + b*x + c*x^2)*Sqrt
[d + f*x^2]), x], x] /; FreeQ[{a, b, c, d, f, A, B, C}, x] && NeQ[b^2 - 4*a
```

```
*c, 0]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+cx^2}}{x^2\left(d+ex+fx^2\right)}\,dx = \int \left(\frac{\sqrt{a+cx^2}}{dx^2}-\frac{e\sqrt{a+cx^2}}{d^2x}+\frac{\left(e^2-df+efx\right)\sqrt{a+cx^2}}{d^2\left(d+ex+fx^2\right)}\right)dx$$

$$= \frac{\int \frac{\left(e^2-df+efx\right)\sqrt{a+cx^2}}{d+ex+fx^2}\,dx}{d^2}+\frac{\int \frac{\sqrt{a+cx^2}}{x^2}\,dx}{d}-\frac{e\int \frac{\sqrt{a+cx^2}}{x}\,dx}{d^2}$$

$$= \frac{e\sqrt{a+cx^2}}{d^2}-\frac{\sqrt{a+cx^2}}{dx}+\frac{c\int \frac{1}{\sqrt{a+cx^2}}\,dx}{d}-\frac{e\,\text{Subst}\left(\int \frac{\sqrt{a+cx}}{x}\,dx,x,x^2\right)}{2d^2}+\frac{\int \frac{af\left(e^2-df\right)}{\sqrt{a+}}}{}$$

$$= -\frac{\sqrt{a+cx^2}}{dx}-\frac{c\int \frac{1}{\sqrt{a+cx^2}}\,dx}{d}+\frac{c\,\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx,x,\frac{x}{\sqrt{a+cx^2}}\right)}{d}-\frac{(ae)\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+}}\right.}{2d^2}$$

$$= -\frac{\sqrt{a+cx^2}}{dx}+\frac{\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{d}-\frac{c\,\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx,x,\frac{x}{\sqrt{a+cx^2}}\right)}{d}-\frac{(ae)\,\text{Subst}}{}$$

$$= -\frac{\sqrt{a+cx^2}}{dx}+\frac{\sqrt{a}\,e\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d^2}+\frac{\left(f\left(2cd^2+a\left(e^2-2df-e\sqrt{e^2-4df}\right)\right)\right)\text{Sub}}{d^2\sqrt{}}$$

$$= -\frac{\sqrt{a+cx^2}}{dx}-\frac{f\left(2cd^2+a\left(e^2-2df+e\sqrt{e^2-4df}\right)\right)\tanh^{-1}\left(\frac{2af-c\left(e-\sqrt{e^2-4d}\right.}{\sqrt{2}\sqrt{2af^2+c\left(e^2-2df-e\sqrt{e^2-}\right.}}\right.}{\sqrt{2}\,d^2\sqrt{e^2-4df}\sqrt{2af^2+c\left(e^2-2df-e\sqrt{e^2-4df}\right)}}$$

Mathematica [A] time = 3.03, size = 569, normalized size = 1.49

$$\frac{\frac{\left(e\sqrt{e^2-4df}-2df+e^2\right)\left(\sqrt{c}\left(\sqrt{e^2-4df}-e\right)\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)-\sqrt{4af^2-2c\left(e\sqrt{e^2-4df}+2df-e^2\right)}\tanh^{-1}\left(\frac{2af+cx\left(\sqrt{e^2-4df}-e\right)}{\sqrt{a+cx^2}\sqrt{4af^2-2c\left(e\sqrt{e^2-4df}+2df-e^2\right)}}\right)\right)}{f\sqrt{e^2-4df}}+\frac{(-}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + c*x^2]/(x^2*(d + e*x + f*x^2)),x]
```

```
[Out] (2*(e + (e^2 - 2*d*f)/Sqrt[e^2 - 4*d*f])*Sqrt[a + c*x^2] + 2*(e + (-e^2 + 2
*d*f)/Sqrt[e^2 - 4*d*f])*Sqrt[a + c*x^2] - (4*d*(a + c*x^2 - Sqrt[a]*Sqrt[c
]*x*Sqrt[1 + (c*x^2)/a]*ArcSinh[(Sqrt[c]*x)/Sqrt[a]]))/(x*Sqrt[a + c*x^2])
+ ((e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])*(Sqrt[c]*(-e + Sqrt[e^2 - 4*d*f])*Ar
cTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]] - Sqrt[4*a*f^2 - 2*c*(-e^2 + 2*d*f + e*S
qrt[e^2 - 4*d*f])]*ArcTanh[(2*a*f + c*(-e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[4*a
*f^2 - 2*c*(-e^2 + 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])]))/(f*Sqr
t[e^2 - 4*d*f]) + ((e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])*(Sqrt[c]*(e + Sqrt[e
```

```
^2 - 4*d*f])*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]] + Sqrt[4*a*f^2 + 2*c*(e^2
 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])
*x)/(Sqrt[4*a*f^2 + 2*c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2
])]))/(f*Sqrt[e^2 - 4*d*f]) - 4*e*(Sqrt[a + c*x^2] - Sqrt[a]*ArcTanh[Sqrt[a
 + c*x^2]/Sqrt[a]]))/(4*d^2)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2)/x^2/(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2)/x^2/(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 5.11Error index.cc index_gc
d Error: Bad Argument Value
```

maple [B] time = 0.02, size = 3703, normalized size = 9.69

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^(1/2)/x^2/(f*x^2+e*x+d),x)
```

```
[Out] -2*f^2/(e+(-4*d*f+e^2)^(1/2))^2/(-4*d*f+e^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2
)^(1/2))/f)^2*c-4*c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/
f)+2*((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)+2*f/(e+(-4*d
*f+e^2)^(1/2))^2*c^(1/2)*ln((-1/2*c*(e+(-4*d*f+e^2)^(1/2))/f+c*(x+1/2*(e+(-
4*d*f+e^2)^(1/2))/f))/c^(1/2)+((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-c*(e+(-
4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*((-4*d*f+e^2)^(1/2
)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))+2*f/(e+(-4*d*f+e^2)^(1/2))^2/(-4*d
*f+e^2)^(1/2)*c^(1/2)*ln((-1/2*c*(e+(-4*d*f+e^2)^(1/2))/f+c*(x+1/2*(e+(-4*d
*f+e^2)^(1/2))/f))/c^(1/2)+((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-c*(e+(-4*d
*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*((-4*d*f+e^2)^(1/2)*c
*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))*e+2/(e+(-4*d*f+e^2)^(1/2))^2*2^(1/2)/
(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*ln((((-4*d*f+e^2
)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e
+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*
f+c*e^2)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e+(-4*d*f+
e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*((-4*d*f+e^2)^(1/2)*c*e+2*
a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))*c*e+4*f^
2/(e+(-4*d*f+e^2)^(1/2))^2/(-4*d*f+e^2)^(1/2)*2^(1/2)/(((-4*d*f+e^2)^(1/2)*
c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*ln((((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2
*c*d*f+c*e^2)/f^2-c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/
f)+1/2*2^(1/2)*(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*(
4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*
(e+(-4*d*f+e^2)^(1/2))/f)+2*((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/
f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))*a-4*f/(e+(-4*d*f+e^2)^(1/2))^
2/(-4*d*f+e^2)^(1/2)*2^(1/2)/(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2
)/f^2)^(1/2)*ln((((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c*(e+(-
```

```
4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*(((-4*d*f+
e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^
(1/2))/f)^2*c-4*c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)
+2*((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))/(x+1/2*(e+(-4
*d*f+e^2)^(1/2))/f))*c*d+2/(e+(-4*d*f+e^2)^(1/2))^2/(-4*d*f+e^2)^(1/2)*2^(1
/2)/(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*ln((((-4*d*f
+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/
2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*
c*d*f+c*e^2)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e+(-4*
d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*((-4*d*f+e^2)^(1/2)*c*
e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))*c*e^
2+4*f/(-e+(-4*d*f+e^2)^(1/2))/(e+(-4*d*f+e^2)^(1/2))/a/x*(c*x^2+a)^(3/2)-4*
f/(-e+(-4*d*f+e^2)^(1/2))/(e+(-4*d*f+e^2)^(1/2))*c/a*x*(c*x^2+a)^(1/2)-4*f/
(-e+(-4*d*f+e^2)^(1/2))/(e+(-4*d*f+e^2)^(1/2))*c^(1/2)*ln(c^(1/2)*x+(c*x^2+
a)^(1/2))+2*f^2/(-e+(-4*d*f+e^2)^(1/2))^2/(-4*d*f+e^2)^(1/2)*(4*(x-1/2*(-e+
(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+
e^2)^(1/2))/f)+2*(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)
+2*f/(-e+(-4*d*f+e^2)^(1/2))^2*c^(1/2)*ln((-1/2*c*(e-(-4*d*f+e^2)^(1/2))/f+
c*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f))/c^(1/2)+((x-1/2*(-e+(-4*d*f+e^2)^(1/2)
)/f)^2*c-c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+1/2*(
-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))-2*f/(-e+(-4*d*f+
e^2)^(1/2))^2/(-4*d*f+e^2)^(1/2)*c^(1/2)*ln((-1/2*c*(e-(-4*d*f+e^2)^(1/2))/
f+c*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f))/c^(1/2)+((x-1/2*(-e+(-4*d*f+e^2)^(1/
2))/f)^2*c-c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+1/2
*(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))*e+2/(-e+(-4*d*
f+e^2)^(1/2))^2*2^(1/2)/((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^
2)^(1/2)*ln(((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c*(e-(-4*d
*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*((-(-4*d*f+e
^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x-1/2*(-e+(-4*d*f+e^2)^
(1/2))/f)^2*c-4*c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f
)+2*(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))/(x-1/2*(-e+
(-4*d*f+e^2)^(1/2))/f))*c*e-4*f^2/(-e+(-4*d*f+e^2)^(1/2))^2/(-4*d*f+e^2)^(1
/2)*2^(1/2)/((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*ln(
((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c*(e-(-4*d*f+e^2)^(1/2
))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*((-(-4*d*f+e^2)^(1/2)*c*
e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*
c-4*c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+2*(-(-4*d*
f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))/(x-1/2*(-e+(-4*d*f+e^2)
^(1/2))/f))*a+4*f/(-e+(-4*d*f+e^2)^(1/2))^2/(-4*d*f+e^2)^(1/2)*2^(1/2)/((-(
-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*ln(((-(-4*d*f+e^2)^
(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+
(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*
f+c*e^2)/f^2)^(1/2)*(4*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e-(-4*d*f
+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+2*(-(-4*d*f+e^2)^(1/2)*c*e
+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))/(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f))*c*d-
2/(-e+(-4*d*f+e^2)^(1/2))^2/(-4*d*f+e^2)^(1/2)*2^(1/2)/((-(-4*d*f+e^2)^(1/2
)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*ln(((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^
2-2*c*d*f+c*e^2)/f^2-c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/
2))/f)+1/2*2^(1/2)*((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1
/2)*(4*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e-(-4*d*f+e^2)^(1/2))/f*(
x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+2*(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f
+c*e^2)/f^2)^(1/2))/(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f))*c*e^2+16*f^2*e/(-e+(
-4*d*f+e^2)^(1/2))^2/(e+(-4*d*f+e^2)^(1/2))^2*a^(1/2)*ln((2*a+2*a^(1/2)*(c*
x^2+a)^(1/2))/x)-16*f^2*e/(-e+(-4*d*f+e^2)^(1/2))^2/(e+(-4*d*f+e^2)^(1/2))^
2*(c*x^2+a)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^2+a}}{(fx^2+ex+d)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2)/x^2/(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^2 + a)/((f*x^2 + e*x + d)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c\,x^2+a}}{x^2\,\left(f\,x^2+e\,x+d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^2)^(1/2)/(x^2*(d + e*x + f*x^2)),x)
```

```
[Out] int((a + c*x^2)^(1/2)/(x^2*(d + e*x + f*x^2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+cx^2}}{x^2\left(d+ex+fx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**(1/2)/x**2/(f*x**2+e*x+d),x)
```

```
[Out] Integral(sqrt(a + c*x**2)/(x**2*(d + e*x + f*x**2)), x)
```

3.57 $\int \frac{\sqrt{a+cx^2}}{x^3\left(d+ex+fx^2\right)}\,dx$

Optimal. Leaf size=507

$$-\frac{\sqrt{a}\left(e^2-df\right)\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d^3}+\frac{e\sqrt{a+cx^2}}{d^2x}+\frac{f\left(a\left(e^2\sqrt{e^2-4df}-df\sqrt{e^2-4df}-3def+e^3\right)+cd^2\left(\sqrt{e^2-4\right.\right.}{\sqrt{2}\,d^3\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{\right.}}}$$

```
[Out] -1/2*c*arctanh((c*x^2+a)^(1/2)/a^(1/2))/d/a^(1/2)-(-d*f+e^2)*arctanh((c*x^2
+a)^(1/2)/a^(1/2))*a^(1/2)/d^3-1/2*(c*x^2+a)^(1/2)/d/x^2+e*(c*x^2+a)^(1/2)/
d^2/x+1/2*f*arctanh(1/2*(2*a*f-c*x*(e-(-4*d*f+e^2)^(1/2)))*2^(1/2)/(c*x^2+a
)^(1/2)/(2*a*f^2+(-(-4*d*f+e^2)^(1/2)*e-2*d*f+e^2)*c)^(1/2))*(c*d^2*(e+(-4*
d*f+e^2)^(1/2))+a*(e^3-3*d*e*f+e^2*(-4*d*f+e^2)^(1/2)-d*f*(-4*d*f+e^2)^(1/2
)))/d^3*2^(1/2)/(-4*d*f+e^2)^(1/2)/(2*a*f^2+(-(-4*d*f+e^2)^(1/2)*e-2*d*f+e^
2)*c)^(1/2)-1/2*f*arctanh(1/2*(2*a*f-c*x*(e+(-4*d*f+e^2)^(1/2)))*2^(1/2)/(c
*x^2+a)^(1/2)/(2*a*f^2+((-4*d*f+e^2)^(1/2)*e-2*d*f+e^2)*c)^(1/2))*(c*d^2*(e
-(-4*d*f+e^2)^(1/2))+a*(e^3-3*d*e*f-e^2*(-4*d*f+e^2)^(1/2)+d*f*(-4*d*f+e^2)
^(1/2)))/d^3*2^(1/2)/(-4*d*f+e^2)^(1/2)/(2*a*f^2+((-4*d*f+e^2)^(1/2)*e-2*d*
f+e^2)*c)^(1/2)
```

Rubi [A] time = 1.88, antiderivative size = 507, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 13, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.482, Rules used = {6728, 266, 47, 63, 208, 277, 217, 206, 50, 1020, 1080, 1034, 725}

$$\frac{f\left(a\left(e^2\sqrt{e^2-4df}-df\sqrt{e^2-4df}-3def+e^3\right)+cd^2\left(\sqrt{e^2-4df}+e\right)\right)\tanh^{-1}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2d\right.}}\right.}{\sqrt{2}\,d^3\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + c*x^2]/(x^3*(d + e*x + f*x^2)),x]
```

```
[Out] -Sqrt[a + c*x^2]/(2*d*x^2) + (e*Sqrt[a + c*x^2])/(d^2*x) + (f*(c*d^2*(e + S
qrt[e^2 - 4*d*f]) + a*(e^3 - 3*d*e*f + e^2*Sqrt[e^2 - 4*d*f] - d*f*Sqrt[e^2
 - 4*d*f]))*ArcTanh[(2*a*f - c*(e - Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a
*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[2]*d
^3*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])])
 - (f*(c*d^2*(e - Sqrt[e^2 - 4*d*f]) + a*(e^3 - 3*d*e*f - e^2*Sqrt[e^2 - 4*
d*f] + d*f*Sqrt[e^2 - 4*d*f]))*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x
)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c
*x^2])])/(Sqrt[2]*d^3*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*S
qrt[e^2 - 4*d*f])]) - (c*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])/(2*Sqrt[a]*d) -
(Sqrt[a]*(e^2 - d*f)*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])/d^3
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 1020

```
Int[((g_.) + (h_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f
_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(h*(a + c*x^2)^p*(d + e*x + f*x^2)^(q +
 1))/(2*f*(p + q + 1)), x] + Dist[1/(2*f*(p + q + 1)), Int[(a + c*x^2)^(p -
 1)*(d + e*x + f*x^2)^q*Simp[a*h*e*p - a*(h*e - 2*g*f)*(p + q + 1) - 2*h*p*
(c*d - a*f)*x - (h*c*e*p + c*(h*e - 2*g*f)*(p + q + 1))*x^2, x], x], x] /;
FreeQ[{a, c, d, e, f, g, h, q}, x] && NeQ[e^2 - 4*d*f, 0] && GtQ[p, 0] && N
```

```
eQ[p + q + 1, 0]
```

Rule 1034

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f
_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(
b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Dist[(2*c*g -
h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a,
 b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1080

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*Sqrt[(d_.) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqrt[d + f*x^2],
 x], x] + Dist[1/c, Int[(A*c - a*C + (B*c - b*C)*x)/((a + b*x + c*x^2)*Sqrt
[d + f*x^2]), x], x] /; FreeQ[{a, b, c, d, f, A, B, C}, x] && NeQ[b^2 - 4*a
*c, 0]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+cx^2}}{x^3\left(d+ex+fx^2\right)}\,dx = \int\left(\frac{\sqrt{a+cx^2}}{dx^3}-\frac{e\sqrt{a+cx^2}}{d^2x^2}+\frac{\left(e^2-df\right)\sqrt{a+cx^2}}{d^3x}+\frac{\left(-e\left(e^2-2df\right)-f\left(e^2-df\right)x\right)}{d^3\left(d+ex+fx^2\right)}\right.$$

$$=\frac{\int\frac{\left(-e\left(e^2-2df\right)-f\left(e^2-df\right)x\right)\sqrt{a+cx^2}}{d+ex+fx^2}\,dx}{d^3}+\frac{\int\frac{\sqrt{a+cx^2}}{x^3}\,dx}{d}-\frac{e\int\frac{\sqrt{a+cx^2}}{x^2}\,dx}{d^2}+\frac{\left(e^2-df\right)\int\frac{\sqrt{a+c}}{x}}{d^3}$$

$$=-\frac{\left(e^2-df\right)\sqrt{a+cx^2}}{d^3}+\frac{e\sqrt{a+cx^2}}{d^2x}+\frac{\text{Subst}\left(\int\frac{\sqrt{a+cx}}{x^2}\,dx,x,x^2\right)}{2d}-\frac{(ce)\int\frac{1}{\sqrt{a+cx^2}}\,dx}{d^2}$$

$$=-\frac{\sqrt{a+cx^2}}{2dx^2}+\frac{e\sqrt{a+cx^2}}{d^2x}+\frac{c\,\text{Subst}\left(\int\frac{1}{x\sqrt{a+cx}}\,dx,x,x^2\right)}{4d}+\frac{(ce)\int\frac{1}{\sqrt{a+cx^2}}\,dx}{d^2}-\frac{(ce)\,\text{S}}{}$$

$$=-\frac{\sqrt{a+cx^2}}{2dx^2}+\frac{e\sqrt{a+cx^2}}{d^2x}-\frac{\sqrt{c}\,e\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{d^2}+\frac{\text{Subst}\left(\int\frac{1}{-\frac{a}{c}+\frac{x^2}{c}}\,dx,x,\sqrt{a+cx^2}\right.}{2d}$$

$$=-\frac{\sqrt{a+cx^2}}{2dx^2}+\frac{e\sqrt{a+cx^2}}{d^2x}-\frac{c\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{2\sqrt{a}\,d}-\frac{\sqrt{a}\left(e^2-df\right)\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d^3}+$$

$$=-\frac{\sqrt{a+cx^2}}{2dx^2}+\frac{e\sqrt{a+cx^2}}{d^2x}+\frac{f\left(cd^2\left(e+\sqrt{e^2-4df}\right)+a\left(e^3-3def+e^2\sqrt{e^2-4df}\right.\right.}{\sqrt{2}\,d^3\sqrt{e^2-4df}\,\sqrt{2af^2}}$$

Mathematica [A] time = 2.61, size = 642, normalized size = 1.27

$$-\frac{\frac{2d^2\left(cx^2\sqrt{\frac{cx^2}{a}+1}\tanh^{-1}\left(\sqrt{\frac{cx^2}{a}+1}\right)+a+cx^2\right)}{x^2\sqrt{a+cx^2}}+\frac{\left(\frac{e(e^2-3df)}{\sqrt{e^2-4df}}-df+e^2\right)\left(\sqrt{4af^2-2c\left(e\sqrt{e^2-4df}+2df-e^2\right)}\tanh^{-1}\left(\frac{2af+cx\left(\sqrt{e^2-4df}-e\right)}{\sqrt{a+cx^2}\sqrt{4af^2-2c\left(e\sqrt{e^2-4df}+2df-e^2\right)}}\right)\right)}{f}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + c*x^2]/(x^3*(d + e*x + f*x^2)),x]
```

```
[Out] (-2*(e^2 - d*f - (e*(e^2 - 3*d*f))/Sqrt[e^2 - 4*d*f])*Sqrt[a + c*x^2] - 2*(
e^2 - d*f + (e*(e^2 - 3*d*f))/Sqrt[e^2 - 4*d*f])*Sqrt[a + c*x^2] + (4*d*e*(
a + c*x^2 - Sqrt[a]*Sqrt[c]*x*Sqrt[1 + (c*x^2)/a]*ArcSinh[(Sqrt[c]*x)/Sqrt[
a]]))/(x*Sqrt[a + c*x^2]) + ((e^2 - d*f + (e*(e^2 - 3*d*f))/Sqrt[e^2 - 4*d*
f])*(-(Sqrt[c]*(-e + Sqrt[e^2 - 4*d*f])*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]
]) + Sqrt[4*a*f^2 - 2*c*(-e^2 + 2*d*f + e*Sqrt[e^2 - 4*d*f])]*ArcTanh[(2*a*
f + c*(-e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[4*a*f^2 - 2*c*(-e^2 + 2*d*f + e*Sqr
t[e^2 - 4*d*f])]*Sqrt[a + c*x^2])]))/f + ((e^2 - d*f - (e*(e^2 - 3*d*f))/Sq
rt[e^2 - 4*d*f])*(Sqrt[c]*(e + Sqrt[e^2 - 4*d*f])*ArcTanh[(Sqrt[c]*x)/Sqrt[
a + c*x^2]] + Sqrt[4*a*f^2 + 2*c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*ArcTa
nh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[4*a*f^2 + 2*c*(e^2 - 2*d*f +
 e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])]))/f + 4*(e^2 - d*f)*(Sqrt[a + c*x^
2] - Sqrt[a]*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]]) - (2*d^2*(a + c*x^2 + c*x^2*
Sqrt[1 + (c*x^2)/a]*ArcTanh[Sqrt[1 + (c*x^2)/a]]))/(x^2*Sqrt[a + c*x^2]))/(
4*d^3)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2)/x^3/(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2)/x^3/(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.02, size = 3993, normalized size = 7.88

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^(1/2)/x^3/(f*x^2+e*x+d),x)
```

```
[Out] 4*f^3/(e+(-4*d*f+e^2)^(1/2))^3/(-4*d*f+e^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)
^(1/2))/f)^2*c-4*c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f
)+2*((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)-4*f^2/(e+(-4*
d*f+e^2)^(1/2))^3*c^(1/2)*ln((-1/2*c*(e+(-4*d*f+e^2)^(1/2))/f+c*(x+1/2*(e+(
```

```
-4*d*f+e^2)^(1/2))/f))/c^(1/2)+((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-c*(e+(
-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*((-4*d*f+e^2)^(1/
2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))-4*f^2/(e+(-4*d*f+e^2)^(1/2))^3/(-
4*d*f+e^2)^(1/2)*c^(1/2)*ln((-1/2*c*(e+(-4*d*f+e^2)^(1/2))/f+c*(x+1/2*(e+(-
4*d*f+e^2)^(1/2))/f))/c^(1/2)+((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-c*(e+(-
4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*((-4*d*f+e^2)^(1/2
)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))*e-4*f/(e+(-4*d*f+e^2)^(1/2))^3*2^(
1/2)/(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*ln((((-4*d*
f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1
/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2
*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e+(-4
*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*((-4*d*f+e^2)^(1/2)*c
*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))*c*e
-8*f^3/(e+(-4*d*f+e^2)^(1/2))^3/(-4*d*f+e^2)^(1/2)*2^(1/2)/(((-4*d*f+e^2)^(
1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*ln((((-4*d*f+e^2)^(1/2)*c*e+2*a*
f^2-2*c*d*f+c*e^2)/f^2-c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1
/2))/f)+1/2*2^(1/2)*(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1
/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e+(-4*d*f+e^2)^(1/2))/f*(x
+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*
e^2)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))*a+8*f^2/(e+(-4*d*f+e^2)^
(1/2))^3/(-4*d*f+e^2)^(1/2)*2^(1/2)/(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*
f+c*e^2)/f^2)^(1/2)*ln((((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-
c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*(((
-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*
f+e^2)^(1/2))/f)^2*c-4*c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1
/2))/f)+2*((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))/(x+1/2
*(e+(-4*d*f+e^2)^(1/2))/f))*c*d-4*f/(e+(-4*d*f+e^2)^(1/2))^3/(-4*d*f+e^2)^(
1/2)*2^(1/2)/(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*ln(
(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c*(e+(-4*d*f+e^2)^(1/2)
)/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*(((-4*d*f+e^2)^(1/2)*c*e+2
*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*
c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*((-4*d*f+e^2)
^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))
/f))*c*e^2+16*f^2*e/(-e+(-4*d*f+e^2)^(1/2))^2/(e+(-4*d*f+e^2)^(1/2))^2/a/x*
(c*x^2+a)^(3/2)-16*f^2*e/(-e+(-4*d*f+e^2)^(1/2))^2/(e+(-4*d*f+e^2)^(1/2))^2
*c/a*x*(c*x^2+a)^(1/2)-16*f^2*e/(-e+(-4*d*f+e^2)^(1/2))^2/(e+(-4*d*f+e^2)^(
1/2))^2*c^(1/2)*ln(c^(1/2)*x+(c*x^2+a)^(1/2))+2*f/(-e+(-4*d*f+e^2)^(1/2))/(
e+(-4*d*f+e^2)^(1/2))/a/x^2*(c*x^2+a)^(3/2)+2*f/(-e+(-4*d*f+e^2)^(1/2))/(e+
(-4*d*f+e^2)^(1/2))*c/a^(1/2)*ln((2*a+2*a^(1/2)*(c*x^2+a)^(1/2))/x)-2*f/(-e
+(-4*d*f+e^2)^(1/2))/(e+(-4*d*f+e^2)^(1/2))*c/a*(c*x^2+a)^(1/2)+4*f^3/(-e+(
-4*d*f+e^2)^(1/2))^3/(-4*d*f+e^2)^(1/2)*(4*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f
)^2*c-4*c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+2*(-(-
4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)+4*f^2/(-e+(-4*d*f+e^
2)^(1/2))^3*c^(1/2)*ln((-1/2*c*(e-(-4*d*f+e^2)^(1/2))/f+c*(x-1/2*(-e+(-4*d*
f+e^2)^(1/2))/f))/c^(1/2)+((x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-c*(e-(-4*d
*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+1/2*(-(-4*d*f+e^2)^(1/2)
*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))-4*f^2/(-e+(-4*d*f+e^2)^(1/2))^3/(-4
*d*f+e^2)^(1/2)*c^(1/2)*ln((-1/2*c*(e-(-4*d*f+e^2)^(1/2))/f+c*(x-1/2*(-e+(-
4*d*f+e^2)^(1/2))/f))/c^(1/2)+((x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-c*(e-(
-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+1/2*(-(-4*d*f+e^2)^(
1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))*e+4*f/(-e+(-4*d*f+e^2)^(1/2))^3
*2^(1/2)/((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*ln(((-
(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c*(e-(-4*d*f+e^2)^(1/2))/
f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*((-(-4*d*f+e^2)^(1/2)*c*e+2
*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-4
*c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+2*(-(-4*d*f+e
^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))/(x-1/2*(-e+(-4*d*f+e^2)^(1
/2))/f))*c*e-8*f^3/(-e+(-4*d*f+e^2)^(1/2))^3/(-4*d*f+e^2)^(1/2)*2^(1/2)/((-
(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*ln(((-(-4*d*f+e^2)
```

```
^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e
+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d
*f+c*e^2)/f^2)^(1/2)*(4*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e-(-4*d*
f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+2*(-(-4*d*f+e^2)^(1/2)*c*
e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))/(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f))*a+8
*f^2/(-e+(-4*d*f+e^2)^(1/2))^3/(-4*d*f+e^2)^(1/2)*2^(1/2)/((-(-4*d*f+e^2)^(
1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*ln(((-(-4*d*f+e^2)^(1/2)*c*e+2*a
*f^2-2*c*d*f+c*e^2)/f^2-c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^
(1/2))/f)+1/2*2^(1/2)*((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)
^(1/2)*(4*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e-(-4*d*f+e^2)^(1/2))/
f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+2*(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*
d*f+c*e^2)/f^2)^(1/2))/(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f))*c*d-4*f/(-e+(-4*d
*f+e^2)^(1/2))^3/(-4*d*f+e^2)^(1/2)*2^(1/2)/((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f
^2-2*c*d*f+c*e^2)/f^2)^(1/2)*ln(((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c
*e^2)/f^2-c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+1/2*
2^(1/2)*((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x-1
/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(
-4*d*f+e^2)^(1/2))/f)+2*(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2
)^(1/2))/(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f))*c*e^2-64*f^4/(-e+(-4*d*f+e^2)^(
1/2))^3/(e+(-4*d*f+e^2)^(1/2))^3*a^(1/2)*ln((2*a+2*a^(1/2)*(c*x^2+a)^(1/2))
/x)*d+64*f^3/(-e+(-4*d*f+e^2)^(1/2))^3/(e+(-4*d*f+e^2)^(1/2))^3*a^(1/2)*ln(
(2*a+2*a^(1/2)*(c*x^2+a)^(1/2))/x)*e^2+64*f^4/(-e+(-4*d*f+e^2)^(1/2))^3/(e+
(-4*d*f+e^2)^(1/2))^3*(c*x^2+a)^(1/2)*d-64*f^3/(-e+(-4*d*f+e^2)^(1/2))^3/(e
+(-4*d*f+e^2)^(1/2))^3*(c*x^2+a)^(1/2)*e^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^2+a}}{\left(fx^2+ex+d\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2)/x^3/(f*x^2+e*x+d),x, algorithm="maxima")
[Out] integrate(sqrt(c*x^2 + a)/((f*x^2 + e*x + d)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c\,x^2+a}}{x^3\,\left(f\,x^2+e\,x+d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^2)^(1/2)/(x^3*(d + e*x + f*x^2)),x)
[Out] int((a + c*x^2)^(1/2)/(x^3*(d + e*x + f*x^2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+cx^2}}{x^3\left(d+ex+fx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**(1/2)/x**3/(f*x**2+e*x+d),x)
[Out] Integral(sqrt(a + c*x**2)/(x**3*(d + e*x + f*x**2)), x)
```

3.58 $\int \frac{x^2(a+cx^2)^{3/2}}{d+ex+fx^2}\,dx$

Optimal. Leaf size=795

$$-\frac{(4e-3fx)\left(cx^2+a\right)^{3/2}}{12f^2}-\frac{\left(8e\left(af^2+c\left(e^2-2df\right)\right)-f\left(3af^2+4c\left(e^2-df\right)\right)x\right)\sqrt{cx^2+a}}{8f^4}+\frac{\left(3a^2f^4+12ac\left(e^2-\right.\right.}{}$$

```
[Out] -1/12*(-3*f*x+4*e)*(c*x^2+a)^(3/2)/f^2+1/8*(3*a^2*f^4+12*a*c*f^2*(-d*f+e^2)
+8*c^2*(d^2*f^2-3*d*e^2*f+e^4))*arctanh(x*c^(1/2)/(c*x^2+a)^(1/2))/f^5/c^(1
/2)-1/8*(8*e*(a*f^2+c*(-2*d*f+e^2))-f*(3*a*f^2+4*c*(-d*f+e^2))*x)*(c*x^2+a)
^(1/2)/f^4-1/2*arctanh(1/2*(2*a*f-c*x*(e-(-4*d*f+e^2)^(1/2)))*2^(1/2)/(c*x^
2+a)^(1/2)/(2*a*f^2+(-(-4*d*f+e^2)^(1/2)*e-2*d*f+e^2)*c)^(1/2))*(a^2*f^4*(-
(-4*d*f+e^2)^(1/2)*e-2*d*f+e^2)+2*a*c*f^2*(e^4-4*d*e^2*f+2*d^2*f^2-e^3*(-4*
d*f+e^2)^(1/2)+2*d*e*f*(-4*d*f+e^2)^(1/2))+c^2*(e^6-6*d*e^4*f+9*d^2*e^2*f^2
-2*d^3*f^3-e^5*(-4*d*f+e^2)^(1/2)+4*d*e^3*f*(-4*d*f+e^2)^(1/2)-3*d^2*e*f^2*
(-4*d*f+e^2)^(1/2)))/f^5*2^(1/2)/(-4*d*f+e^2)^(1/2)/(2*a*f^2+(-(-4*d*f+e^2)
^(1/2)*e-2*d*f+e^2)*c)^(1/2)+1/2*arctanh(1/2*(2*a*f-c*x*(e+(-4*d*f+e^2)^(1/
2)))*2^(1/2)/(c*x^2+a)^(1/2)/(2*a*f^2+((-4*d*f+e^2)^(1/2)*e-2*d*f+e^2)*c)^(
1/2))*(a^2*f^4*((-4*d*f+e^2)^(1/2)*e-2*d*f+e^2)+2*a*c*f^2*(e^4-4*d*e^2*f+2*
d^2*f^2+e^3*(-4*d*f+e^2)^(1/2)-2*d*e*f*(-4*d*f+e^2)^(1/2))+c^2*(e^6-6*d*e^4
*f+9*d^2*e^2*f^2-2*d^3*f^3+e^5*(-4*d*f+e^2)^(1/2)-4*d*e^3*f*(-4*d*f+e^2)^(1
/2)+3*d^2*e*f^2*(-4*d*f+e^2)^(1/2)))/f^5*2^(1/2)/(-4*d*f+e^2)^(1/2)/(2*a*f^
2+((-4*d*f+e^2)^(1/2)*e-2*d*f+e^2)*c)^(1/2)
```

Rubi [A] time = 4.26, antiderivative size = 795, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {1069, 1068, 1080, 217, 206, 1034, 725}

$$-\frac{(4e-3fx)\left(cx^2+a\right)^{3/2}}{12f^2}-\frac{\left(8e\left(af^2+c\left(e^2-2df\right)\right)-f\left(3af^2+4c\left(e^2-df\right)\right)x\right)\sqrt{cx^2+a}}{8f^4}+\frac{\left(3a^2f^4+12ac\left(e^2-\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + c*x^2)^(3/2))/(d + e*x + f*x^2),x]
```

```
[Out] -((8*e*(a*f^2 + c*(e^2 - 2*d*f)) - f*(3*a*f^2 + 4*c*(e^2 - d*f))*x)*Sqrt[a
 + c*x^2])/(8*f^4) - ((4*e - 3*f*x)*(a + c*x^2)^(3/2))/(12*f^2) + ((3*a^2*f^
4 + 12*a*c*f^2*(e^2 - d*f) + 8*c^2*(e^4 - 3*d*e^2*f + d^2*f^2))*ArcTanh[(Sq
rt[c]*x)/Sqrt[a + c*x^2]])/(8*Sqrt[c]*f^5) - ((a^2*f^4*(e^2 - 2*d*f - e*Sqr
t[e^2 - 4*d*f]) + 2*a*c*f^2*(e^4 - 4*d*e^2*f + 2*d^2*f^2 - e^3*Sqrt[e^2 - 4
*d*f] + 2*d*e*f*Sqrt[e^2 - 4*d*f]) + c^2*(e^6 - 6*d*e^4*f + 9*d^2*e^2*f^2 -
 2*d^3*f^3 - e^5*Sqrt[e^2 - 4*d*f] + 4*d*e^3*f*Sqrt[e^2 - 4*d*f] - 3*d^2*e*
f^2*Sqrt[e^2 - 4*d*f]))*ArcTanh[(2*a*f - c*(e - Sqrt[e^2 - 4*d*f])*x)/(Sqrt
[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])]
)/(Sqrt[2]*f^5*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2
 - 4*d*f])]) + ((a^2*f^4*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + 2*a*c*f^2*(e
^4 - 4*d*e^2*f + 2*d^2*f^2 + e^3*Sqrt[e^2 - 4*d*f] - 2*d*e*f*Sqrt[e^2 - 4*d
*f]) + c^2*(e^6 - 6*d*e^4*f + 9*d^2*e^2*f^2 - 2*d^3*f^3 + e^5*Sqrt[e^2 - 4*
d*f] - 4*d*e^3*f*Sqrt[e^2 - 4*d*f] + 3*d^2*e*f^2*Sqrt[e^2 - 4*d*f]))*ArcTan
h[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*
d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[2]*f^5*Sqrt[e^2 - 4*d*
f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 1034

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f
_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(
b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Dist[(2*c*g -
h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a,
 b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1068

```
Int[((a_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)*((d_) +
(e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((B*c*f*(2*p + 2*q + 3)
+ C*(-(c*e*(2*p + q + 2))) + 2*c*C*f*(p + q + 1)*x)*(a + c*x^2)^p*(d + e*x
+ f*x^2)^(q + 1))/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3)), x] - Dist[1/(2*c*f
^2*(p + q + 1)*(2*p + 2*q + 3)), Int[(a + c*x^2)^(p - 1)*(d + e*x + f*x^2)^
q*Simp[p*(-(a*e))*(C*(c*e)*(q + 1) - c*(C*e - B*f)*(2*p + 2*q + 3)) + (p +
q + 1)*(a*c*(C*(2*d*f - e^2*(2*p + q + 2)) + f*(B*e - 2*A*f)*(2*p + 2*q + 3
))) + (2*p*(c*d - a*f)*(C*(c*e)*(q + 1) - c*(C*e - B*f)*(2*p + 2*q + 3)) +
(p + q + 1)*(C*e*f*p*(-4*a*c)))*x + (p*(c*e)*(C*(c*e)*(q + 1) - c*(C*e - B*
f)*(2*p + 2*q + 3)) + (p + q + 1)*(C*f^2*p*(-4*a*c) - c^2*(C*(e^2 - 4*d*f)*
(2*p + q + 2) + f*(2*C*d - B*e + 2*A*f)*(2*p + 2*q + 3))))*x^2, x], x], x]
/; FreeQ[{a, c, d, e, f, A, B, C, q}, x] && NeQ[e^2 - 4*d*f, 0] && GtQ[p, 0
] && NeQ[p + q + 1, 0] && NeQ[2*p + 2*q + 3, 0] &&  !IGtQ[p, 0] &&  !IGtQ[q
, 0]
```

Rule 1069

```
Int[((a_) + (c_.)*(x_)^2)^(p_)*((A_.) + (C_.)*(x_)^2)*((d_) + (e_.)*(x_) +
(f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((C*(-(c*e*(2*p + q + 2))) + 2*c*C*f*
(p + q + 1)*x)*(a + c*x^2)^p*(d + e*x + f*x^2)^(q + 1))/(2*c*f^2*(p + q + 1
)*(2*p + 2*q + 3)), x] - Dist[1/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3)), Int[
(a + c*x^2)^(p - 1)*(d + e*x + f*x^2)^q*Simp[p*(-(a*e))*(C*(c*e)*(q + 1) -
c*(C*e)*(2*p + 2*q + 3)) + (p + q + 1)*(a*c*(C*(2*d*f - e^2*(2*p + q + 2))
+ f*(-2*A*f)*(2*p + 2*q + 3))) + (2*p*(c*d - a*f)*(C*(c*e)*(q + 1) - c*(C*e
)*(2*p + 2*q + 3)) + (p + q + 1)*(C*e*f*p*(-4*a*c)))*x + (p*(c*e)*(C*(c*e)*
(q + 1) - c*(C*e)*(2*p + 2*q + 3)) + (p + q + 1)*(C*f^2*p*(-4*a*c) - c^2*(C
*(e^2 - 4*d*f)*(2*p + q + 2) + f*(2*C*d + 2*A*f)*(2*p + 2*q + 3))))*x^2, x]
, x], x] /; FreeQ[{a, c, d, e, f, A, C, q}, x] && NeQ[e^2 - 4*d*f, 0] && Gt
Q[p, 0] && NeQ[p + q + 1, 0] && NeQ[2*p + 2*q + 3, 0] &&  !IGtQ[p, 0] &&  !
IGtQ[q, 0]
```

Rule 1080

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
```

```
*Sqrt[(d_.) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqrt[d + f*x^2],
 x], x] + Dist[1/c, Int[(A*c - a*C + (B*c - b*C)*x)/((a + b*x + c*x^2)*Sqrt
[d + f*x^2]), x], x] /; FreeQ[{a, b, c, d, f, A, B, C}, x] && NeQ[b^2 - 4*a
*c, 0]
```

Rubi steps

$$\int \frac{x^2\left(a+cx^2\right)^{3/2}}{d+ex+fx^2}\,dx=-\frac{(4e-3fx)\left(a+cx^2\right)^{3/2}}{12f^2}-\frac{\int \frac{\sqrt{a+cx^2}\left(3acdf-3ce(4cd-af)x-3c\left(3af^2+4c\left(e^2-df\right)\right)x^2\right)}{d+ex+fx^2}\,dx}{12cf^2}$$

$$=-\frac{\left(8e\left(af^2+c\left(e^2-2df\right)\right)-f\left(3af^2+4c\left(e^2-df\right)\right)x\right)\sqrt{a+cx^2}}{8f^4}-\frac{(4e-3fx)\left(a+cx^2\right.}{12f^2}$$

$$=-\frac{\left(8e\left(af^2+c\left(e^2-2df\right)\right)-f\left(3af^2+4c\left(e^2-df\right)\right)x\right)\sqrt{a+cx^2}}{8f^4}-\frac{(4e-3fx)\left(a+cx^2\right.}{12f^2}$$

$$=-\frac{\left(8e\left(af^2+c\left(e^2-2df\right)\right)-f\left(3af^2+4c\left(e^2-df\right)\right)x\right)\sqrt{a+cx^2}}{8f^4}-\frac{(4e-3fx)\left(a+cx^2\right.}{12f^2}$$

$$=-\frac{\left(8e\left(af^2+c\left(e^2-2df\right)\right)-f\left(3af^2+4c\left(e^2-df\right)\right)x\right)\sqrt{a+cx^2}}{8f^4}-\frac{(4e-3fx)\left(a+cx^2\right.}{12f^2}$$

$$=-\frac{\left(8e\left(af^2+c\left(e^2-2df\right)\right)-f\left(3af^2+4c\left(e^2-df\right)\right)x\right)\sqrt{a+cx^2}}{8f^4}-\frac{(4e-3fx)\left(a+cx^2\right.}{12f^2}$$

Mathematica [A] time = 3.45, size = 793, normalized size = 1.00

$$3f\sqrt{a+cx^2}\left(\frac{3a^{3/2}\sinh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\sqrt{c}\sqrt{\frac{cx^2}{a}+1}}+5ax+2cx^3\right)-\frac{3\left(\frac{2df-e^2}{\sqrt{e^2-4df}}+e\right)\left(\frac{2\left(2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)\right)\left(-\sqrt{4af^2-2ce\sqrt{e^2-4df}-4cdf+2ce^2}\tanh^{-1}\left(\cdots\right.\right.}{\cdots}\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + c*x^2)^(3/2))/(d + e*x + f*x^2),x]
```

```
[Out] (-4*(e + (e^2 - 2*d*f)/Sqrt[e^2 - 4*d*f])*(a + c*x^2)^(3/2) - 4*(e + (-e^2
+ 2*d*f)/Sqrt[e^2 - 4*d*f])*(a + c*x^2)^(3/2) + 3*f*Sqrt[a + c*x^2]*(5*a*x
+ 2*c*x^3 + (3*a^(3/2)*ArcSinh[(Sqrt[c]*x)/Sqrt[a]])/(Sqrt[c]*Sqrt[1 + (c*x
^2)/a])) - (3*(e + (-e^2 + 2*d*f)/Sqrt[e^2 - 4*d*f])*((2*Sqrt[c]*(-e + Sqrt
[e^2 - 4*d*f])*Sqrt[a + c*x^2]*(Sqrt[c]*x*Sqrt[1 + (c*x^2)/a] + Sqrt[a]*Arc
Sinh[(Sqrt[c]*x)/Sqrt[a]]))/Sqrt[1 + (c*x^2)/a] + (2*(2*a*f^2 + c*(e^2 - 2*
d*f - e*Sqrt[e^2 - 4*d*f]))*(2*f*Sqrt[a + c*x^2] + Sqrt[c]*(-e + Sqrt[e^2 -
 4*d*f])*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]] - Sqrt[2*c*e^2 - 4*c*d*f + 4*
a*f^2 - 2*c*e*Sqrt[e^2 - 4*d*f]]*ArcTanh[(2*a*f + c*(-e + Sqrt[e^2 - 4*d*f]
)*x)/(Sqrt[4*a*f^2 - 2*c*(-e^2 + 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x
```

```
^2])]))/f^2))/(2*f) + (3*(e + (e^2 - 2*d*f)/Sqrt[e^2 - 4*d*f])*((2*Sqrt[c]*
(e + Sqrt[e^2 - 4*d*f])*Sqrt[a + c*x^2]*(Sqrt[c]*x*Sqrt[1 + (c*x^2)/a] + Sq
rt[a]*ArcSinh[(Sqrt[c]*x)/Sqrt[a]]))/Sqrt[1 + (c*x^2)/a] + (2*(2*a*f^2 + c*
(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]))*(-2*f*Sqrt[a + c*x^2] + Sqrt[c]*(e + S
qrt[e^2 - 4*d*f])*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]] + Sqrt[4*a*f^2 + 2*c
*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*
d*f])*x)/(Sqrt[4*a*f^2 + 2*c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a +
c*x^2])]))/f^2))/(2*f))/(24*f^2)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] sage2
```

maple [B] time = 0.03, size = 19148, normalized size = 24.09

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x)
```

```
[Out] result too large to display
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for mo
re details)Is 4*d*f-e^2 positive, negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\,{\left(c\,x^2+a\right)}^{3/2}}{f\,x^2+e\,x+d} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + c*x^2)^(3/2))/(d + e*x + f*x^2),x)
```

```
[Out] int((x^2*(a + c*x^2)^(3/2))/(d + e*x + f*x^2), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c*x**2+a)**(3/2)/(f*x**2+e*x+d),x)
```

```
[Out] Timed out
```

3.59 $\int \frac{x(a+cx^2)^{3/2}}{d+ex+fx^2}\,dx$

Optimal. Leaf size=553

$$-\frac{\left(2cdef\left(2af^2+c\left(e^2-2df\right)\right)-\left(e-\sqrt{e^2-4df}\right)\left(a^2f^4+2acf^2\left(e^2-df\right)+c^2\left(d^2f^2-3de^2f+e^4\right)\right)\right)\tanh^{-1}\left(\frac{}{\sqrt{2}}\right.}{\sqrt{2}\,f^4\sqrt{e^2-4df}\,\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}$$

```
[Out] 1/3*(c*x^2+a)^(3/2)/f-1/2*e*(3*a*f^2+2*c*(-2*d*f+e^2))*arctanh(x*c^(1/2)/(c
*x^2+a)^(1/2))*c^(1/2)/f^4+1/2*(2*a*f^2+2*c*(-d*f+e^2)-c*e*f*x)*(c*x^2+a)^(
1/2)/f^3-1/2*arctanh(1/2*(2*a*f-c*x*(e-(-4*d*f+e^2)^(1/2)))*2^(1/2)/(c*x^2+
a)^(1/2)/(2*a*f^2+(-(-4*d*f+e^2)^(1/2)*e-2*d*f+e^2)*c)^(1/2))*(2*c*d*e*f*(2
*a*f^2+c*(-2*d*f+e^2))-(a^2*f^4+2*a*c*f^2*(-d*f+e^2)+c^2*(d^2*f^2-3*d*e^2*f
+e^4))*(e-(-4*d*f+e^2)^(1/2)))/f^4*2^(1/2)/(-4*d*f+e^2)^(1/2)/(2*a*f^2+(-(-
4*d*f+e^2)^(1/2)*e-2*d*f+e^2)*c)^(1/2)+1/2*arctanh(1/2*(2*a*f-c*x*(e+(-4*d*
f+e^2)^(1/2)))*2^(1/2)/(c*x^2+a)^(1/2)/(2*a*f^2+((-4*d*f+e^2)^(1/2)*e-2*d*f
+e^2)*c)^(1/2))*(2*c*d*e*f*(2*a*f^2+c*(-2*d*f+e^2))-(a^2*f^4+2*a*c*f^2*(-d*
f+e^2)+c^2*(d^2*f^2-3*d*e^2*f+e^4))*(e+(-4*d*f+e^2)^(1/2)))/f^4*2^(1/2)/(-4
*d*f+e^2)^(1/2)/(2*a*f^2+((-4*d*f+e^2)^(1/2)*e-2*d*f+e^2)*c)^(1/2)
```

Rubi [A] time = 2.43, antiderivative size = 553, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {1020, 1068, 1080, 217, 206, 1034, 725}

$$-\frac{\left(2cdef\left(2af^2+c\left(e^2-2df\right)\right)-\left(e-\sqrt{e^2-4df}\right)\left(a^2f^4+2acf^2\left(e^2-df\right)+c^2\left(d^2f^2-3de^2f+e^4\right)\right)\right)\tanh^{-1}\left(\frac{}{\sqrt{2}}\right.}{\sqrt{2}\,f^4\sqrt{e^2-4df}\,\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + c*x^2)^(3/2))/(d + e*x + f*x^2),x]
```

```
[Out] ((2*(a*f^2 + c*(e^2 - d*f)) - c*e*f*x)*Sqrt[a + c*x^2])/(2*f^3) + (a + c*x^
2)^(3/2)/(3*f) - (Sqrt[c]*e*(3*a*f^2 + 2*c*(e^2 - 2*d*f))*ArcTanh[(Sqrt[c]*
x)/Sqrt[a + c*x^2]])/(2*f^4) - ((2*c*d*e*f*(2*a*f^2 + c*(e^2 - 2*d*f)) - (e
 - Sqrt[e^2 - 4*d*f])*(a^2*f^4 + 2*a*c*f^2*(e^2 - d*f) + c^2*(e^4 - 3*d*e^2
*f + d^2*f^2)))*ArcTanh[(2*a*f - c*(e - Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt
[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[
2]*f^4*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f
])]) + ((2*c*d*e*f*(2*a*f^2 + c*(e^2 - 2*d*f)) - (e + Sqrt[e^2 - 4*d*f])*(a
^2*f^4 + 2*a*c*f^2*(e^2 - d*f) + c^2*(e^4 - 3*d*e^2*f + d^2*f^2)))*ArcTanh[
(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*
f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[2]*f^4*Sqrt[e^2 - 4*d*f]
*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 1020

```
Int[((g_.) + (h_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f
_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(h*(a + c*x^2)^p*(d + e*x + f*x^2)^(q +
 1))/(2*f*(p + q + 1)), x] + Dist[1/(2*f*(p + q + 1)), Int[(a + c*x^2)^(p -
 1)*(d + e*x + f*x^2)^q*Simp[a*h*e*p - a*(h*e - 2*g*f)*(p + q + 1) - 2*h*p*
(c*d - a*f)*x - (h*c*e*p + c*(h*e - 2*g*f)*(p + q + 1))*x^2, x], x], x] /;
FreeQ[{a, c, d, e, f, g, h, q}, x] && NeQ[e^2 - 4*d*f, 0] && GtQ[p, 0] && N
eQ[p + q + 1, 0]
```

Rule 1034

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f
_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(
b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Dist[(2*c*g -
h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a,
 b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1068

```
Int[((a_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)*((d_) +
(e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((B*c*f*(2*p + 2*q + 3)
+ C*(-(c*e*(2*p + q + 2))) + 2*c*C*f*(p + q + 1)*x)*(a + c*x^2)^p*(d + e*x
+ f*x^2)^(q + 1))/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3)), x] - Dist[1/(2*c*f
^2*(p + q + 1)*(2*p + 2*q + 3)), Int[(a + c*x^2)^(p - 1)*(d + e*x + f*x^2)^
q*Simp[p*(-(a*e))*(C*(c*e)*(q + 1) - c*(C*e - B*f)*(2*p + 2*q + 3)) + (p +
q + 1)*(a*c*(C*(2*d*f - e^2*(2*p + q + 2)) + f*(B*e - 2*A*f)*(2*p + 2*q + 3
))) + (2*p*(c*d - a*f)*(C*(c*e)*(q + 1) - c*(C*e - B*f)*(2*p + 2*q + 3)) +
(p + q + 1)*(C*e*f*p*(-4*a*c)))*x + (p*(c*e)*(C*(c*e)*(q + 1) - c*(C*e - B*
f)*(2*p + 2*q + 3)) + (p + q + 1)*(C*f^2*p*(-4*a*c) - c^2*(C*(e^2 - 4*d*f)*
(2*p + q + 2) + f*(2*C*d - B*e + 2*A*f)*(2*p + 2*q + 3))))*x^2, x], x], x]
/; FreeQ[{a, c, d, e, f, A, B, C, q}, x] && NeQ[e^2 - 4*d*f, 0] && GtQ[p, 0
] && NeQ[p + q + 1, 0] && NeQ[2*p + 2*q + 3, 0] &&  !IGtQ[p, 0] &&  !IGtQ[q
, 0]
```

Rule 1080

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*Sqrt[(d_.) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqrt[d + f*x^2],
 x], x] + Dist[1/c, Int[(A*c - a*C + (B*c - b*C)*x)/((a + b*x + c*x^2)*Sqrt
[d + f*x^2]), x], x] /; FreeQ[{a, b, c, d, f, A, B, C}, x] && NeQ[b^2 - 4*a
*c, 0]
```

Rubi steps

$$\int \frac{x\left(a+cx^2\right)^{3/2}}{d+ex+fx^2}\,dx = \frac{\left(a+cx^2\right)^{3/2}}{3f} + \frac{\int \frac{\sqrt{a+cx^2}\left(-3(cd-af)x-3cex^2\right)}{d+ex+fx^2}\,dx}{3f}$$

$$= \frac{\left(2\left(af^2+c\left(e^2-df\right)\right)-cefx\right)\sqrt{a+cx^2}}{2f^3} + \frac{\left(a+cx^2\right)^{3/2}}{3f} - \frac{\int \frac{-3ac^2def+3c\left(ace^2f+2(cd-af)\left(ce^2-cdf\right.\right.}{\sqrt{a+cx^2}\left(d+\right.}}{6cf}$$

$$= \frac{\left(2\left(af^2+c\left(e^2-df\right)\right)-cefx\right)\sqrt{a+cx^2}}{2f^3} + \frac{\left(a+cx^2\right)^{3/2}}{3f} - \frac{\int \frac{-3ac^2def^2-3c^2de\left(3af^2+2c\left(e^2-2df\right)\right)+}{}}{}$$

$$= \frac{\left(2\left(af^2+c\left(e^2-df\right)\right)-cefx\right)\sqrt{a+cx^2}}{2f^3} + \frac{\left(a+cx^2\right)^{3/2}}{3f} - \frac{\left(ce\left(3af^2+2c\left(e^2-2df\right)\right)\right)\text{Sub}}{2f^4}$$

$$= \frac{\left(2\left(af^2+c\left(e^2-df\right)\right)-cefx\right)\sqrt{a+cx^2}}{2f^3} + \frac{\left(a+cx^2\right)^{3/2}}{3f} - \frac{\sqrt{c}\,e\left(3af^2+2c\left(e^2-2df\right)\right)\tanh}{2f^4}$$

$$= \frac{\left(2\left(af^2+c\left(e^2-df\right)\right)-cefx\right)\sqrt{a+cx^2}}{2f^3} + \frac{\left(a+cx^2\right)^{3/2}}{3f} - \frac{\sqrt{c}\,e\left(3af^2+2c\left(e^2-2df\right)\right)\tanh}{2f^4}$$

Mathematica [A] time = 2.01, size = 755, normalized size = 1.37

$$\frac{8f^3\left(a+cx^2\right)^{5/2}\sqrt{\frac{cx^2}{a}+1}\left(\sqrt{e^2-4df}-e\right)+8f^3\left(a+cx^2\right)^{5/2}\sqrt{\frac{cx^2}{a}+1}\left(\sqrt{e^2-4df}+e\right)+3\left(e-\sqrt{e^2-4df}\right)\left(2\sqrt{\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + c*x^2)^(3/2))/(d + e*x + f*x^2),x]
```

```
[Out] (8*f^3*(-e + Sqrt[e^2 - 4*d*f])*(a + c*x^2)^(5/2)*Sqrt[1 + (c*x^2)/a] + 8*f
^3*(e + Sqrt[e^2 - 4*d*f])*(a + c*x^2)^(5/2)*Sqrt[1 + (c*x^2)/a] + 3*(e - S
qrt[e^2 - 4*d*f])*(2*Sqrt[c]*f^2*(e - Sqrt[e^2 - 4*d*f])*Sqrt[a + c*x^2]*(a
*Sqrt[c]*x*(1 + (c*x^2)/a)^(3/2) + Sqrt[a]*(a + c*x^2)*ArcSinh[(Sqrt[c]*x)/
Sqrt[a]]) - a*(4*a*f^2 + c*(e - Sqrt[e^2 - 4*d*f])^2)*(1 + (c*x^2)/a)^(3/2)
*(2*f*Sqrt[a + c*x^2] + Sqrt[c]*(-e + Sqrt[e^2 - 4*d*f])*ArcTanh[(Sqrt[c]*x
)/Sqrt[a + c*x^2]] - Sqrt[2*c*e^2 - 4*c*d*f + 4*a*f^2 - 2*c*e*Sqrt[e^2 - 4*
d*f]]*ArcTanh[(2*a*f + c*(-e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[4*a*f^2 - 2*c*(-
e^2 + 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])) - 3*(e + Sqrt[e^2 -
 4*d*f])*(2*Sqrt[c]*f^2*(e + Sqrt[e^2 - 4*d*f])*Sqrt[a + c*x^2]*(a*Sqrt[c]*
x*(1 + (c*x^2)/a)^(3/2) + Sqrt[a]*(a + c*x^2)*ArcSinh[(Sqrt[c]*x)/Sqrt[a]])
 - a*(4*a*f^2 + c*(e + Sqrt[e^2 - 4*d*f])^2)*(1 + (c*x^2)/a)^(3/2)*(2*f*Sqr
t[a + c*x^2] - Sqrt[c]*(e + Sqrt[e^2 - 4*d*f])*ArcTanh[(Sqrt[c]*x)/Sqrt[a +
 c*x^2]] - Sqrt[4*a*f^2 + 2*c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*ArcTanh[
(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[4*a*f^2 + 2*c*(e^2 - 2*d*f + e*
Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])))/(48*a*f^4*Sqrt[e^2 - 4*d*f]*(1 + (
c*x^2)/a)^(3/2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] sage2
```

maple [B] time = 0.02, size = 14709, normalized size = 26.60

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x)
```

```
[Out] result too large to display
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for mo
re details)Is 4*d*f-e^2 positive, negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\,{\left(c\,x^2+a\right)}^{3/2}}{f\,x^2+e\,x+d} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + c*x^2)^(3/2))/(d + e*x + f*x^2),x)
```

```
[Out] int((x*(a + c*x^2)^(3/2))/(d + e*x + f*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \left(a + cx^{2}\right)^{\frac{3}{2}}}{d + ex + fx^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x**2+a)**(3/2)/(f*x**2+e*x+d),x)
```

```
[Out] Integral(x*(a + c*x**2)**(3/2)/(d + e*x + f*x**2), x)
```

3.60 $\int \frac{\left(a+cx^2\right)^{3/2}}{d+ex+fx^2}\,dx$

Optimal. Leaf size=484

$$-\frac{\left(ce\left(e-\sqrt{e^2-4df}\right)\left(2af^2+c\left(e^2-2df\right)\right)-2f\left(-a^2f^3+2acdf^2+c^2d\left(e^2-df\right)\right)\right)\tanh^{-1}\left(\frac{2af-cx\left(e-\sqrt{e^2}\right.}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\right.}}\right.}{\sqrt{2}f^3\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}$$

```
[Out] 1/2*(3*a*f^2+2*c*(-d*f+e^2))*arctanh(x*c^(1/2)/(c*x^2+a)^(1/2))*c^(1/2)/f^3
-1/2*c*(-f*x+2*e)*(c*x^2+a)^(1/2)/f^2-1/2*arctanh(1/2*(2*a*f-c*x*(e-(-4*d*f
+e^2)^(1/2)))*2^(1/2)/(c*x^2+a)^(1/2)/(2*a*f^2+(-(-4*d*f+e^2)^(1/2)*e-2*d*f
+e^2)*c)^(1/2))*(-2*f*(2*a*c*d*f^2-a^2*f^3+c^2*d*(-d*f+e^2))+c*e*(2*a*f^2+c
*(-2*d*f+e^2))*(e-(-4*d*f+e^2)^(1/2)))/f^3*2^(1/2)/(-4*d*f+e^2)^(1/2)/(2*a*
f^2+(-(-4*d*f+e^2)^(1/2)*e-2*d*f+e^2)*c)^(1/2)+1/2*arctanh(1/2*(2*a*f-c*x*(
e+(-4*d*f+e^2)^(1/2)))*2^(1/2)/(c*x^2+a)^(1/2)/(2*a*f^2+((-4*d*f+e^2)^(1/2)
*e-2*d*f+e^2)*c)^(1/2))*(-2*f*(2*a*c*d*f^2-a^2*f^3+c^2*d*(-d*f+e^2))+c*e*(2
*a*f^2+c*(-2*d*f+e^2))*(e+(-4*d*f+e^2)^(1/2)))/f^3*2^(1/2)/(-4*d*f+e^2)^(1/
2)/(2*a*f^2+((-4*d*f+e^2)^(1/2)*e-2*d*f+e^2)*c)^(1/2)
```

Rubi [A] time = 4.24, antiderivative size = 482, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {979, 1080, 217, 206, 1034, 725}

$$\frac{\left(-2a^2f^4-ce\left(e-\sqrt{e^2-4df}\right)\left(2af^2+c\left(e^2-2df\right)\right)+4acdf^3+2c^2df\left(e^2-df\right)\right)\tanh^{-1}\left(\frac{2af-cx\left(e-\sqrt{e^2-4}\right.}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2}\right.}}\right.}{\sqrt{2}f^3\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)^(3/2)/(d + e*x + f*x^2),x]
```

```
[Out] -(c*(2*e - f*x)*Sqrt[a + c*x^2])/(2*f^2) + (Sqrt[c]*(3*a*f^2 + 2*c*(e^2 - d
*f))*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(2*f^3) + ((4*a*c*d*f^3 - 2*a^2*
f^4 + 2*c^2*d*f*(e^2 - d*f) - c*e*(e - Sqrt[e^2 - 4*d*f])*(2*a*f^2 + c*(e^2
 - 2*d*f)))*ArcTanh[(2*a*f - c*(e - Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a
*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[2]*f
^3*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])])
 - ((4*a*c*d*f^3 - 2*a^2*f^4 + 2*c^2*d*f*(e^2 - d*f) - c*e*(e + Sqrt[e^2 -
4*d*f])*(2*a*f^2 + c*(e^2 - 2*d*f)))*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d
*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt
[a + c*x^2])])/(Sqrt[2]*f^3*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f
 + e*Sqrt[e^2 - 4*d*f])])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 979

```
Int[((a_.) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x
_Symbol] :> -Simp[(c*(e*(2*p + q) - 2*f*(p + q)*x)*(a + c*x^2)^(p - 1)*(d +
 e*x + f*x^2)^(q + 1))/(2*f^2*(p + q)*(2*p + 2*q + 1)), x] - Dist[1/(2*f^2*
(p + q)*(2*p + 2*q + 1)), Int[(a + c*x^2)^(p - 2)*(d + e*x + f*x^2)^q*Simp[
-(a*c*e^2*(1 - p)*(2*p + q)) + a*(p + q)*(-2*a*f^2*(2*p + 2*q + 1) + c*(2*d
*f - e^2*(2*p + q))) + (2*(c*d - a*f)*(c*e)*(1 - p)*(2*p + q) + 4*a*c*e*f*(
1 - p)*(p + q))*x + (p*c^2*e^2*(1 - p) - c*(p + q)*(2*a*f^2*(4*p + 2*q - 1)
 + c*(2*d*f*(1 - 2*p) + e^2*(3*p + q - 1))))*x^2, x], x], x] /; FreeQ[{a, c
, d, e, f, q}, x] && NeQ[e^2 - 4*d*f, 0] && GtQ[p, 1] && NeQ[p + q, 0] && N
eQ[2*p + 2*q + 1, 0] &&  !IGtQ[p, 0] &&  !IGtQ[q, 0]
```

Rule 1034

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f
_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(
b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Dist[(2*c*g -
h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a,
 b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1080

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*Sqrt[(d_.) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqrt[d + f*x^2],
 x], x] + Dist[1/c, Int[(A*c - a*C + (B*c - b*C)*x)/((a + b*x + c*x^2)*Sqrt
[d + f*x^2]), x], x] /; FreeQ[{a, b, c, d, f, A, B, C}, x] && NeQ[b^2 - 4*a
*c, 0]
```

Rubi steps

$$\int \frac{\left(a+cx^2\right)^{3/2}}{d+ex+fx^2}\,dx = -\frac{c(2e-fx)\sqrt{a+cx^2}}{2f^2} - \frac{\int \frac{af(cd-2af)-ce(2cd-af)x-c\left(3af^2+2c\left(e^2-df\right)\right)x^2}{\sqrt{a+cx^2}\left(d+ex+fx^2\right)}\,dx}{2f^2}$$

$$= -\frac{c(2e-fx)\sqrt{a+cx^2}}{2f^2} - \frac{\int \frac{af^2(cd-2af)+cd\left(3af^2+2c\left(e^2-df\right)\right)+\left(-cef(2cd-af)+ce\left(3af^2+2c\left(e^2-df\right)\right)\right)x}{\sqrt{a+cx^2}\left(d+ex+fx^2\right)}\,dx}{2f^3}$$

$$= -\frac{c(2e-fx)\sqrt{a+cx^2}}{2f^2} + \frac{\left(c\left(3af^2+2c\left(e^2-df\right)\right)\right)\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x}{\sqrt{a+cx^2}}\right)}{2f^3} - \frac{\left(2f\left(a\right.\right.\cdots}{\cdots}$$

$$= -\frac{c(2e-fx)\sqrt{a+cx^2}}{2f^2} + \frac{\sqrt{c}\left(3af^2+2c\left(e^2-df\right)\right)\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{2f^3} + \frac{\left(2f\left(af^2(cd-2a\right.\right.\cdots}{\cdots}$$

$$= -\frac{c(2e-fx)\sqrt{a+cx^2}}{2f^2} + \frac{\sqrt{c}\left(3af^2+2c\left(e^2-df\right)\right)\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{2f^3} - \frac{\left(ce\left(e-\sqrt{e^2-4d\cdots}\right.\right.}{\cdots}$$

Mathematica [A] time = 1.07, size = 603, normalized size = 1.25

$$\frac{2\left(2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)\right)\left(-\sqrt{4af^2-2ce\sqrt{e^2-4df}-4cdf+2ce^2}\tanh^{-1}\left(\frac{2af+cx\left(\sqrt{e^2-4df}-e\right)}{\sqrt{a+cx^2}\sqrt{4af^2-2c\left(e\sqrt{e^2-4df}+2df-e^2\right)}}\right)+\sqrt{c}\left(\sqrt{e^2-4df}-e\right)\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx}}\right.\right.}{f^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)^(3/2)/(d + e*x + f*x^2),x]
```

```
[Out] ((2*Sqrt[c]*(-e + Sqrt[e^2 - 4*d*f])*Sqrt[a + c*x^2]*(Sqrt[c]*x*Sqrt[1 + (c
*x^2)/a] + Sqrt[a]*ArcSinh[(Sqrt[c]*x)/Sqrt[a]]))/Sqrt[1 + (c*x^2)/a] + (2*
Sqrt[c]*(e + Sqrt[e^2 - 4*d*f])*Sqrt[a + c*x^2]*(Sqrt[c]*x*Sqrt[1 + (c*x^2)
/a] + Sqrt[a]*ArcSinh[(Sqrt[c]*x)/Sqrt[a]]))/Sqrt[1 + (c*x^2)/a] + (2*(2*a*
f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]))*(2*f*Sqrt[a + c*x^2] + Sqrt[c]
*(-e + Sqrt[e^2 - 4*d*f])*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]] - Sqrt[2*c*e
^2 - 4*c*d*f + 4*a*f^2 - 2*c*e*Sqrt[e^2 - 4*d*f]]*ArcTanh[(2*a*f + c*(-e +
Sqrt[e^2 - 4*d*f])*x)/(Sqrt[4*a*f^2 - 2*c*(-e^2 + 2*d*f + e*Sqrt[e^2 - 4*d*
f])]*Sqrt[a + c*x^2])]))/f^2 + (2*(2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 -
4*d*f]))*(-2*f*Sqrt[a + c*x^2] + Sqrt[c]*(e + Sqrt[e^2 - 4*d*f])*ArcTanh[(S
qrt[c]*x)/Sqrt[a + c*x^2]] + Sqrt[4*a*f^2 + 2*c*(e^2 - 2*d*f + e*Sqrt[e^2 -
 4*d*f])]*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[4*a*f^2 + 2*c
*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])]))/f^2)/(8*f*Sqrt[e^
2 - 4*d*f])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] sage2
```

maple [B] time = 0.01, size = 8954, normalized size = 18.50

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^(3/2)/(f*x^2+e*x+d),x)
```

```
[Out] result too large to display
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for mo
re details)Is 4*d*f-e^2 positive, negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,x^2+a\right)^{3/2}}{f\,x^2+e\,x+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^2)^(3/2)/(d + e*x + f*x^2),x)
```

```
[Out] int((a + c*x^2)^(3/2)/(d + e*x + f*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+cx^2\right)^{\frac{3}{2}}}{d+ex+fx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**(3/2)/(f*x**2+e*x+d),x)
```

```
[Out] Integral((a + c*x**2)**(3/2)/(d + e*x + f*x**2), x)
```

3.61 $\int \frac{\left(a+cx^2\right)^{3/2}}{x\left(d+ex+fx^2\right)}\,dx$

Optimal. Leaf size=496

$$-\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d}-\frac{\left(2ef\left(c^2d^2-a^2f^2\right)-\left(e-\sqrt{e^2-4df}\right)\left(c^2de^2-f(cd-af)^2\right)\right)\tanh^{-1}\left(\frac{2af-cx\left(e-\sqrt{\right.}}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\right.}}\right.}{\sqrt{2}df^2\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}$$

```
[Out] -c^(3/2)*e*arctanh(x*c^(1/2)/(c*x^2+a)^(1/2))/f^2-a^(3/2)*arctanh((c*x^2+a)
^(1/2)/a^(1/2))/d+a*(c*x^2+a)^(1/2)/d+(-a*f+c*d)*(c*x^2+a)^(1/2)/d/f-1/2*ar
ctanh(1/2*(2*a*f-c*x*(e-(-4*d*f+e^2)^(1/2)))*2^(1/2)/(c*x^2+a)^(1/2)/(2*a*f
^2+(-(-4*d*f+e^2)^(1/2)*e-2*d*f+e^2)*c)^(1/2))*(2*e*f*(-a^2*f^2+c^2*d^2)-(c
^2*d*e^2-f*(-a*f+c*d)^2)*(e-(-4*d*f+e^2)^(1/2)))/d/f^2*2^(1/2)/(-4*d*f+e^2)
^(1/2)/(2*a*f^2+(-(-4*d*f+e^2)^(1/2)*e-2*d*f+e^2)*c)^(1/2)+1/2*arctanh(1/2*
(2*a*f-c*x*(e+(-4*d*f+e^2)^(1/2)))*2^(1/2)/(c*x^2+a)^(1/2)/(2*a*f^2+((-4*d*
f+e^2)^(1/2)*e-2*d*f+e^2)*c)^(1/2))*(2*e*f*(-a^2*f^2+c^2*d^2)-(c^2*d*e^2-f*
(-a*f+c*d)^2)*(e+(-4*d*f+e^2)^(1/2)))/d/f^2*2^(1/2)/(-4*d*f+e^2)^(1/2)/(2*a
*f^2+((-4*d*f+e^2)^(1/2)*e-2*d*f+e^2)*c)^(1/2)
```

Rubi [A] time = 2.57, antiderivative size = 496, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 11, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.407, Rules used = {6728, 266, 50, 63, 208, 1020, 1080, 217, 206, 1034, 725}

$$-\frac{\left(2ef\left(c^2d^2-a^2f^2\right)-\left(e-\sqrt{e^2-4df}\right)\left(c^2de^2-f(cd-af)^2\right)\right)\tanh^{-1}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}df^2\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}+\frac{(2ef}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)^(3/2)/(x*(d + e*x + f*x^2)),x]
```

```
[Out] (a*Sqrt[a + c*x^2])/d + ((c*d - a*f)*Sqrt[a + c*x^2])/(d*f) - (c^(3/2)*e*Ar
cTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/f^2 - ((2*e*f*(c^2*d^2 - a^2*f^2) - (c^
2*d*e^2 - f*(c*d - a*f)^2)*(e - Sqrt[e^2 - 4*d*f]))*ArcTanh[(2*a*f - c*(e -
 Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2
- 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[2]*d*f^2*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2
 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]) + ((2*e*f*(c^2*d^2 - a^2*f^2) -
(c^2*d*e^2 - f*(c*d - a*f)^2)*(e + Sqrt[e^2 - 4*d*f]))*ArcTanh[(2*a*f - c*(
e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e
^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[2]*d*f^2*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*
f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]) - (a^(3/2)*ArcTanh[Sqrt[a + c
*x^2]/Sqrt[a]])/d
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 1020

```
Int[((g_.) + (h_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f
_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(h*(a + c*x^2)^p*(d + e*x + f*x^2)^(q +
 1))/(2*f*(p + q + 1)), x] + Dist[1/(2*f*(p + q + 1)), Int[(a + c*x^2)^(p -
 1)*(d + e*x + f*x^2)^q*Simp[a*h*e*p - a*(h*e - 2*g*f)*(p + q + 1) - 2*h*p*
(c*d - a*f)*x - (h*c*e*p + c*(h*e - 2*g*f)*(p + q + 1))*x^2, x], x], x] /;
FreeQ[{a, c, d, e, f, g, h, q}, x] && NeQ[e^2 - 4*d*f, 0] && GtQ[p, 0] && N
eQ[p + q + 1, 0]
```

Rule 1034

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f
_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(
b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Dist[(2*c*g -
h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a,
 b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1080

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*Sqrt[(d_.) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqrt[d + f*x^2],
 x], x] + Dist[1/c, Int[(A*c - a*C + (B*c - b*C)*x)/((a + b*x + c*x^2)*Sqrt
[d + f*x^2]), x], x] /; FreeQ[{a, b, c, d, f, A, B, C}, x] && NeQ[b^2 - 4*a
*c, 0]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\left(a+cx^2\right)^{3/2}}{x\left(d+ex+fx^2\right)}dx = \int \left(\frac{\left(a+cx^2\right)^{3/2}}{dx}+\frac{(-e-fx)\left(a+cx^2\right)^{3/2}}{d\left(d+ex+fx^2\right)}\right)dx$$

$$= \frac{\int \frac{\left(a+cx^2\right)^{3/2}}{x}dx}{d}+\frac{\int \frac{(-e-fx)\left(a+cx^2\right)^{3/2}}{d+ex+fx^2}dx}{d}$$

$$= -\frac{\left(a+cx^2\right)^{3/2}}{3d}+\frac{\text{Subst}\left(\int \frac{(a+cx)^{3/2}}{x}dx,x,x^2\right)}{2d}+\frac{\int \frac{\left(-3aef+3f(cd-af)x\right)\sqrt{a+cx^2}}{d+ex+fx^2}dx}{3df}$$

$$= \frac{(cd-af)\sqrt{a+cx^2}}{df}+\frac{a\,\text{Subst}\left(\int \frac{\sqrt{a+cx}}{x}dx,x,x^2\right)}{2d}+\frac{\int \frac{-3a^2ef^2-3f(cd-af)^2x-3c^2defx^2}{\sqrt{a+cx^2}\left(d+ex+fx^2\right)}dx}{3df^2}$$

$$= \frac{a\sqrt{a+cx^2}}{d}+\frac{(cd-af)\sqrt{a+cx^2}}{df}+\frac{a^2\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+cx}}dx,x,x^2\right)}{2d}+\frac{\int \frac{3c^2d^2ef-3a^2ef^3+(3c^2}{\sqrt{a+cx^2}\,(d}}{3d}$$

$$= \frac{a\sqrt{a+cx^2}}{d}+\frac{(cd-af)\sqrt{a+cx^2}}{df}+\frac{a^2\,\text{Subst}\left(\int \frac{1}{-\frac{a}{c}+\frac{x^2}{c}}dx,x,\sqrt{a+cx^2}\right)}{cd}-\frac{\left(c^2e\right)\text{Subst}(}{}$$

$$= \frac{a\sqrt{a+cx^2}}{d}+\frac{(cd-af)\sqrt{a+cx^2}}{df}-\frac{c^{3/2}e\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{f^2}-\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d}-\frac{(2e}{}$$

$$= \frac{a\sqrt{a+cx^2}}{d}+\frac{(cd-af)\sqrt{a+cx^2}}{df}-\frac{c^{3/2}e\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{f^2}-\frac{\left(2ef\left(c^2d^2-a^2f^2\right)-\left(c^2de\right.\right.}{\sqrt{}}$$

Mathematica [A] time = 1.58, size = 746, normalized size = 1.50

$$-\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d}-\frac{c^{3/2}e\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{f^2}+\frac{a\sqrt{af^2+\frac{1}{2}c\left(e\sqrt{e^2-4df}-2df+e^2\right)}\tanh^{-1}\left(\frac{2af-cx\left(\sqrt{e^2-}\right.}{\sqrt{a+cx^2}\sqrt{4af^2+2c\left(e\sqrt{}\right.}}\right.}{2df}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)^(3/2)/(x*(d + e*x + f*x^2)),x]
```

```
[Out] (c*Sqrt[a + c*x^2])/f - (c^(3/2)*e*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/f^
2 - ((c*d*(-e + Sqrt[e^2 - 4*d*f]) - a*f*(e + Sqrt[e^2 - 4*d*f]))*Sqrt[4*a*
f^2 - 2*c*(-e^2 + 2*d*f + e*Sqrt[e^2 - 4*d*f])]*ArcTanh[(2*a*f + c*(-e + Sq
rt[e^2 - 4*d*f])*x)/(Sqrt[4*a*f^2 - 2*c*(-e^2 + 2*d*f + e*Sqrt[e^2 - 4*d*f]
)]*Sqrt[a + c*x^2])])/(4*d*f^2*Sqrt[e^2 - 4*d*f]) - (c*Sqrt[a*f^2 + (c*(e^2
 - 2*d*f + e*Sqrt[e^2 - 4*d*f]))/2]*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*
f])*x)/(Sqrt[4*a*f^2 + 2*c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*
x^2])])/(2*f^2) + (a*Sqrt[a*f^2 + (c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]))/2
]*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[4*a*f^2 + 2*c*(e^2 -
2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(2*d*f) - (c*e*Sqrt[4*a*f^
```

```
2 + 2*c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*ArcTanh[(2*a*f - c*(e + Sqrt[e
^2 - 4*d*f])*x)/(Sqrt[4*a*f^2 + 2*c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sq
rt[a + c*x^2])])/(4*f^2*Sqrt[e^2 - 4*d*f]) - (a*e*Sqrt[4*a*f^2 + 2*c*(e^2 -
 2*d*f + e*Sqrt[e^2 - 4*d*f])]*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x
)/(Sqrt[4*a*f^2 + 2*c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])
])/(4*d*f*Sqrt[e^2 - 4*d*f]) - (a^(3/2)*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])/d
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)/x/(f*x^2+e*x+d),x, algorithm="fricas")
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)/x/(f*x^2+e*x+d),x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [B] time = 0.02, size = 9728, normalized size = 19.61

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^(3/2)/x/(f*x^2+e*x+d),x)
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^2+a\right)^{\frac{3}{2}}}{\left(fx^2+ex+d\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)/x/(f*x^2+e*x+d),x, algorithm="maxima")
[Out] integrate((c*x^2 + a)^(3/2)/((f*x^2 + e*x + d)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(c\,x^2+a\right)}^{3/2}}{x\,\left(f\,x^2+e\,x+d\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^2)^(3/2)/(x*(d + e*x + f*x^2)),x)
[Out] int((a + c*x^2)^(3/2)/(x*(d + e*x + f*x^2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+c x^2\right)^{\frac{3}{2}}}{x\left(d+e x+f x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**(3/2)/x/(f*x**2+e*x+d),x)
```

```
[Out] Integral((a + c*x**2)**(3/2)/(x*(d + e*x + f*x**2)), x)
```

3.62 $\int \frac{\left(a+cx^2\right)^{3/2}}{x^2\left(d+ex+fx^2\right)}\,dx$

Optimal. Leaf size=604

$$\frac{a^{3/2}e\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d^2}-\frac{\left(a^2f^2\left(e\sqrt{e^2-4df}-2df+e^2\right)+4acd^2f^2+c^2d^2\left(-e\sqrt{e^2-4df}-2df+e^2\right)\right)\tanh^{-1}\left(\right.}{\sqrt{2}\,d^2f\sqrt{e^2-4df}\,\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e\right.}}$$

```
[Out] -(c*x^2+a)^(3/2)/d/x+a^(3/2)*e*arctanh((c*x^2+a)^(1/2)/a^(1/2))/d^2+3/2*a*a
rctanh(x*c^(1/2)/(c*x^2+a)^(1/2))*c^(1/2)/d+1/2*(-3*a*f+2*c*d)*arctanh(x*c^
(1/2)/(c*x^2+a)^(1/2))*c^(1/2)/d/f-a*e*(c*x^2+a)^(1/2)/d^2+3/2*c*x*(c*x^2+a
)^(1/2)/d+1/2*(-c*d*x+2*a*e)*(c*x^2+a)^(1/2)/d^2-1/2*arctanh(1/2*(2*a*f-c*x
*(e-(-4*d*f+e^2)^(1/2)))*2^(1/2)/(c*x^2+a)^(1/2)/(2*a*f^2+(-(-4*d*f+e^2)^(1
/2)*e-2*d*f+e^2)*c)^(1/2))*(4*a*c*d^2*f^2+c^2*d^2*(-(-4*d*f+e^2)^(1/2)*e-2*
d*f+e^2)+a^2*f^2*((-4*d*f+e^2)^(1/2)*e-2*d*f+e^2))/d^2/f*2^(1/2)/(-4*d*f+e^
2)^(1/2)/(2*a*f^2+(-(-4*d*f+e^2)^(1/2)*e-2*d*f+e^2)*c)^(1/2)+1/2*arctanh(1/
2*(2*a*f-c*x*(e+(-4*d*f+e^2)^(1/2)))*2^(1/2)/(c*x^2+a)^(1/2)/(2*a*f^2+((-4*
d*f+e^2)^(1/2)*e-2*d*f+e^2)*c)^(1/2))*(4*a*c*d^2*f^2+a^2*f^2*(-(-4*d*f+e^2)
^(1/2)*e-2*d*f+e^2)+c^2*d^2*((-4*d*f+e^2)^(1/2)*e-2*d*f+e^2))/d^2/f*2^(1/2)
/(-4*d*f+e^2)^(1/2)/(2*a*f^2+((-4*d*f+e^2)^(1/2)*e-2*d*f+e^2)*c)^(1/2)
```

Rubi [A] time = 2.81, antiderivative size = 604, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 14, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.518, Rules used = {6728, 277, 195, 217, 206, 266, 50, 63, 208, 1020, 1068, 1080, 1034, 725}

$$-\frac{\left(a^2f^2\left(e\sqrt{e^2-4df}-2df+e^2\right)+4acd^2f^2+c^2d^2\left(-e\sqrt{e^2-4df}-2df+e^2\right)\right)\tanh^{-1}\left(\frac{2af-cx\left(e-\sqrt{e^2-4\right.}}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2\right.}}}\right.}{\sqrt{2}\,d^2f\sqrt{e^2-4df}\,\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)^(3/2)/(x^2*(d + e*x + f*x^2)),x]
```

```
[Out] -((a*e*Sqrt[a + c*x^2])/d^2) + (3*c*x*Sqrt[a + c*x^2])/(2*d) + ((2*a*e - c*
d*x)*Sqrt[a + c*x^2])/(2*d^2) - (a + c*x^2)^(3/2)/(d*x) + (3*a*Sqrt[c]*ArcT
anh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(2*d) + (Sqrt[c]*(2*c*d - 3*a*f)*ArcTanh[
(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(2*d*f) - ((4*a*c*d^2*f^2 + c^2*d^2*(e^2 - 2*
d*f - e*Sqrt[e^2 - 4*d*f]) + a^2*f^2*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]))*A
rcTanh[(2*a*f - c*(e - Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2
 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[2]*d^2*f*Sqrt[e^2
 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]) + ((4*a*c*
d^2*f^2 + a^2*f^2*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + c^2*d^2*(e^2 - 2*d*
f + e*Sqrt[e^2 - 4*d*f]))*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sq
rt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2]
)])/(Sqrt[2]*d^2*f*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt
[e^2 - 4*d*f])]) + (a^(3/2)*e*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])/d^2
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 1020

```
Int[((g_.) + (h_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f
_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(h*(a + c*x^2)^p*(d + e*x + f*x^2)^(q +
 1))/(2*f*(p + q + 1)), x] + Dist[1/(2*f*(p + q + 1)), Int[(a + c*x^2)^(p -
 1)*(d + e*x + f*x^2)^q*Simp[a*h*e*p - a*(h*e - 2*g*f)*(p + q + 1) - 2*h*p*
(c*d - a*f)*x - (h*c*e*p + c*(h*e - 2*g*f)*(p + q + 1))*x^2, x], x], x] /;
FreeQ[{a, c, d, e, f, g, h, q}, x] && NeQ[e^2 - 4*d*f, 0] && GtQ[p, 0] && N
```

```
eQ[p + q + 1, 0]
```

Rule 1034

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f
_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(
b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Dist[(2*c*g -
h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a,
 b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1068

```
Int[((a_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)*((d_) +
(e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((B*c*f*(2*p + 2*q + 3)
+ C*(-(c*e*(2*p + q + 2))) + 2*c*C*f*(p + q + 1)*x)*(a + c*x^2)^p*(d + e*x
+ f*x^2)^(q + 1))/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3)), x] - Dist[1/(2*c*f
^2*(p + q + 1)*(2*p + 2*q + 3)), Int[(a + c*x^2)^(p - 1)*(d + e*x + f*x^2)^
q*Simp[p*(-(a*e))*(C*(c*e)*(q + 1) - c*(C*e - B*f)*(2*p + 2*q + 3)) + (p +
q + 1)*(a*c*(C*(2*d*f - e^2*(2*p + q + 2)) + f*(B*e - 2*A*f)*(2*p + 2*q + 3
))) + (2*p*(c*d - a*f)*(C*(c*e)*(q + 1) - c*(C*e - B*f)*(2*p + 2*q + 3)) +
(p + q + 1)*(C*e*f*p*(-4*a*c)))*x + (p*(c*e)*(C*(c*e)*(q + 1) - c*(C*e - B*
f)*(2*p + 2*q + 3)) + (p + q + 1)*(C*f^2*p*(-4*a*c) - c^2*(C*(e^2 - 4*d*f)*
(2*p + q + 2) + f*(2*C*d - B*e + 2*A*f)*(2*p + 2*q + 3))))*x^2, x], x], x]
/; FreeQ[{a, c, d, e, f, A, B, C, q}, x] && NeQ[e^2 - 4*d*f, 0] && GtQ[p, 0
] && NeQ[p + q + 1, 0] && NeQ[2*p + 2*q + 3, 0] &&  !IGtQ[p, 0] &&  !IGtQ[q
, 0]
```

Rule 1080

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*Sqrt[(d_.) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqrt[d + f*x^2],
 x], x] + Dist[1/c, Int[(A*c - a*C + (B*c - b*C)*x)/((a + b*x + c*x^2)*Sqrt
[d + f*x^2]), x], x] /; FreeQ[{a, b, c, d, f, A, B, C}, x] && NeQ[b^2 - 4*a
*c, 0]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\left(a+cx^2\right)^{3/2}}{x^2\left(d+ex+fx^2\right)}\,dx=\int \left(\frac{\left(a+cx^2\right)^{3/2}}{dx^2}-\frac{e\left(a+cx^2\right)^{3/2}}{d^2x}+\frac{\left(e^2-df+efx\right)\left(a+cx^2\right)^{3/2}}{d^2\left(d+ex+fx^2\right)}\right)dx$$

$$=\frac{\int \frac{\left(e^2-df+efx\right)\left(a+cx^2\right)^{3/2}}{d+ex+fx^2}\,dx}{d^2}+\frac{\int \frac{\left(a+cx^2\right)^{3/2}}{x^2}\,dx}{d}-\frac{e\int \frac{\left(a+cx^2\right)^{3/2}}{x}\,dx}{d^2}$$

$$=\frac{e\left(a+cx^2\right)^{3/2}}{3d^2}-\frac{\left(a+cx^2\right)^{3/2}}{dx}+\frac{(3c)\int \sqrt{a+cx^2}\,dx}{d}-\frac{e\,\text{Subst}\left(\int \frac{(a+cx)^{3/2}}{x}\,dx,x,x^2\right)}{2d^2}+$$

$$=\frac{3cx\sqrt{a+cx^2}}{2d}+\frac{(2ae-cdx)\sqrt{a+cx^2}}{2d^2}-\frac{\left(a+cx^2\right)^{3/2}}{dx}+\frac{(3ac)\int \frac{1}{\sqrt{a+cx^2}}\,dx}{2d}-\frac{(ae)\,\text{Subst}}{}$$

$$=-\frac{ae\sqrt{a+cx^2}}{d^2}+\frac{3cx\sqrt{a+cx^2}}{2d}+\frac{(2ae-cdx)\sqrt{a+cx^2}}{2d^2}-\frac{\left(a+cx^2\right)^{3/2}}{dx}+\frac{(3ac)\,\text{Subst}\left(\int\right.}{}$$

$$=-\frac{ae\sqrt{a+cx^2}}{d^2}+\frac{3cx\sqrt{a+cx^2}}{2d}+\frac{(2ae-cdx)\sqrt{a+cx^2}}{2d^2}-\frac{\left(a+cx^2\right)^{3/2}}{dx}+\frac{3a\sqrt{c}\,\tanh^{-1}\left(\right.}{2d}$$

$$=-\frac{ae\sqrt{a+cx^2}}{d^2}+\frac{3cx\sqrt{a+cx^2}}{2d}+\frac{(2ae-cdx)\sqrt{a+cx^2}}{2d^2}-\frac{\left(a+cx^2\right)^{3/2}}{dx}+\frac{3a\sqrt{c}\,\tanh^{-1}\left(\right.}{2d}$$

$$=-\frac{ae\sqrt{a+cx^2}}{d^2}+\frac{3cx\sqrt{a+cx^2}}{2d}+\frac{(2ae-cdx)\sqrt{a+cx^2}}{2d^2}-\frac{\left(a+cx^2\right)^{3/2}}{dx}+\frac{3a\sqrt{c}\,\tanh^{-1}\left(\right.}{2d}$$

Mathematica [C] time = 4.16, size = 885, normalized size = 1.47

$$\frac{-x\left(2\sqrt{a}\sqrt{c}\,df\sqrt{e^2-4df}\,\sqrt{cx^2+a}\,\sinh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)+\sqrt{\frac{cx^2}{a}+1}\left(-2c\sqrt{4af^2+2c\left(e^2+\sqrt{e^2-4df}\,e-2df\right)}\,\tanh^{-1}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)^(3/2)/(x^2*(d + e*x + f*x^2)),x]
```

```
[Out] (-(x*(2*Sqrt[a]*Sqrt[c]*d*f*Sqrt[e^2 - 4*d*f]*Sqrt[a + c*x^2]*ArcSinh[(Sqrt
[c]*x)/Sqrt[a]] + Sqrt[1 + (c*x^2)/a]*(2*c*d*f*Sqrt[e^2 - 4*d*f]*x*Sqrt[a +
 c*x^2] - 4*Sqrt[c]*d*(c*d - a*f)*Sqrt[e^2 - 4*d*f]*ArcTanh[(Sqrt[c]*x)/Sqr
t[a + c*x^2]] + (2*c*d^2 + a*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]))*Sqrt[4*a*
f^2 - 2*c*(-e^2 + 2*d*f + e*Sqrt[e^2 - 4*d*f])]*ArcTanh[(2*a*f + c*(-e + Sq
rt[e^2 - 4*d*f])*x)/(Sqrt[4*a*f^2 - 2*c*(-e^2 + 2*d*f + e*Sqrt[e^2 - 4*d*f]
)]*Sqrt[a + c*x^2])] + Sqrt[2]*a*e*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2
- 2*d*f + e*Sqrt[e^2 - 4*d*f])]*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*
x)/(Sqrt[4*a*f^2 + 2*c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2]
)] - 2*c*d^2*Sqrt[4*a*f^2 + 2*c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*ArcTan
h[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[4*a*f^2 + 2*c*(e^2 - 2*d*f +
e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])] - a*e^2*Sqrt[4*a*f^2 + 2*c*(e^2 - 2
*d*f + e*Sqrt[e^2 - 4*d*f])]*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/
(Sqrt[4*a*f^2 + 2*c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])]
+ 2*a*d*f*Sqrt[4*a*f^2 + 2*c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*ArcTanh[(
```

```
2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[4*a*f^2 + 2*c*(e^2 - 2*d*f + e*S
qrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])] - 4*a^(3/2)*e*f*Sqrt[e^2 - 4*d*f]*ArcT
anh[Sqrt[a + c*x^2]/Sqrt[a]]))) - 4*a*d*f*Sqrt[e^2 - 4*d*f]*Sqrt[a + c*x^2]
*Hypergeometric2F1[-3/2, -1/2, 1/2, -((c*x^2)/a)])/(4*d^2*f*Sqrt[e^2 - 4*d*
f]*x*Sqrt[1 + (c*x^2)/a])
```

fricas [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)/x^2/(f*x^2+e*x+d),x, algorithm="fricas")
[Out] Timed out
```

giac [**F**] time = 0.00, size = 0, normalized size = 0.00

$$sage_2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)/x^2/(f*x^2+e*x+d),x, algorithm="giac")
[Out] sage2
```

maple [**B**] time = 0.02, size = 9912, normalized size = 16.41

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^(3/2)/x^2/(f*x^2+e*x+d),x)
[Out] result too large to display
```

maxima [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^2+a\right)^{\frac{3}{2}}}{\left(fx^2+ex+d\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)/x^2/(f*x^2+e*x+d),x, algorithm="maxima")
[Out] integrate((c*x^2 + a)^(3/2)/((f*x^2 + e*x + d)*x^2), x)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,x^2+a\right)^{3/2}}{x^2\,\left(f\,x^2+e\,x+d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^2)^(3/2)/(x^2*(d + e*x + f*x^2)),x)
[Out] int((a + c*x^2)^(3/2)/(x^2*(d + e*x + f*x^2)), x)
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+cx^2\right)^{\frac{3}{2}}}{x^2\left(d+ex+fx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**(3/2)/x**2/(f*x**2+e*x+d),x)
```

```
[Out] Integral((a + c*x**2)**(3/2)/(x**2*(d + e*x + f*x**2)), x)
```

3.63 $\int \frac{\left(a+cx^2\right)^{3/2}}{x^3\left(d+ex+fx^2\right)}\,dx$

Optimal. Leaf size=668

$$-\frac{a^{3/2}\left(e^2-df\right)\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d^3}+\frac{\left(a^2f\left(e^2\sqrt{e^2-4df}-df\sqrt{e^2-4df}-3def+e^3\right)+2acd^2f\left(\sqrt{e^2-4df}+e\right)}{\sqrt{2}\,d^3\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\right.}}$$

```
[Out] -1/2*(c*x^2+a)^(3/2)/d/x^2+e*(c*x^2+a)^(3/2)/d^2/x-a^(3/2)*(-d*f+e^2)*arcta
nh((c*x^2+a)^(1/2)/a^(1/2))/d^3-3/2*c*arctanh((c*x^2+a)^(1/2)/a^(1/2))*a^(1
/2)/d+3/2*c*(c*x^2+a)^(1/2)/d+a*(-d*f+e^2)*(c*x^2+a)^(1/2)/d^3-3/2*c*e*x*(c
*x^2+a)^(1/2)/d^2-1/2*(2*c*d^2+2*a*(-d*f+e^2)-c*d*e*x)*(c*x^2+a)^(1/2)/d^3+
1/2*arctanh(1/2*(2*a*f-c*x*(e-(-4*d*f+e^2)^(1/2)))*2^(1/2)/(c*x^2+a)^(1/2)/
(2*a*f^2+(-(-4*d*f+e^2)^(1/2)*e-2*d*f+e^2)*c)^(1/2))*(c^2*d^3*(e-(-4*d*f+e^
2)^(1/2))+2*a*c*d^2*f*(e+(-4*d*f+e^2)^(1/2))+a^2*f*(e^3-3*d*e*f+e^2*(-4*d*f
+e^2)^(1/2)-d*f*(-4*d*f+e^2)^(1/2)))/d^3*2^(1/2)/(-4*d*f+e^2)^(1/2)/(2*a*f^
2+(-(-4*d*f+e^2)^(1/2)*e-2*d*f+e^2)*c)^(1/2)-1/2*arctanh(1/2*(2*a*f-c*x*(e+
(-4*d*f+e^2)^(1/2)))*2^(1/2)/(c*x^2+a)^(1/2)/(2*a*f^2+((-4*d*f+e^2)^(1/2)*e
-2*d*f+e^2)*c)^(1/2))*(2*a*c*d^2*f*(e-(-4*d*f+e^2)^(1/2))+c^2*d^3*(e+(-4*d*
f+e^2)^(1/2))+a^2*f*(e^3-3*d*e*f-e^2*(-4*d*f+e^2)^(1/2)+d*f*(-4*d*f+e^2)^(1
/2)))/d^3*2^(1/2)/(-4*d*f+e^2)^(1/2)/(2*a*f^2+((-4*d*f+e^2)^(1/2)*e-2*d*f+e
^2)*c)^(1/2)
```

Rubi [A] time = 3.46, antiderivative size = 668, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 15, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {6728, 266, 47, 50, 63, 208, 277, 195, 217, 206, 1020, 1068, 1080, 1034, 725}

$$\frac{\left(a^2f\left(e^2\sqrt{e^2-4df}-df\sqrt{e^2-4df}-3def+e^3\right)+2acd^2f\left(\sqrt{e^2-4df}+e\right)+c^2d^3\left(e-\sqrt{e^2-4df}\right)\right)\tanh^{-1}\left(\right.}{\sqrt{2}\,d^3\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)^(3/2)/(x^3*(d + e*x + f*x^2)),x]
```

```
[Out] (3*c*Sqrt[a + c*x^2])/(2*d) + (a*(e^2 - d*f)*Sqrt[a + c*x^2])/d^3 - (3*c*e*
x*Sqrt[a + c*x^2])/(2*d^2) - ((2*(c*d^2 + a*(e^2 - d*f)) - c*d*e*x)*Sqrt[a
+ c*x^2])/(2*d^3) - (a + c*x^2)^(3/2)/(2*d*x^2) + (e*(a + c*x^2)^(3/2))/(d^
2*x) + ((c^2*d^3*(e - Sqrt[e^2 - 4*d*f]) + 2*a*c*d^2*f*(e + Sqrt[e^2 - 4*d*
f]) + a^2*f*(e^3 - 3*d*e*f + e^2*Sqrt[e^2 - 4*d*f] - d*f*Sqrt[e^2 - 4*d*f])
)*ArcTanh[(2*a*f - c*(e - Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(
e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[2]*d^3*Sqrt[e^
2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]) - ((2*a*c
*d^2*f*(e - Sqrt[e^2 - 4*d*f]) + c^2*d^3*(e + Sqrt[e^2 - 4*d*f]) + a^2*f*(e
^3 - 3*d*e*f - e^2*Sqrt[e^2 - 4*d*f] + d*f*Sqrt[e^2 - 4*d*f]))*ArcTanh[(2*a
*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f +
e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[2]*d^3*Sqrt[e^2 - 4*d*f]*Sqr
t[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]) - (3*Sqrt[a]*c*ArcTanh[
Sqrt[a + c*x^2]/Sqrt[a]])/(2*d) - (a^(3/2)*(e^2 - d*f)*ArcTanh[Sqrt[a + c*x
^2]/Sqrt[a]])/d^3
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
```

```
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
```

```
nomialQ[a, b, c, n, m, p, x]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 1020

```
Int[((g_.) + (h_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f
_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(h*(a + c*x^2)^p*(d + e*x + f*x^2)^(q +
 1))/(2*f*(p + q + 1)), x] + Dist[1/(2*f*(p + q + 1)), Int[(a + c*x^2)^(p -
 1)*(d + e*x + f*x^2)^q*Simp[a*h*e*p - a*(h*e - 2*g*f)*(p + q + 1) - 2*h*p*
(c*d - a*f)*x - (h*c*e*p + c*(h*e - 2*g*f)*(p + q + 1))*x^2, x], x], x] /;
FreeQ[{a, c, d, e, f, g, h, q}, x] && NeQ[e^2 - 4*d*f, 0] && GtQ[p, 0] && N
eQ[p + q + 1, 0]
```

Rule 1034

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f
_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(
b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Dist[(2*c*g -
h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a,
 b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1068

```
Int[((a_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)*((d_) +
(e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((B*c*f*(2*p + 2*q + 3)
+ C*(-(c*e*(2*p + q + 2))) + 2*c*C*f*(p + q + 1)*x)*(a + c*x^2)^p*(d + e*x
+ f*x^2)^(q + 1))/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3)), x] - Dist[1/(2*c*f
^2*(p + q + 1)*(2*p + 2*q + 3)), Int[(a + c*x^2)^(p - 1)*(d + e*x + f*x^2)^
q*Simp[p*(-(a*e))*(C*(c*e)*(q + 1) - c*(C*e - B*f)*(2*p + 2*q + 3)) + (p +
q + 1)*(a*c*(C*(2*d*f - e^2*(2*p + q + 2)) + f*(B*e - 2*A*f)*(2*p + 2*q + 3
))) + (2*p*(c*d - a*f)*(C*(c*e)*(q + 1) - c*(C*e - B*f)*(2*p + 2*q + 3)) +
(p + q + 1)*(C*e*f*p*(-4*a*c)))*x + (p*(c*e)*(C*(c*e)*(q + 1) - c*(C*e - B*
f)*(2*p + 2*q + 3)) + (p + q + 1)*(C*f^2*p*(-4*a*c) - c^2*(C*(e^2 - 4*d*f)*
(2*p + q + 2) + f*(2*C*d - B*e + 2*A*f)*(2*p + 2*q + 3))))*x^2, x], x], x]
/; FreeQ[{a, c, d, e, f, A, B, C, q}, x] && NeQ[e^2 - 4*d*f, 0] && GtQ[p, 0
] && NeQ[p + q + 1, 0] && NeQ[2*p + 2*q + 3, 0] &&  !IGtQ[p, 0] &&  !IGtQ[q
, 0]
```

Rule 1080

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*Sqrt[(d_.) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqrt[d + f*x^2],
 x], x] + Dist[1/c, Int[(A*c - a*C + (B*c - b*C)*x)/((a + b*x + c*x^2)*Sqrt
[d + f*x^2]), x], x] /; FreeQ[{a, b, c, d, f, A, B, C}, x] && NeQ[b^2 - 4*a
*c, 0]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{(a+cx^2)^{3/2}}{x^3(d+ex+fx^2)}\,dx = \int\left(\frac{(a+cx^2)^{3/2}}{dx^3} - \frac{e(a+cx^2)^{3/2}}{d^2x^2} + \frac{(e^2-df)(a+cx^2)^{3/2}}{d^3x} + \frac{(-e(e^2-2df)-f(e^2-d\ldots}{d^3(d+ex+\ldots}\right.$$

$$= \frac{\int \frac{(-e(e^2-2df)-f(e^2-df)x)(a+cx^2)^{3/2}}{d+ex+fx^2}dx}{d^3} + \frac{\int \frac{(a+cx^2)^{3/2}}{x^3}dx}{d} - \frac{e\int \frac{(a+cx^2)^{3/2}}{x^2}dx}{d^2} + \frac{(e^2-df)\int \frac{(a+\ldots}{}}{d^3}$$

$$= -\frac{(e^2-df)(a+cx^2)^{3/2}}{3d^3} + \frac{e(a+cx^2)^{3/2}}{d^2x} + \frac{\text{Subst}\left(\int \frac{(a+cx)^{3/2}}{x^2}dx, x, x^2\right)}{2d} - \frac{(3ce)\int\sqrt{a+\ldots}}{d^2}$$

$$= -\frac{3cex\sqrt{a+cx^2}}{2d^2} - \frac{(2(cd^2+a(e^2-df))-cdex)\sqrt{a+cx^2}}{2d^3} - \frac{(a+cx^2)^{3/2}}{2dx^2} + \frac{e(a+cx^2\ldots}{d^2x}$$

$$= \frac{3c\sqrt{a+cx^2}}{2d} + \frac{a(e^2-df)\sqrt{a+cx^2}}{d^3} - \frac{3cex\sqrt{a+cx^2}}{2d^2} - \frac{(2(cd^2+a(e^2-df))-cdex)\ldots}{2d^3}$$

$$= \frac{3c\sqrt{a+cx^2}}{2d} + \frac{a(e^2-df)\sqrt{a+cx^2}}{d^3} - \frac{3cex\sqrt{a+cx^2}}{2d^2} - \frac{(2(cd^2+a(e^2-df))-cdex)\ldots}{2d^3}$$

$$= \frac{3c\sqrt{a+cx^2}}{2d} + \frac{a(e^2-df)\sqrt{a+cx^2}}{d^3} - \frac{3cex\sqrt{a+cx^2}}{2d^2} - \frac{(2(cd^2+a(e^2-df))-cdex)\ldots}{2d^3}$$

$$= \frac{3c\sqrt{a+cx^2}}{2d} + \frac{a(e^2-df)\sqrt{a+cx^2}}{d^3} - \frac{3cex\sqrt{a+cx^2}}{2d^2} - \frac{(2(cd^2+a(e^2-df))-cdex)\ldots}{2d^3}$$

Mathematica [**C**] time = 3.22, size = 904, normalized size = 1.35

$$\frac{\frac{6cd^2\,{}_2F_1\left(2,\frac{5}{2};\frac{7}{2};\frac{cx^2}{a}+1\right)(cx^2+a)^{5/2}}{a^2} - 5\left(e^2-\frac{(e^2-3df)e}{\sqrt{e^2-4df}}-df\right)(cx^2+a)^{3/2} - 5\left(e^2+\frac{(e^2-3df)e}{\sqrt{e^2-4df}}-df\right)(cx^2+a)^{3/2} + \frac{30ade\,{}_2F_1\left(-\frac{3}{2}\ldots\right)}{x}\ldots}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)^(3/2)/(x^3*(d + e*x + f*x^2)),x]
```

```
[Out] (-5*(e^2 - d*f - (e*(e^2 - 3*d*f))/Sqrt[e^2 - 4*d*f])*(a + c*x^2)^(3/2) - 5
*(e^2 - d*f + (e*(e^2 - 3*d*f))/Sqrt[e^2 - 4*d*f])*(a + c*x^2)^(3/2) + (15*
(-e^2 + d*f - (e*(e^2 - 3*d*f))/Sqrt[e^2 - 4*d*f])*((2*Sqrt[c]*(-e + Sqrt[e
^2 - 4*d*f])*Sqrt[a + c*x^2]*(Sqrt[c]*x*Sqrt[1 + (c*x^2)/a] + Sqrt[a]*ArcSi
nh[(Sqrt[c]*x)/Sqrt[a]]))/Sqrt[1 + (c*x^2)/a] + (2*(2*a*f^2 + c*(e^2 - 2*d*
f - e*Sqrt[e^2 - 4*d*f]))*(2*f*Sqrt[a + c*x^2] + Sqrt[c]*(-e + Sqrt[e^2 - 4
*d*f])*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]] - Sqrt[2*c*e^2 - 4*c*d*f + 4*a*
f^2 - 2*c*e*Sqrt[e^2 - 4*d*f]]*ArcTanh[(2*a*f + c*(-e + Sqrt[e^2 - 4*d*f])*
x)/(Sqrt[4*a*f^2 - 2*c*(-e^2 + 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2
])]))/f^2))/(8*f) - (15*(-e^2 + d*f + (e*(e^2 - 3*d*f))/Sqrt[e^2 - 4*d*f])*
```

```
((2*Sqrt[c]*(e + Sqrt[e^2 - 4*d*f])*Sqrt[a + c*x^2]*(Sqrt[c]*x*Sqrt[1 + (c*
x^2)/a] + Sqrt[a]*ArcSinh[(Sqrt[c]*x)/Sqrt[a]]))/Sqrt[1 + (c*x^2)/a] + (2*(
2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]))*(-2*f*Sqrt[a + c*x^2] + Sq
rt[c]*(e + Sqrt[e^2 - 4*d*f])*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]] + Sqrt[4
*a*f^2 + 2*c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*ArcTanh[(2*a*f - c*(e + S
qrt[e^2 - 4*d*f])*x)/(Sqrt[4*a*f^2 + 2*c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]
)]*Sqrt[a + c*x^2])]))/f^2))/(8*f) + 10*(e^2 - d*f)*(Sqrt[a + c*x^2]*(4*a +
 c*x^2) - 3*a^(3/2)*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]]) + (30*a*d*e*Sqrt[a +
c*x^2]*Hypergeometric2F1[-3/2, -1/2, 1/2, -((c*x^2)/a)])/(x*Sqrt[1 + (c*x^2
)/a]) + (6*c*d^2*(a + c*x^2)^(5/2)*Hypergeometric2F1[2, 5/2, 7/2, 1 + (c*x^
2)/a])/a^2)/(30*d^3)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)/x^3/(f*x^2+e*x+d),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)/x^3/(f*x^2+e*x+d),x, algorithm="giac")
[Out] sage2
```

maple [B] time = 0.02, size = 10298, normalized size = 15.42

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^(3/2)/x^3/(f*x^2+e*x+d),x)
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^2 + a\right)^{\frac{3}{2}}}{\left(fx^2 + ex + d\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)/x^3/(f*x^2+e*x+d),x, algorithm="maxima")
[Out] integrate((c*x^2 + a)^(3/2)/((f*x^2 + e*x + d)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,x^2+a\right)^{3/2}}{x^3\,\left(f\,x^2+e\,x+d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^2)^(3/2)/(x^3*(d + e*x + f*x^2)),x)
```

```
[Out] int((a + c*x^2)^(3/2)/(x^3*(d + e*x + f*x^2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**(3/2)/x**3/(f*x**2+e*x+d),x)
```

```
[Out] Timed out
```

3.64 $\int \frac{x^3}{\sqrt{a+cx^2}\left(d+ex+fx^2\right)}\,dx$

Optimal. Leaf size=380

$$-\frac{\left(2def-\left(e^2-df\right)\left(e-\sqrt{e^2-4df}\right)\right)\tanh^{-1}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}f^2\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}+\frac{\left(2def-\left(e^2-df\right)\left(\sqrt{e^2-\right.\right.}{\sqrt{2}f^2\sqrt{e^2-}}$$

```
[Out] -e*arctanh(x*c^(1/2)/(c*x^2+a)^(1/2))/f^2/c^(1/2)+(c*x^2+a)^(1/2)/c/f-1/2*a
rctanh(1/2*(2*a*f-c*x*(e-(-4*d*f+e^2)^(1/2)))*2^(1/2)/(c*x^2+a)^(1/2)/(2*a*
f^2+(-(-4*d*f+e^2)^(1/2)*e-2*d*f+e^2)*c)^(1/2))*(2*d*e*f-(-d*f+e^2)*(e-(-4*
d*f+e^2)^(1/2)))/f^2*2^(1/2)/(-4*d*f+e^2)^(1/2)/(2*a*f^2+(-(-4*d*f+e^2)^(1/
2)*e-2*d*f+e^2)*c)^(1/2)+1/2*arctanh(1/2*(2*a*f-c*x*(e+(-4*d*f+e^2)^(1/2)))
*2^(1/2)/(c*x^2+a)^(1/2)/(2*a*f^2+((-4*d*f+e^2)^(1/2)*e-2*d*f+e^2)*c)^(1/2)
)*(2*d*e*f-(-d*f+e^2)*(e+(-4*d*f+e^2)^(1/2)))/f^2*2^(1/2)/(-4*d*f+e^2)^(1/2
)/(2*a*f^2+((-4*d*f+e^2)^(1/2)*e-2*d*f+e^2)*c)^(1/2)
```

Rubi [A] time = 1.17, antiderivative size = 380, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6728, 217, 206, 261, 1034, 725}

$$-\frac{\left(2def-\left(e^2-df\right)\left(e-\sqrt{e^2-4df}\right)\right)\tanh^{-1}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}f^2\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}+\frac{\left(2def-\left(e^2-df\right)\left(\sqrt{e^2-\right.\right.}{\sqrt{2}f^2\sqrt{e^2-}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(Sqrt[a + c*x^2]*(d + e*x + f*x^2)),x]
```

```
[Out] Sqrt[a + c*x^2]/(c*f) - (e*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(Sqrt[c]*f
^2) - ((2*d*e*f - (e^2 - d*f)*(e - Sqrt[e^2 - 4*d*f]))*ArcTanh[(2*a*f - c*(
e - Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e
^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[2]*f^2*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^
2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]) + ((2*d*e*f - (e^2 - d*f)*(e +
Sqrt[e^2 - 4*d*f]))*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*
Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(S
qrt[2]*f^2*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4
*d*f])])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 1034

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f
_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(
b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Dist[(2*c*g -
h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a,
 b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{x^3}{\sqrt{a+cx^2}\left(d+ex+fx^2\right)}\,dx = \int \left(-\frac{e}{f^2\sqrt{a+cx^2}}+\frac{x}{f\sqrt{a+cx^2}}+\frac{de+\left(e^2-df\right)x}{f^2\sqrt{a+cx^2}\left(d+ex+fx^2\right)}\right)dx$$

$$=\frac{\int \frac{de+\left(e^2-df\right)x}{\sqrt{a+cx^2}\left(d+ex+fx^2\right)}\,dx}{f^2}-\frac{e\int \frac{1}{\sqrt{a+cx^2}}\,dx}{f^2}+\frac{\int \frac{x}{\sqrt{a+cx^2}}\,dx}{f}$$

$$=\frac{\sqrt{a+cx^2}}{cf}-\frac{e\,\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx,x,\frac{x}{\sqrt{a+cx^2}}\right)}{f^2}+\frac{\left(2def-\left(e^2-df\right)\left(e-\sqrt{e^2-4d\right.\right.}{f^2\sqrt{e^2-}}$$

$$=\frac{\sqrt{a+cx^2}}{cf}-\frac{e\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}\,f^2}-\frac{\left(2def-\left(e^2-df\right)\left(e-\sqrt{e^2-4df}\right)\right)\text{Subst}\left(\int\right.}{f^2\sqrt{e^2}}$$

$$=\frac{\sqrt{a+cx^2}}{cf}-\frac{e\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}\,f^2}-\frac{\left(2def-\left(e^2-df\right)\left(e-\sqrt{e^2-4df}\right)\right)\tanh^{-1}\left(\right.}{\sqrt{2}\,f^2\sqrt{e^2-4df}\,\sqrt{2af^2+c\left(e^2-\right.}}$$

Mathematica [A] time = 1.33, size = 378, normalized size = 0.99

$$-\frac{\frac{\sqrt{2}\left(\left(e^2-df\right)\left(\sqrt{e^2-4df}-e\right)+2def\right)\tanh^{-1}\left(\frac{2af+cx\left(\sqrt{e^2-4df}-e\right)}{\sqrt{a+cx^2}\sqrt{4af^2-2c\left(e\sqrt{e^2-4df}+2df-e^2\right)}}\right)}{\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}+\frac{\sqrt{2}\left(e^2\sqrt{e^2-4df}-df\sqrt{e^2-4df}-3def+e^3\right)\tanh^{-1}\left(\frac{2af-c}{\sqrt{a+cx^2}\sqrt{4af^2}}\right.}{\sqrt{e^2-4df}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}}{2f^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(Sqrt[a + c*x^2]*(d + e*x + f*x^2)),x]
```

```
[Out] -1/2*((-2*f*Sqrt[a + c*x^2])/c + (2*e*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])
/Sqrt[c] + (Sqrt[2]*(2*d*e*f + (e^2 - d*f)*(-e + Sqrt[e^2 - 4*d*f]))*ArcTan
h[(2*a*f + c*(-e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[4*a*f^2 - 2*c*(-e^2 + 2*d*f
```

```
+ e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2
+ c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]) + (Sqrt[2]*(e^3 - 3*d*e*f + e^2*S
qrt[e^2 - 4*d*f] - d*f*Sqrt[e^2 - 4*d*f])*ArcTanh[(2*a*f - c*(e + Sqrt[e^2
- 4*d*f])*x)/(Sqrt[4*a*f^2 + 2*c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[
a + c*x^2])])/(Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2
 - 4*d*f])]))/f^2
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 11.34sym2poly/r2sym(const g
en & e,const index_m & i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.03, size = 2397, normalized size = 6.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x)
```

```
[Out] (c*x^2+a)^(1/2)/c/f-1/f^2*e*ln(c^(1/2)*x+(c*x^2+a)^(1/2))/c^(1/2)+1/2/f^2*2
^(1/2)/(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*ln((((-4*
d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c*(e+(-4*d*f+e^2)^(1/2))/f*(x
+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2
-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e+(
-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*((-4*d*f+e^2)^(1/2)
*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))*d
-1/2/f^3*2^(1/2)/(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)
*ln((((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c*(e+(-4*d*f+e^2)^(
1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*(((-4*d*f+e^2)^(1/2)*c
*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*
c-4*c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*((-4*d*f+
e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1
/2))/f))*e^2+3/2/f^2/(-4*d*f+e^2)^(1/2)*2^(1/2)/(((-4*d*f+e^2)^(1/2)*c*e+2*
a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*ln((((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f
+c*e^2)/f^2-c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2
*2^(1/2)*(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x+1
/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4
*d*f+e^2)^(1/2))/f)+2*((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(
1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))*d*e-1/2/f^3/(-4*d*f+e^2)^(1/2)*2^(1
/2)/(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*ln((((-4*d*f
+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/
2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*
c*d*f+c*e^2)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e+(-4*
d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*((-4*d*f+e^2)^(1/2)*c*
```

```
e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))*e^3+
1/2/f^2*2^(1/2)/((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)
*ln(((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c*(e-(-4*d*f+e^2)^
(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*((-(-4*d*f+e^2)^(1/2
)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f
)^2*c-4*c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+2*(-(-
4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))/(x-1/2*(-e+(-4*d*f+
e^2)^(1/2))/f))*d-1/2/f^3*2^(1/2)/((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f
+c*e^2)/f^2)^(1/2)*ln(((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-
c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*((
-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x-1/2*(-e+(-4
*d*f+e^2)^(1/2))/f)^2*c-4*c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2
)^(1/2))/f)+2*(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))/(
x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f))*e^2-3/2/f^2/(-4*d*f+e^2)^(1/2)*2^(1/2)/((
-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*ln(((-(-4*d*f+e^2
)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-
e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*
d*f+c*e^2)/f^2)^(1/2)*(4*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e-(-4*d
*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+2*(-(-4*d*f+e^2)^(1/2)*c
*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))/(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f))*d*
e+1/2/f^3/(-4*d*f+e^2)^(1/2)*2^(1/2)/((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*
d*f+c*e^2)/f^2)^(1/2)*ln(((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f
^2-c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)
*((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x-1/2*(-e+
(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+
e^2)^(1/2))/f)+2*(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)
)/(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f))*e^3
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for mo
re details)Is 4*d*f-e^2 positive, negative or zero?
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3}{\sqrt{c\,x^2+a}\,\left(f\,x^2+e\,x+d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((a + c*x^2)^(1/2)*(d + e*x + f*x^2)),x)
```

```
[Out] int(x^3/((a + c*x^2)^(1/2)*(d + e*x + f*x^2)), x)
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{a+cx^2}\,\left(d+ex+fx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(f*x**2+e*x+d)/(c*x**2+a)**(1/2),x)
```

```
[Out] Integral(x**3/(sqrt(a + c*x**2)*(d + e*x + f*x**2)), x)
```

3.65 $\int \frac{x^2}{\sqrt{a+cx^2}\left(d+ex+fx^2\right)} \, dx$

Optimal. Leaf size=344

$$-\frac{\left(-e\sqrt{e^2-4df}-2df+e^2\right)\tanh^{-1}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}f\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}-\frac{\left(2df-e\left(\sqrt{e^2-4df}+e\right)\right)\tanh^{-1}\left(\cdots\right.}{\sqrt{2}f\sqrt{e^2-4df}\sqrt{2af^2+c\cdots}}$$

```
[Out] arctanh(x*c^(1/2)/(c*x^2+a)^(1/2))/f/c^(1/2)-1/2*arctanh(1/2*(2*a*f-c*x*(e-
(-4*d*f+e^2)^(1/2)))*2^(1/2)/(c*x^2+a)^(1/2)/(2*a*f^2+(-(-4*d*f+e^2)^(1/2)*
e-2*d*f+e^2)*c)^(1/2))*(-(-4*d*f+e^2)^(1/2)*e-2*d*f+e^2)/f*2^(1/2)/(-4*d*f+
e^2)^(1/2)/(2*a*f^2+(-(-4*d*f+e^2)^(1/2)*e-2*d*f+e^2)*c)^(1/2)-1/2*arctanh(
1/2*(2*a*f-c*x*(e+(-4*d*f+e^2)^(1/2)))*2^(1/2)/(c*x^2+a)^(1/2)/(2*a*f^2+((-
4*d*f+e^2)^(1/2)*e-2*d*f+e^2)*c)^(1/2))*(2*d*f-e*(e+(-4*d*f+e^2)^(1/2)))/f*
2^(1/2)/(-4*d*f+e^2)^(1/2)/(2*a*f^2+((-4*d*f+e^2)^(1/2)*e-2*d*f+e^2)*c)^(1/
2)
```

Rubi [A] time = 0.54, antiderivative size = 344, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1081, 217, 206, 1034, 725}

$$-\frac{\left(-e\sqrt{e^2-4df}-2df+e^2\right)\tanh^{-1}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}f\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}-\frac{\left(2df-e\left(\sqrt{e^2-4df}+e\right)\right)\tanh^{-1}\left(\cdots\right.}{\sqrt{2}f\sqrt{e^2-4df}\sqrt{2af^2+c\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(Sqrt[a + c*x^2]*(d + e*x + f*x^2)),x]
```

```
[Out] ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]]/(Sqrt[c]*f) - ((e^2 - 2*d*f - e*Sqrt[e
^2 - 4*d*f])*ArcTanh[(2*a*f - c*(e - Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*
a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[2]*
f*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])])
- ((2*d*f - e*(e + Sqrt[e^2 - 4*d*f]))*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4
*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sq
rt[a + c*x^2])])/(Sqrt[2]*f*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f
 + e*Sqrt[e^2 - 4*d*f])])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 1034

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f
_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(
b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Dist[(2*c*g -
h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a,
 b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1081

```
Int[((A_.) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
 (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqrt[d + f*x^2], x], x] + Dis
t[1/c, Int[(A*c - a*C - b*C*x)/((a + b*x + c*x^2)*Sqrt[d + f*x^2]), x], x]
/; FreeQ[{a, b, c, d, f, A, C}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x^2}{\sqrt{a+cx^2}\left(d+ex+fx^2\right)}\,dx = \frac{\int \frac{1}{\sqrt{a+cx^2}}\,dx}{f} + \frac{\int \frac{-d-ex}{\sqrt{a+cx^2}\left(d+ex+fx^2\right)}\,dx}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x}{\sqrt{a+cx^2}}\right)}{f} + \frac{\left(e^2-2df-e\sqrt{e^2-4df}\right)\int \frac{1}{\left(e-\sqrt{e^2-4df}+2fx\right)\sqrt{a+c}}}{f\sqrt{e^2-4df}}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}\,f} - \frac{\left(e^2-2df-e\sqrt{e^2-4df}\right)\text{Subst}\left(\int \frac{1}{4af^2+c\left(e-\sqrt{e^2-4df}\right)^2-x^2}\,dx,\right.}{f\sqrt{e^2-4df}}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}\,f} - \frac{\left(e^2-2df-e\sqrt{e^2-4df}\right)\tanh^{-1}\left(\frac{2af-c\left(e-\sqrt{e^2-4df}\right)x}{\sqrt{2}\sqrt{2af^2+c\left(e^2-2df-e\sqrt{e^2-4df}\right.}}\right.}{\sqrt{2}\,f\sqrt{e^2-4df}\sqrt{2af^2+c\left(e^2-2df-e\sqrt{e^2-4df}\right.}}$$

Mathematica [A] time = 0.70, size = 334, normalized size = 0.97

$$\frac{\frac{\sqrt{2}\left(e\sqrt{e^2-4df}+2df-e^2\right)\tanh^{-1}\left(\frac{2af+cx\left(\sqrt{e^2-4df}-e\right)}{\sqrt{a+cx^2}\sqrt{4af^2-2c\left(e\sqrt{e^2-4df}+2df-e^2\right)}}\right)}{\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}} + \frac{\sqrt{2}\left(e\sqrt{e^2-4df}-2df+e^2\right)\tanh^{-1}\left(\frac{2af-cx\left(\sqrt{e^2-4df}+e\right)}{\sqrt{a+cx^2}\sqrt{4af^2+2c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{e^2-4df}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}} +}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(Sqrt[a + c*x^2]*(d + e*x + f*x^2)),x]
```

```
[Out] ((2*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/Sqrt[c] + (Sqrt[2]*(-e^2 + 2*d*f
+ e*Sqrt[e^2 - 4*d*f])*ArcTanh[(2*a*f + c*(-e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt
[4*a*f^2 - 2*c*(-e^2 + 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(Sq
rt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]) + (S
qrt[2]*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])*ArcTanh[(2*a*f - c*(e + Sqrt[e^2
 - 4*d*f])*x)/(Sqrt[4*a*f^2 + 2*c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt
[a + c*x^2])])/(Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^
2 - 4*d*f])]))/(2*f)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 6.03Error index.cc index_gc
d Error: Bad Argument Value
```

maple [B] time = 0.02, size = 1796, normalized size = 5.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x)
```

```
[Out] 1/f*ln(c^(1/2)*x+(c*x^2+a)^(1/2))/c^(1/2)+1/2/f^2*2^(1/2)/((2*a*f^2-2*c*d*f
+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*ln((-(e+(-4*d*f+e^2)^(1/2))*(x+1/
2*(e+(-4*d*f+e^2)^(1/2))/f)*c/f+(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c
*e)/f^2+1/2*2^(1/2)*((2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1
/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*(e+(-4*d*f+e^2)^(1/2))*(x+1/2
*(e+(-4*d*f+e^2)^(1/2))/f)*c/f+2*(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*
c*e)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))*e-1/(-4*d*f+e^2)^(1/2)/f
*2^(1/2)/((2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*ln((-(e
+(-4*d*f+e^2)^(1/2))*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)*c/f+(2*a*f^2-2*c*d*f+
c*e^2+(-4*d*f+e^2)^(1/2)*c*e)/f^2+1/2*2^(1/2)*((2*a*f^2-2*c*d*f+c*e^2+(-4*d
*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*(e+
(-4*d*f+e^2)^(1/2))*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)*c/f+2*(2*a*f^2-2*c*d*f
+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)
)*d+1/2/(-4*d*f+e^2)^(1/2)/f^2*2^(1/2)/((2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)
^(1/2)*c*e)/f^2)^(1/2)*ln((-(e+(-4*d*f+e^2)^(1/2))*(x+1/2*(e+(-4*d*f+e^2)^(
1/2))/f)*c/f+(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)/f^2+1/2*2^(1/2)
*((2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*(4*(x+1/2*(e+(-
4*d*f+e^2)^(1/2))/f)^2*c-4*(e+(-4*d*f+e^2)^(1/2))*(x+1/2*(e+(-4*d*f+e^2)^(1
/2))/f)*c/f+2*(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2))/(x
+1/2*(e+(-4*d*f+e^2)^(1/2))/f))*e^2+1/2/f^2*2^(1/2)/((2*a*f^2-2*c*d*f+c*e^2
-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*ln((-(e-(-4*d*f+e^2)^(1/2))*(x-1/2*(-e+
(-4*d*f+e^2)^(1/2))/f)*c/f+(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)/f
^2+1/2*2^(1/2)*((2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*(
4*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*(e-(-4*d*f+e^2)^(1/2))*(x-1/2*(-e
+(-4*d*f+e^2)^(1/2))/f)*c/f+2*(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e
)/f^2)^(1/2))/(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f))*e+1/(-4*d*f+e^2)^(1/2)/f*2
^(1/2)/((2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*ln((-(e-(
-4*d*f+e^2)^(1/2))*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)*c/f+(2*a*f^2-2*c*d*f+c
*e^2-(-4*d*f+e^2)^(1/2)*c*e)/f^2+1/2*2^(1/2)*((2*a*f^2-2*c*d*f+c*e^2-(-4*d*
f+e^2)^(1/2)*c*e)/f^2)^(1/2)*(4*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*(e-
(-4*d*f+e^2)^(1/2))*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)*c/f+2*(2*a*f^2-2*c*d*
f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2))/(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/
f))*d-1/2/(-4*d*f+e^2)^(1/2)/f^2*2^(1/2)/((2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^
2)^(1/2)*c*e)/f^2)^(1/2)*ln((-(e-(-4*d*f+e^2)^(1/2))*(x-1/2*(-e+(-4*d*f+e^2
)^(1/2))/f)*c/f+(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)/f^2+1/2*2^(1
/2)*((2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*(4*(x-1/2*(-
e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*(e-(-4*d*f+e^2)^(1/2))*(x-1/2*(-e+(-4*d*f+e^
```

```
2)^(1/2))/f)*c/f+2*(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2
))/(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f))*e^2
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for mo
re details)Is 4*d*f-e^2 positive, negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2}{\sqrt{c\,x^2+a}\,\left(f\,x^2+e\,x+d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a + c*x^2)^(1/2)*(d + e*x + f*x^2)),x)
```

```
[Out] int(x^2/((a + c*x^2)^(1/2)*(d + e*x + f*x^2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2}}{\sqrt{a + cx^{2}}\left(d + ex + fx^{2}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(f*x**2+e*x+d)/(c*x**2+a)**(1/2),x)
```

```
[Out] Integral(x**2/(sqrt(a + c*x**2)*(d + e*x + f*x**2)), x)
```

3.66 $\int \frac{x}{\sqrt{a+cx^2}\left(d+ex+fx^2\right)} \, dx$

Optimal. Leaf size=294

$$\frac{\left(e-\sqrt{e^2-4df}\right)\tanh^{-1}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}-\frac{\left(\sqrt{e^2-4df}+e\right)\tanh^{-1}\left(\frac{2af-cx\left(\sqrt{e^2-4d\ldots}\right.}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(e\sqrt{e^2\ldots}\right.}}\right.}{\sqrt{2}\sqrt{e^2-4df}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2\ldots\right.}}$$

```
[Out] 1/2*arctanh(1/2*(2*a*f-c*x*(e-(-4*d*f+e^2)^(1/2)))*2^(1/2)/(c*x^2+a)^(1/2)/
(2*a*f^2+(-(-4*d*f+e^2)^(1/2)*e-2*d*f+e^2)*c)^(1/2))*(e-(-4*d*f+e^2)^(1/2))
*2^(1/2)/(-4*d*f+e^2)^(1/2)/(2*a*f^2+(-(-4*d*f+e^2)^(1/2)*e-2*d*f+e^2)*c)^(
1/2)-1/2*arctanh(1/2*(2*a*f-c*x*(e+(-4*d*f+e^2)^(1/2)))*2^(1/2)/(c*x^2+a)^(
1/2)/(2*a*f^2+((-4*d*f+e^2)^(1/2)*e-2*d*f+e^2)*c)^(1/2))*(e+(-4*d*f+e^2)^(1
/2))*2^(1/2)/(-4*d*f+e^2)^(1/2)/(2*a*f^2+((-4*d*f+e^2)^(1/2)*e-2*d*f+e^2)*c
)^(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 294, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {1034, 725, 206}

$$\frac{\left(e-\sqrt{e^2-4df}\right)\tanh^{-1}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}-\frac{\left(\sqrt{e^2-4df}+e\right)\tanh^{-1}\left(\frac{2af-cx\left(\sqrt{e^2-4d\ldots}\right.}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(e\sqrt{e^2\ldots}\right.}}\right.}{\sqrt{2}\sqrt{e^2-4df}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2\ldots\right.}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(Sqrt[a + c*x^2]*(d + e*x + f*x^2)),x]
```

```
[Out] ((e - Sqrt[e^2 - 4*d*f])*ArcTanh[(2*a*f - c*(e - Sqrt[e^2 - 4*d*f])*x)/(Sqr
t[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])
])/(Sqrt[2]*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 -
4*d*f])]) - ((e + Sqrt[e^2 - 4*d*f])*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d
*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt
[a + c*x^2])])/(Sqrt[2]*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e
*Sqrt[e^2 - 4*d*f])])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 1034

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f
_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(
b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Dist[(2*c*g -
h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a,
 b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x}{\sqrt{a+cx^2}\left(d+ex+fx^2\right)}dx = -\left(\left(-1-\frac{e}{\sqrt{e^2-4df}}\right)\int \frac{1}{\left(e+\sqrt{e^2-4df}+2fx\right)\sqrt{a+cx^2}}dx\right)+\left(1-\frac{e}{\sqrt{e^2-4d}}\right.$$

$$=\left(-1+\frac{e}{\sqrt{e^2-4df}}\right)\mathrm{Subst}\left(\int \frac{1}{4af^2+c\left(e-\sqrt{e^2-4df}\right)^2-x^2}dx,x,\frac{2af-c\left(e\right.}{\sqrt{a}}\right.$$

$$=-\frac{\left(1-\frac{e}{\sqrt{e^2-4df}}\right)\tanh^{-1}\left(\frac{2af-c\left(e-\sqrt{e^2-4df}\right)x}{\sqrt{2}\sqrt{2af^2+c\left(e^2-2df-e\sqrt{e^2-4df}\right)}\sqrt{a+cx^2}}\right)}{\sqrt{2}\sqrt{2af^2+c\left(e^2-2df-e\sqrt{e^2-4df}\right)}}-\frac{\left(1+\frac{e}{\sqrt{e^2-4df}}\right)\tan}{\sqrt{2}\sqrt{2a}}$$

Mathematica [A] time = 0.32, size = 275, normalized size = 0.94

$$\frac{\sqrt{2}\left(-\frac{\left(\sqrt{e^2-4df}-e\right)\tanh^{-1}\left(\frac{2af+cx\left(\sqrt{e^2-4df}-e\right)}{\sqrt{a+cx^2}\sqrt{4af^2-2c\left(e\sqrt{e^2-4df}+2df-e^2\right)}}\right)}{2\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}-\frac{\left(\sqrt{e^2-4df}+e\right)\tanh^{-1}\left(\frac{2af-cx\left(\sqrt{e^2-4df}+e\right)}{\sqrt{a+cx^2}\sqrt{4af^2+2c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{2\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{e^2-4df}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(Sqrt[a + c*x^2]*(d + e*x + f*x^2)),x]
```

```
[Out] (Sqrt[2]*(-1/2*((-e + Sqrt[e^2 - 4*d*f])*ArcTanh[(2*a*f + c*(-e + Sqrt[e^2
- 4*d*f])*x)/(Sqrt[4*a*f^2 - 2*c*(-e^2 + 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt
[a + c*x^2])])/Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])] - ((e
+ Sqrt[e^2 - 4*d*f])*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[4*
a*f^2 + 2*c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(2*Sqrt
[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])])))/Sqrt[e^2 - 4*d*f]
```

fricas [B] time = 3.46, size = 5085, normalized size = 17.30

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(2)*sqrt((2*c*d^2 + a*e^2 - 2*a*d*f + (c^2*d^2*e^2 + a*c*e^4 - 4*a
^2*d*f^3 + (8*a*c*d^2 + a^2*e^2)*f^2 - 2*(2*c^2*d^3 + 3*a*c*d*e^2)*f)*sqrt(
a^2*e^2/(c^4*d^4*e^2 + 2*a*c^3*d^2*e^4 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (16*a^
3*c*d^2 + a^4*e^2)*f^4 - 12*(2*a^2*c^2*d^3 + a^3*c*d*e^2)*f^3 + 2*(8*a*c^3*
d^4 + 11*a^2*c^2*d^2*e^2 + a^3*c*e^4)*f^2 - 4*(c^4*d^5 + 3*a*c^3*d^3*e^2 +
2*a^2*c^2*d*e^4)*f)))/(c^2*d^2*e^2 + a*c*e^4 - 4*a^2*d*f^3 + (8*a*c*d^2 + a
^2*e^2)*f^2 - 2*(2*c^2*d^3 + 3*a*c*d*e^2)*f))*log((4*a*c*d^2*e*x - 2*a^2*d*
e^2 + sqrt(2)*(a^2*e^4 - 4*a^2*d*e^2*f - (2*c^3*d^4*e^2 + 3*a*c^2*d^2*e^4 +
 a^2*c*e^6 + 8*a^3*d^2*f^4 - 6*(4*a^2*c*d^3 + a^3*d*e^2)*f^3 + (24*a*c^2*d^
4 + 22*a^2*c*d^2*e^2 + a^3*e^4)*f^2 - 2*(4*c^3*d^5 + 9*a*c^2*d^3*e^2 + 4*a^
2*c*d*e^4)*f)*sqrt(a^2*e^2/(c^4*d^4*e^2 + 2*a*c^3*d^2*e^4 + a^2*c^2*e^6 - 4
*a^4*d*f^5 + (16*a^3*c*d^2 + a^4*e^2)*f^4 - 12*(2*a^2*c^2*d^3 + a^3*c*d*e^2
)*f^3 + 2*(8*a*c^3*d^4 + 11*a^2*c^2*d^2*e^2 + a^3*c*e^4)*f^2 - 4*(c^4*d^5 +
 3*a*c^3*d^3*e^2 + 2*a^2*c^2*d*e^4)*f)))*sqrt(c*x^2 + a)*sqrt((2*c*d^2 + a*
e^2 - 2*a*d*f + (c^2*d^2*e^2 + a*c*e^4 - 4*a^2*d*f^3 + (8*a*c*d^2 + a^2*e^2
)*f^2 - 2*(2*c^2*d^3 + 3*a*c*d*e^2)*f)*sqrt(a^2*e^2/(c^4*d^4*e^2 + 2*a*c^3*
```

```
d^2*e^4 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (16*a^3*c*d^2 + a^4*e^2)*f^4 - 12*(2*
a^2*c^2*d^3 + a^3*c*d*e^2)*f^3 + 2*(8*a*c^3*d^4 + 11*a^2*c^2*d^2*e^2 + a^3*
c*e^4)*f^2 - 4*(c^4*d^5 + 3*a*c^3*d^3*e^2 + 2*a^2*c^2*d*e^4)*f)))/(c^2*d^2*
e^2 + a*c*e^4 - 4*a^2*d*f^3 + (8*a*c*d^2 + a^2*e^2)*f^2 - 2*(2*c^2*d^3 + 3*
a*c*d*e^2)*f)) + 2*(a*c^2*d^3*e^2 + a^2*c*d*e^4 - 4*a^3*d^2*f^3 + (8*a^2*c*
d^3 + a^3*d*e^2)*f^2 - 2*(2*a*c^2*d^4 + 3*a^2*c*d^2*e^2)*f)*sqrt(a^2*e^2/(c
^4*d^4*e^2 + 2*a*c^3*d^2*e^4 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (16*a^3*c*d^2 +
a^4*e^2)*f^4 - 12*(2*a^2*c^2*d^3 + a^3*c*d*e^2)*f^3 + 2*(8*a*c^3*d^4 + 11*a
^2*c^2*d^2*e^2 + a^3*c*e^4)*f^2 - 4*(c^4*d^5 + 3*a*c^3*d^3*e^2 + 2*a^2*c^2*
d*e^4)*f)))/x) + 1/4*sqrt(2)*sqrt((2*c*d^2 + a*e^2 - 2*a*d*f + (c^2*d^2*e^2
 + a*c*e^4 - 4*a^2*d*f^3 + (8*a*c*d^2 + a^2*e^2)*f^2 - 2*(2*c^2*d^3 + 3*a*c
*d*e^2)*f)*sqrt(a^2*e^2/(c^4*d^4*e^2 + 2*a*c^3*d^2*e^4 + a^2*c^2*e^6 - 4*a^
4*d*f^5 + (16*a^3*c*d^2 + a^4*e^2)*f^4 - 12*(2*a^2*c^2*d^3 + a^3*c*d*e^2)*f
^3 + 2*(8*a*c^3*d^4 + 11*a^2*c^2*d^2*e^2 + a^3*c*e^4)*f^2 - 4*(c^4*d^5 + 3*
a*c^3*d^3*e^2 + 2*a^2*c^2*d*e^4)*f)))/(c^2*d^2*e^2 + a*c*e^4 - 4*a^2*d*f^3
+ (8*a*c*d^2 + a^2*e^2)*f^2 - 2*(2*c^2*d^3 + 3*a*c*d*e^2)*f))*log((4*a*c*d^
2*e*x - 2*a^2*d*e^2 - sqrt(2)*(a^2*e^4 - 4*a^2*d*e^2*f - (2*c^3*d^4*e^2 + 3
*a*c^2*d^2*e^4 + a^2*c*e^6 + 8*a^3*d^2*f^4 - 6*(4*a^2*c*d^3 + a^3*d*e^2)*f^
3 + (24*a*c^2*d^4 + 22*a^2*c*d^2*e^2 + a^3*e^4)*f^2 - 2*(4*c^3*d^5 + 9*a*c^
2*d^3*e^2 + 4*a^2*c*d*e^4)*f)*sqrt(a^2*e^2/(c^4*d^4*e^2 + 2*a*c^3*d^2*e^4 +
 a^2*c^2*e^6 - 4*a^4*d*f^5 + (16*a^3*c*d^2 + a^4*e^2)*f^4 - 12*(2*a^2*c^2*d
^3 + a^3*c*d*e^2)*f^3 + 2*(8*a*c^3*d^4 + 11*a^2*c^2*d^2*e^2 + a^3*c*e^4)*f^
2 - 4*(c^4*d^5 + 3*a*c^3*d^3*e^2 + 2*a^2*c^2*d*e^4)*f)))*sqrt(c*x^2 + a)*sq
rt((2*c*d^2 + a*e^2 - 2*a*d*f + (c^2*d^2*e^2 + a*c*e^4 - 4*a^2*d*f^3 + (8*a
*c*d^2 + a^2*e^2)*f^2 - 2*(2*c^2*d^3 + 3*a*c*d*e^2)*f)*sqrt(a^2*e^2/(c^4*d^
4*e^2 + 2*a*c^3*d^2*e^4 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (16*a^3*c*d^2 + a^4*e
^2)*f^4 - 12*(2*a^2*c^2*d^3 + a^3*c*d*e^2)*f^3 + 2*(8*a*c^3*d^4 + 11*a^2*c^
2*d^2*e^2 + a^3*c*e^4)*f^2 - 4*(c^4*d^5 + 3*a*c^3*d^3*e^2 + 2*a^2*c^2*d*e^4
)*f)))/(c^2*d^2*e^2 + a*c*e^4 - 4*a^2*d*f^3 + (8*a*c*d^2 + a^2*e^2)*f^2 - 2
*(2*c^2*d^3 + 3*a*c*d*e^2)*f)) + 2*(a*c^2*d^3*e^2 + a^2*c*d*e^4 - 4*a^3*d^2
*f^3 + (8*a^2*c*d^3 + a^3*d*e^2)*f^2 - 2*(2*a*c^2*d^4 + 3*a^2*c*d^2*e^2)*f)
*sqrt(a^2*e^2/(c^4*d^4*e^2 + 2*a*c^3*d^2*e^4 + a^2*c^2*e^6 - 4*a^4*d*f^5 +
(16*a^3*c*d^2 + a^4*e^2)*f^4 - 12*(2*a^2*c^2*d^3 + a^3*c*d*e^2)*f^3 + 2*(8*
a*c^3*d^4 + 11*a^2*c^2*d^2*e^2 + a^3*c*e^4)*f^2 - 4*(c^4*d^5 + 3*a*c^3*d^3*
e^2 + 2*a^2*c^2*d*e^4)*f)))/x) - 1/4*sqrt(2)*sqrt((2*c*d^2 + a*e^2 - 2*a*d*
f - (c^2*d^2*e^2 + a*c*e^4 - 4*a^2*d*f^3 + (8*a*c*d^2 + a^2*e^2)*f^2 - 2*(2
*c^2*d^3 + 3*a*c*d*e^2)*f)*sqrt(a^2*e^2/(c^4*d^4*e^2 + 2*a*c^3*d^2*e^4 + a^
2*c^2*e^6 - 4*a^4*d*f^5 + (16*a^3*c*d^2 + a^4*e^2)*f^4 - 12*(2*a^2*c^2*d^3
+ a^3*c*d*e^2)*f^3 + 2*(8*a*c^3*d^4 + 11*a^2*c^2*d^2*e^2 + a^3*c*e^4)*f^2 -
 4*(c^4*d^5 + 3*a*c^3*d^3*e^2 + 2*a^2*c^2*d*e^4)*f)))/(c^2*d^2*e^2 + a*c*e^
4 - 4*a^2*d*f^3 + (8*a*c*d^2 + a^2*e^2)*f^2 - 2*(2*c^2*d^3 + 3*a*c*d*e^2)*f
))*log((4*a*c*d^2*e*x - 2*a^2*d*e^2 + sqrt(2)*(a^2*e^4 - 4*a^2*d*e^2*f + (2
*c^3*d^4*e^2 + 3*a*c^2*d^2*e^4 + a^2*c*e^6 + 8*a^3*d^2*f^4 - 6*(4*a^2*c*d^3
 + a^3*d*e^2)*f^3 + (24*a*c^2*d^4 + 22*a^2*c*d^2*e^2 + a^3*e^4)*f^2 - 2*(4*
c^3*d^5 + 9*a*c^2*d^3*e^2 + 4*a^2*c*d*e^4)*f)*sqrt(a^2*e^2/(c^4*d^4*e^2 + 2
*a*c^3*d^2*e^4 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (16*a^3*c*d^2 + a^4*e^2)*f^4 -
 12*(2*a^2*c^2*d^3 + a^3*c*d*e^2)*f^3 + 2*(8*a*c^3*d^4 + 11*a^2*c^2*d^2*e^2
 + a^3*c*e^4)*f^2 - 4*(c^4*d^5 + 3*a*c^3*d^3*e^2 + 2*a^2*c^2*d*e^4)*f)))*sq
rt(c*x^2 + a)*sqrt((2*c*d^2 + a*e^2 - 2*a*d*f - (c^2*d^2*e^2 + a*c*e^4 - 4*
a^2*d*f^3 + (8*a*c*d^2 + a^2*e^2)*f^2 - 2*(2*c^2*d^3 + 3*a*c*d*e^2)*f)*sqrt
(a^2*e^2/(c^4*d^4*e^2 + 2*a*c^3*d^2*e^4 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (16*a
^3*c*d^2 + a^4*e^2)*f^4 - 12*(2*a^2*c^2*d^3 + a^3*c*d*e^2)*f^3 + 2*(8*a*c^3
*d^4 + 11*a^2*c^2*d^2*e^2 + a^3*c*e^4)*f^2 - 4*(c^4*d^5 + 3*a*c^3*d^3*e^2 +
 2*a^2*c^2*d*e^4)*f)))/(c^2*d^2*e^2 + a*c*e^4 - 4*a^2*d*f^3 + (8*a*c*d^2 +
a^2*e^2)*f^2 - 2*(2*c^2*d^3 + 3*a*c*d*e^2)*f)) - 2*(a*c^2*d^3*e^2 + a^2*c*d
*e^4 - 4*a^3*d^2*f^3 + (8*a^2*c*d^3 + a^3*d*e^2)*f^2 - 2*(2*a*c^2*d^4 + 3*a
^2*c*d^2*e^2)*f)*sqrt(a^2*e^2/(c^4*d^4*e^2 + 2*a*c^3*d^2*e^4 + a^2*c^2*e^6
- 4*a^4*d*f^5 + (16*a^3*c*d^2 + a^4*e^2)*f^4 - 12*(2*a^2*c^2*d^3 + a^3*c*d*
e^2)*f^3 + 2*(8*a*c^3*d^4 + 11*a^2*c^2*d^2*e^2 + a^3*c*e^4)*f^2 - 4*(c^4*d^
```

```
5 + 3*a*c^3*d^3*e^2 + 2*a^2*c^2*d*e^4)*f)))/x) + 1/4*sqrt(2)*sqrt((2*c*d^2
+ a*e^2 - 2*a*d*f - (c^2*d^2*e^2 + a*c*e^4 - 4*a^2*d*f^3 + (8*a*c*d^2 + a^2
*e^2)*f^2 - 2*(2*c^2*d^3 + 3*a*c*d*e^2)*f)*sqrt(a^2*e^2/(c^4*d^4*e^2 + 2*a*
c^3*d^2*e^4 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (16*a^3*c*d^2 + a^4*e^2)*f^4 - 12
*(2*a^2*c^2*d^3 + a^3*c*d*e^2)*f^3 + 2*(8*a*c^3*d^4 + 11*a^2*c^2*d^2*e^2 +
a^3*c*e^4)*f^2 - 4*(c^4*d^5 + 3*a*c^3*d^3*e^2 + 2*a^2*c^2*d*e^4)*f)))/(c^2*
d^2*e^2 + a*c*e^4 - 4*a^2*d*f^3 + (8*a*c*d^2 + a^2*e^2)*f^2 - 2*(2*c^2*d^3
+ 3*a*c*d*e^2)*f))*log((4*a*c*d^2*e*x - 2*a^2*d*e^2 - sqrt(2)*(a^2*e^4 - 4*
a^2*d*e^2*f + (2*c^3*d^4*e^2 + 3*a*c^2*d^2*e^4 + a^2*c*e^6 + 8*a^3*d^2*f^4
- 6*(4*a^2*c*d^3 + a^3*d*e^2)*f^3 + (24*a*c^2*d^4 + 22*a^2*c*d^2*e^2 + a^3*
e^4)*f^2 - 2*(4*c^3*d^5 + 9*a*c^2*d^3*e^2 + 4*a^2*c*d*e^4)*f)*sqrt(a^2*e^2/
(c^4*d^4*e^2 + 2*a*c^3*d^2*e^4 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (16*a^3*c*d^2
+ a^4*e^2)*f^4 - 12*(2*a^2*c^2*d^3 + a^3*c*d*e^2)*f^3 + 2*(8*a*c^3*d^4 + 11
*a^2*c^2*d^2*e^2 + a^3*c*e^4)*f^2 - 4*(c^4*d^5 + 3*a*c^3*d^3*e^2 + 2*a^2*c^
2*d*e^4)*f)))*sqrt(c*x^2 + a)*sqrt((2*c*d^2 + a*e^2 - 2*a*d*f - (c^2*d^2*e^
2 + a*c*e^4 - 4*a^2*d*f^3 + (8*a*c*d^2 + a^2*e^2)*f^2 - 2*(2*c^2*d^3 + 3*a*
c*d*e^2)*f)*sqrt(a^2*e^2/(c^4*d^4*e^2 + 2*a*c^3*d^2*e^4 + a^2*c^2*e^6 - 4*a
^4*d*f^5 + (16*a^3*c*d^2 + a^4*e^2)*f^4 - 12*(2*a^2*c^2*d^3 + a^3*c*d*e^2)*
f^3 + 2*(8*a*c^3*d^4 + 11*a^2*c^2*d^2*e^2 + a^3*c*e^4)*f^2 - 4*(c^4*d^5 + 3
*a*c^3*d^3*e^2 + 2*a^2*c^2*d*e^4)*f)))/(c^2*d^2*e^2 + a*c*e^4 - 4*a^2*d*f^3
 + (8*a*c*d^2 + a^2*e^2)*f^2 - 2*(2*c^2*d^3 + 3*a*c*d*e^2)*f)) - 2*(a*c^2*d
^3*e^2 + a^2*c*d*e^4 - 4*a^3*d^2*f^3 + (8*a^2*c*d^3 + a^3*d*e^2)*f^2 - 2*(2
*a*c^2*d^4 + 3*a^2*c*d^2*e^2)*f)*sqrt(a^2*e^2/(c^4*d^4*e^2 + 2*a*c^3*d^2*e^
4 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (16*a^3*c*d^2 + a^4*e^2)*f^4 - 12*(2*a^2*c^
2*d^3 + a^3*c*d*e^2)*f^3 + 2*(8*a*c^3*d^4 + 11*a^2*c^2*d^2*e^2 + a^3*c*e^4)
*f^2 - 4*(c^4*d^5 + 3*a*c^3*d^3*e^2 + 2*a^2*c^2*d*e^4)*f)))/x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 1.77index.cc index_m operat
or + Error: Bad Argument Value
```

maple [B] time = 0.01, size = 1172, normalized size = 3.99

$$\frac{\sqrt{2}\, e \ln\left(\frac{-\frac{\left(e-\sqrt{-4df+e^2}\right)\left(x-\frac{-e+\sqrt{-4df+e^2}}{2f}\right)c}{f}+\frac{2af^2-2cdf+c e^2-\sqrt{-4df+e^2}\, ce}{f^2}+\frac{\sqrt{2}\sqrt{\frac{2af^2-2cdf+c e^2-\sqrt{-4df+e^2}\, ce}{f^2}}\sqrt{4\left(x-\frac{-e+\sqrt{-4df+e^2}}{2f}\right)^2 c-\frac{4\left(e-\sqrt{-4df+e^2}\right.}{}}}{2}}{x-\frac{-e+\sqrt{-4df+e^2}}{2f}}\right)}{2\sqrt{-4df+e^2}\,\sqrt{\frac{2af^2-2cdf+c e^2-\sqrt{-4df+e^2}\, ce}{f^2}}\, f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x)
```

```
[Out] -1/2/(-4*d*f+e^2)^(1/2)/f*2^(1/2)/((2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2
)*c*e)/f^2)^(1/2)*ln((-(e+(-4*d*f+e^2)^(1/2))*(x+1/2*(e+(-4*d*f+e^2)^(1/2))
/f)*c/f+(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)/f^2+1/2*2^(1/2)*((2*
a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f
+e^2)^(1/2))/f)^2*c-4*(e+(-4*d*f+e^2)^(1/2))*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/
```

```
f)*c/f+2*(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2))/(x+1/2*
(e+(-4*d*f+e^2)^(1/2))/f))*e-1/2/f*2^(1/2)/((2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+
e^2)^(1/2)*c*e)/f^2)^(1/2)*ln((-(e+(-4*d*f+e^2)^(1/2))*(x+1/2*(e+(-4*d*f+e^
2)^(1/2))/f)*c/f+(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)/f^2+1/2*2^(
1/2)*((2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*(4*(x+1/2*(
e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*(e+(-4*d*f+e^2)^(1/2))*(x+1/2*(e+(-4*d*f+e^2
)^(1/2))/f)*c/f+2*(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)
)/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))-1/2/f*2^(1/2)/((2*a*f^2-2*c*d*f+c*e^2-(
-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*ln((-(e-(-4*d*f+e^2)^(1/2))*(x-1/2*(-e+(-
4*d*f+e^2)^(1/2))/f)*c/f+(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)/f^2
+1/2*2^(1/2)*((2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*(4*
(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*(e-(-4*d*f+e^2)^(1/2))*(x-1/2*(-e+(
-4*d*f+e^2)^(1/2))/f)*c/f+2*(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)/
f^2)^(1/2))/(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f))+1/2/(-4*d*f+e^2)^(1/2)/f*2^(
1/2)/((2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*ln((-(e-(-4
*d*f+e^2)^(1/2))*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)*c/f+(2*a*f^2-2*c*d*f+c*e
^2-(-4*d*f+e^2)^(1/2)*c*e)/f^2+1/2*2^(1/2)*((2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+
e^2)^(1/2)*c*e)/f^2)^(1/2)*(4*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*(e-(-
4*d*f+e^2)^(1/2))*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)*c/f+2*(2*a*f^2-2*c*d*f+
c*e^2-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2))/(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)
)*e
```

maxima [F(-2)]   time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for mo
re details)Is 4*d*f-e^2 positive, negative or zero?
```

mupad [F]   time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x}{\sqrt{c\,x^2+a}\,\left(f\,x^2+e\,x+d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a + c*x^2)^(1/2)*(d + e*x + f*x^2)),x)
```

```
[Out] int(x/((a + c*x^2)^(1/2)*(d + e*x + f*x^2)), x)
```

sympy [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{a+cx^2}\,\left(d+ex+fx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(f*x**2+e*x+d)/(c*x**2+a)**(1/2),x)
```

```
[Out] Integral(x/(sqrt(a + c*x**2)*(d + e*x + f*x**2)), x)
```

3.67 $\int \frac{1}{\sqrt{a+cx^2}\left(d+ex+fx^2\right)}\,dx$

Optimal. Leaf size=266

$$\frac{\sqrt{2}f\tanh^{-1}\left(\frac{2af-cx\left(\sqrt{e^2-4df}+e\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{e^2-4df}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}-\frac{\sqrt{2}f\tanh^{-1}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}$$

```
[Out] -f*arctanh(1/2*(2*a*f-c*x*(e-(-4*d*f+e^2)^(1/2)))*2^(1/2)/(c*x^2+a)^(1/2)/(
2*a*f^2+(-(-4*d*f+e^2)^(1/2)*e-2*d*f+e^2)*c)^(1/2))*2^(1/2)/(-4*d*f+e^2)^(1
/2)/(2*a*f^2+(-(-4*d*f+e^2)^(1/2)*e-2*d*f+e^2)*c)^(1/2)+f*arctanh(1/2*(2*a*
f-c*x*(e+(-4*d*f+e^2)^(1/2)))*2^(1/2)/(c*x^2+a)^(1/2)/(2*a*f^2+((-4*d*f+e^2
)^(1/2)*e-2*d*f+e^2)*c)^(1/2))*2^(1/2)/(-4*d*f+e^2)^(1/2)/(2*a*f^2+((-4*d*f
+e^2)^(1/2)*e-2*d*f+e^2)*c)^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 266, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {985, 725, 206}

$$\frac{\sqrt{2}f\tanh^{-1}\left(\frac{2af-cx\left(\sqrt{e^2-4df}+e\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{e^2-4df}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}-\frac{\sqrt{2}f\tanh^{-1}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + c*x^2]*(d + e*x + f*x^2)),x]
```

```
[Out] -((Sqrt[2]*f*ArcTanh[(2*a*f - c*(e - Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*
a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[e^2
 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])])) + (Sqrt[2
]*f*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c
*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[e^2 - 4*d*f]
*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 985

```
Int[1/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f_.)*(x_)^2]), x_Sym
bol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[1/((b - q + 2*c*x)
*Sqrt[d + f*x^2]), x], x] - Dist[(2*c)/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f
*x^2]), x], x]] /; FreeQ[{a, b, c, d, f}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ
[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+cx^2}\left(d+ex+fx^2\right)}\,dx = \frac{(2f)\int \frac{1}{\left(e-\sqrt{e^2-4df}+2fx\right)\sqrt{a+cx^2}}\,dx}{\sqrt{e^2-4df}} - \frac{(2f)\int \frac{1}{\left(e+\sqrt{e^2-4df}+2fx\right)\sqrt{a+cx^2}}\,dx}{\sqrt{e^2-4df}}$$

$$= -\frac{(2f)\,\text{Subst}\left(\int \frac{1}{4af^2+c\left(e-\sqrt{e^2-4df}\right)^2-x^2}\,dx, x, \frac{2af-c\left(e-\sqrt{e^2-4df}\right)x}{\sqrt{a+cx^2}}\right)}{\sqrt{e^2-4df}} + \frac{(2f)\,\text{Subst}\left(\right.}{}$$

$$= -\frac{\sqrt{2}f\tanh^{-1}\left(\frac{2af-c\left(e-\sqrt{e^2-4df}\right)x}{\sqrt{2}\sqrt{2af^2+c\left(e^2-2df-e\sqrt{e^2-4df}\right)}\sqrt{a+cx^2}}\right)}{\sqrt{e^2-4df}\sqrt{2af^2+c\left(e^2-2df-e\sqrt{e^2-4df}\right)}} + \frac{\sqrt{2}f\tanh^{-1}\left(\frac{}{\sqrt{2}\sqrt{2af^2}}\right.}{\sqrt{e^2-4df}\sqrt{2af^2+c}}$$

Mathematica [**A**] time = 0.29, size = 247, normalized size = 0.93

$$\frac{2\sqrt{2}f\left(\frac{\tanh^{-1}\left(\frac{2af-cx\left(\sqrt{e^2-4df}+e\right)}{\sqrt{a+cx^2}\sqrt{4af^2+2c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{2\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}} - \frac{\tanh^{-1}\left(\frac{2af+cx\left(\sqrt{e^2-4df}-e\right)}{\sqrt{a+cx^2}\sqrt{4af^2-2c\left(e\sqrt{e^2-4df}+2df-e^2\right)}}\right)}{2\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{e^2-4df}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + c*x^2]*(d + e*x + f*x^2)),x]
```

```
[Out] (2*Sqrt[2]*f*(-1/2*ArcTanh[(2*a*f + c*(-e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[4*a
*f^2 - 2*c*(-e^2 + 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])]/Sqrt[2*a
*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])] + ArcTanh[(2*a*f - c*(e + Sqr
t[e^2 - 4*d*f])*x)/(Sqrt[4*a*f^2 + 2*c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]
*Sqrt[a + c*x^2])]/(2*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]
)))/Sqrt[e^2 - 4*d*f]
```

fricas [**B**] time = 3.45, size = 5073, normalized size = 19.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(2)*sqrt((c*e^2 - 2*c*d*f + 2*a*f^2 + (c^2*d^2*e^2 + a*c*e^4 - 4*a
^2*d*f^3 + (8*a*c*d^2 + a^2*e^2)*f^2 - 2*(2*c^2*d^3 + 3*a*c*d*e^2)*f)*sqrt(
c^2*e^2/(c^4*d^4*e^2 + 2*a*c^3*d^2*e^4 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (16*a^
3*c*d^2 + a^4*e^2)*f^4 - 12*(2*a^2*c^2*d^3 + a^3*c*d*e^2)*f^3 + 2*(8*a*c^3*
d^4 + 11*a^2*c^2*d^2*e^2 + a^3*c*e^4)*f^2 - 4*(c^4*d^5 + 3*a*c^3*d^3*e^2 +
2*a^2*c^2*d*e^4)*f)))/(c^2*d^2*e^2 + a*c*e^4 - 4*a^2*d*f^3 + (8*a*c*d^2 + a
^2*e^2)*f^2 - 2*(2*c^2*d^3 + 3*a*c*d*e^2)*f))*log((4*c^2*d*e*f*x - 2*a*c*e^
2*f + sqrt(2)*(c^2*d*e^3 + 4*a*c*d*e*f^2 - (4*c^2*d^2*e + a*c*e^3)*f - (c^3
*d^3*e^3 + a*c^2*d*e^5 - 4*a^3*d*e*f^4 + (4*a^2*c*d^2*e + a^3*e^3)*f^3 + (4
*a*c^2*d^3*e - 5*a^2*c*d*e^3)*f^2 - (4*c^3*d^4*e + 5*a*c^2*d^2*e^3 - a^2*c*
e^5)*f)*sqrt(c^2*e^2/(c^4*d^4*e^2 + 2*a*c^3*d^2*e^4 + a^2*c^2*e^6 - 4*a^4*d
*f^5 + (16*a^3*c*d^2 + a^4*e^2)*f^4 - 12*(2*a^2*c^2*d^3 + a^3*c*d*e^2)*f^3
+ 2*(8*a*c^3*d^4 + 11*a^2*c^2*d^2*e^2 + a^3*c*e^4)*f^2 - 4*(c^4*d^5 + 3*a*c
^3*d^3*e^2 + 2*a^2*c^2*d*e^4)*f)))*sqrt(c*x^2 + a)*sqrt((c*e^2 - 2*c*d*f +
2*a*f^2 + (c^2*d^2*e^2 + a*c*e^4 - 4*a^2*d*f^3 + (8*a*c*d^2 + a^2*e^2)*f^2
```

```
- 2*(2*c^2*d^3 + 3*a*c*d*e^2)*f)*sqrt(c^2*e^2/(c^4*d^4*e^2 + 2*a*c^3*d^2*e^
4 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (16*a^3*c*d^2 + a^4*e^2)*f^4 - 12*(2*a^2*c^
2*d^3 + a^3*c*d*e^2)*f^3 + 2*(8*a*c^3*d^4 + 11*a^2*c^2*d^2*e^2 + a^3*c*e^4)
*f^2 - 4*(c^4*d^5 + 3*a*c^3*d^3*e^2 + 2*a^2*c^2*d*e^4)*f)))/(c^2*d^2*e^2 +
a*c*e^4 - 4*a^2*d*f^3 + (8*a*c*d^2 + a^2*e^2)*f^2 - 2*(2*c^2*d^3 + 3*a*c*d*
e^2)*f)) + 2*(4*a^3*d*f^4 - (8*a^2*c*d^2 + a^3*e^2)*f^3 + 2*(2*a*c^2*d^3 +
3*a^2*c*d*e^2)*f^2 - (a*c^2*d^2*e^2 + a^2*c*e^4)*f)*sqrt(c^2*e^2/(c^4*d^4*e
^2 + 2*a*c^3*d^2*e^4 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (16*a^3*c*d^2 + a^4*e^2)
*f^4 - 12*(2*a^2*c^2*d^3 + a^3*c*d*e^2)*f^3 + 2*(8*a*c^3*d^4 + 11*a^2*c^2*d
^2*e^2 + a^3*c*e^4)*f^2 - 4*(c^4*d^5 + 3*a*c^3*d^3*e^2 + 2*a^2*c^2*d*e^4)*f
)))/x) + 1/4*sqrt(2)*sqrt((c*e^2 - 2*c*d*f + 2*a*f^2 + (c^2*d^2*e^2 + a*c*e
^4 - 4*a^2*d*f^3 + (8*a*c*d^2 + a^2*e^2)*f^2 - 2*(2*c^2*d^3 + 3*a*c*d*e^2)*
f)*sqrt(c^2*e^2/(c^4*d^4*e^2 + 2*a*c^3*d^2*e^4 + a^2*c^2*e^6 - 4*a^4*d*f^5
+ (16*a^3*c*d^2 + a^4*e^2)*f^4 - 12*(2*a^2*c^2*d^3 + a^3*c*d*e^2)*f^3 + 2*(
8*a*c^3*d^4 + 11*a^2*c^2*d^2*e^2 + a^3*c*e^4)*f^2 - 4*(c^4*d^5 + 3*a*c^3*d^
3*e^2 + 2*a^2*c^2*d*e^4)*f)))/(c^2*d^2*e^2 + a*c*e^4 - 4*a^2*d*f^3 + (8*a*c
*d^2 + a^2*e^2)*f^2 - 2*(2*c^2*d^3 + 3*a*c*d*e^2)*f))*log((4*c^2*d*e*f*x -
2*a*c*e^2*f - sqrt(2)*(c^2*d*e^3 + 4*a*c*d*e*f^2 - (4*c^2*d^2*e + a*c*e^3)*
f - (c^3*d^3*e^3 + a*c^2*d*e^5 - 4*a^3*d*e*f^4 + (4*a^2*c*d^2*e + a^3*e^3)*
f^3 + (4*a*c^2*d^3*e - 5*a^2*c*d*e^3)*f^2 - (4*c^3*d^4*e + 5*a*c^2*d^2*e^3
- a^2*c*e^5)*f)*sqrt(c^2*e^2/(c^4*d^4*e^2 + 2*a*c^3*d^2*e^4 + a^2*c^2*e^6 -
 4*a^4*d*f^5 + (16*a^3*c*d^2 + a^4*e^2)*f^4 - 12*(2*a^2*c^2*d^3 + a^3*c*d*e
^2)*f^3 + 2*(8*a*c^3*d^4 + 11*a^2*c^2*d^2*e^2 + a^3*c*e^4)*f^2 - 4*(c^4*d^5
 + 3*a*c^3*d^3*e^2 + 2*a^2*c^2*d*e^4)*f)))*sqrt(c*x^2 + a)*sqrt((c*e^2 - 2*
c*d*f + 2*a*f^2 + (c^2*d^2*e^2 + a*c*e^4 - 4*a^2*d*f^3 + (8*a*c*d^2 + a^2*e
^2)*f^2 - 2*(2*c^2*d^3 + 3*a*c*d*e^2)*f)*sqrt(c^2*e^2/(c^4*d^4*e^2 + 2*a*c^
3*d^2*e^4 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (16*a^3*c*d^2 + a^4*e^2)*f^4 - 12*(
2*a^2*c^2*d^3 + a^3*c*d*e^2)*f^3 + 2*(8*a*c^3*d^4 + 11*a^2*c^2*d^2*e^2 + a^
3*c*e^4)*f^2 - 4*(c^4*d^5 + 3*a*c^3*d^3*e^2 + 2*a^2*c^2*d*e^4)*f)))/(c^2*d^
2*e^2 + a*c*e^4 - 4*a^2*d*f^3 + (8*a*c*d^2 + a^2*e^2)*f^2 - 2*(2*c^2*d^3 +
3*a*c*d*e^2)*f)) + 2*(4*a^3*d*f^4 - (8*a^2*c*d^2 + a^3*e^2)*f^3 + 2*(2*a*c^
2*d^3 + 3*a^2*c*d*e^2)*f^2 - (a*c^2*d^2*e^2 + a^2*c*e^4)*f)*sqrt(c^2*e^2/(c
^4*d^4*e^2 + 2*a*c^3*d^2*e^4 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (16*a^3*c*d^2 +
a^4*e^2)*f^4 - 12*(2*a^2*c^2*d^3 + a^3*c*d*e^2)*f^3 + 2*(8*a*c^3*d^4 + 11*a
^2*c^2*d^2*e^2 + a^3*c*e^4)*f^2 - 4*(c^4*d^5 + 3*a*c^3*d^3*e^2 + 2*a^2*c^2*
d*e^4)*f)))/x) - 1/4*sqrt(2)*sqrt((c*e^2 - 2*c*d*f + 2*a*f^2 - (c^2*d^2*e^2
 + a*c*e^4 - 4*a^2*d*f^3 + (8*a*c*d^2 + a^2*e^2)*f^2 - 2*(2*c^2*d^3 + 3*a*c
*d*e^2)*f)*sqrt(c^2*e^2/(c^4*d^4*e^2 + 2*a*c^3*d^2*e^4 + a^2*c^2*e^6 - 4*a^
4*d*f^5 + (16*a^3*c*d^2 + a^4*e^2)*f^4 - 12*(2*a^2*c^2*d^3 + a^3*c*d*e^2)*f
^3 + 2*(8*a*c^3*d^4 + 11*a^2*c^2*d^2*e^2 + a^3*c*e^4)*f^2 - 4*(c^4*d^5 + 3*
a*c^3*d^3*e^2 + 2*a^2*c^2*d*e^4)*f)))/(c^2*d^2*e^2 + a*c*e^4 - 4*a^2*d*f^3
+ (8*a*c*d^2 + a^2*e^2)*f^2 - 2*(2*c^2*d^3 + 3*a*c*d*e^2)*f))*log((4*c^2*d*
e*f*x - 2*a*c*e^2*f + sqrt(2)*(c^2*d*e^3 + 4*a*c*d*e*f^2 - (4*c^2*d^2*e + a
*c*e^3)*f + (c^3*d^3*e^3 + a*c^2*d*e^5 - 4*a^3*d*e*f^4 + (4*a^2*c*d^2*e + a
^3*e^3)*f^3 + (4*a*c^2*d^3*e - 5*a^2*c*d*e^3)*f^2 - (4*c^3*d^4*e + 5*a*c^2*
d^2*e^3 - a^2*c*e^5)*f)*sqrt(c^2*e^2/(c^4*d^4*e^2 + 2*a*c^3*d^2*e^4 + a^2*c
^2*e^6 - 4*a^4*d*f^5 + (16*a^3*c*d^2 + a^4*e^2)*f^4 - 12*(2*a^2*c^2*d^3 + a
^3*c*d*e^2)*f^3 + 2*(8*a*c^3*d^4 + 11*a^2*c^2*d^2*e^2 + a^3*c*e^4)*f^2 - 4*
(c^4*d^5 + 3*a*c^3*d^3*e^2 + 2*a^2*c^2*d*e^4)*f)))*sqrt(c*x^2 + a)*sqrt((c*
e^2 - 2*c*d*f + 2*a*f^2 - (c^2*d^2*e^2 + a*c*e^4 - 4*a^2*d*f^3 + (8*a*c*d^2
 + a^2*e^2)*f^2 - 2*(2*c^2*d^3 + 3*a*c*d*e^2)*f)*sqrt(c^2*e^2/(c^4*d^4*e^2
+ 2*a*c^3*d^2*e^4 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (16*a^3*c*d^2 + a^4*e^2)*f^
4 - 12*(2*a^2*c^2*d^3 + a^3*c*d*e^2)*f^3 + 2*(8*a*c^3*d^4 + 11*a^2*c^2*d^2*
e^2 + a^3*c*e^4)*f^2 - 4*(c^4*d^5 + 3*a*c^3*d^3*e^2 + 2*a^2*c^2*d*e^4)*f)))
/(c^2*d^2*e^2 + a*c*e^4 - 4*a^2*d*f^3 + (8*a*c*d^2 + a^2*e^2)*f^2 - 2*(2*c^
2*d^3 + 3*a*c*d*e^2)*f)) - 2*(4*a^3*d*f^4 - (8*a^2*c*d^2 + a^3*e^2)*f^3 + 2
*(2*a*c^2*d^3 + 3*a^2*c*d*e^2)*f^2 - (a*c^2*d^2*e^2 + a^2*c*e^4)*f)*sqrt(c^
2*e^2/(c^4*d^4*e^2 + 2*a*c^3*d^2*e^4 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (16*a^3*
c*d^2 + a^4*e^2)*f^4 - 12*(2*a^2*c^2*d^3 + a^3*c*d*e^2)*f^3 + 2*(8*a*c^3*d^
```

```
4 + 11*a^2*c^2*d^2*e^2 + a^3*c*e^4)*f^2 - 4*(c^4*d^5 + 3*a*c^3*d^3*e^2 + 2*
a^2*c^2*d*e^4)*f)))/x) + 1/4*sqrt(2)*sqrt((c*e^2 - 2*c*d*f + 2*a*f^2 - (c^2
*d^2*e^2 + a*c*e^4 - 4*a^2*d*f^3 + (8*a*c*d^2 + a^2*e^2)*f^2 - 2*(2*c^2*d^3
 + 3*a*c*d*e^2)*f)*sqrt(c^2*e^2/(c^4*d^4*e^2 + 2*a*c^3*d^2*e^4 + a^2*c^2*e^
6 - 4*a^4*d*f^5 + (16*a^3*c*d^2 + a^4*e^2)*f^4 - 12*(2*a^2*c^2*d^3 + a^3*c*
d*e^2)*f^3 + 2*(8*a*c^3*d^4 + 11*a^2*c^2*d^2*e^2 + a^3*c*e^4)*f^2 - 4*(c^4*
d^5 + 3*a*c^3*d^3*e^2 + 2*a^2*c^2*d*e^4)*f)))/(c^2*d^2*e^2 + a*c*e^4 - 4*a^
2*d*f^3 + (8*a*c*d^2 + a^2*e^2)*f^2 - 2*(2*c^2*d^3 + 3*a*c*d*e^2)*f))*log((
4*c^2*d*e*f*x - 2*a*c*e^2*f - sqrt(2)*(c^2*d*e^3 + 4*a*c*d*e*f^2 - (4*c^2*d
^2*e + a*c*e^3)*f + (c^3*d^3*e^3 + a*c^2*d*e^5 - 4*a^3*d*e*f^4 + (4*a^2*c*d
^2*e + a^3*e^3)*f^3 + (4*a*c^2*d^3*e - 5*a^2*c*d*e^3)*f^2 - (4*c^3*d^4*e +
5*a*c^2*d^2*e^3 - a^2*c*e^5)*f)*sqrt(c^2*e^2/(c^4*d^4*e^2 + 2*a*c^3*d^2*e^4
 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (16*a^3*c*d^2 + a^4*e^2)*f^4 - 12*(2*a^2*c^2
*d^3 + a^3*c*d*e^2)*f^3 + 2*(8*a*c^3*d^4 + 11*a^2*c^2*d^2*e^2 + a^3*c*e^4)*
f^2 - 4*(c^4*d^5 + 3*a*c^3*d^3*e^2 + 2*a^2*c^2*d*e^4)*f)))*sqrt(c*x^2 + a)*
sqrt((c*e^2 - 2*c*d*f + 2*a*f^2 - (c^2*d^2*e^2 + a*c*e^4 - 4*a^2*d*f^3 + (8
*a*c*d^2 + a^2*e^2)*f^2 - 2*(2*c^2*d^3 + 3*a*c*d*e^2)*f)*sqrt(c^2*e^2/(c^4*
d^4*e^2 + 2*a*c^3*d^2*e^4 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (16*a^3*c*d^2 + a^4
*e^2)*f^4 - 12*(2*a^2*c^2*d^3 + a^3*c*d*e^2)*f^3 + 2*(8*a*c^3*d^4 + 11*a^2*
c^2*d^2*e^2 + a^3*c*e^4)*f^2 - 4*(c^4*d^5 + 3*a*c^3*d^3*e^2 + 2*a^2*c^2*d*e
^4)*f)))/(c^2*d^2*e^2 + a*c*e^4 - 4*a^2*d*f^3 + (8*a*c*d^2 + a^2*e^2)*f^2 -
 2*(2*c^2*d^3 + 3*a*c*d*e^2)*f)) - 2*(4*a^3*d*f^4 - (8*a^2*c*d^2 + a^3*e^2)
*f^3 + 2*(2*a*c^2*d^3 + 3*a^2*c*d*e^2)*f^2 - (a*c^2*d^2*e^2 + a^2*c*e^4)*f)
*sqrt(c^2*e^2/(c^4*d^4*e^2 + 2*a*c^3*d^2*e^4 + a^2*c^2*e^6 - 4*a^4*d*f^5 +
(16*a^3*c*d^2 + a^4*e^2)*f^4 - 12*(2*a^2*c^2*d^3 + a^3*c*d*e^2)*f^3 + 2*(8*
a*c^3*d^4 + 11*a^2*c^2*d^2*e^2 + a^3*c*e^4)*f^2 - 4*(c^4*d^5 + 3*a*c^3*d^3*
e^2 + 2*a^2*c^2*d*e^4)*f)))/x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 1.45Error index.cc index_gc
d Error: Bad Argument Value
```

maple [B] time = 0.01, size = 589, normalized size = 2.21

$$-\frac{\sqrt{2}\,\ln\left(\frac{-\frac{\left(e-\sqrt{-4df+e^2}\right)\left(x-\frac{-e+\sqrt{-4df+e^2}}{2f}\right)c}{f}+\frac{2af^2-2cdf+ce^2-\sqrt{-4df+e^2}\,ce}{f^2}+\frac{\sqrt{2}\,\sqrt{\frac{2af^2-2cdf+ce^2-\sqrt{-4df+e^2}\,ce}{f^2}}\,\sqrt{4\left(x-\frac{-e+\sqrt{-4df+e^2}}{2f}\right)^2c-\frac{4\left(e-\sqrt{-4d\cdots}\right.}{\cdots}}}{2}}{x-\frac{-e+\sqrt{-4df+e^2}}{2f}}\right.}{\sqrt{-4df+e^2}\,\sqrt{\frac{2af^2-2cdf+ce^2-\sqrt{-4df+e^2}\,ce}{f^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x)
```

```
[Out] 1/(-4*d*f+e^2)^(1/2)*2^(1/2)/((2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e
)/f^2)^(1/2)*ln((-(e+(-4*d*f+e^2)^(1/2))*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)*c
/f+(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)/f^2+1/2*2^(1/2)*((2*a*f^2
-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)
```

```
^(1/2))/f)^2*c-4*(e+(-4*d*f+e^2)^(1/2))*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)*c/
f+2*(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2))/(x+1/2*(e+(-
4*d*f+e^2)^(1/2))/f))-1/(-4*d*f+e^2)^(1/2)*2^(1/2)/((2*a*f^2-2*c*d*f+c*e^2-
(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*ln((-(e-(-4*d*f+e^2)^(1/2))*(x-1/2*(-e+(
-4*d*f+e^2)^(1/2))/f)*c/f+(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)/f^
2+1/2*2^(1/2)*((2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*(4
*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*(e-(-4*d*f+e^2)^(1/2))*(x-1/2*(-e+
(-4*d*f+e^2)^(1/2))/f)*c/f+2*(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)
/f^2)^(1/2))/(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f))
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for mo
re details)Is 4*d*f-e^2 positive, negative or zero?
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{c\,x^2+a}\,\left(f\,x^2+e\,x+d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + c*x^2)^(1/2)*(d + e*x + f*x^2)),x)
```

```
[Out] int(1/((a + c*x^2)^(1/2)*(d + e*x + f*x^2)), x)
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+cx^2}\,\left(d+ex+fx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(f*x**2+e*x+d)/(c*x**2+a)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a + c*x**2)*(d + e*x + f*x**2)), x)
```

3.68 $\int \frac{1}{x\sqrt{a+cx^2}\left(d+ex+fx^2\right)}\,dx$

Optimal. Leaf size=330

$$\frac{f\left(\sqrt{e^2-4df}+e\right)\tanh^{-1}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}d\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}-\frac{f\left(e-\sqrt{e^2-4df}\right)\tanh^{-1}\left(\frac{2af-cx\left(\sqrt{e^2\cdots}\right.}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c(\cdots}}\right.}{\sqrt{2}d\sqrt{e^2-4df}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}\cdots\right.}}$$

```
[Out] -arctanh((c*x^2+a)^(1/2)/a^(1/2))/d/a^(1/2)+1/2*f*arctanh(1/2*(2*a*f-c*x*(e
-(-4*d*f+e^2)^(1/2)))*2^(1/2)/(c*x^2+a)^(1/2)/(2*a*f^2+(-(-4*d*f+e^2)^(1/2)
*e-2*d*f+e^2)*c)^(1/2))*(e+(-4*d*f+e^2)^(1/2))/d*2^(1/2)/(-4*d*f+e^2)^(1/2)
/(2*a*f^2+(-(-4*d*f+e^2)^(1/2)*e-2*d*f+e^2)*c)^(1/2)-1/2*f*arctanh(1/2*(2*a
*f-c*x*(e+(-4*d*f+e^2)^(1/2)))*2^(1/2)/(c*x^2+a)^(1/2)/(2*a*f^2+((-4*d*f+e^
2)^(1/2)*e-2*d*f+e^2)*c)^(1/2))*(e-(-4*d*f+e^2)^(1/2))/d*2^(1/2)/(-4*d*f+e^
2)^(1/2)/(2*a*f^2+((-4*d*f+e^2)^(1/2)*e-2*d*f+e^2)*c)^(1/2)
```

Rubi [A] time = 0.82, antiderivative size = 330, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {6728, 266, 63, 208, 1034, 725, 206}

$$\frac{f\left(\sqrt{e^2-4df}+e\right)\tanh^{-1}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}d\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}-\frac{f\left(e-\sqrt{e^2-4df}\right)\tanh^{-1}\left(\frac{2af-cx\left(\sqrt{e^2\cdots}\right.}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c(\cdots}}\right.}{\sqrt{2}d\sqrt{e^2-4df}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}\cdots\right.}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[a + c*x^2]*(d + e*x + f*x^2)),x]
```

```
[Out] (f*(e + Sqrt[e^2 - 4*d*f])*ArcTanh[(2*a*f - c*(e - Sqrt[e^2 - 4*d*f])*x)/(S
qrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2
])])/(Sqrt[2]*d*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^
2 - 4*d*f])]) - (f*(e - Sqrt[e^2 - 4*d*f])*ArcTanh[(2*a*f - c*(e + Sqrt[e^2
 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])
]*Sqrt[a + c*x^2])])/(Sqrt[2]*d*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2
*d*f + e*Sqrt[e^2 - 4*d*f])]) - ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]]/(Sqrt[a]*d
)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 1034

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f
_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(
b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Dist[(2*c*g -
h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a,
 b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{x\sqrt{a+cx^2}\left(d+ex+fx^2\right)}dx = \int \left(\frac{1}{dx\sqrt{a+cx^2}} + \frac{-e-fx}{d\sqrt{a+cx^2}\left(d+ex+fx^2\right)}\right)dx$$

$$= \frac{\int \frac{1}{x\sqrt{a+cx^2}}dx}{d} + \frac{\int \frac{-e-fx}{\sqrt{a+cx^2}\left(d+ex+fx^2\right)}dx}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+cx}}dx, x, x^2\right)}{2d} - \frac{\left(f\left(1-\frac{e}{\sqrt{e^2-4df}}\right)\right)\int \frac{1}{\left(e+\sqrt{e^2-4df}+2fx\right)\sqrt{a+cx^2}}dx}{d} - \cdots$$

$$= \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{c}+\frac{x^2}{c}}dx, x, \sqrt{a+cx^2}\right)}{cd} + \frac{\left(f\left(1-\frac{e}{\sqrt{e^2-4df}}\right)\right)\text{Subst}\left(\int \frac{1}{4af^2+c\left(e+\sqrt{e^2-4\cdots}\right.}\cdots\right.}{d}$$

$$= \frac{f\left(1+\frac{e}{\sqrt{e^2-4df}}\right)\tanh^{-1}\left(\frac{2af-c\left(e-\sqrt{e^2-4df}\right)x}{\sqrt{2}\sqrt{2af^2+c\left(e^2-2df-e\sqrt{e^2-4df}\right)}\sqrt{a+cx^2}}\right)}{\sqrt{2}\,d\sqrt{2af^2+c\left(e^2-2df-e\sqrt{e^2-4df}\right)}} + \frac{f\left(1-\frac{e}{\sqrt{e^2-4df}}\right)\cdots}{\sqrt{2}\,d\sqrt{\cdots}}$$

Mathematica [A] time = 0.74, size = 319, normalized size = 0.97

$$\frac{\frac{\sqrt{2}f\left(\sqrt{e^2-4df}+e\right)\tanh^{-1}\left(\frac{2af+cx\left(\sqrt{e^2-4df}-e\right)}{\sqrt{a+cx^2}\sqrt{4af^2-2c\left(e\sqrt{e^2-4df}+2df-e^2\right)}}\right)}{\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}} + \frac{\sqrt{2}f\left(\sqrt{e^2-4df}-e\right)\tanh^{-1}\left(\frac{2af-cx\left(\sqrt{e^2-4df}+e\right)}{\sqrt{a+cx^2}\sqrt{4af^2+2c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{e^2-4df}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}} - \frac{2\tanh^{-1}\left(\cdots\right)}{\sqrt{a}\cdots}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[a + c*x^2]*(d + e*x + f*x^2)),x]
```

```
[Out] ((Sqrt[2]*f*(e + Sqrt[e^2 - 4*d*f])*ArcTanh[(2*a*f + c*(-e + Sqrt[e^2 - 4*d
*f])*x)/(Sqrt[4*a*f^2 - 2*c*(-e^2 + 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a +
c*x^2])])/(Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4
*d*f])]) + (Sqrt[2]*f*(-e + Sqrt[e^2 - 4*d*f])*ArcTanh[(2*a*f - c*(e + Sqrt
[e^2 - 4*d*f])*x)/(Sqrt[4*a*f^2 + 2*c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*
Sqrt[a + c*x^2])])/(Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqr
t[e^2 - 4*d*f])]) - (2*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])/Sqrt[a])/(2*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.02, size = 681, normalized size = 2.06

$$-\frac{2\sqrt{2}\, f \ln\left(\frac{-\frac{\left(e-\sqrt{-4df+e^2}\right)\left(x-\frac{-e+\sqrt{-4df+e^2}}{2f}\right)c}{f}+\frac{2af^2-2cdf+ce^2-\sqrt{-4df+e^2}\,ce}{f^2}+\frac{\sqrt{2}\sqrt{\frac{2af^2-2cdf+ce^2-\sqrt{-4df+e^2}\,ce}{f^2}}\sqrt{4\left(x-\frac{-e+\sqrt{-4df+e^2}}{2f}\right)^2 c-\frac{4\left(e-\sqrt{\cdots}\right.}{\cdots}}}{2}}{x-\frac{-e+\sqrt{-4df+e^2}}{2f}}\right.}{\left(-e+\sqrt{-4df+e^2}\right)\sqrt{-4df+e^2}\sqrt{\frac{2af^2-2cdf+ce^2-\sqrt{-4df+e^2}\,ce}{f^2}}\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x)
```

```
[Out] -2*f/(e+(-4*d*f+e^2)^(1/2))/(-4*d*f+e^2)^(1/2)*2^(1/2)/((2*a*f^2-2*c*d*f+c*
e^2+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*ln((-(e+(-4*d*f+e^2)^(1/2))*(x+1/2*(
e+(-4*d*f+e^2)^(1/2))/f)*c/f+(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)
/f^2+1/2*2^(1/2)*((2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)
*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*(e+(-4*d*f+e^2)^(1/2))*(x+1/2*(e
+(-4*d*f+e^2)^(1/2))/f)*c/f+2*(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e
)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))-2*f/(-e+(-4*d*f+e^2)^(1/2))
/(-4*d*f+e^2)^(1/2)*2^(1/2)/((2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)
/f^2)^(1/2)*ln((-(e-(-4*d*f+e^2)^(1/2))*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)*c
/f+(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)/f^2+1/2*2^(1/2)*((2*a*f^2
-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*(4*(x-1/2*(-e+(-4*d*f+e^2
)^(1/2))/f)^2*c-4*(e-(-4*d*f+e^2)^(1/2))*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)*
c/f+2*(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2))/(x-1/2*(-e
+(-4*d*f+e^2)^(1/2))/f))+4*f/(-e+(-4*d*f+e^2)^(1/2))/(e+(-4*d*f+e^2)^(1/2))
/a^(1/2)*ln((2*a+2*a^(1/2)*(c*x^2+a)^(1/2))/x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^2 + a}\left(fx^2 + ex + d\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^2 + a)*(f*x^2 + e*x + d)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x\,\sqrt{c\,x^2 + a}\,\left(f\,x^2 + e\,x + d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + c*x^2)^(1/2)*(d + e*x + f*x^2)),x)
```

```
[Out] int(1/(x*(a + c*x^2)^(1/2)*(d + e*x + f*x^2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{a + cx^2}\,\left(d + ex + fx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(f*x**2+e*x+d)/(c*x**2+a)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(a + c*x**2)*(d + e*x + f*x**2)), x)
```

3.69 $\int \frac{1}{x^2 \sqrt{a+cx^2} \left(d+ex+fx^2\right)} \, dx$

Optimal. Leaf size=367

$$-\frac{f\left(e\sqrt{e^2-4df}-2df+e^2\right)\tanh^{-1}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}\,d^2\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}+\frac{f\left(-e\sqrt{e^2-4df}-2df+e^2\right)\tanh^{-1}}{\sqrt{2}\,d^2\sqrt{e^2-4df}\sqrt{2af^2+}}$$

[Out] e*arctanh((c*x^2+a)^(1/2)/a^(1/2))/d^2/a^(1/2)-(c*x^2+a)^(1/2)/a/d/x-1/2*f*arctanh(1/2*(2*a*f-c*x*(e-(-4*d*f+e^2)^(1/2)))*2^(1/2)/(c*x^2+a)^(1/2)/(2*a*f^2+(-(-4*d*f+e^2)^(1/2)*e-2*d*f+e^2)*c)^(1/2))*((-4*d*f+e^2)^(1/2)*e-2*d*f+e^2)/d^2*2^(1/2)/(-4*d*f+e^2)^(1/2)/(2*a*f^2+(-(-4*d*f+e^2)^(1/2)*e-2*d*f+e^2)*c)^(1/2)+1/2*f*arctanh(1/2*(2*a*f-c*x*(e+(-4*d*f+e^2)^(1/2)))*2^(1/2)/(c*x^2+a)^(1/2)/(2*a*f^2+((-4*d*f+e^2)^(1/2)*e-2*d*f+e^2)*c)^(1/2))*(-(-4*d*f+e^2)^(1/2)*e-2*d*f+e^2)/d^2*2^(1/2)/(-4*d*f+e^2)^(1/2)/(2*a*f^2+((-4*d*f+e^2)^(1/2)*e-2*d*f+e^2)*c)^(1/2)

Rubi [A] time = 1.20, antiderivative size = 367, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {6728, 264, 266, 63, 208, 1034, 725, 206}

$$-\frac{f\left(e\sqrt{e^2-4df}-2df+e^2\right)\tanh^{-1}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}\,d^2\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}+\frac{f\left(-e\sqrt{e^2-4df}-2df+e^2\right)\tanh^{-1}}{\sqrt{2}\,d^2\sqrt{e^2-4df}\sqrt{2af^2+}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[a + c*x^2]*(d + e*x + f*x^2)),x]
```

```
[Out] -(Sqrt[a + c*x^2]/(a*d*x)) - (f*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])*ArcTanh
[(2*a*f - c*(e - Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d
*f - e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[2]*d^2*Sqrt[e^2 - 4*d*f
]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]) + (f*(e^2 - 2*d*f
- e*Sqrt[e^2 - 4*d*f])*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[
2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])
/(Sqrt[2]*d^2*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2
- 4*d*f])]) + (e*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])/(Sqrt[a]*d^2)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 1034

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f
_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(
b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Dist[(2*c*g -
h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a,
 b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{x^2\sqrt{a+cx^2}\left(d+ex+fx^2\right)}\,dx = \int \left(\frac{1}{dx^2\sqrt{a+cx^2}} - \frac{e}{d^2x\sqrt{a+cx^2}} + \frac{e^2-df+efx}{d^2\sqrt{a+cx^2}\left(d+ex+fx^2\right)}\right)dx$$

$$= \frac{\int \frac{e^2-df+efx}{\sqrt{a+cx^2}\left(d+ex+fx^2\right)}\,dx}{d^2} + \frac{\int \frac{1}{x^2\sqrt{a+cx^2}}\,dx}{d} - \frac{e\int \frac{1}{x\sqrt{a+cx^2}}\,dx}{d^2}$$

$$= -\frac{\sqrt{a+cx^2}}{adx} - \frac{e\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+cx}}\,dx, x, x^2\right)}{2d^2} - \frac{\left(f\left(e^2-2df-e\sqrt{e^2-4df}\right)\right)\int}{d^2\sqrt{e^2-4}}$$

$$= -\frac{\sqrt{a+cx^2}}{adx} - \frac{e\,\text{Subst}\left(\int \frac{1}{-\frac{a}{c}+\frac{x^2}{c}}\,dx, x, \sqrt{a+cx^2}\right)}{cd^2} + \frac{\left(f\left(e^2-2df-e\sqrt{e^2-4d\right.\right.}}{}$$

$$= -\frac{\sqrt{a+cx^2}}{adx} - \frac{f\left(e^2-2df+e\sqrt{e^2-4df}\right)\tanh^{-1}\left(\frac{2af-c\left(e-\sqrt{e^2-4df}\right)x}{\sqrt{2}\sqrt{2af^2+c\left(e^2-2df-e\sqrt{e^2-4df}\right)}}\right.}{\sqrt{2}\,d^2\sqrt{e^2-4df}\sqrt{2af^2+c\left(e^2-2df-e\sqrt{e^2-4df}\right)}}$$

Mathematica [A] time = 0.84, size = 356, normalized size = 0.97

$$-\frac{\frac{\sqrt{2}f\left(e\sqrt{e^2-4df}-2df+e^2\right)\tanh^{-1}\left(\frac{2af+cx\left(\sqrt{e^2-4df}-e\right)}{\sqrt{a+cx^2}\sqrt{4af^2-2c\left(e\sqrt{e^2-4df}+2df-e^2\right)}}\right)}{\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}+\frac{\sqrt{2}f\left(e\sqrt{e^2-4df}+2df-e^2\right)\tanh^{-1}\left(\frac{2af-cx\left(\sqrt{e^2-4df}+e\right)}{\sqrt{a+cx^2}\sqrt{4af^2+2c\left(e\sqrt{e^2-4df}-2df\cdots\right.}}\right.}{\sqrt{e^2-4df}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}}{2d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Sqrt[a + c*x^2]*(d + e*x + f*x^2)),x]
```

```
[Out] -1/2*((2*d*Sqrt[a + c*x^2])/(a*x) + (Sqrt[2]*f*(e^2 - 2*d*f + e*Sqrt[e^2 -
4*d*f])*ArcTanh[(2*a*f + c*(-e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[4*a*f^2 - 2*c*
(-e^2 + 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[e^2 - 4*d*f]
*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]) + (Sqrt[2]*f*(-e^2
+ 2*d*f + e*Sqrt[e^2 - 4*d*f])*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x
)/(Sqrt[4*a*f^2 + 2*c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])
])/(Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]
) - (2*e*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])/Sqrt[a])/d^2
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 9.45Error index.cc index_gc
d Error: Bad Argument Value
```

maple [B] time = 0.02, size = 736, normalized size = 2.01

$$\frac{16ef^2\ln\left(\frac{2a+2\sqrt{c x^2+a}\,\sqrt{a}}{x}\right)}{\left(-e+\sqrt{-4df+e^2}\right)^2\left(e+\sqrt{-4df+e^2}\right)^2\sqrt{a}}-\frac{4\sqrt{2}\,f^2\ln\left(\frac{-\frac{\left(e-\sqrt{-4df+e^2}\right)\left(x-\frac{-e+\sqrt{-4df+e^2}}{2f}\right)c}{f}+\frac{2af^2-2cdf+ce^2-\sqrt{-4df+e^2}\,ce}{f^2}+\cdots}{\cdots}\right.}{\left(-e+\sqrt{-4df+\cdots}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x)
```

```
[Out] 4*f^2/(e+(-4*d*f+e^2)^(1/2))^2/(-4*d*f+e^2)^(1/2)*2^(1/2)/((2*a*f^2-2*c*d*f
+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*ln((-(e+(-4*d*f+e^2)^(1/2))*(x+1/
```

```
2*(e+(-4*d*f+e^2)^(1/2))/f)*c/f+(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c
*e)/f^2+1/2*2^(1/2)*((2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1
/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*(e+(-4*d*f+e^2)^(1/2))*(x+1/2
*(e+(-4*d*f+e^2)^(1/2))/f)*c/f+2*(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*
c*e)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))+4*f/(-e+(-4*d*f+e^2)^(1/
2))/(e+(-4*d*f+e^2)^(1/2))/a/x*(c*x^2+a)^(1/2)-4*f^2/(-e+(-4*d*f+e^2)^(1/2)
)^2/(-4*d*f+e^2)^(1/2)*2^(1/2)/((2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c
*e)/f^2)^(1/2)*ln((-(e-(-4*d*f+e^2)^(1/2))*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f
)*c/f+(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)/f^2+1/2*2^(1/2)*((2*a*
f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*(4*(x-1/2*(-e+(-4*d*f+
e^2)^(1/2))/f)^2*c-4*(e-(-4*d*f+e^2)^(1/2))*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/
f)*c/f+2*(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2))/(x-1/2*
(-e+(-4*d*f+e^2)^(1/2))/f))+16*f^2*e/(-e+(-4*d*f+e^2)^(1/2))^2/(e+(-4*d*f+e
^2)^(1/2))^2/a^(1/2)*ln((2*a+2*(c*x^2+a)^(1/2)*a^(1/2))/x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^2 + a}\left(fx^2 + ex + d\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(c*x^2 + a)*(f*x^2 + e*x + d)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^2\,\sqrt{c\,x^2+a}\,\left(f\,x^2+e\,x+d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + c*x^2)^(1/2)*(d + e*x + f*x^2)),x)
[Out] int(1/(x^2*(a + c*x^2)^(1/2)*(d + e*x + f*x^2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{2}\sqrt{a + cx^{2}}\left(d + ex + fx^{2}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(f*x**2+e*x+d)/(c*x**2+a)**(1/2),x)
[Out] Integral(1/(x**2*sqrt(a + c*x**2)*(d + e*x + f*x**2)), x)
```

3.70 $\int \frac{1}{x^3\sqrt{a+cx^2}\left(d+ex+fx^2\right)}\,dx$

Optimal. Leaf size=457

$$\frac{c\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{2a^{3/2}d}-\frac{\left(e^2-df\right)\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}\,d^3}+\frac{f\left(-\left(e^2-df\right)\left(e-\sqrt{e^2-4df}\right)-4def+2e^3\right)\tanh^{-1}\left(\frac{}{\sqrt{2}\,\sqrt{a+c}}\right.}{\sqrt{2}\,d^3\sqrt{e^2-4df}\,\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}\right.}}$$

```
[Out] 1/2*c*arctanh((c*x^2+a)^(1/2)/a^(1/2))/a^(3/2)/d-(-d*f+e^2)*arctanh((c*x^2+
a)^(1/2)/a^(1/2))/d^3/a^(1/2)-1/2*(c*x^2+a)^(1/2)/a/d/x^2+e*(c*x^2+a)^(1/2)
/a/d^2/x+1/2*f*arctanh(1/2*(2*a*f-c*x*(e-(-4*d*f+e^2)^(1/2)))*2^(1/2)/(c*x^
2+a)^(1/2)/(2*a*f^2+(-(-4*d*f+e^2)^(1/2)*e-2*d*f+e^2)*c)^(1/2))*(2*e^3-4*d*
e*f-(-d*f+e^2)*(e-(-4*d*f+e^2)^(1/2)))/d^3*2^(1/2)/(-4*d*f+e^2)^(1/2)/(2*a*
f^2+(-(-4*d*f+e^2)^(1/2)*e-2*d*f+e^2)*c)^(1/2)-1/2*f*arctanh(1/2*(2*a*f-c*x
*(e+(-4*d*f+e^2)^(1/2)))*2^(1/2)/(c*x^2+a)^(1/2)/(2*a*f^2+((-4*d*f+e^2)^(1/
2)*e-2*d*f+e^2)*c)^(1/2))*(2*e^3-4*d*e*f-(-d*f+e^2)*(e+(-4*d*f+e^2)^(1/2)))
/d^3*2^(1/2)/(-4*d*f+e^2)^(1/2)/(2*a*f^2+((-4*d*f+e^2)^(1/2)*e-2*d*f+e^2)*c
)^(1/2)
```

Rubi [A] time = 1.86, antiderivative size = 457, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6728, 266, 51, 63, 208, 264, 1034, 725, 206}

$$\frac{c\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{2a^{3/2}d}+\frac{f\left(-\left(e^2-df\right)\left(e-\sqrt{e^2-4df}\right)-4def+2e^3\right)\tanh^{-1}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\,\sqrt{a+cx^2}\,\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}\,d^3\sqrt{e^2-4df}\,\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*Sqrt[a + c*x^2]*(d + e*x + f*x^2)),x]
```

```
[Out] -Sqrt[a + c*x^2]/(2*a*d*x^2) + (e*Sqrt[a + c*x^2])/(a*d^2*x) + (f*(2*e^3 -
4*d*e*f - (e^2 - d*f)*(e - Sqrt[e^2 - 4*d*f]))*ArcTanh[(2*a*f - c*(e - Sqrt
[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d
*f])]*Sqrt[a + c*x^2])])/(Sqrt[2]*d^3*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e
^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]) - (f*(2*e^3 - 4*d*e*f - (e^2 - d*f)*(e
+ Sqrt[e^2 - 4*d*f]))*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2
]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/
(Sqrt[2]*d^3*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 -
 4*d*f])]) + (c*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])/(2*a^(3/2)*d) - ((e^2 - d
*f)*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])/(Sqrt[a]*d^3)
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 1034

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f
_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(
b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Dist[(2*c*g -
h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a,
 b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{x^3\sqrt{a+cx^2}\left(d+ex+fx^2\right)}dx = \int \left(\frac{1}{dx^3\sqrt{a+cx^2}} - \frac{e}{d^2x^2\sqrt{a+cx^2}} + \frac{e^2-df}{d^3x\sqrt{a+cx^2}} + \frac{-e\left(e^2-2df\right)-f\left(e\right.}{d^3\sqrt{a+cx^2}\left(d+e\right.}\right.$$

$$= \frac{\int \frac{-e\left(e^2-2df\right)-f\left(e^2-df\right)x}{\sqrt{a+cx^2}\left(d+ex+fx^2\right)}dx}{d^3} + \frac{\int \frac{1}{x^3\sqrt{a+cx^2}}dx}{d} - \frac{e\int \frac{1}{x^2\sqrt{a+cx^2}}dx}{d^2} + \frac{\left(e^2-df\right)\int}{d^3}$$

$$= \frac{e\sqrt{a+cx^2}}{ad^2x} + \frac{\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+cx}}dx, x, x^2\right)}{2d} + \frac{\left(e^2-df\right)\text{Subst}\left(\int \frac{1}{x\sqrt{a+cx}}dx\right.}{2d^3}$$

$$= -\frac{\sqrt{a+cx^2}}{2adx^2} + \frac{e\sqrt{a+cx^2}}{ad^2x} - \frac{c\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+cx}}dx, x, x^2\right)}{4ad} + \frac{\left(e^2-df\right)\text{Subs}}{}$$

$$= -\frac{\sqrt{a+cx^2}}{2adx^2} + \frac{e\sqrt{a+cx^2}}{ad^2x} + \frac{f\left(2e^3-4def-\left(e^2-df\right)\left(e-\sqrt{e^2-4df}\right)\right)\text{ta}}{\sqrt{2}\,d^3\sqrt{e^2-4df}\sqrt{2af^2+c\left(\right.}}$$

$$= -\frac{\sqrt{a+cx^2}}{2adx^2} + \frac{e\sqrt{a+cx^2}}{ad^2x} + \frac{f\left(2e^3-4def-\left(e^2-df\right)\left(e-\sqrt{e^2-4df}\right)\right)\text{ta}}{\sqrt{2}\,d^3\sqrt{e^2-4df}\sqrt{2af^2+c\left(\right.}}$$

Mathematica [A] time = 1.57, size = 460, normalized size = 1.01

$$-\frac{\frac{cd^2\sqrt{a+cx^2}\left(\frac{a}{cx^2}-\frac{\tanh^{-1}\left(\sqrt{\frac{cx^2}{a}+1}\right)}{\sqrt{\frac{cx^2}{a}+1}}\right)}{a^2} + \frac{2\left(e^2-df\right)\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}} - \frac{\sqrt{2}f\left(e^2\sqrt{e^2-4df}-df\sqrt{e^2-4df}-3def+e^3\right)\tanh^{-1}\left(\frac{2af+cx\left(\sqrt{e^2-\right.}}{\sqrt{a+cx^2}\sqrt{4af^2-2c\left(e\sqrt{e\right.}}}\right.}{\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}}{2d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*Sqrt[a + c*x^2]*(d + e*x + f*x^2)),x]
```

```
[Out] -1/2*((-2*d*e*Sqrt[a + c*x^2])/(a*x) - (Sqrt[2]*f*(e^3 - 3*d*e*f + e^2*Sqrt
[e^2 - 4*d*f] - d*f*Sqrt[e^2 - 4*d*f])*ArcTanh[(2*a*f + c*(-e + Sqrt[e^2 -
4*d*f])*x)/(Sqrt[4*a*f^2 - 2*c*(-e^2 + 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a
 + c*x^2])])/(Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2
- 4*d*f])]) + (Sqrt[2]*f*(e^3 - 3*d*e*f - e^2*Sqrt[e^2 - 4*d*f] + d*f*Sqrt[
e^2 - 4*d*f])*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[4*a*f^2 +
 2*c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[e^2 - 4*
d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]) + (2*(e^2 - d*f
)*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])/Sqrt[a] + (c*d^2*Sqrt[a + c*x^2]*(a/(c*
x^2) - ArcTanh[Sqrt[1 + (c*x^2)/a]]/Sqrt[1 + (c*x^2)/a]))/a^2)/d^3
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.02, size = 911, normalized size = 1.99

$$-\frac{64 d f^4 \ln\left(\frac{2a+2\sqrt{c x^2+a}\,\sqrt{a}}{x}\right)}{\left(-e+\sqrt{-4df+e^2}\right)^3\left(e+\sqrt{-4df+e^2}\right)^3\sqrt{a}}+\frac{64 e^2 f^3 \ln\left(\frac{2a+2\sqrt{c x^2+a}\,\sqrt{a}}{x}\right)}{\left(-e+\sqrt{-4df+e^2}\right)^3\left(e+\sqrt{-4df+e^2}\right)^3\sqrt{a}}-\frac{8\sqrt{2}\, f^3 \ln\left(\frac{-\frac{(e-\sqrt{-}}{}}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x)
```

```
[Out] -8*f^3/(e+(-4*d*f+e^2)^(1/2))^3/(-4*d*f+e^2)^(1/2)*2^(1/2)/((2*a*f^2-2*c*d*
f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*ln((-(e+(-4*d*f+e^2)^(1/2))*(x+1
/2*(e+(-4*d*f+e^2)^(1/2))/f)*c/f+(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*
c*e)/f^2+1/2*2^(1/2)*((2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(
1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*(e+(-4*d*f+e^2)^(1/2))*(x+1/
2*(e+(-4*d*f+e^2)^(1/2))/f)*c/f+2*(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)
*c*e)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))+16*f^2*e/(-e+(-4*d*f+e^
2)^(1/2))^2/(e+(-4*d*f+e^2)^(1/2))^2/a/x*(c*x^2+a)^(1/2)+2*f/(-e+(-4*d*f+e^
2)^(1/2))/(e+(-4*d*f+e^2)^(1/2))/a/x^2*(c*x^2+a)^(1/2)-2*f/(-e+(-4*d*f+e^2)
^(1/2))/(e+(-4*d*f+e^2)^(1/2))*c/a^(3/2)*ln((2*a+2*(c*x^2+a)^(1/2)*a^(1/2))
/x)-8*f^3/(-e+(-4*d*f+e^2)^(1/2))^3/(-4*d*f+e^2)^(1/2)*2^(1/2)/((2*a*f^2-2*
c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*ln((-(e-(-4*d*f+e^2)^(1/2))*
(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)*c/f+(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^(
1/2)*c*e)/f^2+1/2*2^(1/2)*((2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)/f
^2)^(1/2)*(4*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*(e-(-4*d*f+e^2)^(1/2))
*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)*c/f+2*(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2
)^(1/2)*c*e)/f^2)^(1/2))/(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f))-64*f^4/(-e+(-4*
d*f+e^2)^(1/2))^3/(e+(-4*d*f+e^2)^(1/2))^3/a^(1/2)*ln((2*a+2*(c*x^2+a)^(1/2
)*a^(1/2))/x)*d+64*f^3/(-e+(-4*d*f+e^2)^(1/2))^3/(e+(-4*d*f+e^2)^(1/2))^3/a
^(1/2)*ln((2*a+2*(c*x^2+a)^(1/2)*a^(1/2))/x)*e^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^2+a}\left(fx^2+ex+d\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^2 + a)*(f*x^2 + e*x + d)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^3\,\sqrt{c\,x^2+a}\,\left(f\,x^2+e\,x+d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + c*x^2)^(1/2)*(d + e*x + f*x^2)),x)
```

```
[Out] int(1/(x^3*(a + c*x^2)^(1/2)*(d + e*x + f*x^2)), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\sqrt{a+cx^2}\,\left(d+ex+fx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(f*x**2+e*x+d)/(c*x**2+a)**(1/2),x)
```

```
[Out] Integral(1/(x**3*sqrt(a + c*x**2)*(d + e*x + f*x**2)), x)
```

3.71 $\int \frac{x^3}{\left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}\,dx$

Optimal. Leaf size=499

$$\frac{cex\left(a\left(e^2-2df\right)+cd^2\right)+af\left(a\left(e^2-df\right)+cd^2\right)}{af^2\sqrt{a+cx^2}\left((cd-af)^2+ace^2\right)}-\frac{\left(2adef-\left(e-\sqrt{e^2-4df}\right)\left(a\left(e^2-df\right)+cd^2\right)\right)\tanh^{-1}\left(\frac{}{\sqrt{2}\sqrt{a+c}}\right.}{\sqrt{2}\sqrt{e^2-4df}\left((cd-af)^2+ace^2\right)\sqrt{2af^2+c\left(-e\sqrt{e^2}\right.}}$$

```
[Out] -1/c/f/(c*x^2+a)^(1/2)-e*x/a/f^2/(c*x^2+a)^(1/2)+(a*f*(c*d^2+a*(-d*f+e^2))+
c*e*(c*d^2+a*(-2*d*f+e^2))*x)/a/f^2/(a*c*e^2+(-a*f+c*d)^2)/(c*x^2+a)^(1/2)-
1/2*arctanh(1/2*(2*a*f-c*x*(e-(-4*d*f+e^2)^(1/2)))*2^(1/2)/(c*x^2+a)^(1/2)/
(2*a*f^2+(-(-4*d*f+e^2)^(1/2)*e-2*d*f+e^2)*c)^(1/2))*(2*a*d*e*f-(c*d^2+a*(-
d*f+e^2))*(e-(-4*d*f+e^2)^(1/2)))/(a*c*e^2+(-a*f+c*d)^2)*2^(1/2)/(-4*d*f+e^
2)^(1/2)/(2*a*f^2+(-(-4*d*f+e^2)^(1/2)*e-2*d*f+e^2)*c)^(1/2)+1/2*arctanh(1/
2*(2*a*f-c*x*(e+(-4*d*f+e^2)^(1/2)))*2^(1/2)/(c*x^2+a)^(1/2)/(2*a*f^2+((-4*
d*f+e^2)^(1/2)*e-2*d*f+e^2)*c)^(1/2))*(2*a*d*e*f-(c*d^2+a*(-d*f+e^2))*(e+(-
4*d*f+e^2)^(1/2)))/(a*c*e^2+(-a*f+c*d)^2)*2^(1/2)/(-4*d*f+e^2)^(1/2)/(2*a*f
^2+((-4*d*f+e^2)^(1/2)*e-2*d*f+e^2)*c)^(1/2)
```

Rubi [A] time = 2.11, antiderivative size = 499, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {6728, 191, 261, 1017, 1034, 725, 206}

$$\frac{cex\left(a\left(e^2-2df\right)+cd^2\right)+af\left(a\left(e^2-df\right)+cd^2\right)}{af^2\sqrt{a+cx^2}\left((cd-af)^2+ace^2\right)}-\frac{\left(2adef-\left(e-\sqrt{e^2-4df}\right)\left(a\left(e^2-df\right)+cd^2\right)\right)\tanh^{-1}\left(\frac{}{\sqrt{2}\sqrt{a+c}}\right.}{\sqrt{2}\sqrt{e^2-4df}\left((cd-af)^2+ace^2\right)\sqrt{2af^2+c\left(-e\sqrt{e^2}\right.}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((a + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]
```

```
[Out] -(1/(c*f*Sqrt[a + c*x^2])) - (e*x)/(a*f^2*Sqrt[a + c*x^2]) + (a*f*(c*d^2 +
a*(e^2 - d*f)) + c*e*(c*d^2 + a*(e^2 - 2*d*f))*x)/(a*f^2*(a*c*e^2 + (c*d -
a*f)^2)*Sqrt[a + c*x^2]) - ((2*a*d*e*f - (e - Sqrt[e^2 - 4*d*f])*(c*d^2 + a
*(e^2 - d*f)))*ArcTanh[(2*a*f - c*(e - Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[
2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[2
]*Sqrt[e^2 - 4*d*f]*(a*c*e^2 + (c*d - a*f)^2)*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f
 - e*Sqrt[e^2 - 4*d*f])]) + ((2*a*d*e*f - (e + Sqrt[e^2 - 4*d*f])*(c*d^2 +
a*(e^2 - d*f)))*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt
[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[
2]*Sqrt[e^2 - 4*d*f]*(a*c*e^2 + (c*d - a*f)^2)*Sqrt[2*a*f^2 + c*(e^2 - 2*d*
f + e*Sqrt[e^2 - 4*d*f])])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 1017

```
Int[((g_.) + (h_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f
_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((a + c*x^2)^(p + 1)*(d + e*x + f*x^2)^
(q + 1)*(g*c*(2*a*c*e) + (-(a*h))*(2*c^2*d - c*(2*a*f)) + c*(g*(2*c^2*d - c
*(2*a*f)) - h*(-2*a*c*e))*x))/((-4*a*c)*(a*c*e^2 + (c*d - a*f)^2)*(p + 1)),
 x] + Dist[1/((-4*a*c)*(a*c*e^2 + (c*d - a*f)^2)*(p + 1)), Int[(a + c*x^2)^
(p + 1)*(d + e*x + f*x^2)^q*Simp[(-2*g*c)*((c*d - a*f)^2 - (-(a*e))*(c*e))*
(p + 1) + (2*(g*c*(c*d - a*f) - a*(-(h*c*e))))*(a*f*(p + 1) - c*d*(p + 2))
- e*((g*c)*(2*a*c*e) + (-(a*h))*(2*c^2*d - c*((Plus[2])*a*f)))*(p + q + 2)
- (2*f*((g*c)*(2*a*c*e) + (-(a*h))*(2*c^2*d - c*((Plus[2])*a*f)))*(p + q +
2) - (2*(g*c*(c*d - a*f) - a*(-(h*c*e))))*(-(c*e*(2*p + q + 4))))*x - c*f*(
2*(g*c*(c*d - a*f) - a*(-(h*c*e))))*(2*p + 2*q + 5)*x^2, x], x], x] /; Free
Q[{a, c, d, e, f, g, h, q}, x] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[
a*c*e^2 + (c*d - a*f)^2, 0] &&  !( !IntegerQ[p] && ILtQ[q, -1])
```

Rule 1034

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f
_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(
b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Dist[(2*c*g -
h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a,
 b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{x^3}{\left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}dx = \int \left(-\frac{e}{f^2\left(a+cx^2\right)^{3/2}} + \frac{x}{f\left(a+cx^2\right)^{3/2}} + \frac{de+\left(e^2-df\right)x}{f^2\left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}\right)dx$$

$$= \frac{\int \frac{de+\left(e^2-df\right)x}{\left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}dx}{f^2} - \frac{e\int \frac{1}{\left(a+cx^2\right)^{3/2}}dx}{f^2} + \frac{\int \frac{x}{\left(a+cx^2\right)^{3/2}}dx}{f}$$

$$= -\frac{1}{cf\sqrt{a+cx^2}} - \frac{ex}{af^2\sqrt{a+cx^2}} + \frac{af\left(cd^2+a\left(e^2-df\right)\right)+ce\left(cd^2+a\left(e^2-2a\right.\right.}{af^2\left(ace^2+(cd-af)^2\right)\sqrt{a+cx^2}}$$

$$= -\frac{1}{cf\sqrt{a+cx^2}} - \frac{ex}{af^2\sqrt{a+cx^2}} + \frac{af\left(cd^2+a\left(e^2-df\right)\right)+ce\left(cd^2+a\left(e^2-2a\right.\right.}{af^2\left(ace^2+(cd-af)^2\right)\sqrt{a+cx^2}}$$

$$= -\frac{1}{cf\sqrt{a+cx^2}} - \frac{ex}{af^2\sqrt{a+cx^2}} + \frac{af\left(cd^2+a\left(e^2-df\right)\right)+ce\left(cd^2+a\left(e^2-2a\right.\right.}{af^2\left(ace^2+(cd-af)^2\right)\sqrt{a+cx^2}}$$

$$= -\frac{1}{cf\sqrt{a+cx^2}} - \frac{ex}{af^2\sqrt{a+cx^2}} + \frac{af\left(cd^2+a\left(e^2-df\right)\right)+ce\left(cd^2+a\left(e^2-2a\right.\right.}{af^2\left(ace^2+(cd-af)^2\right)\sqrt{a+cx^2}}$$

Mathematica [A] time = 2.70, size = 577, normalized size = 1.16

$$\frac{\left(-\frac{e\left(e^2-3df\right)}{\sqrt{e^2-4df}}-df+e^2\right)\left(2af+cx\left(e-\sqrt{e^2-4df}\right)\right)}{af^2\sqrt{a+cx^2}\left(4af^2+c\left(e-\sqrt{e^2-4df}\right)^2\right)}+\frac{\left(\frac{e\left(e^2-3df\right)}{\sqrt{e^2-4df}}-df+e^2\right)\left(2af+cx\left(\sqrt{e^2-4df}+e\right)\right)}{af^2\sqrt{a+cx^2}\left(4af^2+c\left(\sqrt{e^2-4df}+e\right)^2\right)}+\frac{\sqrt{2}\left(-e^2\sqrt{\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((a + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]
```

```
[Out] -(1/(c*f*Sqrt[a + c*x^2])) - (e*x)/(a*f^2*Sqrt[a + c*x^2]) + ((e^2 - d*f -
(e*(e^2 - 3*d*f))/Sqrt[e^2 - 4*d*f])*(2*a*f + c*(e - Sqrt[e^2 - 4*d*f])*x))
/(a*f^2*(4*a*f^2 + c*(e - Sqrt[e^2 - 4*d*f])^2)*Sqrt[a + c*x^2]) + ((e^2 -
d*f + (e*(e^2 - 3*d*f))/Sqrt[e^2 - 4*d*f])*(2*a*f + c*(e + Sqrt[e^2 - 4*d*f
])*x))/(a*f^2*(4*a*f^2 + c*(e + Sqrt[e^2 - 4*d*f])^2)*Sqrt[a + c*x^2]) + (S
qrt[2]*(e^3 - 3*d*e*f - e^2*Sqrt[e^2 - 4*d*f] + d*f*Sqrt[e^2 - 4*d*f])*ArcT
anh[(2*a*f + c*(-e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[4*a*f^2 - 2*c*(-e^2 + 2*d*
f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])]])/(Sqrt[e^2 - 4*d*f]*(2*a*f^2 +
c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]))^(3/2)) - (Sqrt[2]*(e^3 - 3*d*e*f + e
^2*Sqrt[e^2 - 4*d*f] - d*f*Sqrt[e^2 - 4*d*f])*ArcTanh[(2*a*f - c*(e + Sqrt[
e^2 - 4*d*f])*x)/(Sqrt[4*a*f^2 + 2*c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*S
qrt[a + c*x^2])]])/(Sqrt[e^2 - 4*d*f]*(2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2
 - 4*d*f]))^(3/2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="giac")
[Out] sage2
```

maple [B] time = 0.03, size = 6124, normalized size = 12.27

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x)
[Out] result too large to display
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for mo
re details)Is 4*d*f-e^2 positive, negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3}{{\left(c\,x^2+a\right)}^{3/2}\,\left(f\,x^2+e\,x+d\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((a + c*x^2)^(3/2)*(d + e*x + f*x^2)),x)
[Out] int(x^3/((a + c*x^2)^(3/2)*(d + e*x + f*x^2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{3}}{\left(a + cx^{2}\right)^{\frac{3}{2}} \left(d + ex + fx^{2}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(c*x**2+a)**(3/2)/(f*x**2+e*x+d),x)
[Out] Integral(x**3/((a + c*x**2)**(3/2)*(d + e*x + f*x**2)), x)
```

3.72 $\int \frac{x^2}{\left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)} dx$

Optimal. Leaf size=410

$$-\frac{f\left(2d(cd-af)+ae\left(e-\sqrt{e^2-4df}\right)\right)\tanh^{-1}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}\sqrt{e^2-4df}\left((cd-af)^2+ace^2\right)\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}+\frac{f\left(2d(cd-af)+ae\left(\sqrt{e^2-\right.\right.}{\sqrt{2}\sqrt{e^2-4df}\left((cd-a\right.}$$

```
[Out] (-a*e-(-a*f+c*d)*x)/(a*c*e^2+(-a*f+c*d)^2)/(c*x^2+a)^(1/2)-1/2*f*arctanh(1/
2*(2*a*f-c*x*(e-(-4*d*f+e^2)^(1/2)))*2^(1/2)/(c*x^2+a)^(1/2)/(2*a*f^2+(-(-4
*d*f+e^2)^(1/2)*e-2*d*f+e^2)*c)^(1/2))*(2*d*(-a*f+c*d)+a*e*(e-(-4*d*f+e^2)^
(1/2)))/(a*c*e^2+(-a*f+c*d)^2)*2^(1/2)/(-4*d*f+e^2)^(1/2)/(2*a*f^2+(-(-4*d*
f+e^2)^(1/2)*e-2*d*f+e^2)*c)^(1/2)+1/2*f*arctanh(1/2*(2*a*f-c*x*(e+(-4*d*f+
e^2)^(1/2)))*2^(1/2)/(c*x^2+a)^(1/2)/(2*a*f^2+((-4*d*f+e^2)^(1/2)*e-2*d*f+e
^2)*c)^(1/2))*(2*d*(-a*f+c*d)+a*e*(e+(-4*d*f+e^2)^(1/2)))/(a*c*e^2+(-a*f+c*
d)^2)*2^(1/2)/(-4*d*f+e^2)^(1/2)/(2*a*f^2+((-4*d*f+e^2)^(1/2)*e-2*d*f+e^2)*
c)^(1/2)
```

Rubi [A] time = 0.71, antiderivative size = 410, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {1063, 1034, 725, 206}

$$-\frac{f\left(2d(cd-af)+ae\left(e-\sqrt{e^2-4df}\right)\right)\tanh^{-1}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}\sqrt{e^2-4df}\left((cd-af)^2+ace^2\right)\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}+\frac{f\left(2d(cd-af)+ae\left(\sqrt{e^2-\right.\right.}{\sqrt{2}\sqrt{e^2-4df}\left((cd-a\right.}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((a + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]
```

```
[Out] -((a*e + (c*d - a*f)*x)/((a*c*e^2 + (c*d - a*f)^2)*Sqrt[a + c*x^2])) - (f*(
2*d*(c*d - a*f) + a*e*(e - Sqrt[e^2 - 4*d*f]))*ArcTanh[(2*a*f - c*(e - Sqrt
[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d
*f])]*Sqrt[a + c*x^2])]])/(Sqrt[2]*Sqrt[e^2 - 4*d*f]*(a*c*e^2 + (c*d - a*f)^
2)*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]) + (f*(2*d*(c*d -
a*f) + a*e*(e + Sqrt[e^2 - 4*d*f]))*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*
f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[
a + c*x^2])]])/(Sqrt[2]*Sqrt[e^2 - 4*d*f]*(a*c*e^2 + (c*d - a*f)^2)*Sqrt[2*a
*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 1034

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f
_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(
```

```
b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Dist[(2*c*g -
h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a,
 b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1063

```
Int[((a_) + (c_.)*(x_)^2)^(p_)*((A_.) + (C_.)*(x_)^2)*((d_) + (e_.)*(x_) +
(f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((a + c*x^2)^(p + 1)*(d + e*x + f*x^2
)^(q + 1)*((A*c - a*C)*(2*a*c*e) + c*(A*(2*c^2*d - c*(2*a*f)) + C*(-2*a*(c*
d - a*f)))*x))/((-4*a*c)*(a*c*e^2 + (c*d - a*f)^2)*(p + 1)), x] + Dist[1/((
-4*a*c)*(a*c*e^2 + (c*d - a*f)^2)*(p + 1)), Int[(a + c*x^2)^(p + 1)*(d + e*
x + f*x^2)^q*Simp[(-2*A*c - 2*a*C)*((c*d - a*f)^2 - (-(a*e))*(c*e))*(p + 1)
 + (2*(A*c*(c*d - a*f) - a*(c*C*d - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) -
e*((A*c - a*C)*(2*a*c*e))*(p + q + 2) - (2*f*((A*c - a*C)*(2*a*c*e))*(p + q
 + 2) - (2*(A*c*(c*d - a*f) - a*(c*C*d - a*C*f)))*(-(c*e*(2*p + q + 4))))*x
 - c*f*(2*(A*c*(c*d - a*f) - a*(c*C*d - a*C*f)))*(2*p + 2*q + 5)*x^2, x], x
], x] /; FreeQ[{a, c, d, e, f, A, C, q}, x] && NeQ[e^2 - 4*d*f, 0] && LtQ[p
, -1] && NeQ[a*c*e^2 + (c*d - a*f)^2, 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]
) &&  !IGtQ[q, 0]
```

Rubi steps

$$\int \frac{x^2}{\left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}\,dx = -\frac{ae+(cd-af)x}{\left(ace^2+(cd-af)^2\right)\sqrt{a+cx^2}} + \frac{\int \frac{2acd(cd-af)-2a^2cefx}{\sqrt{a+cx^2}\left(d+ex+fx^2\right)}\,dx}{2ac\left(ace^2+(cd-af)^2\right)}$$

$$= -\frac{ae+(cd-af)x}{\left(ace^2+(cd-af)^2\right)\sqrt{a+cx^2}} + \frac{\left(f\left(2d(cd-af)+ae\left(e-\sqrt{e^2-4df}\right)\right)\right)}{\sqrt{e^2-4df}\left(ace^2+(c\right.}$$

$$= -\frac{ae+(cd-af)x}{\left(ace^2+(cd-af)^2\right)\sqrt{a+cx^2}} - \frac{\left(f\left(2d(cd-af)+ae\left(e-\sqrt{e^2-4df}\right)\right)\right)}{\sqrt{e^2-4}}$$

$$= -\frac{ae+(cd-af)x}{\left(ace^2+(cd-af)^2\right)\sqrt{a+cx^2}} - \frac{f\left(2d(cd-af)+ae\left(e-\sqrt{e^2-4df}\right)\right)\text{ta}}{\sqrt{2}\sqrt{e^2-4df}\left(ace^2+(cd-af)^2\right)}$$

Mathematica [A] time = 2.07, size = 509, normalized size = 1.24

$$\frac{-\frac{\left(\frac{2df-e^2}{\sqrt{e^2-4df}}+e\right)\left(2af+cx\left(e-\sqrt{e^2-4df}\right)\right)}{a\sqrt{a+cx^2}\left(4af^2+c\left(e-\sqrt{e^2-4df}\right)^2\right)} - \frac{\left(\frac{e^2-2df}{\sqrt{e^2-4df}}+e\right)\left(2af+cx\left(\sqrt{e^2-4df}+e\right)\right)}{a\sqrt{a+cx^2}\left(4af^2+c\left(\sqrt{e^2-4df}+e\right)^2\right)} + \frac{\sqrt{2}f^2\left(e\sqrt{e^2-4df}+2df-e^2\right)\tanh^{-1}\left(\frac{2af+cx\left(\sqrt{e^2-4df}\right.}{\sqrt{a+cx^2}\sqrt{4af^2-2c\left(e\sqrt{e^2-4\right.}}}\right.}{\sqrt{e^2-4df}\left(2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)\right)^{3/2}}}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((a + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]
```

```
[Out] (x/(a*Sqrt[a + c*x^2]) - ((e + (-e^2 + 2*d*f)/Sqrt[e^2 - 4*d*f])*(2*a*f + c
*(e - Sqrt[e^2 - 4*d*f])*x))/(a*(4*a*f^2 + c*(e - Sqrt[e^2 - 4*d*f])^2)*Sqr
t[a + c*x^2]) - ((e + (e^2 - 2*d*f)/Sqrt[e^2 - 4*d*f])*(2*a*f + c*(e + Sqrt
[e^2 - 4*d*f])*x))/(a*(4*a*f^2 + c*(e + Sqrt[e^2 - 4*d*f])^2)*Sqrt[a + c*x^
2]) + (Sqrt[2]*f^2*(-e^2 + 2*d*f + e*Sqrt[e^2 - 4*d*f])*ArcTanh[(2*a*f + c*
```

```
(-e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[4*a*f^2 - 2*c*(-e^2 + 2*d*f + e*Sqrt[e^2
- 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[e^2 - 4*d*f]*(2*a*f^2 + c*(e^2 - 2*d*f
- e*Sqrt[e^2 - 4*d*f]))^(3/2)) + (Sqrt[2]*f^2*(e^2 - 2*d*f + e*Sqrt[e^2 - 4
*d*f])*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[4*a*f^2 + 2*c*(e
^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[e^2 - 4*d*f]*(2
*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]))^(3/2)))/f
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$sage_2$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] sage2
```

maple [B] time = 0.02, size = 4752, normalized size = 11.59

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x)
```

```
[Out] 1/f*x/a/(c*x^2+a)^(1/2)-1/(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)/((
x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-(e+(-4*d*f+e^2)^(1/2))*(x+1/2*(e+(-4*d*
f+e^2)^(1/2))/f)*c/f+1/2*(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)/f^2
)^(1/2)*e+2/(-4*d*f+e^2)^(1/2)*f/(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*
c*e)/((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-(e+(-4*d*f+e^2)^(1/2))*(x+1/2*(e
+(-4*d*f+e^2)^(1/2))/f)*c/f+1/2*(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c
*e)/f^2)^(1/2)*d-1/(-4*d*f+e^2)^(1/2)/(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(
1/2)*c*e)/((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-(e+(-4*d*f+e^2)^(1/2))*(x+1
/2*(e+(-4*d*f+e^2)^(1/2))/f)*c/f+1/2*(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1
/2)*c*e)/f^2)^(1/2)*e^2-2*(-4*d*f+e^2)^(1/2)/f*c^2/(2*a*f^2-2*c*d*f+c*e^2+(
-4*d*f+e^2)^(1/2)*c*e)/(4*a*c-4*c^2/f*d+c^2/f^2*e^2-c^2/f^2*(-4*d*f+e^2))/(
(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-(e+(-4*d*f+e^2)^(1/2))*(x+1/2*(e+(-4*d
*f+e^2)^(1/2))/f)*c/f+1/2*(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)/f^
2)^(1/2)*x*e+4*c^2/(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)/(4*a*c-4*
c^2/f*d+c^2/f^2*e^2-c^2/f^2*(-4*d*f+e^2))/((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)
^2*c-(e+(-4*d*f+e^2)^(1/2))*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)*c/f+1/2*(2*a*f
^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*x*d-4/f*c^2/(2*a*f^2-2*
c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)/(4*a*c-4*c^2/f*d+c^2/f^2*e^2-c^2/f^2*(-
4*d*f+e^2))/((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-(e+(-4*d*f+e^2)^(1/2))*(x
+1/2*(e+(-4*d*f+e^2)^(1/2))/f)*c/f+1/2*(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^
(1/2)*c*e)/f^2)^(1/2)*x*e^2+4/(-4*d*f+e^2)^(1/2)*c^2/(2*a*f^2-2*c*d*f+c*e^2
+(-4*d*f+e^2)^(1/2)*c*e)/(4*a*c-4*c^2/f*d+c^2/f^2*e^2-c^2/f^2*(-4*d*f+e^2))
/((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-(e+(-4*d*f+e^2)^(1/2))*(x+1/2*(e+(-4
*d*f+e^2)^(1/2))/f)*c/f+1/2*(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)/
f^2)^(1/2)*e*x*d-2/(-4*d*f+e^2)^(1/2)/f*c^2/(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+
e^2)^(1/2)*c*e)/(4*a*c-4*c^2/f*d+c^2/f^2*e^2-c^2/f^2*(-4*d*f+e^2))/((x+1/2*
(e+(-4*d*f+e^2)^(1/2))/f)^2*c-(e+(-4*d*f+e^2)^(1/2))*(x+1/2*(e+(-4*d*f+e^2)
```

```
^(1/2))/f)*c/f+1/2*(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2
)*e^3*x+1/(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)*2^(1/2)/((2*a*f^2-
2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*ln((-(e+(-4*d*f+e^2)^(1/2)
)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)*c/f+(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^
(1/2)*c*e)/f^2+1/2*2^(1/2)*((2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)/
f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*(e+(-4*d*f+e^2)^(1/2))
*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)*c/f+2*(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)
^(1/2)*c*e)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))*e-2/(-4*d*f+e^2)^
(1/2)*f/(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)*2^(1/2)/((2*a*f^2-2*
c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*ln((-(e+(-4*d*f+e^2)^(1/2))*
(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)*c/f+(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1
/2)*c*e)/f^2+1/2*2^(1/2)*((2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)/f^
2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*(e+(-4*d*f+e^2)^(1/2))*(
x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)*c/f+2*(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(
1/2)*c*e)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))*d+1/(-4*d*f+e^2)^(1
/2)/(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)*2^(1/2)/((2*a*f^2-2*c*d*
f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*ln((-(e+(-4*d*f+e^2)^(1/2))*(x+1
/2*(e+(-4*d*f+e^2)^(1/2))/f)*c/f+(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*
c*e)/f^2+1/2*2^(1/2)*((2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(
1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*(e+(-4*d*f+e^2)^(1/2))*(x+1/
2*(e+(-4*d*f+e^2)^(1/2))/f)*c/f+2*(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)
*c*e)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))*e^2-1/(2*a*f^2-2*c*d*f+
c*e^2-(-4*d*f+e^2)^(1/2)*c*e)/((x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-(e-(-4
*d*f+e^2)^(1/2))*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)*c/f+1/2*(2*a*f^2-2*c*d*f
+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*e-2/(-4*d*f+e^2)^(1/2)*f/(2*a*f^2
-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)/((x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2
*c-(e-(-4*d*f+e^2)^(1/2))*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)*c/f+1/2*(2*a*f^
2-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*d+1/(-4*d*f+e^2)^(1/2)/(
2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)/((x-1/2*(-e+(-4*d*f+e^2)^(1/2
))/f)^2*c-(e-(-4*d*f+e^2)^(1/2))*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)*c/f+1/2*
(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*e^2+2*(-4*d*f+e^2
)^(1/2)/f*c^2/(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)/(4*a*c-4*c^2/f
*d+c^2/f^2*e^2-c^2/f^2*(-4*d*f+e^2))/((x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c
-(e-(-4*d*f+e^2)^(1/2))*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)*c/f+1/2*(2*a*f^2-
2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*x*e+4*c^2/(2*a*f^2-2*c*d*f
+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)/(4*a*c-4*c^2/f*d+c^2/f^2*e^2-c^2/f^2*(-4*d*f
+e^2))/((x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-(e-(-4*d*f+e^2)^(1/2))*(x-1/2
*(-e+(-4*d*f+e^2)^(1/2))/f)*c/f+1/2*(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/
2)*c*e)/f^2)^(1/2)*x*d-4/f*c^2/(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c*
e)/(4*a*c-4*c^2/f*d+c^2/f^2*e^2-c^2/f^2*(-4*d*f+e^2))/((x-1/2*(-e+(-4*d*f+e
^2)^(1/2))/f)^2*c-(e-(-4*d*f+e^2)^(1/2))*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)*
c/f+1/2*(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*x*e^2-4/(
-4*d*f+e^2)^(1/2)*c^2/(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)/(4*a*c
-4*c^2/f*d+c^2/f^2*e^2-c^2/f^2*(-4*d*f+e^2))/((x-1/2*(-e+(-4*d*f+e^2)^(1/2)
)/f)^2*c-(e-(-4*d*f+e^2)^(1/2))*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)*c/f+1/2*(
2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*e*x*d+2/(-4*d*f+e^
2)^(1/2)/f*c^2/(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)/(4*a*c-4*c^2/
f*d+c^2/f^2*e^2-c^2/f^2*(-4*d*f+e^2))/((x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*
c-(e-(-4*d*f+e^2)^(1/2))*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)*c/f+1/2*(2*a*f^2
-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*e^3*x+1/(2*a*f^2-2*c*d*f+
c*e^2-(-4*d*f+e^2)^(1/2)*c*e)*2^(1/2)/((2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^
(1/2)*c*e)/f^2)^(1/2)*ln((-(e-(-4*d*f+e^2)^(1/2))*(x-1/2*(-e+(-4*d*f+e^2)^(
1/2))/f)*c/f+(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)/f^2+1/2*2^(1/2)
*((2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*(4*(x-1/2*(-e+(
-4*d*f+e^2)^(1/2))/f)^2*c-4*(e-(-4*d*f+e^2)^(1/2))*(x-1/2*(-e+(-4*d*f+e^2)^
(1/2))/f)*c/f+2*(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2))/
(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f))*e+2/(-4*d*f+e^2)^(1/2)*f/(2*a*f^2-2*c*d*
f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)*2^(1/2)/((2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2
)^(1/2)*c*e)/f^2)^(1/2)*ln((-(e-(-4*d*f+e^2)^(1/2))*(x-1/2*(-e+(-4*d*f+e^2)
```

```
^(1/2))/f)*c/f+(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)/f^2+1/2*2^(1/
2)*((2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*(4*(x-1/2*(-e
+(-4*d*f+e^2)^(1/2))/f)^2*c-4*(e-(-4*d*f+e^2)^(1/2))*(x-1/2*(-e+(-4*d*f+e^2
)^(1/2))/f)*c/f+2*(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)
)/(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f))*d-1/(-4*d*f+e^2)^(1/2)/(2*a*f^2-2*c*d*
f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)*2^(1/2)/((2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2
)^(1/2)*c*e)/f^2)^(1/2)*ln((-(e-(-4*d*f+e^2)^(1/2))*(x-1/2*(-e+(-4*d*f+e^2)
^(1/2))/f)*c/f+(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)/f^2+1/2*2^(1/
2)*((2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*(4*(x-1/2*(-e
+(-4*d*f+e^2)^(1/2))/f)^2*c-4*(e-(-4*d*f+e^2)^(1/2))*(x-1/2*(-e+(-4*d*f+e^2
)^(1/2))/f)*c/f+2*(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)
)/(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f))*e^2
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for mo
re details)Is 4*d*f-e^2 positive, negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2}{\left(c\,x^2+a\right)^{3/2}\,\left(f\,x^2+e\,x+d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a + c*x^2)^(3/2)*(d + e*x + f*x^2)),x)
```

```
[Out] int(x^2/((a + c*x^2)^(3/2)*(d + e*x + f*x^2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2}}{\left(a + cx^{2}\right)^{\frac{3}{2}} \left(d + ex + fx^{2}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(c*x**2+a)**(3/2)/(f*x**2+e*x+d),x)
```

```
[Out] Integral(x**2/((a + c*x**2)**(3/2)*(d + e*x + f*x**2)), x)
```

3.73 $\int \frac{x}{\left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)} dx$

Optimal. Leaf size=411

$$\frac{f\left(2cde-\left(e-\sqrt{e^2-4df}\right)(cd-af)\right)\tanh^{-1}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}\sqrt{e^2-4df}\left((cd-af)^2+ace^2\right)\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}-\frac{f\left(2cde-\left(\sqrt{e^2-4df}+e\right)\right.}{\sqrt{2}\sqrt{e^2-4df}\left((cd-\right.}$$

```
[Out] (c*e*x+a*f-c*d)/(a*c*e^2+(-a*f+c*d)^2)/(c*x^2+a)^(1/2)+1/2*f*arctanh(1/2*(2
*a*f-c*x*(e-(-4*d*f+e^2)^(1/2)))*2^(1/2)/(c*x^2+a)^(1/2)/(2*a*f^2+(-(-4*d*f
+e^2)^(1/2)*e-2*d*f+e^2)*c)^(1/2))*(2*c*d*e-(-a*f+c*d)*(e-(-4*d*f+e^2)^(1/2
)))/(a*c*e^2+(-a*f+c*d)^2)*2^(1/2)/(-4*d*f+e^2)^(1/2)/(2*a*f^2+(-(-4*d*f+e^
2)^(1/2)*e-2*d*f+e^2)*c)^(1/2)-1/2*f*arctanh(1/2*(2*a*f-c*x*(e+(-4*d*f+e^2)
^(1/2)))*2^(1/2)/(c*x^2+a)^(1/2)/(2*a*f^2+((-4*d*f+e^2)^(1/2)*e-2*d*f+e^2)*
c)^(1/2))*(2*c*d*e-(-a*f+c*d)*(e+(-4*d*f+e^2)^(1/2)))/(a*c*e^2+(-a*f+c*d)^2
)*2^(1/2)/(-4*d*f+e^2)^(1/2)/(2*a*f^2+((-4*d*f+e^2)^(1/2)*e-2*d*f+e^2)*c)^(
1/2)
```

Rubi [A] time = 0.83, antiderivative size = 411, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {1017, 1034, 725, 206}

$$\frac{f\left(2cde-\left(e-\sqrt{e^2-4df}\right)(cd-af)\right)\tanh^{-1}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}\sqrt{e^2-4df}\left((cd-af)^2+ace^2\right)\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}-\frac{f\left(2cde-\left(\sqrt{e^2-4df}+e\right)\right.}{\sqrt{2}\sqrt{e^2-4df}\left((cd-\right.}$$

Antiderivative was successfully verified.

```
[In] Int[x/((a + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]
```

```
[Out] -((c*d - a*f - c*e*x)/((a*c*e^2 + (c*d - a*f)^2)*Sqrt[a + c*x^2])) + (f*(2*
c*d*e - (c*d - a*f)*(e - Sqrt[e^2 - 4*d*f]))*ArcTanh[(2*a*f - c*(e - Sqrt[e
^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f
])]*Sqrt[a + c*x^2])])/(Sqrt[2]*Sqrt[e^2 - 4*d*f]*(a*c*e^2 + (c*d - a*f)^2)
*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]) - (f*(2*c*d*e - (c*
d - a*f)*(e + Sqrt[e^2 - 4*d*f]))*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f]
)*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a
+ c*x^2])])/(Sqrt[2]*Sqrt[e^2 - 4*d*f]*(a*c*e^2 + (c*d - a*f)^2)*Sqrt[2*a*f
^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 1017

```
Int[((g_.) + (h_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f
_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((a + c*x^2)^(p + 1)*(d + e*x + f*x^2)^
(q + 1)*(g*c*(2*a*c*e) + (-(a*h))*(2*c^2*d - c*(2*a*f)) + c*(g*(2*c^2*d - c
```

```
*(2*a*f)) - h*(-2*a*c*e))*x))/((-4*a*c)*(a*c*e^2 + (c*d - a*f)^2)*(p + 1)),
 x] + Dist[1/((-4*a*c)*(a*c*e^2 + (c*d - a*f)^2)*(p + 1)), Int[(a + c*x^2)^
(p + 1)*(d + e*x + f*x^2)^q*Simp[(-2*g*c)*((c*d - a*f)^2 - (-(a*e))*(c*e))*
(p + 1) + (2*(g*c*(c*d - a*f) - a*(-(h*c*e))))*(a*f*(p + 1) - c*d*(p + 2))
- e*((g*c)*(2*a*c*e) + (-(a*h))*(2*c^2*d - c*((Plus[2])*a*f)))*(p + q + 2)
- (2*f*((g*c)*(2*a*c*e) + (-(a*h))*(2*c^2*d - c*((Plus[2])*a*f)))*(p + q +
2) - (2*(g*c*(c*d - a*f) - a*(-(h*c*e))))*(-(c*e*(2*p + q + 4))))*x - c*f*(
2*(g*c*(c*d - a*f) - a*(-(h*c*e))))*(2*p + 2*q + 5)*x^2, x], x], x] /; Free
Q[{a, c, d, e, f, g, h, q}, x] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[
a*c*e^2 + (c*d - a*f)^2, 0] &&  !( !IntegerQ[p] && ILtQ[q, -1])
```

Rule 1034

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f
_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(
b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Dist[(2*c*g -
h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a,
 b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x}{\left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}dx = -\frac{cd-af-cex}{\left(ace^2+(cd-af)^2\right)\sqrt{a+cx^2}}+\frac{\int \frac{-2ac^2de-2acf(cd-af)x}{\sqrt{a+cx^2}\left(d+ex+fx^2\right)}dx}{2ac\left(ace^2+(cd-af)^2\right)}$$

$$= -\frac{cd-af-cex}{\left(ace^2+(cd-af)^2\right)\sqrt{a+cx^2}}-\frac{\left(f\left(2cde-(cd-af)\left(e-\sqrt{e^2-4df}\right)\right)\right)\int \frac{}{(}}{\sqrt{e^2-4df}\left(ace^2+(cd-\right.}$$

$$= -\frac{cd-af-cex}{\left(ace^2+(cd-af)^2\right)\sqrt{a+cx^2}}+\frac{\left(f\left(2cde-(cd-af)\left(e-\sqrt{e^2-4df}\right)\right)\right)\text{Sul}}{\sqrt{e^2-4df}}$$

$$= -\frac{cd-af-cex}{\left(ace^2+(cd-af)^2\right)\sqrt{a+cx^2}}+\frac{f\left(2cde-(cd-af)\left(e-\sqrt{e^2-4df}\right)\right)\tanh}{\sqrt{2}\sqrt{e^2-4df}\left(ace^2+(cd-af)^2\right)\sqrt{2}}$$

Mathematica [A] time = 0.81, size = 457, normalized size = 1.11

$$\frac{\left(1-\frac{e}{\sqrt{e^2-4df}}\right)\left(2af+cx\left(e-\sqrt{e^2-4df}\right)\right)}{a\sqrt{a+cx^2}\left(4af^2+c\left(e-\sqrt{e^2-4df}\right)^2\right)}+\frac{\left(\frac{e}{\sqrt{e^2-4df}}+1\right)\left(2af+cx\left(\sqrt{e^2-4df}+e\right)\right)}{a\sqrt{a+cx^2}\left(4af^2+c\left(\sqrt{e^2-4df}+e\right)^2\right)}+\frac{\sqrt{2}f^2\left(e-\sqrt{e^2-4df}\right)\text{t}}{\sqrt{e^2-4df}\left(2af^2\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((a + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]
```

```
[Out] ((1 - e/Sqrt[e^2 - 4*d*f])*(2*a*f + c*(e - Sqrt[e^2 - 4*d*f])*x))/(a*(4*a*f
^2 + c*(e - Sqrt[e^2 - 4*d*f])^2)*Sqrt[a + c*x^2]) + ((1 + e/Sqrt[e^2 - 4*d
*f])*(2*a*f + c*(e + Sqrt[e^2 - 4*d*f])*x))/(a*(4*a*f^2 + c*(e + Sqrt[e^2 -
 4*d*f])^2)*Sqrt[a + c*x^2]) + (Sqrt[2]*f^2*(e - Sqrt[e^2 - 4*d*f])*ArcTanh
[(2*a*f + c*(-e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[4*a*f^2 - 2*c*(-e^2 + 2*d*f +
 e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[e^2 - 4*d*f]*(2*a*f^2 + c*(
e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]))^(3/2)) - (Sqrt[2]*f^2*(e + Sqrt[e^2 - 4
```

```
*d*f])*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[4*a*f^2 + 2*c*(e
^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])]])/(Sqrt[e^2 - 4*d*f]*(2
*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]))^(3/2))
```

fricas [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [**F**] time = 0.00, size = 0, normalized size = 0.00

$sage_2$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] sage2
```

maple [**B**] time = 0.02, size = 3000, normalized size = 7.30

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x)
```

```
[Out] 1/(-4*d*f+e^2)^(1/2)*f*e/(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)/((x
+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-(e+(-4*d*f+e^2)^(1/2))*(x+1/2*(e+(-4*d*f
+e^2)^(1/2))/f)*c/f+1/2*(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)/f^2)
^(1/2)+4*e*c^2/(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)/(4*a*c-4*c^2/
f*d+c^2/f^2*e^2-c^2/f^2*(-4*d*f+e^2))/((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c
-(e+(-4*d*f+e^2)^(1/2))*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)*c/f+1/2*(2*a*f^2-2
*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*x+2/(-4*d*f+e^2)^(1/2)*c^2/
(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)/(4*a*c-4*c^2/f*d+c^2/f^2*e^2
-c^2/f^2*(-4*d*f+e^2))/((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-(e+(-4*d*f+e^2
)^(1/2))*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)*c/f+1/2*(2*a*f^2-2*c*d*f+c*e^2+(-
4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*e^2*x-1/(-4*d*f+e^2)^(1/2)*f*e/(2*a*f^2-2*
c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)*2^(1/2)/((2*a*f^2-2*c*d*f+c*e^2+(-4*d*f
+e^2)^(1/2)*c*e)/f^2)^(1/2)*ln((-(e+(-4*d*f+e^2)^(1/2))*(x+1/2*(e+(-4*d*f+e
^2)^(1/2))/f)*c/f+(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)/f^2+1/2*2^
(1/2)*((2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*(4*(x+1/2*
(e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*(e+(-4*d*f+e^2)^(1/2))*(x+1/2*(e+(-4*d*f+e^
2)^(1/2))/f)*c/f+2*(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2
))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))+f/(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^
(1/2)*c*e)/((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-(e+(-4*d*f+e^2)^(1/2))*(x+
1/2*(e+(-4*d*f+e^2)^(1/2))/f)*c/f+1/2*(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(
1/2)*c*e)/f^2)^(1/2)+2*(-4*d*f+e^2)^(1/2)*c^2/(2*a*f^2-2*c*d*f+c*e^2+(-4*d*
f+e^2)^(1/2)*c*e)/(4*a*c-4*c^2/f*d+c^2/f^2*e^2-c^2/f^2*(-4*d*f+e^2))/((x+1/
2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-(e+(-4*d*f+e^2)^(1/2))*(x+1/2*(e+(-4*d*f+e^
2)^(1/2))/f)*c/f+1/2*(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1
/2)*x-f/(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)*2^(1/2)/((2*a*f^2-2*
c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*ln((-(e+(-4*d*f+e^2)^(1/2))*
(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)*c/f+(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1
/2)*c*e)/f^2+1/2*2^(1/2)*((2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)/f^
2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*(e+(-4*d*f+e^2)^(1/2))*(
x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)*c/f+2*(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(
```

```
1/2)*c*e)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))+f/(2*a*f^2-2*c*d*f+
c*e^2-(-4*d*f+e^2)^(1/2)*c*e)/((x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-(e-(-4
*d*f+e^2)^(1/2))*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)*c/f+1/2*(2*a*f^2-2*c*d*f
+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)-2*(-4*d*f+e^2)^(1/2)*c^2/(2*a*f^2
-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)/(4*a*c-4*c^2/f*d+c^2/f^2*e^2-c^2/f^2
*(-4*d*f+e^2))/((x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-(e-(-4*d*f+e^2)^(1/2)
)*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)*c/f+1/2*(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+
e^2)^(1/2)*c*e)/f^2)^(1/2)*x+4*c^2/(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2
)*c*e)/(4*a*c-4*c^2/f*d+c^2/f^2*e^2-c^2/f^2*(-4*d*f+e^2))/((x-1/2*(-e+(-4*d
*f+e^2)^(1/2))/f)^2*c-(e-(-4*d*f+e^2)^(1/2))*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))
/f)*c/f+1/2*(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*e*x-f
/(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)*2^(1/2)/((2*a*f^2-2*c*d*f+c
*e^2-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*ln((-(e-(-4*d*f+e^2)^(1/2))*(x-1/2*
(-e+(-4*d*f+e^2)^(1/2))/f)*c/f+(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c*
e)/f^2+1/2*2^(1/2)*((2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/
2)*(4*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*(e-(-4*d*f+e^2)^(1/2))*(x-1/2
*(-e+(-4*d*f+e^2)^(1/2))/f)*c/f+2*(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)
*c*e)/f^2)^(1/2))/(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f))-1/(-4*d*f+e^2)^(1/2)*f
*e/(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)/((x-1/2*(-e+(-4*d*f+e^2)^
(1/2))/f)^2*c-(e-(-4*d*f+e^2)^(1/2))*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)*c/f+
1/2*(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)-2/(-4*d*f+e^2
)^(1/2)*c^2/(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)/(4*a*c-4*c^2/f*d
+c^2/f^2*e^2-c^2/f^2*(-4*d*f+e^2))/((x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-(
e-(-4*d*f+e^2)^(1/2))*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)*c/f+1/2*(2*a*f^2-2*
c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*e^2*x+1/(-4*d*f+e^2)^(1/2)*f
*e/(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)*2^(1/2)/((2*a*f^2-2*c*d*f
+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*ln((-(e-(-4*d*f+e^2)^(1/2))*(x-1/
2*(-e+(-4*d*f+e^2)^(1/2))/f)*c/f+(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*
c*e)/f^2+1/2*2^(1/2)*((2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(
1/2)*(4*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*(e-(-4*d*f+e^2)^(1/2))*(x-1
/2*(-e+(-4*d*f+e^2)^(1/2))/f)*c/f+2*(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/
2)*c*e)/f^2)^(1/2))/(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for mo
re details)Is 4*d*f-e^2 positive, negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x}{\left(c\,x^2+a\right)^{3/2}\,\left(f\,x^2+e\,x+d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a + c*x^2)^(3/2)*(d + e*x + f*x^2)),x)
```

```
[Out] int(x/((a + c*x^2)^(3/2)*(d + e*x + f*x^2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(a + cx^{2}\right)^{\frac{3}{2}} \left(d + ex + fx^{2}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x**2+a)**(3/2)/(f*x**2+e*x+d),x)
```

```
[Out] Integral(x/((a + c*x**2)**(3/2)*(d + e*x + f*x**2)), x)
```

3.74 $\int \frac{1}{\left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}\,dx$

Optimal. Leaf size=416

$$-\frac{f\left(2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)\right)\tanh^{-1}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}\sqrt{e^2-4df}\left((cd-af)^2+ace^2\right)\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}+\frac{f\left(2af^2+c\left(-e\sqrt{e^2-4df}\right.\right.}{\sqrt{2}\sqrt{e^2-4df}\left((cd-\right.}$$

```
[Out] c*(a*e+(-a*f+c*d)*x)/a/(a*c*e^2+(-a*f+c*d)^2)/(c*x^2+a)^(1/2)-1/2*f*arctanh
(1/2*(2*a*f-c*x*(e-(-4*d*f+e^2)^(1/2)))*2^(1/2)/(c*x^2+a)^(1/2)/(2*a*f^2+(-
(-4*d*f+e^2)^(1/2)*e-2*d*f+e^2)*c)^(1/2))*(2*a*f^2+((-4*d*f+e^2)^(1/2)*e-2*
d*f+e^2)*c)/(a*c*e^2+(-a*f+c*d)^2)*2^(1/2)/(-4*d*f+e^2)^(1/2)/(2*a*f^2+(-(-
4*d*f+e^2)^(1/2)*e-2*d*f+e^2)*c)^(1/2)+1/2*f*arctanh(1/2*(2*a*f-c*x*(e+(-4*
d*f+e^2)^(1/2)))*2^(1/2)/(c*x^2+a)^(1/2)/(2*a*f^2+((-4*d*f+e^2)^(1/2)*e-2*d
*f+e^2)*c)^(1/2))*(2*a*f^2+(-(-4*d*f+e^2)^(1/2)*e-2*d*f+e^2)*c)/(a*c*e^2+(-
a*f+c*d)^2)*2^(1/2)/(-4*d*f+e^2)^(1/2)/(2*a*f^2+((-4*d*f+e^2)^(1/2)*e-2*d*f
+e^2)*c)^(1/2)
```

Rubi [A] time = 0.62, antiderivative size = 416, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {976, 1034, 725, 206}

$$-\frac{f\left(2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)\right)\tanh^{-1}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}\sqrt{e^2-4df}\left((cd-af)^2+ace^2\right)\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}+\frac{f\left(2af^2+c\left(-e\sqrt{e^2-4df}\right.\right.}{\sqrt{2}\sqrt{e^2-4df}\left((cd-\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]
```

```
[Out] (c*(a*e + (c*d - a*f)*x))/(a*(a*c*e^2 + (c*d - a*f)^2)*Sqrt[a + c*x^2]) - (
f*(2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]))*ArcTanh[(2*a*f - c*(e -
 Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2
- 4*d*f])]*Sqrt[a + c*x^2])]])/(Sqrt[2]*Sqrt[e^2 - 4*d*f]*(a*c*e^2 + (c*d -
a*f)^2)*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]) + (f*(2*a*f^
2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]))*ArcTanh[(2*a*f - c*(e + Sqrt[e^2
 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])
]*Sqrt[a + c*x^2])]])/(Sqrt[2]*Sqrt[e^2 - 4*d*f]*(a*c*e^2 + (c*d - a*f)^2)*S
qrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 976

```
Int[((a_.) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x
_Symbol] :> Simp[((2*a*c^2*e + c*(2*c^2*d - c*(2*a*f))*x)*(a + c*x^2)^(p +
1)*(d + e*x + f*x^2)^(q + 1))/((-4*a*c)*(a*c*e^2 + (c*d - a*f)^2)*(p + 1)),
```

```
x] - Dist[1/((-4*a*c)*(a*c*e^2 + (c*d - a*f)^2)*(p + 1)), Int[(a + c*x^2)^
(p + 1)*(d + e*x + f*x^2)^q*Simp[2*c*((c*d - a*f)^2 - (-(a*e))*(c*e))*(p +
1) - (2*c^2*d - c*(2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(-2*a*c^2*e)*(p
+ q + 2) + (2*f*(2*a*c^2*e)*(p + q + 2) - (2*c^2*d - c*(2*a*f))*(-(c*e*(2*p
 + q + 4))))*x + c*f*(2*c^2*d - c*(2*a*f))*(2*p + 2*q + 5)*x^2, x], x], x]
/; FreeQ[{a, c, d, e, f, q}, x] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ
[a*c*e^2 + (c*d - a*f)^2, 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[
q, 0]
```

Rule 1034

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f
_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(
b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Dist[(2*c*g -
h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a,
 b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{\left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}\,dx = \frac{c(ae+(cd-af)x)}{a\left(ace^2+(cd-af)^2\right)\sqrt{a+cx^2}} - \frac{\int \frac{-2ac\left(af^2+c\left(e^2-df\right)\right)-2ac^2efx}{\sqrt{a+cx^2}\left(d+ex+fx^2\right)}\,dx}{2ac\left(ace^2+(cd-af)^2\right)}$$

$$= \frac{c(ae+(cd-af)x)}{a\left(ace^2+(cd-af)^2\right)\sqrt{a+cx^2}} - \frac{\left(f\left(2af^2+c\left(e^2-2df-e\sqrt{e^2-4df}\right)\right)\right)}{\sqrt{e^2-4df}\left(ace^2+(c\cdots\right.}$$

$$= \frac{c(ae+(cd-af)x)}{a\left(ace^2+(cd-af)^2\right)\sqrt{a+cx^2}} + \frac{\left(f\left(2af^2+c\left(e^2-2df-e\sqrt{e^2-4df}\right)\right)\right)}{\sqrt{e^2-4\cdots}}$$

$$= \frac{c(ae+(cd-af)x)}{a\left(ace^2+(cd-af)^2\right)\sqrt{a+cx^2}} - \frac{f\left(2af^2+c\left(e^2-2df+e\sqrt{e^2-4df}\right)\right)\,\mathrm{ta}\cdots}{\sqrt{2}\sqrt{e^2-4df}\left(ace^2+(cd-af)^2\right)}$$

Mathematica [A] time = 2.06, size = 320, normalized size = 0.77

$$\frac{c(a(e-fx)+cdx)}{a\sqrt{a+cx^2}\left(a^2f^2+ac\left(e^2-2df\right)+c^2d^2\right)} - \frac{2\sqrt{2}f^3\tanh^{-1}\left(\frac{2af+cx\left(\sqrt{e^2-4df}-e\right)}{\sqrt{a+cx^2}\sqrt{4af^2-2c\left(e\sqrt{e^2-4df}+2df-e^2\right)}}\right)}{\sqrt{e^2-4df}\left(2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)\right)^{3/2}} + \frac{2\sqrt{2}f^3\tanh\cdots}{\sqrt{e^2-4df}\left(2\cdots\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]
```

```
[Out] (c*(c*d*x + a*(e - f*x)))/(a*(c^2*d^2 + a^2*f^2 + a*c*(e^2 - 2*d*f))*Sqrt[a
 + c*x^2]) - (2*Sqrt[2]*f^3*ArcTanh[(2*a*f + c*(-e + Sqrt[e^2 - 4*d*f])*x)/
(Sqrt[4*a*f^2 - 2*c*(-e^2 + 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])]
)/(Sqrt[e^2 - 4*d*f]*(2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]))^(3/2
)) + (2*Sqrt[2]*f^3*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[4*a
*f^2 + 2*c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[e^
2 - 4*d*f]*(2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]))^(3/2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

sage$_2$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] sage2
```

maple [B] time = 0.02, size = 1713, normalized size = 4.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x)
```

```
[Out] -2/(-4*d*f+e^2)^(1/2)/(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)*f^2/((
x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-(e+(-4*d*f+e^2)^(1/2))*(x+1/2*(e+(-4*d*
f+e^2)^(1/2))/f)*c/f+1/2*(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)/f^2
)^(1/2)-4*c^2*f/(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)/(4*a*c-4*c^2
/f*d+c^2/f^2*e^2-c^2/f^2*(-4*d*f+e^2))/((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*
c-(e+(-4*d*f+e^2)^(1/2))*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)*c/f+1/2*(2*a*f^2-
2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*x-4/(-4*d*f+e^2)^(1/2)*c^2
*f/(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)/(4*a*c-4*c^2/f*d+c^2/f^2*
e^2-c^2/f^2*(-4*d*f+e^2))/((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-(e+(-4*d*f+
e^2)^(1/2))*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)*c/f+1/2*(2*a*f^2-2*c*d*f+c*e^2
+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*e*x+2/(-4*d*f+e^2)^(1/2)/(2*a*f^2-2*c*d
*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)*f^2*2^(1/2)/((2*a*f^2-2*c*d*f+c*e^2+(-4*d*
f+e^2)^(1/2)*c*e)/f^2)^(1/2)*ln((-(e+(-4*d*f+e^2)^(1/2))*(x+1/2*(e+(-4*d*f+
e^2)^(1/2))/f)*c/f+(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)/f^2+1/2*2
^(1/2)*((2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*(4*(x+1/2
*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*(e+(-4*d*f+e^2)^(1/2))*(x+1/2*(e+(-4*d*f+e
^2)^(1/2))/f)*c/f+2*(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/
2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))+2/(-4*d*f+e^2)^(1/2)/(2*a*f^2-2*c*d*f
+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)*f^2/((x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-(
e-(-4*d*f+e^2)^(1/2))*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)*c/f+1/2*(2*a*f^2-2*
c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)-4*c^2*f/(2*a*f^2-2*c*d*f+c*e
^2-(-4*d*f+e^2)^(1/2)*c*e)/(4*a*c-4*c^2/f*d+c^2/f^2*e^2-c^2/f^2*(-4*d*f+e^2
))/((x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-(e-(-4*d*f+e^2)^(1/2))*(x-1/2*(-e
+(-4*d*f+e^2)^(1/2))/f)*c/f+1/2*(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c
*e)/f^2)^(1/2)*x+4/(-4*d*f+e^2)^(1/2)*c^2*f/(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+
e^2)^(1/2)*c*e)/(4*a*c-4*c^2/f*d+c^2/f^2*e^2-c^2/f^2*(-4*d*f+e^2))/((x-1/2*
(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-(e-(-4*d*f+e^2)^(1/2))*(x-1/2*(-e+(-4*d*f+e^
2)^(1/2))/f)*c/f+1/2*(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1
/2)*e*x-2/(-4*d*f+e^2)^(1/2)/(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)
*f^2*2^(1/2)/((2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*ln(
(-(e-(-4*d*f+e^2)^(1/2))*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)*c/f+(2*a*f^2-2*c
*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)/f^2+1/2*2^(1/2)*((2*a*f^2-2*c*d*f+c*e^2-
(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*(4*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c
-4*(e-(-4*d*f+e^2)^(1/2))*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)*c/f+2*(2*a*f^2-
2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2))/(x-1/2*(-e+(-4*d*f+e^2)^(
1/2))/f))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for mo
re details)Is 4*d*f-e^2 positive, negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(c\,x^2+a\right)^{3/2}\,\left(f\,x^2+e\,x+d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + c*x^2)^(3/2)*(d + e*x + f*x^2)),x)
```

```
[Out] int(1/((a + c*x^2)^(3/2)*(d + e*x + f*x^2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+cx^2\right)^{\frac{3}{2}}\left(d+ex+fx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**2+a)**(3/2)/(f*x**2+e*x+d),x)
```

```
[Out] Integral(1/((a + c*x**2)**(3/2)*(d + e*x + f*x**2)), x)
```

3.75 $\int \frac{1}{x\left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}\,dx$

Optimal. Leaf size=526

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{a^{3/2}d}-\frac{a\left(af^2+c\left(e^2-df\right)\right)+c^2dex}{ad\sqrt{a+cx^2}\left((cd-af)^2+ace^2\right)}+\frac{f\left(2e\left(af^2+c\left(e^2-2df\right)\right)-\left(e-\sqrt{e^2-4df}\right)\left(af^2+c\left(e^2-\right.\right.\right.}{\sqrt{2}\,d\sqrt{e^2-4df}\left((cd-af)^2+ace^2\right)\sqrt{2a}}$$

```
[Out] -arctanh((c*x^2+a)^(1/2)/a^(1/2))/a^(3/2)/d+1/a/d/(c*x^2+a)^(1/2)+(-a*(a*f^
2+c*(-d*f+e^2))-c^2*d*e*x)/a/d/(a*c*e^2+(-a*f+c*d)^2)/(c*x^2+a)^(1/2)+1/2*f
*arctanh(1/2*(2*a*f-c*x*(e-(-4*d*f+e^2)^(1/2)))*2^(1/2)/(c*x^2+a)^(1/2)/(2*
a*f^2+(-(-4*d*f+e^2)^(1/2)*e-2*d*f+e^2)*c)^(1/2))*(2*e*(a*f^2+c*(-2*d*f+e^2
))-(a*f^2+c*(-d*f+e^2))*(e-(-4*d*f+e^2)^(1/2)))/d/(a*c*e^2+(-a*f+c*d)^2)*2^
(1/2)/(-4*d*f+e^2)^(1/2)/(2*a*f^2+(-(-4*d*f+e^2)^(1/2)*e-2*d*f+e^2)*c)^(1/2
)-1/2*f*arctanh(1/2*(2*a*f-c*x*(e+(-4*d*f+e^2)^(1/2)))*2^(1/2)/(c*x^2+a)^(1
/2)/(2*a*f^2+((-4*d*f+e^2)^(1/2)*e-2*d*f+e^2)*c)^(1/2))*(2*e*(a*f^2+c*(-2*d
*f+e^2))-(a*f^2+c*(-d*f+e^2))*(e+(-4*d*f+e^2)^(1/2)))/d/(a*c*e^2+(-a*f+c*d)
^2)*2^(1/2)/(-4*d*f+e^2)^(1/2)/(2*a*f^2+((-4*d*f+e^2)^(1/2)*e-2*d*f+e^2)*c)
^(1/2)
```

Rubi [A] time = 2.18, antiderivative size = 526, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6728, 266, 51, 63, 208, 1017, 1034, 725, 206}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{a^{3/2}d}-\frac{a\left(af^2+c\left(e^2-df\right)\right)+c^2dex}{ad\sqrt{a+cx^2}\left((cd-af)^2+ace^2\right)}+\frac{f\left(2e\left(af^2+c\left(e^2-2df\right)\right)-\left(e-\sqrt{e^2-4df}\right)\left(af^2+c\left(e^2-\right.\right.\right.}{\sqrt{2}\,d\sqrt{e^2-4df}\left((cd-af)^2+ace^2\right)\sqrt{2a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]
```

```
[Out] 1/(a*d*Sqrt[a + c*x^2]) - (a*(a*f^2 + c*(e^2 - d*f)) + c^2*d*e*x)/(a*d*(a*c
*e^2 + (c*d - a*f)^2)*Sqrt[a + c*x^2]) + (f*(2*e*(a*f^2 + c*(e^2 - 2*d*f))
- (e - Sqrt[e^2 - 4*d*f])*(a*f^2 + c*(e^2 - d*f)))*ArcTanh[(2*a*f - c*(e -
Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 -
 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[2]*d*Sqrt[e^2 - 4*d*f]*(a*c*e^2 + (c*d -
 a*f)^2)*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]) - (f*(2*e*(
a*f^2 + c*(e^2 - 2*d*f)) - (e + Sqrt[e^2 - 4*d*f])*(a*f^2 + c*(e^2 - d*f)))
*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e
^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[2]*d*Sqrt[e^2 -
 4*d*f]*(a*c*e^2 + (c*d - a*f)^2)*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^
2 - 4*d*f])]) - ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]]/(a^(3/2)*d)
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 1017

```
Int[((g_.) + (h_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f
_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((a + c*x^2)^(p + 1)*(d + e*x + f*x^2)^
(q + 1)*(g*c*(2*a*c*e) + (-(a*h))*(2*c^2*d - c*(2*a*f)) + c*(g*(2*c^2*d - c
*(2*a*f)) - h*(-2*a*c*e))*x))/((-4*a*c)*(a*c*e^2 + (c*d - a*f)^2)*(p + 1)),
 x] + Dist[1/((-4*a*c)*(a*c*e^2 + (c*d - a*f)^2)*(p + 1)), Int[(a + c*x^2)^
(p + 1)*(d + e*x + f*x^2)^q*Simp[(-2*g*c)*((c*d - a*f)^2 - (-(a*e))*(c*e))*
(p + 1) + (2*(g*c*(c*d - a*f) - a*(-(h*c*e))))*(a*f*(p + 1) - c*d*(p + 2))
- e*((g*c)*(2*a*c*e) + (-(a*h))*(2*c^2*d - c*((Plus[2])*a*f)))*(p + q + 2)
- (2*f*((g*c)*(2*a*c*e) + (-(a*h))*(2*c^2*d - c*((Plus[2])*a*f)))*(p + q +
2) - (2*(g*c*(c*d - a*f) - a*(-(h*c*e))))*(-(c*e*(2*p + q + 4))))*x - c*f*(
2*(g*c*(c*d - a*f) - a*(-(h*c*e))))*(2*p + 2*q + 5)*x^2, x], x], x] /; Free
Q[{a, c, d, e, f, g, h, q}, x] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[
a*c*e^2 + (c*d - a*f)^2, 0] &&  !( !IntegerQ[p] && ILtQ[q, -1])
```

Rule 1034

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f
_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(
b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Dist[(2*c*g -
h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a,
 b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{x\left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}\,dx = \int \left(\frac{1}{dx\left(a+cx^2\right)^{3/2}} + \frac{-e-fx}{d\left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}\right)dx$$

$$= \frac{\int \frac{1}{x\left(a+cx^2\right)^{3/2}}\,dx}{d} + \frac{\int \frac{-e-fx}{\left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}\,dx}{d}$$

$$= -\frac{a\left(af^2+c\left(e^2-df\right)\right)+c^2dex}{ad\left(ace^2+(cd-af)^2\right)\sqrt{a+cx^2}} + \frac{\text{Subst}\left(\int \frac{1}{x(a+cx)^{3/2}}\,dx, x, x^2\right)}{2d} + \frac{\int \frac{-2ace(\cdots}{\cdots}}{\cdots}$$

$$= \frac{1}{ad\sqrt{a+cx^2}} - \frac{a\left(af^2+c\left(e^2-df\right)\right)+c^2dex}{ad\left(ace^2+(cd-af)^2\right)\sqrt{a+cx^2}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+cx}}\,dx, x, x^{\cdots}\right.}{2ad}$$

$$= \frac{1}{ad\sqrt{a+cx^2}} - \frac{a\left(af^2+c\left(e^2-df\right)\right)+c^2dex}{ad\left(ace^2+(cd-af)^2\right)\sqrt{a+cx^2}} + \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{c}+\frac{x^2}{c}}\,dx, x, \sqrt{\cdots}\right.}{acd}$$

$$= \frac{1}{ad\sqrt{a+cx^2}} - \frac{a\left(af^2+c\left(e^2-df\right)\right)+c^2dex}{ad\left(ace^2+(cd-af)^2\right)\sqrt{a+cx^2}} + \frac{f\left(2e\left(af^2+c\left(e^2-2df\right)\cdots\right.\right.}{\sqrt{2}\,d\sqrt{\cdots}}$$

Mathematica [C] time = 3.50, size = 497, normalized size = 0.94

$$\frac{-\frac{f\left(\frac{e}{\sqrt{e^2-4df}}+1\right)\left(2af+cx\left(e-\sqrt{e^2-4df}\right)\right)}{a\sqrt{a+cx^2}\left(4af^2+c\left(e-\sqrt{e^2-4df}\right)^2\right)} - \frac{f\left(1-\frac{e}{\sqrt{e^2-4df}}\right)\left(2af+cx\left(\sqrt{e^2-4df}+e\right)\right)}{a\sqrt{a+cx^2}\left(4af^2+c\left(\sqrt{e^2-4df}+e\right)^2\right)} + \frac{\sqrt{2}f^3\left(\sqrt{e^2-4df}+e\right)\tanh^{-1}\left(\frac{2af+cx\left(\sqrt{e^2-4df}-e\right)}{\sqrt{a+cx^2}\sqrt{4af^2-2c\left(e\sqrt{e^2-4df}+2df-\cdots\right)}}\right.}{\sqrt{e^2-4df}\left(2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)\right)^{3/2}}\cdots}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]
```

```
[Out] (-((f*(1 + e/Sqrt[e^2 - 4*d*f])*(2*a*f + c*(e - Sqrt[e^2 - 4*d*f])*x))/(a*(
4*a*f^2 + c*(e - Sqrt[e^2 - 4*d*f])^2)*Sqrt[a + c*x^2])) - (f*(1 - e/Sqrt[e
^2 - 4*d*f])*(2*a*f + c*(e + Sqrt[e^2 - 4*d*f])*x))/(a*(4*a*f^2 + c*(e + Sq
rt[e^2 - 4*d*f])^2)*Sqrt[a + c*x^2]) + (Sqrt[2]*f^3*(e + Sqrt[e^2 - 4*d*f])
*ArcTanh[(2*a*f + c*(-e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[4*a*f^2 - 2*c*(-e^2 +
 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])]])/(Sqrt[e^2 - 4*d*f]*(2*a*f
^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]))^(3/2)) + (Sqrt[2]*f^3*(-e + Sqr
t[e^2 - 4*d*f])*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[4*a*f^2
 + 2*c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])]])/(Sqrt[e^2 -
4*d*f]*(2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]))^(3/2)) + Hypergeom
etric2F1[-1/2, 1, 1/2, 1 + (c*x^2)/a]/(a*Sqrt[a + c*x^2]))/d
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.02, size = 1945, normalized size = 3.70

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x)
```

```
[Out] 4*f^3/(e+(-4*d*f+e^2)^(1/2))/(-4*d*f+e^2)^(1/2)/(2*a*f^2-2*c*d*f+c*e^2+(-4*
d*f+e^2)^(1/2)*c*e)/((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-(e+(-4*d*f+e^2)^(
1/2))*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)*c/f+1/2*(2*a*f^2-2*c*d*f+c*e^2+(-4*d
*f+e^2)^(1/2)*c*e)/f^2)^(1/2)+8*f^2/(e+(-4*d*f+e^2)^(1/2))*c^2/(2*a*f^2-2*c
*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)/(4*a*c-4*c^2*d/f+c^2*e^2/f^2-(-4*d*f+e^2
)*c^2/f^2)/((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-(e+(-4*d*f+e^2)^(1/2))*(x+
1/2*(e+(-4*d*f+e^2)^(1/2))/f)*c/f+1/2*(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(
1/2)*c*e)/f^2)^(1/2)*x+8*f^2/(e+(-4*d*f+e^2)^(1/2))/(-4*d*f+e^2)^(1/2)*c^2/
(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)/(4*a*c-4*c^2*d/f+c^2*e^2/f^2
-(-4*d*f+e^2)*c^2/f^2)/((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-(e+(-4*d*f+e^2
)^(1/2))*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)*c/f+1/2*(2*a*f^2-2*c*d*f+c*e^2+(-
4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*e*x-4*f^3/(e+(-4*d*f+e^2)^(1/2))/(-4*d*f+e
^2)^(1/2)/(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)*2^(1/2)/((2*a*f^2-
2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*ln((-(e+(-4*d*f+e^2)^(1/2)
)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)*c/f+(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^
(1/2)*c*e)/f^2+1/2*2^(1/2)*((2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)/
f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*(e+(-4*d*f+e^2)^(1/2))
*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)*c/f+2*(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)
^(1/2)*c*e)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))+4*f^3/(-e+(-4*d*f
+e^2)^(1/2))/(-4*d*f+e^2)^(1/2)/(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c
*e)/((x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-(e-(-4*d*f+e^2)^(1/2))*(x-1/2*(-
e+(-4*d*f+e^2)^(1/2))/f)*c/f+1/2*(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*
c*e)/f^2)^(1/2)-8*f^2/(-e+(-4*d*f+e^2)^(1/2))*c^2/(2*a*f^2-2*c*d*f+c*e^2-(-
4*d*f+e^2)^(1/2)*c*e)/(4*a*c-4*c^2*d/f+c^2*e^2/f^2-(-4*d*f+e^2)*c^2/f^2)/((
x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-(e-(-4*d*f+e^2)^(1/2))*(x-1/2*(-e+(-4*
d*f+e^2)^(1/2))/f)*c/f+1/2*(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)/f
^2)^(1/2)*x+8*f^2/(-e+(-4*d*f+e^2)^(1/2))/(-4*d*f+e^2)^(1/2)*c^2/(2*a*f^2-2
*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)/(4*a*c-4*c^2*d/f+c^2*e^2/f^2-(-4*d*f+e
^2)*c^2/f^2)/((x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-(e-(-4*d*f+e^2)^(1/2))*
(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)*c/f+1/2*(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^
2)^(1/2)*c*e)/f^2)^(1/2)*e*x-4*f^3/(-e+(-4*d*f+e^2)^(1/2))/(-4*d*f+e^2)^(1/
2)/(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)*2^(1/2)/((2*a*f^2-2*c*d*f
+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*ln((-(e-(-4*d*f+e^2)^(1/2))*(x-1/
2*(-e+(-4*d*f+e^2)^(1/2))/f)*c/f+(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*
c*e)/f^2+1/2*2^(1/2)*((2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(
1/2)*(4*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*(e-(-4*d*f+e^2)^(1/2))*(x-1
/2*(-e+(-4*d*f+e^2)^(1/2))/f)*c/f+2*(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/
2)*c*e)/f^2)^(1/2))/(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f))-4*f/(-e+(-4*d*f+e^2)
^(1/2))/(e+(-4*d*f+e^2)^(1/2))/a/(c*x^2+a)^(1/2)+4*f/(-e+(-4*d*f+e^2)^(1/2)
)/(e+(-4*d*f+e^2)^(1/2))/a^(3/2)*ln((2*a+2*(c*x^2+a)^(1/2)*a^(1/2))/x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(cx^2+a\right)^{\frac{3}{2}}\left(fx^2+ex+d\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^2 + a)^(3/2)*(f*x^2 + e*x + d)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x\left(c\,x^2+a\right)^{3/2}\,\left(f\,x^2+e\,x+d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + c*x^2)^(3/2)*(d + e*x + f*x^2)),x)
```

```
[Out] int(1/(x*(a + c*x^2)^(3/2)*(d + e*x + f*x^2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(a+cx^2\right)^{\frac{3}{2}}\left(d+ex+fx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x**2+a)**(3/2)/(f*x**2+e*x+d),x)
```

```
[Out] Integral(1/(x*(a + c*x**2)**(3/2)*(d + e*x + f*x**2)), x)
```

3.76 $\int \frac{1}{x^2(a+cx^2)^{3/2}(d+ex+fx^2)}\,dx$

Optimal. Leaf size=618

$$\frac{e\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{a^{3/2}d^2}-\frac{2cx}{a^2d\sqrt{a+cx^2}}+\frac{cdx\left(af^2+c\left(e^2-df\right)\right)+ae\left(af^2+c\left(e^2-2df\right)\right)}{ad^2\sqrt{a+cx^2}\left((cd-af)^2+ace^2\right)}+\frac{f\left(e\left(e-\sqrt{e^2-4df}\right)\left(a\right.\right.}{}$$

```
[Out] e*arctanh((c*x^2+a)^(1/2)/a^(1/2))/a^(3/2)/d^2-e/a/d^2/(c*x^2+a)^(1/2)-1/a/
d/x/(c*x^2+a)^(1/2)-2*c*x/a^2/d/(c*x^2+a)^(1/2)+(a*e*(a*f^2+c*(-2*d*f+e^2))
+c*d*(a*f^2+c*(-d*f+e^2))*x)/a/d^2/(a*c*e^2+(-a*f+c*d)^2)/(c*x^2+a)^(1/2)+1
/2*f*arctanh(1/2*(2*a*f-c*x*(e-(-4*d*f+e^2)^(1/2)))*2^(1/2)/(c*x^2+a)^(1/2)
/(2*a*f^2+(-(-4*d*f+e^2)^(1/2)*e-2*d*f+e^2)*c)^(1/2))*(-2*a*f^2*(-d*f+e^2)-
2*c*(d^2*f^2-3*d*e^2*f+e^4)+e*(a*f^2+c*(-2*d*f+e^2))*(e-(-4*d*f+e^2)^(1/2))
)/d^2/(a*c*e^2+(-a*f+c*d)^2)*2^(1/2)/(-4*d*f+e^2)^(1/2)/(2*a*f^2+(-(-4*d*f+
e^2)^(1/2)*e-2*d*f+e^2)*c)^(1/2)-1/2*f*arctanh(1/2*(2*a*f-c*x*(e+(-4*d*f+e^
2)^(1/2)))*2^(1/2)/(c*x^2+a)^(1/2)/(2*a*f^2+((-4*d*f+e^2)^(1/2)*e-2*d*f+e^2
)*c)^(1/2))*(-2*a*f^2*(-d*f+e^2)-2*c*(d^2*f^2-3*d*e^2*f+e^4)+e*(a*f^2+c*(-2
*d*f+e^2))*(e+(-4*d*f+e^2)^(1/2)))/d^2/(a*c*e^2+(-a*f+c*d)^2)*2^(1/2)/(-4*d
*f+e^2)^(1/2)/(2*a*f^2+((-4*d*f+e^2)^(1/2)*e-2*d*f+e^2)*c)^(1/2)
```

Rubi [A] time = 2.28, antiderivative size = 618, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.407, Rules used = {6728, 271, 191, 266, 51, 63, 208, 1017, 1034, 725, 206}

$$\frac{e\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{a^{3/2}d^2}-\frac{2cx}{a^2d\sqrt{a+cx^2}}+\frac{cdx\left(af^2+c\left(e^2-df\right)\right)+ae\left(af^2+c\left(e^2-2df\right)\right)}{ad^2\sqrt{a+cx^2}\left((cd-af)^2+ace^2\right)}+\frac{f\left(e\left(e-\sqrt{e^2-4df}\right)\left(a\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]
```

```
[Out] -(e/(a*d^2*Sqrt[a + c*x^2])) - 1/(a*d*x*Sqrt[a + c*x^2]) - (2*c*x)/(a^2*d*S
qrt[a + c*x^2]) + (a*e*(a*f^2 + c*(e^2 - 2*d*f)) + c*d*(a*f^2 + c*(e^2 - d*
f))*x)/(a*d^2*(a*c*e^2 + (c*d - a*f)^2)*Sqrt[a + c*x^2]) + (f*(e*(e - Sqrt[
e^2 - 4*d*f])*(a*f^2 + c*(e^2 - 2*d*f)) - 2*(a*f^2*(e^2 - d*f) + c*(e^4 - 3
*d*e^2*f + d^2*f^2)))*ArcTanh[(2*a*f - c*(e - Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2
]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/
(Sqrt[2]*d^2*Sqrt[e^2 - 4*d*f]*(a*c*e^2 + (c*d - a*f)^2)*Sqrt[2*a*f^2 + c*(
e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]) - (f*(e*(e + Sqrt[e^2 - 4*d*f])*(a*f^2
 + c*(e^2 - 2*d*f)) - 2*(a*f^2*(e^2 - d*f) + c*(e^4 - 3*d*e^2*f + d^2*f^2))
)*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(
e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[2]*d^2*Sqrt[e^
2 - 4*d*f]*(a*c*e^2 + (c*d - a*f)^2)*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt
[e^2 - 4*d*f])]) + (e*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])/(a^(3/2)*d^2)
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 1017

```
Int[((g_.) + (h_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f
_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((a + c*x^2)^(p + 1)*(d + e*x + f*x^2)^
(q + 1)*(g*c*(2*a*c*e) + (-(a*h))*(2*c^2*d - c*(2*a*f)) + c*(g*(2*c^2*d - c
*(2*a*f)) - h*(-2*a*c*e))*x))/((-4*a*c)*(a*c*e^2 + (c*d - a*f)^2)*(p + 1)),
 x] + Dist[1/((-4*a*c)*(a*c*e^2 + (c*d - a*f)^2)*(p + 1)), Int[(a + c*x^2)^
(p + 1)*(d + e*x + f*x^2)^q*Simp[(-2*g*c)*((c*d - a*f)^2 - (-(a*e))*(c*e))*
(p + 1) + (2*(g*c*(c*d - a*f) - a*(-(h*c*e))))*(a*f*(p + 1) - c*d*(p + 2))
- e*((g*c)*(2*a*c*e) + (-(a*h))*(2*c^2*d - c*((Plus[2])*a*f)))*(p + q + 2)
- (2*f*((g*c)*(2*a*c*e) + (-(a*h))*(2*c^2*d - c*((Plus[2])*a*f)))*(p + q +
2) - (2*(g*c*(c*d - a*f) - a*(-(h*c*e))))*(-(c*e*(2*p + q + 4))))*x - c*f*(
2*(g*c*(c*d - a*f) - a*(-(h*c*e))))*(2*p + 2*q + 5)*x^2, x], x], x] /; Free
Q[{a, c, d, e, f, g, h, q}, x] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[
a*c*e^2 + (c*d - a*f)^2, 0] &&  !( !IntegerQ[p] && ILtQ[q, -1])
```

Rule 1034

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f
_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(
b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Dist[(2*c*g -
h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a,
 b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{x^2\left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}\,dx = \int \left(\frac{1}{dx^2\left(a+cx^2\right)^{3/2}} - \frac{e}{d^2x\left(a+cx^2\right)^{3/2}} + \frac{e^2-df+efx}{d^2\left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}\right.$$

$$= \frac{\int \frac{e^2-df+efx}{\left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}\,dx}{d^2} + \frac{\int \frac{1}{x^2\left(a+cx^2\right)^{3/2}}\,dx}{d} - \frac{e\int \frac{1}{x\left(a+cx^2\right)^{3/2}}\,dx}{d^2}$$

$$= -\frac{1}{adx\sqrt{a+cx^2}} + \frac{ae\left(af^2+c\left(e^2-2df\right)\right)+cd\left(af^2+c\left(e^2-df\right)\right)x}{ad^2\left(ace^2+(cd-af)^2\right)\sqrt{a+cx^2}} - \cdots$$

$$= -\frac{e}{ad^2\sqrt{a+cx^2}} - \frac{1}{adx\sqrt{a+cx^2}} - \frac{2cx}{a^2d\sqrt{a+cx^2}} + \frac{ae\left(af^2+c\left(e^2-2d\right.\right.}{ad^2\left(ace^2+\right.}$$

$$= -\frac{e}{ad^2\sqrt{a+cx^2}} - \frac{1}{adx\sqrt{a+cx^2}} - \frac{2cx}{a^2d\sqrt{a+cx^2}} + \frac{ae\left(af^2+c\left(e^2-2d\right.\right.}{ad^2\left(ace^2+\right.}$$

$$= -\frac{e}{ad^2\sqrt{a+cx^2}} - \frac{1}{adx\sqrt{a+cx^2}} - \frac{2cx}{a^2d\sqrt{a+cx^2}} + \frac{ae\left(af^2+c\left(e^2-2d\right.\right.}{ad^2\left(ace^2+\right.}$$

Mathematica [C] time = 3.60, size = 557, normalized size = 0.90

$$-\frac{\frac{d\left(a+2cx^2\right)}{a^2x\sqrt{a+cx^2}} - \frac{f\left(\frac{e^2-2df}{\sqrt{e^2-4df}}+e\right)\left(2af+cx\left(e-\sqrt{e^2-4df}\right)\right)}{a\sqrt{a+cx^2}\left(4af^2+c\left(e-\sqrt{e^2-4df}\right)^2\right)} - \frac{f\left(\frac{2df-e^2}{\sqrt{e^2-4df}}+e\right)\left(2af+cx\left(\sqrt{e^2-4df}+e\right)\right)}{a\sqrt{a+cx^2}\left(4af^2+c\left(\sqrt{e^2-4df}+e\right)^2\right)} + \frac{\sqrt{2}f^3\left(e\sqrt{e^2-4df}-2df+e^2\right)\tanh^{-1}\left(\cdots\right.}{\sqrt{e^2-4df}\left(2af^2+c\left(-e\sqrt{e^2-\cdots}\right.\right.}}{d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]
```

```
[Out] -((-((f*(e + (e^2 - 2*d*f)/Sqrt[e^2 - 4*d*f])*(2*a*f + c*(e - Sqrt[e^2 - 4*
d*f])*x))/(a*(4*a*f^2 + c*(e - Sqrt[e^2 - 4*d*f])^2)*Sqrt[a + c*x^2])) - (f
*(e + (-e^2 + 2*d*f)/Sqrt[e^2 - 4*d*f])*(2*a*f + c*(e + Sqrt[e^2 - 4*d*f])*
x))/(a*(4*a*f^2 + c*(e + Sqrt[e^2 - 4*d*f])^2)*Sqrt[a + c*x^2]) + (d*(a + 2
*c*x^2))/(a^2*x*Sqrt[a + c*x^2]) + (Sqrt[2]*f^3*(e^2 - 2*d*f + e*Sqrt[e^2 -
```

```
 4*d*f])*ArcTanh[(2*a*f + c*(-e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[4*a*f^2 - 2*c
*(-e^2 + 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[e^2 - 4*d*f
]*(2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]))^(3/2)) + (Sqrt[2]*f^3*(
-e^2 + 2*d*f + e*Sqrt[e^2 - 4*d*f])*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*
f])*x)/(Sqrt[4*a*f^2 + 2*c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*
x^2])])/(Sqrt[e^2 - 4*d*f]*(2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])
)^(3/2)) + (e*Hypergeometric2F1[-1/2, 1, 1/2, 1 + (c*x^2)/a])/(a*Sqrt[a + c
*x^2]))/d^2)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

sage$_2$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] sage2
```

maple [B] time = 0.02, size = 2046, normalized size = 3.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x)
```

```
[Out] -8*f^4/(e+(-4*d*f+e^2)^(1/2))^2/(-4*d*f+e^2)^(1/2)/(2*a*f^2-2*c*d*f+c*e^2+(
-4*d*f+e^2)^(1/2)*c*e)/((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-(e+(-4*d*f+e^2
)^(1/2))*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)*c/f+1/2*(2*a*f^2-2*c*d*f+c*e^2+(-
4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)-16*f^3/(e+(-4*d*f+e^2)^(1/2))^2*c^2/(2*a*f
^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)/(4*a*c-4*c^2*d/f+c^2*e^2/f^2-(-4*d
*f+e^2)*c^2/f^2)/((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-(e+(-4*d*f+e^2)^(1/2
))*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)*c/f+1/2*(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+
e^2)^(1/2)*c*e)/f^2)^(1/2)*x-16*f^3/(e+(-4*d*f+e^2)^(1/2))^2/(-4*d*f+e^2)^(
1/2)*c^2/(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)/(4*a*c-4*c^2*d/f+c^
2*e^2/f^2-(-4*d*f+e^2)*c^2/f^2)/((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-(e+(-
4*d*f+e^2)^(1/2))*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)*c/f+1/2*(2*a*f^2-2*c*d*f
+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*e*x+8*f^4/(e+(-4*d*f+e^2)^(1/2))^
2/(-4*d*f+e^2)^(1/2)/(2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)*2^(1/2)
/((2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*ln((-(e+(-4*d*f
+e^2)^(1/2))*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)*c/f+(2*a*f^2-2*c*d*f+c*e^2+(-
4*d*f+e^2)^(1/2)*c*e)/f^2+1/2*2^(1/2)*((2*a*f^2-2*c*d*f+c*e^2+(-4*d*f+e^2)^
(1/2)*c*e)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*(e+(-4*d*f+
e^2)^(1/2))*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)*c/f+2*(2*a*f^2-2*c*d*f+c*e^2+(
-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))+4*f/(-
e+(-4*d*f+e^2)^(1/2))/(e+(-4*d*f+e^2)^(1/2))/a/x/(c*x^2+a)^(1/2)+8*f/(-e+(-
4*d*f+e^2)^(1/2))/(e+(-4*d*f+e^2)^(1/2))*c/a^2*x/(c*x^2+a)^(1/2)+8*f^4/(-e+
(-4*d*f+e^2)^(1/2))^2/(-4*d*f+e^2)^(1/2)/(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2
)^(1/2)*c*e)/((x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-(e-(-4*d*f+e^2)^(1/2))*
(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)*c/f+1/2*(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^
2)^(1/2)*c*e)/f^2)^(1/2)-16*f^3/(-e+(-4*d*f+e^2)^(1/2))^2*c^2/(2*a*f^2-2*c*
```

```
d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)/(4*a*c-4*c^2*d/f+c^2*e^2/f^2-(-4*d*f+e^2)
*c^2/f^2)/((x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-(e-(-4*d*f+e^2)^(1/2))*(x-
1/2*(-e+(-4*d*f+e^2)^(1/2))/f)*c/f+1/2*(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^
(1/2)*c*e)/f^2)^(1/2)*x+16*f^3/(-e+(-4*d*f+e^2)^(1/2))^2/(-4*d*f+e^2)^(1/2)
*c^2/(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)/(4*a*c-4*c^2*d/f+c^2*e^
2/f^2-(-4*d*f+e^2)*c^2/f^2)/((x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-(e-(-4*d
*f+e^2)^(1/2))*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)*c/f+1/2*(2*a*f^2-2*c*d*f+c
*e^2-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*e*x-8*f^4/(-e+(-4*d*f+e^2)^(1/2))^2
/(-4*d*f+e^2)^(1/2)/(2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)*2^(1/2)/
((2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*ln((-(e-(-4*d*f+
e^2)^(1/2))*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)*c/f+(2*a*f^2-2*c*d*f+c*e^2-(-
4*d*f+e^2)^(1/2)*c*e)/f^2+1/2*2^(1/2)*((2*a*f^2-2*c*d*f+c*e^2-(-4*d*f+e^2)^
(1/2)*c*e)/f^2)^(1/2)*(4*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*(e-(-4*d*f
+e^2)^(1/2))*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)*c/f+2*(2*a*f^2-2*c*d*f+c*e^2
-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2))/(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f))-16*
f^2*e/(-e+(-4*d*f+e^2)^(1/2))^2/(e+(-4*d*f+e^2)^(1/2))^2/a/(c*x^2+a)^(1/2)+
16*f^2*e/(-e+(-4*d*f+e^2)^(1/2))^2/(e+(-4*d*f+e^2)^(1/2))^2/a^(3/2)*ln((2*a
+2*(c*x^2+a)^(1/2)*a^(1/2))/x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(cx^2 + a\right)^{\frac{3}{2}}\left(fx^2 + ex + d\right)x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="maxima")
[Out] integrate(1/((c*x^2 + a)^(3/2)*(f*x^2 + e*x + d)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^2\,{\left(c\,x^2+a\right)}^{3/2}\,\left(f\,x^2+e\,x+d\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + c*x^2)^(3/2)*(d + e*x + f*x^2)),x)
[Out] int(1/(x^2*(a + c*x^2)^(3/2)*(d + e*x + f*x^2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(a + cx^2\right)^{\frac{3}{2}}\left(d + ex + fx^2\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(c*x**2+a)**(3/2)/(f*x**2+e*x+d),x)
[Out] Integral(1/(x**2*(a + c*x**2)**(3/2)*(d + e*x + f*x**2)), x)
```

3.77 $\int \frac{x^3\sqrt{a+bx+cx^2}}{d-fx^2}\,dx$

Optimal. Leaf size=392

$$-\frac{b\left(b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{16c^{5/2}f}+\frac{b(b+2cx)\sqrt{a+bx+cx^2}}{8c^2f}-\frac{d\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}\,\tanh^{-1}\left(\frac{-2a\sqrt{f}+x(}{2\sqrt{a+bx+cx^2}}\right.}{2f^{5/2}}$$

```
[Out] -1/3*(c*x^2+b*x+a)^(3/2)/c/f-1/16*b*(-4*a*c+b^2)*arctanh(1/2*(2*c*x+b)/c^(1
/2)/(c*x^2+b*x+a)^(1/2))/c^(5/2)/f-1/2*b*d*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c
*x^2+b*x+a)^(1/2))/f^2/c^(1/2)-d*(c*x^2+b*x+a)^(1/2)/f^2+1/8*b*(2*c*x+b)*(c
*x^2+b*x+a)^(1/2)/c^2/f-1/2*d*arctanh(1/2*(b*d^(1/2)-2*a*f^(1/2)+x*(2*c*d^(
1/2)-b*f^(1/2)))/(c*x^2+b*x+a)^(1/2)/(c*d+a*f-b*d^(1/2)*f^(1/2))^(1/2))*(c*
d+a*f-b*d^(1/2)*f^(1/2))^(1/2)/f^(5/2)+1/2*d*arctanh(1/2*(b*d^(1/2)+2*a*f^(
1/2)+x*(2*c*d^(1/2)+b*f^(1/2)))/(c*x^2+b*x+a)^(1/2)/(c*d+a*f+b*d^(1/2)*f^(1
/2))^(1/2))*(c*d+a*f+b*d^(1/2)*f^(1/2))^(1/2)/f^(5/2)
```

Rubi [A] time = 0.95, antiderivative size = 392, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 9, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.321, Rules used = {6725, 640, 612, 621, 206, 1021, 1078, 1033, 724}

$$-\frac{b\left(b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{16c^{5/2}f}+\frac{b(b+2cx)\sqrt{a+bx+cx^2}}{8c^2f}-\frac{d\sqrt{a+bx+cx^2}}{f^2}-\frac{bd\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}f^2}-\ldots$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Sqrt[a + b*x + c*x^2])/(d - f*x^2),x]
```

```
[Out] -((d*Sqrt[a + b*x + c*x^2])/f^2) + (b*(b + 2*c*x)*Sqrt[a + b*x + c*x^2])/(8
*c^2*f) - (a + b*x + c*x^2)^(3/2)/(3*c*f) - (b*d*ArcTanh[(b + 2*c*x)/(2*Sqr
t[c]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[c]*f^2) - (b*(b^2 - 4*a*c)*ArcTanh[(b
 + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(16*c^(5/2)*f) - (d*Sqrt[c*d
- b*Sqrt[d]*Sqrt[f] + a*f]*ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d]
- b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2
])])/(2*f^(5/2)) + (d*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*ArcTanh[(b*Sqrt[d
] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt
[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*f^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1021

```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (f
_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(h*(a + b*x + c*x^2)^p*(d + f*x^2)^(q +
 1))/(2*f*(p + q + 1)), x] - Dist[1/(2*f*(p + q + 1)), Int[(a + b*x + c*x^2
)^(p - 1)*(d + f*x^2)^q*Simp[h*p*(b*d) + a*(-2*g*f)*(p + q + 1) + (2*h*p*(c
*d - a*f) + b*(-2*g*f)*(p + q + 1))*x + (h*p*(-(b*f)) + c*(-2*g*f)*(p + q +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, g, h, q}, x] && NeQ[b^2 - 4*a
*c, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0]
```

Rule 1033

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q
), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q
), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f
, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]
```

Rule 1078

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
 (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqrt[d + e*x + f
*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + B*c*x)/((a + c*x^2)*Sqrt[d + e*x
 + f*x^2]), x], x] /; FreeQ[{a, c, d, e, f, A, B, C}, x] && NeQ[e^2 - 4*d*f
, 0]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{x^3\sqrt{a+bx+cx^2}}{d-fx^2}\,dx = \int \left(-\frac{x\sqrt{a+bx+cx^2}}{f} + \frac{dx\sqrt{a+bx+cx^2}}{f\left(d-fx^2\right)}\right)dx$$

$$= -\frac{\int x\sqrt{a+bx+cx^2}\,dx}{f} + \frac{d\int \frac{x\sqrt{a+bx+cx^2}}{d-fx^2}\,dx}{f}$$

$$= -\frac{d\sqrt{a+bx+cx^2}}{f^2} - \frac{\left(a+bx+cx^2\right)^{3/2}}{3cf} + \frac{d\int \frac{\frac{bd}{2}+(cd+af)x+\frac{1}{2}bfx^2}{\sqrt{a+bx+cx^2}\left(d-fx^2\right)}\,dx}{f^2} + \frac{b\int \sqrt{a+bx+cx^2}}{2cf}$$

$$= -\frac{d\sqrt{a+bx+cx^2}}{f^2} + \frac{b(b+2cx)\sqrt{a+bx+cx^2}}{8c^2f} - \frac{\left(a+bx+cx^2\right)^{3/2}}{3cf} - \frac{d\int \frac{-bdf-f(cd+af)}{\sqrt{a+bx+cx^2}\left(d-f\right.}}{f^3}$$

$$= -\frac{d\sqrt{a+bx+cx^2}}{f^2} + \frac{b(b+2cx)\sqrt{a+bx+cx^2}}{8c^2f} - \frac{\left(a+bx+cx^2\right)^{3/2}}{3cf} - \frac{(bd)\,\text{Subst}\left(\int \frac{1}{4c-}\right.}{}$$

$$= -\frac{d\sqrt{a+bx+cx^2}}{f^2} + \frac{b(b+2cx)\sqrt{a+bx+cx^2}}{8c^2f} - \frac{\left(a+bx+cx^2\right)^{3/2}}{3cf} - \frac{bd\tanh^{-1}\left(\frac{b+}{2\sqrt{c}\sqrt{a}}\right.}{2\sqrt{c}f^2}$$

$$= -\frac{d\sqrt{a+bx+cx^2}}{f^2} + \frac{b(b+2cx)\sqrt{a+bx+cx^2}}{8c^2f} - \frac{\left(a+bx+cx^2\right)^{3/2}}{3cf} - \frac{bd\tanh^{-1}\left(\frac{b+}{2\sqrt{c}\sqrt{a}}\right.}{2\sqrt{c}f^2}$$

Mathematica [A] time = 0.89, size = 327, normalized size = 0.83

$$\frac{-\frac{2\sqrt{f}\sqrt{a+x(b+cx)}\left(2cf(4a+bx)-3b^2f+8c^2\left(3d+fx^2\right)\right)}{c^2} - \frac{3b\sqrt{f}\left(-4acf+b^2f+8c^2d\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)}{c^{5/2}} + 24d\sqrt{af+b\sqrt{d}\sqrt{f}+cd}\,\text{ta}}{48f^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Sqrt[a + b*x + c*x^2])/(d - f*x^2),x]
```

```
[Out] ((-2*Sqrt[f]*Sqrt[a + x*(b + c*x)]*(-3*b^2*f + 2*c*f*(4*a + b*x) + 8*c^2*(3
*d + f*x^2)))/c^2 - (3*b*Sqrt[f]*(8*c^2*d + b^2*f - 4*a*c*f)*ArcTanh[(b + 2
*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/c^(5/2) + 24*d*Sqrt[c*d + b*Sqrt[
d]*Sqrt[f] + a*f]*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + 2*c*Sqrt[d]*x + b*Sqrt
[f]*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])] - 24*
d*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*ArcTanh[(-2*a*Sqrt[f] + 2*c*Sqrt[d]*x
 + b*(Sqrt[d] - Sqrt[f]*x))/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a +
 x*(b + c*x)])])/(48*f^(5/2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [B] time = 0.02, size = 1817, normalized size = 4.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x)
```

```
[Out] -1/3*(c*x^2+b*x+a)^(3/2)/c/f+1/4/f*b/c*x*(c*x^2+b*x+a)^(1/2)+1/8/f*b^2/c^2*
(c*x^2+b*x+a)^(1/2)+1/4/f*b/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1
/2))*a-1/16/f*b^3/c^(5/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-1/2/f
^2*d*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+
c*d-(d*f)^(1/2)*b)/f)^(1/2)+1/2/f^3*d*ln((1/2/f*(b*f-2*(d*f)^(1/2)*c)+c*(x+
(d*f)^(1/2)/f))/c^(1/2)+((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*
f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))*c^(1/2)*(d*f)^(1/2)-1/4/f^2
*d*ln((1/2/f*(b*f-2*(d*f)^(1/2)*c)+c*(x+(d*f)^(1/2)/f))/c^(1/2)+((x+(d*f)^(
1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*
b)/f)^(1/2))/c^(1/2)*b-1/2/f^3*d/((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a
*f+c*d-(d*f)^(1/2)*b)/f+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+2*((a*f+c
*d-(d*f)^(1/2)*b)/f)^(1/2)*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+
(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))/(x+(d*f)^(1/2)/f))*(d*f)
^(1/2)*b+1/2/f^2*d/((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d-(d*f)^(
1/2)*b)/f+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+2*((a*f+c*d-(d*f)^(1/2)
*b)/f)^(1/2)*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)
/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))/(x+(d*f)^(1/2)/f))*a+1/2/f^3*d^2/((a*f
+c*d-(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d-(d*f)^(1/2)*b)/f+(b*f-2*(d*f)^(
1/2)*c)*(x+(d*f)^(1/2)/f)/f+2*((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*((x+(d*f)^(
1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*
b)/f)^(1/2))/(x+(d*f)^(1/2)/f))*c-1/2/f^2*d*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(
d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)-1/2/f^3*
d*ln((1/2*(b*f+2*(d*f)^(1/2)*c)/f+c*(x-(d*f)^(1/2)/f))/c^(1/2)+((x-(d*f)^(1
/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b
)/f)^(1/2))*c^(1/2)*(d*f)^(1/2)-1/4/f^2*d*ln((1/2*(b*f+2*(d*f)^(1/2)*c)/f+c
*(x-(d*f)^(1/2)/f))/c^(1/2)+((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x
-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))/c^(1/2)*b+1/2/f^3*d/((a
*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d+(d*f)^(1/2)*b)/f+(b*f+2*(d*f)
^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+2*((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*((x-(d*f)
^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2
)*b)/f)^(1/2))/(x-(d*f)^(1/2)/f))*(d*f)^(1/2)*b+1/2/f^2*d/((a*f+c*d+(d*f)^(
1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d+(d*f)^(1/2)*b)/f+(b*f+2*(d*f)^(1/2)*c)*(x-(
d*f)^(1/2)/f)/f+2*((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+
(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))
/(x-(d*f)^(1/2)/f))*a+1/2/f^3*d^2/((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(
a*f+c*d+(d*f)^(1/2)*b)/f+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+2*((a*f+
c*d+(d*f)^(1/2)*b)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x
-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))/(x-(d*f)^(1/2)/f))*c
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(((c*sqrt(4*d*f))/(2*f^2)>0)', see `
assume?` for more details)Is ((c*sqrt(4*d*f))/(2*f^2)    +b/(2*f))    ^2
 -(c*((b*sqrt(4*d*f))                  /(2*f)                  +(c*d)/f+a))
     /f^2 positive, negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\,\sqrt{c\,x^2+b\,x+a}}{d-f\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*x + c*x^2)^(1/2))/(d - f*x^2),x)
```

```
[Out] int((x^3*(a + b*x + c*x^2)^(1/2))/(d - f*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^3\sqrt{a+bx+cx^2}}{-d+fx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c*x**2+b*x+a)**(1/2)/(-f*x**2+d),x)
```

```
[Out] -Integral(x**3*sqrt(a + b*x + c*x**2)/(-d + f*x**2), x)
```

3.78 $\int \frac{x^2\sqrt{a+bx+cx^2}}{d-fx^2}\,dx$

Optimal. Leaf size=316

$$-\frac{\left(4acf+b^2(-f)+8c^2d\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{3/2}f^2}+\frac{\sqrt{d}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right.}}\right.}{2f^2}$$

```
[Out] -1/8*(4*a*c*f-b^2*f+8*c^2*d)*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1
/2))/c^(3/2)/f^2-1/4*(2*c*x+b)*(c*x^2+b*x+a)^(1/2)/c/f+1/2*arctanh(1/2*(b*d
^(1/2)-2*a*f^(1/2)+x*(2*c*d^(1/2)-b*f^(1/2)))/(c*x^2+b*x+a)^(1/2)/(c*d+a*f-
b*d^(1/2)*f^(1/2))^(1/2))*d^(1/2)*(c*d+a*f-b*d^(1/2)*f^(1/2))^(1/2)/f^2+1/2
*arctanh(1/2*(b*d^(1/2)+2*a*f^(1/2)+x*(2*c*d^(1/2)+b*f^(1/2)))/(c*x^2+b*x+a
)^(1/2)/(c*d+a*f+b*d^(1/2)*f^(1/2))^(1/2))*d^(1/2)*(c*d+a*f+b*d^(1/2)*f^(1/
2))^(1/2)/f^2
```

Rubi [A] time = 0.49, antiderivative size = 316, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {1071, 1078, 621, 206, 1033, 724}

$$-\frac{\left(4acf+b^2(-f)+8c^2d\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{3/2}f^2}+\frac{\sqrt{d}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right.}}\right.}{2f^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sqrt[a + b*x + c*x^2])/(d - f*x^2),x]
```

```
[Out] -((b + 2*c*x)*Sqrt[a + b*x + c*x^2])/(4*c*f) - ((8*c^2*d - b^2*f + 4*a*c*f)
*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(8*c^(3/2)*f^2) +
(Sqrt[d]*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[
f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sq
rt[a + b*x + c*x^2])])/(2*f^2) + (Sqrt[d]*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*
f]*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[
c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*f^2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1033

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q
```

```
), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q
), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f
, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]
```

Rule 1071

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (C_.)*(x_)^2)*((d_) +
(f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((C*(b*f*p) + 2*c*C*f*(p + q + 1)*x)*
(a + b*x + c*x^2)^p*(d + f*x^2)^(q + 1))/(2*c*f^2*(p + q + 1)*(2*p + 2*q +
3)), x] - Dist[1/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3)), Int[(a + b*x + c*x^
2)^(p - 1)*(d + f*x^2)^q*Simp[p*(b*d)*(C*(-(b*f))*(q + 1)) + (p + q + 1)*(b
^2*C*d*f*p + a*c*(C*(2*d*f) + f*(-2*A*f)*(2*p + 2*q + 3))) + (2*p*(c*d - a*
f)*(C*(-(b*f))*(q + 1)) + (p + q + 1)*(-(b*c*(C*(-4*d*f)*(2*p + q + 2) + f*
(2*C*d + 2*A*f)*(2*p + 2*q + 3)))))*x + (p*(-(b*f))*(C*(-(b*f))*(q + 1)) +
(p + q + 1)*(C*f^2*p*(b^2 - 4*a*c) - c^2*(C*(-4*d*f)*(2*p + q + 2) + f*(2*C
*d + 2*A*f)*(2*p + 2*q + 3))))*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, A,
C, q}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0] && NeQ[2
*p + 2*q + 3, 0] &&  !IGtQ[p, 0] &&  !IGtQ[q, 0]
```

Rule 1078

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
 (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqrt[d + e*x + f
*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + B*c*x)/((a + c*x^2)*Sqrt[d + e*x
 + f*x^2]), x], x] /; FreeQ[{a, c, d, e, f, A, B, C}, x] && NeQ[e^2 - 4*d*f
, 0]
```

Rubi steps

$$\int \frac{x^2\sqrt{a+bx+cx^2}}{d-fx^2}\,dx = -\frac{(b+2cx)\sqrt{a+bx+cx^2}}{4cf} - \frac{\int \frac{-\frac{1}{4}\left(b^2+4ac\right)df-2bcdfx-\frac{1}{4}f\left(8c^2d-b^2f+4acf\right)x^2}{\sqrt{a+bx+cx^2}\left(d-fx^2\right)}\,dx}{2cf^2}$$

$$= -\frac{(b+2cx)\sqrt{a+bx+cx^2}}{4cf} + \frac{\int \frac{\frac{1}{4}\left(b^2+4ac\right)df^2+\frac{1}{4}df\left(8c^2d-b^2f+4acf\right)+2bcdf^2x}{\sqrt{a+bx+cx^2}\left(d-fx^2\right)}\,dx}{2cf^3} - \frac{\left(8c^2d-b^2f\right.}{}$$

$$= -\frac{(b+2cx)\sqrt{a+bx+cx^2}}{4cf} - \frac{\left(\sqrt{d}\left(cd-b\sqrt{d}\sqrt{f}+af\right)\right)\int \frac{1}{\left(-\sqrt{d}\sqrt{f}-fx\right)\sqrt{a+bx+cx^2}}\,dx}{2f^{3/2}} + \ldots$$

$$= -\frac{(b+2cx)\sqrt{a+bx+cx^2}}{4cf} - \frac{\left(8c^2d-b^2f+4acf\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{3/2}f^2} + \frac{\left(\sqrt{d}\left(cd-\right.\right.}{}$$

$$= -\frac{(b+2cx)\sqrt{a+bx+cx^2}}{4cf} - \frac{\left(8c^2d-b^2f+4acf\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{3/2}f^2} + \frac{\sqrt{d}\sqrt{cd-}}{}$$

Mathematica [A] time = 0.49, size = 302, normalized size = 0.96

$$\frac{\left(-4acf+b^2f-8c^2d\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)-2\sqrt{c}\left(-2c\sqrt{d}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}\tanh^{-1}\left(\frac{2a\sqrt{f}+b\sqrt{d}+b\sqrt{f}x+2c}{2\sqrt{a+x(b+cx)}\sqrt{af+b\sqrt{d}\sqrt{}}}\right.\right.}{8c^{3/2}f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sqrt[a + b*x + c*x^2])/(d - f*x^2),x]
```

```
[Out] ((-8*c^2*d + b^2*f - 4*a*c*f)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b
+ c*x)])] - 2*Sqrt[c]*(f*(b + 2*c*x)*Sqrt[a + x*(b + c*x)] - 2*c*Sqrt[d]*Sq
rt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + 2*c*Sq
rt[d]*x + b*Sqrt[f]*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b
 + c*x)])] - 2*c*Sqrt[d]*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*ArcTanh[(-2*a*
Sqrt[f] + 2*c*Sqrt[d]*x + b*(Sqrt[d] - Sqrt[f]*x))/(2*Sqrt[c*d - b*Sqrt[d]*
Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])])))/(8*c^(3/2)*f^2)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [B] time = 0.02, size = 1810, normalized size = 5.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x)
```

```
[Out] -1/2/f*x*(c*x^2+b*x+a)^(1/2)-1/4/f/c*(c*x^2+b*x+a)^(1/2)*b-1/2/f/c^(1/2)*ln
((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a+1/8/f/c^(3/2)*ln((c*x+1/2*b)/c^
(1/2)+(c*x^2+b*x+a)^(1/2))*b^2+1/2*d/(d*f)^(1/2)/f*((x+(d*f)^(1/2)/f)^2*c+(
b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)-1
/2*d/f^2*ln(((x+(d*f)^(1/2)/f)*c+1/2*(b*f-2*(d*f)^(1/2)*c)/f)/c^(1/2)+((x+(
d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^
(1/2)*b)/f)^(1/2))*c^(1/2)+1/4*d/(d*f)^(1/2)/f*ln(((x+(d*f)^(1/2)/f)*c+1/2*
(b*f-2*(d*f)^(1/2)*c)/f)/c^(1/2)+((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*
c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))/c^(1/2)*b+1/2*d/f^
2/((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d-(d*f)^(1/2)*b)/f+(b*f-2*
(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+2*((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*((x+
(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)
^(1/2)*b)/f)^(1/2))/(x+(d*f)^(1/2)/f))*b-1/2*d/(d*f)^(1/2)/f/((a*f+c*d-(d*f
)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d-(d*f)^(1/2)*b)/f+(b*f-2*(d*f)^(1/2)*c)*(
x+(d*f)^(1/2)/f)/f+2*((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*((x+(d*f)^(1/2)/f)^2
*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/
2))/(x+(d*f)^(1/2)/f))*a-1/2*d^2/(d*f)^(1/2)/f^2/((a*f+c*d-(d*f)^(1/2)*b)/f
)^(1/2)*ln((2*(a*f+c*d-(d*f)^(1/2)*b)/f+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2
)/f)/f+2*((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d
*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))/(x+(d*f)
^(1/2)/f))*c-1/2*d/(d*f)^(1/2)/f*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*
c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)-1/2*d/f^2*ln(((x-(d
*f)^(1/2)/f)*c+1/2*(b*f+2*(d*f)^(1/2)*c)/f)/c^(1/2)+((x-(d*f)^(1/2)/f)^2*c+
(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))
*c^(1/2)-1/4*d/(d*f)^(1/2)/f*ln(((x-(d*f)^(1/2)/f)*c+1/2*(b*f+2*(d*f)^(1/2)
```

```
*c)/f)/c^(1/2)+((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/
f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))/c^(1/2)*b+1/2*d/f^2/((a*f+c*d+(d*f)^
(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d+(d*f)^(1/2)*b)/f+(b*f+2*(d*f)^(1/2)*c)*(x-
(d*f)^(1/2)/f)/f+2*((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c
+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)
)/(x-(d*f)^(1/2)/f))*b+1/2*d/(d*f)^(1/2)/f/((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2
)*ln((2*(a*f+c*d+(d*f)^(1/2)*b)/f+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f
+2*((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1
/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))/(x-(d*f)^(1/2)
/f))*a+1/2*d^2/(d*f)^(1/2)/f^2/((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f
+c*d+(d*f)^(1/2)*b)/f+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+2*((a*f+c*d
+(d*f)^(1/2)*b)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d
*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))/(x-(d*f)^(1/2)/f))*c
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(((c*sqrt(4*d*f))/(2*f^2)>0)', see `
assume?` for more details)Is ((c*sqrt(4*d*f))/(2*f^2)    +b/(2*f))    ^2
 -(c*((b*sqrt(4*d*f))                  /(2*f)                 +(c*d)/f+a))
     /f^2 positive, negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\,\sqrt{c\,x^2+b\,x+a}}{d-f\,x^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*x + c*x^2)^(1/2))/(d - f*x^2),x)
```

```
[Out] int((x^2*(a + b*x + c*x^2)^(1/2))/(d - f*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^{2}\sqrt{a+bx+cx^{2}}}{-d+fx^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c*x**2+b*x+a)**(1/2)/(-f*x**2+d),x)
```

```
[Out] -Integral(x**2*sqrt(a + b*x + c*x**2)/(-d + f*x**2), x)
```

3.79 $\int \frac{x\sqrt{a+bx+cx^2}}{d-fx^2}\,dx$

Optimal. Leaf size=282

$$-\frac{\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}\,\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2f^{3/2}}+\frac{\sqrt{af+b\sqrt{d}\sqrt{f}+cd}\,\tanh^{-1}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}\cdots\right.}{2\sqrt{a+bx+cx^2}\cdots}\right.}{2f^{3/2}}$$

```
[Out] -1/2*b*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/f/c^(1/2)-(c*x^2+
b*x+a)^(1/2)/f-1/2*arctanh(1/2*(b*d^(1/2)-2*a*f^(1/2)+x*(2*c*d^(1/2)-b*f^(1
/2)))/(c*x^2+b*x+a)^(1/2)/(c*d+a*f-b*d^(1/2)*f^(1/2))^(1/2))*(c*d+a*f-b*d^(
1/2)*f^(1/2))^(1/2)/f^(3/2)+1/2*arctanh(1/2*(b*d^(1/2)+2*a*f^(1/2)+x*(2*c*d
^(1/2)+b*f^(1/2)))/(c*x^2+b*x+a)^(1/2)/(c*d+a*f+b*d^(1/2)*f^(1/2))^(1/2))*(
c*d+a*f+b*d^(1/2)*f^(1/2))^(1/2)/f^(3/2)
```

Rubi [A] time = 0.29, antiderivative size = 282, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1021, 1078, 621, 206, 1033, 724}

$$-\frac{\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}\,\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2f^{3/2}}+\frac{\sqrt{af+b\sqrt{d}\sqrt{f}+cd}\,\tanh^{-1}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}\cdots\right.}{2\sqrt{a+bx+cx^2}\cdots}\right.}{2f^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sqrt[a + b*x + c*x^2])/(d - f*x^2),x]
```

```
[Out] -(Sqrt[a + b*x + c*x^2]/f) - (b*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x
 + c*x^2])])/(2*Sqrt[c]*f) - (Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*ArcTanh[(
b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt
[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*f^(3/2)) + (Sqrt[c*d + b*Sqr
t[d]*Sqrt[f] + a*f]*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqr
t[f])*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2
*f^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1021

```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (f
_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(h*(a + b*x + c*x^2)^p*(d + f*x^2)^(q +
```

```
1))/(2*f*(p + q + 1)), x] - Dist[1/(2*f*(p + q + 1)), Int[(a + b*x + c*x^2
)^(p - 1)*(d + f*x^2)^q*Simp[h*p*(b*d) + a*(-2*g*f)*(p + q + 1) + (2*h*p*(c
*d - a*f) + b*(-2*g*f)*(p + q + 1))*x + (h*p*(-(b*f)) + c*(-2*g*f)*(p + q +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, g, h, q}, x] && NeQ[b^2 - 4*a
*c, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0]
```

Rule 1033

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q
), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q
), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f
, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]
```

Rule 1078

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
 (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqrt[d + e*x + f
*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + B*c*x)/((a + c*x^2)*Sqrt[d + e*x
 + f*x^2]), x], x] /; FreeQ[{a, c, d, e, f, A, B, C}, x] && NeQ[e^2 - 4*d*f
, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x\sqrt{a+bx+cx^2}}{d-fx^2}\,dx &= -\frac{\sqrt{a+bx+cx^2}}{f}+\frac{\int \frac{\frac{bd}{2}+(cd+af)x+\frac{1}{2}bfx^2}{\sqrt{a+bx+cx^2}\left(d-fx^2\right)}\,dx}{f}\\ &= -\frac{\sqrt{a+bx+cx^2}}{f}-\frac{\int \frac{-bdf-f(cd+af)x}{\sqrt{a+bx+cx^2}\left(d-fx^2\right)}\,dx}{f^2}-\frac{b\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{2f}\\ &= -\frac{\sqrt{a+bx+cx^2}}{f}-\frac{b\,\mathrm{Subst}\left(\int \frac{1}{4c-x^2}\,dx,x,\frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{f}+\frac{\left(cd-b\sqrt{d}\sqrt{f}+af\right)\int \frac{}{\left(-\sqrt{d}\sqrt{f}\right.}}{2f}\\ &= -\frac{\sqrt{a+bx+cx^2}}{f}-\frac{b\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}f}-\frac{\left(cd-b\sqrt{d}\sqrt{f}+af\right)\mathrm{Subst}\left(\int \frac{}{4cdf-4b\sqrt{d}}\right.}{}\\ &= -\frac{\sqrt{a+bx+cx^2}}{f}-\frac{b\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}f}-\frac{\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\,\tanh^{-1}\left(\frac{b\sqrt{d}-2a\sqrt{}}{2\sqrt{cd-b\sqrt{d}}}\right.}{2f^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.29, size = 272, normalized size = 0.96

$$\frac{\sqrt{af+b\sqrt{d}\sqrt{f}+cd}\,\tanh^{-1}\left(\frac{2a\sqrt{f}+b\sqrt{d}+b\sqrt{f}x+2c\sqrt{d}x}{2\sqrt{a+x(b+cx)}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)-\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}\,\tanh^{-1}\left(\frac{-2a\sqrt{f}+b\left(\sqrt{d}-\sqrt{f}x\right.}{2\sqrt{a+x(b+cx)}\sqrt{af+b(-}}\right.}{2f^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sqrt[a + b*x + c*x^2])/(d - f*x^2),x]
```

```
[Out] (-2*Sqrt[f]*Sqrt[a + x*(b + c*x)] - (b*Sqrt[f]*ArcTanh[(b + 2*c*x)/(2*Sqrt[
c]*Sqrt[a + x*(b + c*x)])]))/Sqrt[c] + Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*A
rcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + 2*c*Sqrt[d]*x + b*Sqrt[f]*x)/(2*Sqrt[c*d
```

```
+ b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])] - Sqrt[c*d - b*Sqrt[d]*S
qrt[f] + a*f]*ArcTanh[(-2*a*Sqrt[f] + 2*c*Sqrt[d]*x + b*(Sqrt[d] - Sqrt[f]*
x))/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])])/(2*f^(3
/2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [B] time = 0.01, size = 1667, normalized size = 5.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x)
```

```
[Out] -1/2/f*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*
f+c*d-(d*f)^(1/2)*b)/f)^(1/2)+1/2/f^2*ln(((x+(d*f)^(1/2)/f)*c+1/2*(b*f-2*(d
*f)^(1/2)*c)/f)/c^(1/2)+((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*
f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))*c^(1/2)*(d*f)^(1/2)-1/4/f*l
n(((x+(d*f)^(1/2)/f)*c+1/2*(b*f-2*(d*f)^(1/2)*c)/f)/c^(1/2)+((x+(d*f)^(1/2)
/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f
)^(1/2))/c^(1/2)*b-1/2/f^2/((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d
-(d*f)^(1/2)*b)/f+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+2*((a*f+c*d-(d*
f)^(1/2)*b)/f)^(1/2)*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^
(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))/(x+(d*f)^(1/2)/f))*(d*f)^(1/2)
*b+1/2/f/((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d-(d*f)^(1/2)*b)/f+
(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+2*((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/
2)*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*
d-(d*f)^(1/2)*b)/f)^(1/2))/(x+(d*f)^(1/2)/f))*a+1/2/f^2/((a*f+c*d-(d*f)^(1/
2)*b)/f)^(1/2)*ln((2*(a*f+c*d-(d*f)^(1/2)*b)/f+(b*f-2*(d*f)^(1/2)*c)*(x+(d*
f)^(1/2)/f)/f+2*((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*((x+(d*f)^(1/2)/f)^2*c+(b
*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))/(
x+(d*f)^(1/2)/f))*c*d-1/2/f*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x
-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)-1/2/f^2*ln(((x-(d*f)^(1/
2)/f)*c+1/2*(b*f+2*(d*f)^(1/2)*c)/f)/c^(1/2)+((x-(d*f)^(1/2)/f)^2*c+(b*f+2*
(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))*c^(1/2
)*(d*f)^(1/2)-1/4/f*ln(((x-(d*f)^(1/2)/f)*c+1/2*(b*f+2*(d*f)^(1/2)*c)/f)/c^
(1/2)+((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f
+c*d+(d*f)^(1/2)*b)/f)^(1/2))/c^(1/2)*b+1/2/f^2/((a*f+c*d+(d*f)^(1/2)*b)/f)
^(1/2)*ln((2*(a*f+c*d+(d*f)^(1/2)*b)/f+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)
/f)/f+2*((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*
f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))/(x-(d*f)^
(1/2)/f))*(d*f)^(1/2)*b+1/2/f/((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+
```

```
c*d+(d*f)^(1/2)*b)/f+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+2*((a*f+c*d+
(d*f)^(1/2)*b)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*
f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))/(x-(d*f)^(1/2)/f))*a+1/2/f^
2/((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d+(d*f)^(1/2)*b)/f+(b*f+2*
(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+2*((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*((x-
(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)
^(1/2)*b)/f)^(1/2))/(x-(d*f)^(1/2)/f))*c*d
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(((c*sqrt(4*d*f))/(2*f^2)>0)', see `
assume?` for more details)Is ((c*sqrt(4*d*f))/(2*f^2)    +b/(2*f))    ^2
 -(c*((b*sqrt(4*d*f))                   /(2*f)                 +(c*d)/f+a))
     /f^2 positive, negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\,\sqrt{c\,x^2+b\,x+a}}{d-f\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*x + c*x^2)^(1/2))/(d - f*x^2),x)
```

```
[Out] int((x*(a + b*x + c*x^2)^(1/2))/(d - f*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x\sqrt{a+bx+cx^2}}{-d+fx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x**2+b*x+a)**(1/2)/(-f*x**2+d),x)
```

```
[Out] -Integral(x*sqrt(a + b*x + c*x**2)/(-d + f*x**2), x)
```

3.80 $\int \frac{\sqrt{a+bx+cx^2}}{d-fx^2}\,dx$

Optimal. Leaf size=266

$$\frac{\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}\,\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\,\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2\sqrt{d}\,f}+\frac{\sqrt{af+b\sqrt{d}\sqrt{f}+cd}\,\tanh^{-1}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+\right.}{2\sqrt{a+bx+cx^2}\,\sqrt{a}}\right.}{2\sqrt{d}\,f}$$

```
[Out] -arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))*c^(1/2)/f+1/2*arctanh(1
/2*(b*d^(1/2)-2*a*f^(1/2)+x*(2*c*d^(1/2)-b*f^(1/2)))/(c*x^2+b*x+a)^(1/2)/(c
*d+a*f-b*d^(1/2)*f^(1/2))^(1/2))*(c*d+a*f-b*d^(1/2)*f^(1/2))^(1/2)/f/d^(1/2
)+1/2*arctanh(1/2*(b*d^(1/2)+2*a*f^(1/2)+x*(2*c*d^(1/2)+b*f^(1/2)))/(c*x^2+
b*x+a)^(1/2)/(c*d+a*f+b*d^(1/2)*f^(1/2))^(1/2))*(c*d+a*f+b*d^(1/2)*f^(1/2))
^(1/2)/f/d^(1/2)
```

Rubi [A] time = 0.23, antiderivative size = 266, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {990, 621, 206, 1033, 724}

$$\frac{\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}\,\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\,\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2\sqrt{d}\,f}+\frac{\sqrt{af+b\sqrt{d}\sqrt{f}+cd}\,\tanh^{-1}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+\right.}{2\sqrt{a+bx+cx^2}\,\sqrt{a}}\right.}{2\sqrt{d}\,f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x + c*x^2]/(d - f*x^2),x]
```

```
[Out] -((Sqrt[c]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/f) + (Sq
rt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*S
qrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x
+ c*x^2])])/(2*Sqrt[d]*f) + (Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*ArcTanh[(b
*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqrt[
d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[d]*f)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 990

```
Int[Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2]/((d_) + (f_.)*(x_)^2), x_Symbol]
 :> Dist[c/f, Int[1/Sqrt[a + b*x + c*x^2], x], x] - Dist[1/f, Int[(c*d - a*
f - b*f*x)/(Sqrt[a + b*x + c*x^2]*(d + f*x^2)), x], x] /; FreeQ[{a, b, c, d
```

```
, f}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1033

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q
), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q
), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f
, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]
```

Rubi steps

$$\int \frac{\sqrt{a+bx+cx^2}}{d-fx^2}\,dx = \frac{\int \frac{cd+af+bfx}{\sqrt{a+bx+cx^2}\,(d-fx^2)}\,dx}{f} - \frac{c\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{f}$$

$$= -\frac{(2c)\,\mathrm{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{f} + \frac{1}{2}\left(b-\frac{cd+af}{\sqrt{d}\sqrt{f}}\right)\int \frac{1}{\left(-\sqrt{d}\sqrt{f}-fx\right)\sqrt{a+bx+c}$$

$$= -\frac{\sqrt{c}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{f} + \left(-b-\frac{cd+af}{\sqrt{d}\sqrt{f}}\right)\mathrm{Subst}\left(\int \frac{1}{4cdf+4b\sqrt{d}f^{3/2}+4af^2-x^2}$$

$$= -\frac{\sqrt{c}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{f} + \frac{\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\,\tanh^{-1}\left(\frac{b\sqrt{d}-2a\sqrt{f}+\left(2c\sqrt{d}-b\sqrt{f}\right)x}{2\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{d}f}$$

Mathematica [A] time = 0.17, size = 253, normalized size = 0.95

$$\frac{\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}\,\tanh^{-1}\left(\frac{-2a\sqrt{f}+b\sqrt{d}-b\sqrt{f}x+2c\sqrt{d}x}{2\sqrt{a+x(b+cx)}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)+\sqrt{af+b\sqrt{d}\sqrt{f}+cd}\,\tanh^{-1}\left(\frac{2a\sqrt{f}+b\sqrt{d}+b\sqrt{f}}{2\sqrt{a+x(b+cx)}\sqrt{af+}}\right.}{2\sqrt{d}f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x + c*x^2]/(d - f*x^2),x]
```

```
[Out] (-2*Sqrt[c]*Sqrt[d]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])]
+ Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + 2*
c*Sqrt[d]*x - b*Sqrt[f]*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a +
x*(b + c*x)])] + Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*ArcTanh[(b*Sqrt[d] + 2
*a*Sqrt[f] + 2*c*Sqrt[d]*x + b*Sqrt[f]*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] +
 a*f]*Sqrt[a + x*(b + c*x)])])/(2*Sqrt[d]*f)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [B] time = 0.01, size = 1669, normalized size = 6.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x)
```

```
[Out] 1/2/(d*f)^(1/2)*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)
/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)-1/2/f*ln(((x+(d*f)^(1/2)/f)*c+1/2*(b
*f-2*(d*f)^(1/2)*c)/f)/c^(1/2)+((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)
*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))*c^(1/2)+1/4/(d*f)^(1
/2)*ln(((x+(d*f)^(1/2)/f)*c+1/2*(b*f-2*(d*f)^(1/2)*c)/f)/c^(1/2)+((x+(d*f)^
(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)
*b)/f)^(1/2))/c^(1/2)*b+1/2/f/((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+
c*d-(d*f)^(1/2)*b)/f+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+2*((a*f+c*d-
(d*f)^(1/2)*b)/f)^(1/2)*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*
f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))/(x+(d*f)^(1/2)/f))*b-1/2/(d
*f)^(1/2)/((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d-(d*f)^(1/2)*b)/f
+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+2*((a*f+c*d-(d*f)^(1/2)*b)/f)^(1
/2)*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c
*d-(d*f)^(1/2)*b)/f)^(1/2))/(x+(d*f)^(1/2)/f))*a-1/2/(d*f)^(1/2)/f/((a*f+c*
d-(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d-(d*f)^(1/2)*b)/f+(b*f-2*(d*f)^(1/2
)*c)*(x+(d*f)^(1/2)/f)/f+2*((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*((x+(d*f)^(1/2
)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/
f)^(1/2))/(x+(d*f)^(1/2)/f))*c*d-1/2/(d*f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(b*
f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)-1/2
/f*ln(((x-(d*f)^(1/2)/f)*c+1/2*(b*f+2*(d*f)^(1/2)*c)/f)/c^(1/2)+((x-(d*f)^(
1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*
b)/f)^(1/2))*c^(1/2)-1/4/(d*f)^(1/2)*ln(((x-(d*f)^(1/2)/f)*c+1/2*(b*f+2*(d*
f)^(1/2)*c)/f)/c^(1/2)+((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f
)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))/c^(1/2)*b+1/2/f/((a*f+c*d+(d
*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d+(d*f)^(1/2)*b)/f+(b*f+2*(d*f)^(1/2)*c)
*(x-(d*f)^(1/2)/f)/f+2*((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*((x-(d*f)^(1/2)/f)
^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(
1/2))/(x-(d*f)^(1/2)/f))*b+1/2/(d*f)^(1/2)/((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2
)*ln((2*(a*f+c*d+(d*f)^(1/2)*b)/f+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f
+2*((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1
/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))/(x-(d*f)^(1/2)
/f))*a+1/2/(d*f)^(1/2)/f/((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d+(
d*f)^(1/2)*b)/f+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+2*((a*f+c*d+(d*f)
^(1/2)*b)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1
/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))/(x-(d*f)^(1/2)/f))*c*d
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(((c*sqrt(4*d*f))/(2*f^2)>0)', see `
```

```
assume?` for more details)Is ((c*sqrt(4*d*f))/(2*f^2)    +b/(2*f))    ^2
 -(c*((b*sqrt(4*d*f))                     /(2*f)                    +(c*d)/f+a))
     /f^2 positive, negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c\,x^2+b\,x+a}}{d-f\,x^2} \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)^(1/2)/(d - f*x^2),x)
[Out] int((a + b*x + c*x^2)^(1/2)/(d - f*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt{a+bx+cx^2}}{-d+fx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(1/2)/(-f*x**2+d),x)
[Out] -Integral(sqrt(a + b*x + c*x**2)/(-d + f*x**2), x)
```

3.81 $\int \frac{\sqrt{a+bx+cx^2}}{x(d-fx^2)}\,dx$

Optimal. Leaf size=267

$$-\frac{\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}\,\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2d\sqrt{f}}+\frac{\sqrt{af+b\sqrt{d}\sqrt{f}+cd}\,\tanh^{-1}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}\right.}{2\sqrt{a+bx+cx^2}\sqrt{}}\right.}{2d\sqrt{f}}$$

```
[Out] -arctanh(1/2*(b*x+2*a)/a^(1/2)/(c*x^2+b*x+a)^(1/2))*a^(1/2)/d-1/2*arctanh(1
/2*(b*d^(1/2)-2*a*f^(1/2)+x*(2*c*d^(1/2)-b*f^(1/2)))/(c*x^2+b*x+a)^(1/2)/(c
*d+a*f-b*d^(1/2)*f^(1/2))^(1/2))*(c*d+a*f-b*d^(1/2)*f^(1/2))^(1/2)/d/f^(1/2
)+1/2*arctanh(1/2*(b*d^(1/2)+2*a*f^(1/2)+x*(2*c*d^(1/2)+b*f^(1/2)))/(c*x^2+
b*x+a)^(1/2)/(c*d+a*f+b*d^(1/2)*f^(1/2))^(1/2))*(c*d+a*f+b*d^(1/2)*f^(1/2))
^(1/2)/d/f^(1/2)
```

Rubi [A] time = 0.78, antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 9, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.321, Rules used = {6725, 734, 843, 621, 206, 724, 1021, 1078, 1033}

$$-\frac{\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}\,\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2d\sqrt{f}}+\frac{\sqrt{af+b\sqrt{d}\sqrt{f}+cd}\,\tanh^{-1}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}\right.}{2\sqrt{a+bx+cx^2}\sqrt{}}\right.}{2d\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x + c*x^2]/(x*(d - f*x^2)),x]
```

```
[Out] -((Sqrt[a]*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/d) - (Sq
rt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*S
qrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x
+ c*x^2])])/(2*d*Sqrt[f]) + (Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*ArcTanh[(b
*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqrt[
d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*d*Sqrt[f])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
```

```
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1021

```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (f
_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(h*(a + b*x + c*x^2)^p*(d + f*x^2)^(q +
 1))/(2*f*(p + q + 1)), x] - Dist[1/(2*f*(p + q + 1)), Int[(a + b*x + c*x^2
)^(p - 1)*(d + f*x^2)^q*Simp[h*p*(b*d) + a*(-2*g*f)*(p + q + 1) + (2*h*p*(c
*d - a*f) + b*(-2*g*f)*(p + q + 1))*x + (h*p*(-(b*f)) + c*(-2*g*f)*(p + q +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, g, h, q}, x] && NeQ[b^2 - 4*a
*c, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0]
```

Rule 1033

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q
), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q
), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f
, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]
```

Rule 1078

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
 (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqrt[d + e*x + f
*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + B*c*x)/((a + c*x^2)*Sqrt[d + e*x
 + f*x^2]), x], x] /; FreeQ[{a, c, d, e, f, A, B, C}, x] && NeQ[e^2 - 4*d*f
, 0]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+bx+cx^2}}{x\left(d-fx^2\right)}\,dx = \int \left(\frac{\sqrt{a+bx+cx^2}}{dx} - \frac{fx\sqrt{a+bx+cx^2}}{d\left(-d+fx^2\right)}\right)dx$$

$$= \frac{\int \frac{\sqrt{a+bx+cx^2}}{x}\,dx}{d} - \frac{f\int \frac{x\sqrt{a+bx+cx^2}}{-d+fx^2}\,dx}{d}$$

$$= -\frac{\int \frac{-2a-bx}{x\sqrt{a+bx+cx^2}}\,dx}{2d} + \frac{\int \frac{-\frac{bd}{2}-(cd+af)x-\frac{1}{2}bfx^2}{\sqrt{a+bx+cx^2}\left(-d+fx^2\right)}\,dx}{d}$$

$$= \frac{a\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx}{d} + \frac{\int \frac{-bdf+f(-cd-af)x}{\sqrt{a+bx+cx^2}\left(-d+fx^2\right)}\,dx}{df}$$

$$= -\frac{(2a)\,\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+bx}{\sqrt{a+bx+cx^2}}\right)}{d} - \frac{\left(cd-b\sqrt{d}\sqrt{f}+af\right)\int \frac{1}{\left(\sqrt{d}\sqrt{f}+fx\right)\sqrt{a+bx+cx^2}}\,c}{2d}$$

$$= -\frac{\sqrt{a}\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{d} + \frac{\left(cd-b\sqrt{d}\sqrt{f}+af\right)\text{Subst}\left(\int \frac{1}{4cdf-4b\sqrt{d}f^{3/2}+4af^2-x^2}\,dx\right.}{d}$$

$$= -\frac{\sqrt{a}\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{d} - \frac{\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\tanh^{-1}\left(\frac{b\sqrt{d}-2a\sqrt{f}+\left(2c\sqrt{d}-b\sqrt{f}\right)x}{2\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx}}\right.}{2d\sqrt{f}}$$

Mathematica [A] time = 0.30, size = 255, normalized size = 0.96

$$-\frac{\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}\tanh^{-1}\left(\frac{-2a\sqrt{f}+b\sqrt{d}-b\sqrt{f}x+2c\sqrt{d}x}{2\sqrt{a+x(b+cx)}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)-\sqrt{af+b\sqrt{d}\sqrt{f}+cd}\tanh^{-1}\left(\frac{2a\sqrt{f}+b\sqrt{d}}{2\sqrt{a+x(b+cx)}}\right.}{2d\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x + c*x^2]/(x*(d - f*x^2)),x]
```

```
[Out] -1/2*(2*Sqrt[a]*Sqrt[f]*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + x*(b + c*x)
])] + Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f]
+ 2*c*Sqrt[d]*x - b*Sqrt[f]*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[
a + x*(b + c*x)])] - Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*ArcTanh[(b*Sqrt[d]
 + 2*a*Sqrt[f] + 2*c*Sqrt[d]*x + b*Sqrt[f]*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[
f] + a*f]*Sqrt[a + x*(b + c*x)])])/(d*Sqrt[f])
```

fricas [B] time = 26.88, size = 1253, normalized size = 4.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/x/(-f*x^2+d),x, algorithm="fricas")
```

```
[Out] [1/4*(d*sqrt((d^2*f*sqrt(b^2/(d^3*f)) + c*d + a*f)/(d^2*f))*log((2*sqrt(c*x
^2 + b*x + a)*d^2*f*sqrt(b^2/(d^3*f))*sqrt((d^2*f*sqrt(b^2/(d^3*f)) + c*d +
 a*f)/(d^2*f)) + 2*b*c*x + b^2 + (b*d*f*x + 2*a*d*f)*sqrt(b^2/(d^3*f)))/x)
- d*sqrt((d^2*f*sqrt(b^2/(d^3*f)) + c*d + a*f)/(d^2*f))*log(-(2*sqrt(c*x^2
+ b*x + a)*d^2*f*sqrt(b^2/(d^3*f))*sqrt((d^2*f*sqrt(b^2/(d^3*f)) + c*d + a*
f)/(d^2*f)) - 2*b*c*x - b^2 - (b*d*f*x + 2*a*d*f)*sqrt(b^2/(d^3*f)))/x) - d
*sqrt(-(d^2*f*sqrt(b^2/(d^3*f)) - c*d - a*f)/(d^2*f))*log((2*sqrt(c*x^2 + b
*x + a)*d^2*f*sqrt(b^2/(d^3*f))*sqrt(-(d^2*f*sqrt(b^2/(d^3*f)) - c*d - a*f)
```

```
/(d^2*f)) + 2*b*c*x + b^2 - (b*d*f*x + 2*a*d*f)*sqrt(b^2/(d^3*f)))/x) + d*s
qrt(-(d^2*f*sqrt(b^2/(d^3*f)) - c*d - a*f)/(d^2*f))*log(-(2*sqrt(c*x^2 + b*
x + a)*d^2*f*sqrt(b^2/(d^3*f))*sqrt(-(d^2*f*sqrt(b^2/(d^3*f)) - c*d - a*f)/
(d^2*f)) - 2*b*c*x - b^2 + (b*d*f*x + 2*a*d*f)*sqrt(b^2/(d^3*f)))/x) + 2*sq
rt(a)*log(-(8*a*b*x + (b^2 + 4*a*c)*x^2 - 4*sqrt(c*x^2 + b*x + a)*(b*x + 2*
a)*sqrt(a) + 8*a^2)/x^2))/d, 1/4*(d*sqrt((d^2*f*sqrt(b^2/(d^3*f)) + c*d + a
*f)/(d^2*f))*log((2*sqrt(c*x^2 + b*x + a)*d^2*f*sqrt(b^2/(d^3*f))*sqrt((d^2
*f*sqrt(b^2/(d^3*f)) + c*d + a*f)/(d^2*f)) + 2*b*c*x + b^2 + (b*d*f*x + 2*a
*d*f)*sqrt(b^2/(d^3*f)))/x) - d*sqrt((d^2*f*sqrt(b^2/(d^3*f)) + c*d + a*f)/
(d^2*f))*log(-(2*sqrt(c*x^2 + b*x + a)*d^2*f*sqrt(b^2/(d^3*f))*sqrt((d^2*f*
sqrt(b^2/(d^3*f)) + c*d + a*f)/(d^2*f)) - 2*b*c*x - b^2 - (b*d*f*x + 2*a*d*
f)*sqrt(b^2/(d^3*f)))/x) - d*sqrt(-(d^2*f*sqrt(b^2/(d^3*f)) - c*d - a*f)/(d
^2*f))*log((2*sqrt(c*x^2 + b*x + a)*d^2*f*sqrt(b^2/(d^3*f))*sqrt(-(d^2*f*sq
rt(b^2/(d^3*f)) - c*d - a*f)/(d^2*f)) + 2*b*c*x + b^2 - (b*d*f*x + 2*a*d*f)
*sqrt(b^2/(d^3*f)))/x) + d*sqrt(-(d^2*f*sqrt(b^2/(d^3*f)) - c*d - a*f)/(d^2
*f))*log(-(2*sqrt(c*x^2 + b*x + a)*d^2*f*sqrt(b^2/(d^3*f))*sqrt(-(d^2*f*sqr
t(b^2/(d^3*f)) - c*d - a*f)/(d^2*f)) - 2*b*c*x - b^2 + (b*d*f*x + 2*a*d*f)*
sqrt(b^2/(d^3*f)))/x) + 4*sqrt(-a)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(b*x +
2*a)*sqrt(-a)/(a*c*x^2 + a*b*x + a^2)))/d]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/x/(-f*x^2+d),x, algorithm="giac")
```

```
[Out] sage2
```

maple [B] time = 0.02, size = 1764, normalized size = 6.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(1/2)/x/(-f*x^2+d),x)
```

```
[Out] -1/2/d*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*
f+c*d-(d*f)^(1/2)*b)/f)^(1/2)+1/2/d/f*ln(((x+(d*f)^(1/2)/f)*c+1/2*(b*f-2*(d
*f)^(1/2)*c)/f)/c^(1/2)+((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*
f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))*c^(1/2)*(d*f)^(1/2)-1/4/d*l
n(((x+(d*f)^(1/2)/f)*c+1/2*(b*f-2*(d*f)^(1/2)*c)/f)/c^(1/2)+((x+(d*f)^(1/2)
/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f
)^(1/2))/c^(1/2)*b-1/2/d/f/((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d
-(d*f)^(1/2)*b)/f+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+2*((a*f+c*d-(d*
f)^(1/2)*b)/f)^(1/2)*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^
(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))/(x+(d*f)^(1/2)/f))*(d*f)^(1/2)
*b+1/2/d/((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d-(d*f)^(1/2)*b)/f+
(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+2*((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/
2)*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*
d-(d*f)^(1/2)*b)/f)^(1/2))/(x+(d*f)^(1/2)/f))*a+1/2/f/((a*f+c*d-(d*f)^(1/2)
*b)/f)^(1/2)*ln((2*(a*f+c*d-(d*f)^(1/2)*b)/f+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)
^(1/2)/f)/f+2*((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*((x+(d*f)^(1/2)/f)^2*c+(b*f
-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))/(x+
(d*f)^(1/2)/f))*c-1/2/d*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*
f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)-1/2/d/f*ln(((x-(d*f)^(1/2)/f
)*c+1/2*(b*f+2*(d*f)^(1/2)*c)/f)/c^(1/2)+((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f
)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))*c^(1/2)*(d
*f)^(1/2)-1/4/d*ln(((x-(d*f)^(1/2)/f)*c+1/2*(b*f+2*(d*f)^(1/2)*c)/f)/c^(1/2
)+((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d
+(d*f)^(1/2)*b)/f)^(1/2))/c^(1/2)*b+1/2/d/f/((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/
```

```
2)*ln((2*(a*f+c*d+(d*f)^(1/2)*b)/f+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/
f+2*((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(
1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))/(x-(d*f)^(1/2
)/f))*(d*f)^(1/2)*b+1/2/d/((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d+
(d*f)^(1/2)*b)/f+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+2*((a*f+c*d+(d*f
)^(1/2)*b)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(
1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))/(x-(d*f)^(1/2)/f))*a+1/2/f/((a*
f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d+(d*f)^(1/2)*b)/f+(b*f+2*(d*f)^
(1/2)*c)*(x-(d*f)^(1/2)/f)/f+2*((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*((x-(d*f)^
(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)
*b)/f)^(1/2))/(x-(d*f)^(1/2)/f))*c+1/d*(c*x^2+b*x+a)^(1/2)+1/2/d*b*ln((c*x+
1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2)-1/d*a^(1/2)*ln((2*a+b*x+2*a^(1/
2)*(c*x^2+b*x+a)^(1/2))/x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt{cx^2 + bx + a}}{\left(fx^2 - d\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/x/(-f*x^2+d),x, algorithm="maxima")
[Out] -integrate(sqrt(c*x^2 + b*x + a)/((f*x^2 - d)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c\,x^2 + b\,x + a}}{x\,\left(d - f\,x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)^(1/2)/(x*(d - f*x^2)),x)
[Out] int((a + b*x + c*x^2)^(1/2)/(x*(d - f*x^2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt{a + bx + cx^2}}{-dx + fx^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(1/2)/x/(-f*x**2+d),x)
[Out] -Integral(sqrt(a + b*x + c*x**2)/(-d*x + f*x**3), x)
```

3.82 $\int \frac{\sqrt{a+bx+cx^2}}{x^2(d-fx^2)}\,dx$

Optimal. Leaf size=286

$$\frac{\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2d^{3/2}}+\frac{\sqrt{af+b\sqrt{d}\sqrt{f}+cd}\tanh^{-1}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\right.}{2\sqrt{a+bx+cx^2}\sqrt{af+b}}\right.}{2d^{3/2}}$$

```
[Out] -1/2*b*arctanh(1/2*(b*x+2*a)/a^(1/2)/(c*x^2+b*x+a)^(1/2))/d/a^(1/2)-(c*x^2+
b*x+a)^(1/2)/d/x+1/2*arctanh(1/2*(b*d^(1/2)-2*a*f^(1/2)+x*(2*c*d^(1/2)-b*f^
(1/2)))/(c*x^2+b*x+a)^(1/2)/(c*d+a*f-b*d^(1/2)*f^(1/2))^(1/2))*(c*d+a*f-b*d
^(1/2)*f^(1/2))^(1/2)/d^(3/2)+1/2*arctanh(1/2*(b*d^(1/2)+2*a*f^(1/2)+x*(2*c
*d^(1/2)+b*f^(1/2)))/(c*x^2+b*x+a)^(1/2)/(c*d+a*f+b*d^(1/2)*f^(1/2))^(1/2))
*(c*d+a*f+b*d^(1/2)*f^(1/2))^(1/2)/d^(3/2)
```

Rubi [A] time = 0.71, antiderivative size = 286, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 8, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {6725, 732, 843, 621, 206, 724, 990, 1033}

$$\frac{\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2d^{3/2}}+\frac{\sqrt{af+b\sqrt{d}\sqrt{f}+cd}\tanh^{-1}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\right.}{2\sqrt{a+bx+cx^2}\sqrt{af+b}}\right.}{2d^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x + c*x^2]/(x^2*(d - f*x^2)),x]
```

```
[Out] -(Sqrt[a + b*x + c*x^2]/(d*x)) - (b*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a +
 b*x + c*x^2])])/(2*Sqrt[a]*d) + (Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*ArcTa
nh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*
Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*d^(3/2)) + (Sqrt[c*d + b
*Sqrt[d]*Sqrt[f] + a*f]*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b
*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])]
)/(2*d^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 1)), x] - Di
```

```
st[p/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && (IntegerQ[p]
 || LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a,
 b, c, d, e, m, p, x]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 990

```
Int[Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2]/((d_) + (f_.)*(x_)^2), x_Symbol]
 :> Dist[c/f, Int[1/Sqrt[a + b*x + c*x^2], x], x] - Dist[1/f, Int[(c*d - a*
f - b*f*x)/(Sqrt[a + b*x + c*x^2]*(d + f*x^2)), x], x] /; FreeQ[{a, b, c, d
, f}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1033

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q
), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q
), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f
, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+bx+cx^2}}{x^2\left(d-fx^2\right)}\,dx = \int \left(\frac{\sqrt{a+bx+cx^2}}{dx^2}+\frac{f\sqrt{a+bx+cx^2}}{d\left(d-fx^2\right)}\right)dx$$

$$= \frac{\int \frac{\sqrt{a+bx+cx^2}}{x^2}\,dx}{d}+\frac{f\int \frac{\sqrt{a+bx+cx^2}}{d-fx^2}\,dx}{d}$$

$$= -\frac{\sqrt{a+bx+cx^2}}{dx}+\frac{\int \frac{b+2cx}{x\sqrt{a+bx+cx^2}}\,dx}{2d}+\frac{\int \frac{cd+af+bfx}{\sqrt{a+bx+cx^2}\left(d-fx^2\right)}\,dx}{d}-\frac{c\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{d}$$

$$= -\frac{\sqrt{a+bx+cx^2}}{dx}+\frac{b\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx}{2d}+\frac{c\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{d}-\frac{(2c)\,\mathrm{Subst}\left(\int \frac{1}{4c-x^2}\,dx,x,\ldots\right.}{d}$$

$$= -\frac{\sqrt{a+bx+cx^2}}{dx}-\frac{\sqrt{c}\,\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\,\sqrt{a+bx+cx^2}}\right)}{d}-\frac{b\,\mathrm{Subst}\left(\int \frac{1}{4a-x^2}\,dx,x,\frac{2a+bx}{\sqrt{a+bx+cx^2}}\right)}{d}+\ldots$$

$$= -\frac{\sqrt{a+bx+cx^2}}{dx}-\frac{b\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\,\sqrt{a+bx+cx^2}}\right)}{2\sqrt{a}\,d}+\frac{\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\,\tanh^{-1}\left(\frac{b\sqrt{d}-2a\ldots}{2\sqrt{cd-b\sqrt{\ldots}}}\right.}{2d^{3/2}}$$

Mathematica [A] time = 0.43, size = 275, normalized size = 0.96

$$\frac{\sqrt{af+b\sqrt{d}\sqrt{f}+cd}\tanh^{-1}\left(\frac{2a\sqrt{f}+b\sqrt{d}+b\sqrt{f}x+2c\sqrt{d}x}{2\sqrt{a+x(b+cx)}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)+\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}\tanh^{-1}\left(\frac{-2a\sqrt{f}+b\left(\sqrt{d}-\sqrt{f}x\right.}{2\sqrt{a+x(b+cx)}\sqrt{af+b(-}}\right.}{2d^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x + c*x^2]/(x^2*(d - f*x^2)),x]
```

```
[Out] ((-2*Sqrt[d]*Sqrt[a + x*(b + c*x)])/x - (b*Sqrt[d]*ArcTanh[(2*a + b*x)/(2*S
qrt[a]*Sqrt[a + x*(b + c*x)])])/Sqrt[a] + Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*
f]*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + 2*c*Sqrt[d]*x + b*Sqrt[f]*x)/(2*Sqrt[
c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])] + Sqrt[c*d - b*Sqrt[
d]*Sqrt[f] + a*f]*ArcTanh[(-2*a*Sqrt[f] + 2*c*Sqrt[d]*x + b*(Sqrt[d] - Sqrt
[f]*x))/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])])/(2*
d^(3/2))
```

fricas [B] time = 38.76, size = 1094, normalized size = 3.83

$$\left[\frac{adx\sqrt{\frac{d^3\sqrt{\frac{b^2f}{d^5}}+cd+af}{d^3}}\log\left(\frac{2\,bcx+2\,\sqrt{cx^2+bx+a}\,bd\sqrt{\frac{d^3\sqrt{\frac{b^2f}{d^5}}+cd+af}{d^3}}+b^2+\left(bd^2x+2\,ad^2\right)\sqrt{\frac{b^2f}{d^5}}}{x}\right)-adx\sqrt{\frac{d^3\sqrt{\frac{b^2f}{d^5}}+cd+af}{d^3}}\log\left(\frac{2\,bcx-2\,\sqrt{}}{}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/x^2/(-f*x^2+d),x, algorithm="fricas")
```

```
[Out] [1/4*(a*d*x*sqrt((d^3*sqrt(b^2*f/d^5) + c*d + a*f)/d^3)*log((2*b*c*x + 2*sq
rt(c*x^2 + b*x + a)*b*d*sqrt((d^3*sqrt(b^2*f/d^5) + c*d + a*f)/d^3) + b^2 +
 (b*d^2*x + 2*a*d^2)*sqrt(b^2*f/d^5))/x) - a*d*x*sqrt((d^3*sqrt(b^2*f/d^5)
+ c*d + a*f)/d^3)*log((2*b*c*x - 2*sqrt(c*x^2 + b*x + a)*b*d*sqrt((d^3*sqrt
(b^2*f/d^5) + c*d + a*f)/d^3) + b^2 + (b*d^2*x + 2*a*d^2)*sqrt(b^2*f/d^5))/
x) + a*d*x*sqrt(-(d^3*sqrt(b^2*f/d^5) - c*d - a*f)/d^3)*log((2*b*c*x + 2*sq
rt(c*x^2 + b*x + a)*b*d*sqrt(-(d^3*sqrt(b^2*f/d^5) - c*d - a*f)/d^3) + b^2
- (b*d^2*x + 2*a*d^2)*sqrt(b^2*f/d^5))/x) - a*d*x*sqrt(-(d^3*sqrt(b^2*f/d^5
) - c*d - a*f)/d^3)*log((2*b*c*x - 2*sqrt(c*x^2 + b*x + a)*b*d*sqrt(-(d^3*s
qrt(b^2*f/d^5) - c*d - a*f)/d^3) + b^2 - (b*d^2*x + 2*a*d^2)*sqrt(b^2*f/d^5
))/x) + sqrt(a)*b*x*log(-(8*a*b*x + (b^2 + 4*a*c)*x^2 - 4*sqrt(c*x^2 + b*x
+ a)*(b*x + 2*a)*sqrt(a) + 8*a^2)/x^2) - 4*sqrt(c*x^2 + b*x + a)*a)/(a*d*x)
, 1/4*(a*d*x*sqrt((d^3*sqrt(b^2*f/d^5) + c*d + a*f)/d^3)*log((2*b*c*x + 2*s
qrt(c*x^2 + b*x + a)*b*d*sqrt((d^3*sqrt(b^2*f/d^5) + c*d + a*f)/d^3) + b^2
+ (b*d^2*x + 2*a*d^2)*sqrt(b^2*f/d^5))/x) - a*d*x*sqrt((d^3*sqrt(b^2*f/d^5)
 + c*d + a*f)/d^3)*log((2*b*c*x - 2*sqrt(c*x^2 + b*x + a)*b*d*sqrt((d^3*sqr
t(b^2*f/d^5) + c*d + a*f)/d^3) + b^2 + (b*d^2*x + 2*a*d^2)*sqrt(b^2*f/d^5))
/x) + a*d*x*sqrt(-(d^3*sqrt(b^2*f/d^5) - c*d - a*f)/d^3)*log((2*b*c*x + 2*s
qrt(c*x^2 + b*x + a)*b*d*sqrt(-(d^3*sqrt(b^2*f/d^5) - c*d - a*f)/d^3) + b^2
 - (b*d^2*x + 2*a*d^2)*sqrt(b^2*f/d^5))/x) - a*d*x*sqrt(-(d^3*sqrt(b^2*f/d^
5) - c*d - a*f)/d^3)*log((2*b*c*x - 2*sqrt(c*x^2 + b*x + a)*b*d*sqrt(-(d^3*
sqrt(b^2*f/d^5) - c*d - a*f)/d^3) + b^2 - (b*d^2*x + 2*a*d^2)*sqrt(b^2*f/d^
5))/x) + 2*sqrt(-a)*b*x*arctan(1/2*sqrt(c*x^2 + b*x + a)*(b*x + 2*a)*sqrt(-
a)/(a*c*x^2 + a*b*x + a^2)) - 4*sqrt(c*x^2 + b*x + a)*a)/(a*d*x)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/x^2/(-f*x^2+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 0.87sym2poly/r2sym(const ge
n & e,const index_m & i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.02, size = 1819, normalized size = 6.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(1/2)/x^2/(-f*x^2+d),x)
```

```
[Out] 1/2*f/d/(d*f)^(1/2)*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(
1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)-1/2/d*ln(((x+(d*f)^(1/2)/f)*c+1/
2*(b*f-2*(d*f)^(1/2)*c)/f)/c^(1/2)+((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2
)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))*c^(1/2)+1/4*f/d/
(d*f)^(1/2)*ln(((x+(d*f)^(1/2)/f)*c+1/2*(b*f-2*(d*f)^(1/2)*c)/f)/c^(1/2)+((
x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*
f)^(1/2)*b)/f)^(1/2))/c^(1/2)*b+1/2/d/((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*ln(
(2*(a*f+c*d-(d*f)^(1/2)*b)/f+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+2*((
a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c
)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))/(x+(d*f)^(1/2)/f))*
b-1/2*f/d/(d*f)^(1/2)/((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d-(d*f
)^(1/2)*b)/f+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+2*((a*f+c*d-(d*f)^(1
/2)*b)/f)^(1/2)*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)
/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))/(x+(d*f)^(1/2)/f))*a-1/2/(d*f)^(1/2
)/((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d-(d*f)^(1/2)*b)/f+(b*f-2*
(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+2*((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*((x+
(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)
^(1/2)*b)/f)^(1/2))/(x+(d*f)^(1/2)/f))*c-1/d/a/x*(c*x^2+b*x+a)^(3/2)+1/d*b/
a*(c*x^2+b*x+a)^(1/2)-1/2/d*b/a^(1/2)*ln((b*x+2*a+2*(c*x^2+b*x+a)^(1/2)*a^(
1/2))/x)+1/d*c/a*(c*x^2+b*x+a)^(1/2)*x+1/d*c^(1/2)*ln((c*x+1/2*b)/c^(1/2)+(
c*x^2+b*x+a)^(1/2))-1/2*f/d/(d*f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)
^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)-1/2/d*ln(((x
-(d*f)^(1/2)/f)*c+1/2*(b*f+2*(d*f)^(1/2)*c)/f)/c^(1/2)+((x-(d*f)^(1/2)/f)^2
*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/
2))*c^(1/2)-1/4*f/d/(d*f)^(1/2)*ln(((x-(d*f)^(1/2)/f)*c+1/2*(b*f+2*(d*f)^(1
/2)*c)/f)/c^(1/2)+((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/
2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))/c^(1/2)*b+1/2/d/((a*f+c*d+(d*f)^(
1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d+(d*f)^(1/2)*b)/f+(b*f+2*(d*f)^(1/2)*c)*(x-(
d*f)^(1/2)/f)/f+2*((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+
(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))
/(x-(d*f)^(1/2)/f))*b+1/2*f/d/(d*f)^(1/2)/((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)
*ln((2*(a*f+c*d+(d*f)^(1/2)*b)/f+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+
2*((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/
2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))/(x-(d*f)^(1/2)/
f))*a+1/2/(d*f)^(1/2)/((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d+(d*f
)^(1/2)*b)/f+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+2*((a*f+c*d+(d*f)^(1
/2)*b)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)
/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))/(x-(d*f)^(1/2)/f))*c
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt{cx^2 + bx + a}}{\left(fx^2 - d\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/x^2/(-f*x^2+d),x, algorithm="maxima")
```

```
[Out] -integrate(sqrt(c*x^2 + b*x + a)/((f*x^2 - d)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c\,x^2+b\,x+a}}{x^2\,\left(d-f\,x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)^(1/2)/(x^2*(d - f*x^2)),x)
```

```
[Out] int((a + b*x + c*x^2)^(1/2)/(x^2*(d - f*x^2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt{a+bx+cx^2}}{-dx^2+fx^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(1/2)/x**2/(-f*x**2+d),x)
```

```
[Out] -Integral(sqrt(a + b*x + c*x**2)/(-d*x**2 + f*x**4), x)
```

3.83 $\int \frac{\sqrt{a+bx+cx^2}}{x^3(d-fx^2)}\,dx$

Optimal. Leaf size=353

$$\frac{(b^2-4ac)\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{8a^{3/2}d}-\frac{\sqrt{a}\,f\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{d^2}-\frac{\sqrt{f}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}\,\tanh^{-1}\left(\frac{-2a\ldots}{2\sqrt{a+\ldots}}\right)}{2d^2}$$

```
[Out] 1/8*(-4*a*c+b^2)*arctanh(1/2*(b*x+2*a)/a^(1/2)/(c*x^2+b*x+a)^(1/2))/a^(3/2)
/d-f*arctanh(1/2*(b*x+2*a)/a^(1/2)/(c*x^2+b*x+a)^(1/2))*a^(1/2)/d^2-1/4*(b*
x+2*a)*(c*x^2+b*x+a)^(1/2)/a/d/x^2-1/2*arctanh(1/2*(b*d^(1/2)-2*a*f^(1/2)+x
*(2*c*d^(1/2)-b*f^(1/2)))/(c*x^2+b*x+a)^(1/2)/(c*d+a*f-b*d^(1/2)*f^(1/2))^(
1/2))*f^(1/2)*(c*d+a*f-b*d^(1/2)*f^(1/2))^(1/2)/d^2+1/2*arctanh(1/2*(b*d^(1
/2)+2*a*f^(1/2)+x*(2*c*d^(1/2)+b*f^(1/2)))/(c*x^2+b*x+a)^(1/2)/(c*d+a*f+b*d
^(1/2)*f^(1/2))^(1/2))*f^(1/2)*(c*d+a*f+b*d^(1/2)*f^(1/2))^(1/2)/d^2
```

Rubi [A] time = 0.88, antiderivative size = 353, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {6725, 720, 724, 206, 734, 843, 621, 1021, 1078, 1033}

$$\frac{(b^2-4ac)\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{8a^{3/2}d}-\frac{\sqrt{a}\,f\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{d^2}-\frac{\sqrt{f}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}\,\tanh^{-1}\left(\frac{-2a\ldots}{2\sqrt{a+\ldots}}\right)}{2d^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x + c*x^2]/(x^3*(d - f*x^2)),x]
```

```
[Out] -((2*a + b*x)*Sqrt[a + b*x + c*x^2])/(4*a*d*x^2) + ((b^2 - 4*a*c)*ArcTanh[(
2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/(8*a^(3/2)*d) - (Sqrt[a]*f*A
rcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/d^2 - (Sqrt[f]*Sqrt[
c*d - b*Sqrt[d]*Sqrt[f] + a*f]*ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt
[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c
*x^2])])/(2*d^2) + (Sqrt[f]*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*ArcTanh[(b*
Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqrt[d
]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*d^2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 720

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[((d + e*x)^(m + 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
 + c*x^2)^p)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[(p*(b^2 - 4*a*c
))/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*
x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2, 0
] && GtQ[p, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1021

```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (f
_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(h*(a + b*x + c*x^2)^p*(d + f*x^2)^(q +
 1))/(2*f*(p + q + 1)), x] - Dist[1/(2*f*(p + q + 1)), Int[(a + b*x + c*x^2
)^(p - 1)*(d + f*x^2)^q*Simp[h*p*(b*d) + a*(-2*g*f)*(p + q + 1) + (2*h*p*(c
*d - a*f) + b*(-2*g*f)*(p + q + 1))*x + (h*p*(-(b*f)) + c*(-2*g*f)*(p + q +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, g, h, q}, x] && NeQ[b^2 - 4*a
*c, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0]
```

Rule 1033

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q
), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q
), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f
, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]
```

Rule 1078

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
 (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqrt[d + e*x + f
*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + B*c*x)/((a + c*x^2)*Sqrt[d + e*x
 + f*x^2]), x], x] /; FreeQ[{a, c, d, e, f, A, B, C}, x] && NeQ[e^2 - 4*d*f
, 0]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+bx+cx^2}}{x^3\left(d-fx^2\right)}dx = \int \left(\frac{\sqrt{a+bx+cx^2}}{dx^3} + \frac{f\sqrt{a+bx+cx^2}}{d^2x} + \frac{f^2x\sqrt{a+bx+cx^2}}{d^2\left(d-fx^2\right)}\right)dx$$

$$= \frac{\int \frac{\sqrt{a+bx+cx^2}}{x^3}dx}{d} + \frac{f\int \frac{\sqrt{a+bx+cx^2}}{x}dx}{d^2} + \frac{f^2\int \frac{x\sqrt{a+bx+cx^2}}{d-fx^2}dx}{d^2}$$

$$= -\frac{(2a+bx)\sqrt{a+bx+cx^2}}{4adx^2} - \frac{\left(b^2-4ac\right)\int \frac{1}{x\sqrt{a+bx+cx^2}}dx}{8ad} - \frac{f\int \frac{-2a-bx}{x\sqrt{a+bx+cx^2}}dx}{2d^2} + \frac{f\int \frac{\frac{bd}{2}}{\sqrt{\ldots}}}{}$$

$$= -\frac{(2a+bx)\sqrt{a+bx+cx^2}}{4adx^2} - \frac{\int \frac{-bdf-f(cd+af)x}{\sqrt{a+bx+cx^2}\left(d-fx^2\right)}dx}{d^2} + \frac{\left(b^2-4ac\right)\text{Subst}\left(\int \frac{1}{4a-x^2}dx, x, \ldots\right.}{4ad}$$

$$= -\frac{(2a+bx)\sqrt{a+bx+cx^2}}{4adx^2} + \frac{\left(b^2-4ac\right)\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{8a^{3/2}d} - \frac{(2af)\text{Subst}\left(\int \frac{1}{4a-x\ldots}\right.}{d^2}$$

$$= -\frac{(2a+bx)\sqrt{a+bx+cx^2}}{4adx^2} + \frac{\left(b^2-4ac\right)\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{8a^{3/2}d} - \frac{\sqrt{a}f\tanh^{-1}\left(\frac{2a\ldots}{2\sqrt{a}\sqrt{\ldots}}\right.}{d^2}$$

$$= -\frac{(2a+bx)\sqrt{a+bx+cx^2}}{4adx^2} + \frac{\left(b^2-4ac\right)\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{8a^{3/2}d} - \frac{\sqrt{a}f\tanh^{-1}\left(\frac{2a\ldots}{2\sqrt{a}\sqrt{\ldots}}\right.}{d^2}$$

Mathematica [A] time = 0.52, size = 316, normalized size = 0.90

$$\frac{x^2\left(b^2d-4a(2af+cd)\right)\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+x(b+cx)}}\right) - 2\sqrt{a}\left(-2a\sqrt{f}x^2\sqrt{af+b\sqrt{d}\sqrt{f}+cd}\tanh^{-1}\left(\frac{2a\sqrt{f}+b\sqrt{d}+\ldots}{2\sqrt{a+x(b+cx)}\sqrt{\ldots}}\right.\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x + c*x^2]/(x^3*(d - f*x^2)),x]
```

```
[Out] ((b^2*d - 4*a*(c*d + 2*a*f))*x^2*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + x*
(b + c*x)])] - 2*Sqrt[a]*(d*(2*a + b*x)*Sqrt[a + x*(b + c*x)] - 2*a*Sqrt[f]
*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*x^2*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] +
 2*c*Sqrt[d]*x + b*Sqrt[f]*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a
 + x*(b + c*x)])] + 2*a*Sqrt[f]*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*x^2*Arc
Tanh[(-2*a*Sqrt[f] + 2*c*Sqrt[d]*x + b*(Sqrt[d] - Sqrt[f]*x))/(2*Sqrt[c*d -
 b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])]))/(8*a^(3/2)*d^2*x^2)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/x^3/(-f*x^2+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/x^3/(-f*x^2+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 1.36sym2poly/r2sym(const ge
n & e,const index_m & i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.02, size = 1953, normalized size = 5.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(1/2)/x^3/(-f*x^2+d),x)
```

```
[Out] -1/2*f/d^2*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f
+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)+1/2/d^2*ln(((x+(d*f)^(1/2)/f)*c+1/2*(b*f-
2*(d*f)^(1/2)*c)/f)/c^(1/2)+((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x
+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))*c^(1/2)*(d*f)^(1/2)-1/4
*f/d^2*ln(((x+(d*f)^(1/2)/f)*c+1/2*(b*f-2*(d*f)^(1/2)*c)/f)/c^(1/2)+((x+(d*
f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1
/2)*b)/f)^(1/2))/c^(1/2)*b-1/2/d^2/((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*ln((2*
(a*f+c*d-(d*f)^(1/2)*b)/f+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+2*((a*f
+c*d-(d*f)^(1/2)*b)/f)^(1/2)*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(
x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))/(x+(d*f)^(1/2)/f))*(d*
f)^(1/2)*b+1/2*f/d^2/((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d-(d*f)
^(1/2)*b)/f+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+2*((a*f+c*d-(d*f)^(1/
2)*b)/f)^(1/2)*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/
f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))/(x+(d*f)^(1/2)/f))*a+1/2/d/((a*f+c*d
-(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d-(d*f)^(1/2)*b)/f+(b*f-2*(d*f)^(1/2)
*c)*(x+(d*f)^(1/2)/f)/f+2*((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*((x+(d*f)^(1/2)
/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f
)^(1/2))/(x+(d*f)^(1/2)/f))*c-1/2*f/d^2*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)
^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)-1/2/d^2*ln((
(x-(d*f)^(1/2)/f)*c+1/2*(b*f+2*(d*f)^(1/2)*c)/f)/c^(1/2)+((x-(d*f)^(1/2)/f)
^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(
1/2))*c^(1/2)*(d*f)^(1/2)-1/4*f/d^2*ln(((x-(d*f)^(1/2)/f)*c+1/2*(b*f+2*(d*f
)^(1/2)*c)/f)/c^(1/2)+((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)
^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))/c^(1/2)*b+1/2/d^2/((a*f+c*d+(
d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d+(d*f)^(1/2)*b)/f+(b*f+2*(d*f)^(1/2)*c
)*(x-(d*f)^(1/2)/f)/f+2*((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*((x-(d*f)^(1/2)/f
)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^
(1/2))/(x-(d*f)^(1/2)/f))*(d*f)^(1/2)*b+1/2*f/d^2/((a*f+c*d+(d*f)^(1/2)*b)/
f)^(1/2)*ln((2*(a*f+c*d+(d*f)^(1/2)*b)/f+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/
2)/f)/f+2*((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(
d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))/(x-(d*f
)^(1/2)/f))*a+1/2/d/((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d+(d*f)^
(1/2)*b)/f+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+2*((a*f+c*d+(d*f)^(1/2
)*b)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f
)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))/(x-(d*f)^(1/2)/f))*c-1/2/d/a/x^2*(c*x
^2+b*x+a)^(3/2)+1/4/d*b/a^2/x*(c*x^2+b*x+a)^(3/2)-1/4/d*b^2/a^2*(c*x^2+b*x+
a)^(1/2)+1/8/d*b^2/a^(3/2)*ln((b*x+2*a+2*(c*x^2+b*x+a)^(1/2)*a^(1/2))/x)-1/
4/d*b/a^2*c*(c*x^2+b*x+a)^(1/2)*x+1/2/d*c/a*(c*x^2+b*x+a)^(1/2)-1/2/d*c/a^(
1/2)*ln((b*x+2*a+2*(c*x^2+b*x+a)^(1/2)*a^(1/2))/x)+f/d^2*(c*x^2+b*x+a)^(1/2
)+1/2*f/d^2*b*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2)-f/d^2*a^(
1/2)*ln((b*x+2*a+2*(c*x^2+b*x+a)^(1/2)*a^(1/2))/x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt{cx^2 + bx + a}}{\left(fx^2 - d\right)x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/x^3/(-f*x^2+d),x, algorithm="maxima")
[Out] -integrate(sqrt(c*x^2 + b*x + a)/((f*x^2 - d)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c\,x^2+b\,x+a}}{x^3\,\left(d-f\,x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)^(1/2)/(x^3*(d - f*x^2)),x)
[Out] int((a + b*x + c*x^2)^(1/2)/(x^3*(d - f*x^2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt{a+bx+cx^2}}{-dx^3+fx^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(1/2)/x**3/(-f*x**2+d),x)
[Out] -Integral(sqrt(a + b*x + c*x**2)/(-d*x**3 + f*x**5), x)
```

3.84 $\int \frac{x^3\left(a+bx+cx^2\right)^{3/2}}{d-fx^2}\,dx$

Optimal. Leaf size=501

$$\frac{3b\left(b^2-4ac\right)^2\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{256c^{7/2}f}-\frac{3b\left(b^2-4ac\right)(b+2cx)\sqrt{a+bx+cx^2}}{128c^3f}-\frac{d\sqrt{a+bx+cx^2}\left(8acf+b^2f+2b\right.}{8cf^3}$$

```
[Out] -1/3*d*(c*x^2+b*x+a)^(3/2)/f^2+1/16*b*(2*c*x+b)*(c*x^2+b*x+a)^(3/2)/c^2/f-1
/5*(c*x^2+b*x+a)^(5/2)/c/f+3/256*b*(-4*a*c+b^2)^2*arctanh(1/2*(2*c*x+b)/c^(
1/2)/(c*x^2+b*x+a)^(1/2))/c^(7/2)/f-1/16*b*d*(12*a*c*f-b^2*f+24*c^2*d)*arct
anh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(3/2)/f^3-1/2*d*arctanh(1/
2*(b*d^(1/2)-2*a*f^(1/2)+x*(2*c*d^(1/2)-b*f^(1/2)))/(c*x^2+b*x+a)^(1/2)/(c*
d+a*f-b*d^(1/2)*f^(1/2))^(1/2))*(c*d+a*f-b*d^(1/2)*f^(1/2))^(3/2)/f^(7/2)+1
/2*d*arctanh(1/2*(b*d^(1/2)+2*a*f^(1/2)+x*(2*c*d^(1/2)+b*f^(1/2)))/(c*x^2+b
*x+a)^(1/2)/(c*d+a*f+b*d^(1/2)*f^(1/2))^(1/2))*(c*d+a*f+b*d^(1/2)*f^(1/2))^
(3/2)/f^(7/2)-3/128*b*(-4*a*c+b^2)*(2*c*x+b)*(c*x^2+b*x+a)^(1/2)/c^3/f-1/8*
d*(2*b*c*f*x+8*a*c*f+b^2*f+8*c^2*d)*(c*x^2+b*x+a)^(1/2)/c/f^3
```

Rubi [A] time = 1.41, antiderivative size = 501, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {6725, 640, 612, 621, 206, 1021, 1070, 1078, 1033, 724}

$$-\frac{d\sqrt{a+bx+cx^2}\left(8acf+b^2f+2bcfx+8c^2d\right)}{8cf^3}-\frac{bd\left(12acf+b^2(-f)+24c^2d\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{16c^{3/2}f^3}-\frac{3b\left(b^2-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*x + c*x^2)^(3/2))/(d - f*x^2),x]
```

```
[Out] (-3*b*(b^2 - 4*a*c)*(b + 2*c*x)*Sqrt[a + b*x + c*x^2])/(128*c^3*f) - (d*(8*
c^2*d + b^2*f + 8*a*c*f + 2*b*c*f*x)*Sqrt[a + b*x + c*x^2])/(8*c*f^3) - (d*
(a + b*x + c*x^2)^(3/2))/(3*f^2) + (b*(b + 2*c*x)*(a + b*x + c*x^2)^(3/2))/
(16*c^2*f) - (a + b*x + c*x^2)^(5/2)/(5*c*f) + (3*b*(b^2 - 4*a*c)^2*ArcTanh
[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(256*c^(7/2)*f) - (b*d*(24
*c^2*d - b^2*f + 12*a*c*f)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*
x^2])])/(16*c^(3/2)*f^3) - (d*(c*d - b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)*ArcTanh
[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sq
rt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*f^(7/2)) + (d*(c*d + b*Sqr
t[d]*Sqrt[f] + a*f)^(3/2)*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] +
 b*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2]
)])/(2*f^(7/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1021

```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (f
_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(h*(a + b*x + c*x^2)^p*(d + f*x^2)^(q +
 1))/(2*f*(p + q + 1)), x] - Dist[1/(2*f*(p + q + 1)), Int[(a + b*x + c*x^2
)^(p - 1)*(d + f*x^2)^q*Simp[h*p*(b*d) + a*(-2*g*f)*(p + q + 1) + (2*h*p*(c
*d - a*f) + b*(-2*g*f)*(p + q + 1))*x + (h*p*(-(b*f)) + c*(-2*g*f)*(p + q +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, g, h, q}, x] && NeQ[b^2 - 4*a
*c, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0]
```

Rule 1033

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q
), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q
), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f
, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]
```

Rule 1070

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((B*c*f*(2*p + 2*q + 3)
+ C*(b*f*p) + 2*c*C*f*(p + q + 1)*x)*(a + b*x + c*x^2)^p*(d + f*x^2)^(q + 1
))/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3)), x] - Dist[1/(2*c*f^2*(p + q + 1)*
(2*p + 2*q + 3)), Int[(a + b*x + c*x^2)^(p - 1)*(d + f*x^2)^q*Simp[p*(b*d)*
(C*(-(b*f))*(q + 1) - c*(-(B*f))*(2*p + 2*q + 3)) + (p + q + 1)*(b^2*C*d*f*
p + a*c*(C*(2*d*f) + f*(-2*A*f)*(2*p + 2*q + 3))) + (2*p*(c*d - a*f)*(C*(-(
b*f))*(q + 1) - c*(-(B*f))*(2*p + 2*q + 3)) + (p + q + 1)*(-(b*c*(C*(-4*d*f
)*(2*p + q + 2) + f*(2*C*d + 2*A*f)*(2*p + 2*q + 3)))))*x + (p*(-(b*f))*(C*
(-(b*f))*(q + 1) - c*(-(B*f))*(2*p + 2*q + 3)) + (p + q + 1)*(C*f^2*p*(b^2
- 4*a*c) - c^2*(C*(-4*d*f)*(2*p + q + 2) + f*(2*C*d + 2*A*f)*(2*p + 2*q + 3
))))*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, A, B, C, q}, x] && NeQ[b^2 -
4*a*c, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0] && NeQ[2*p + 2*q + 3, 0] &&  !I
GtQ[p, 0] &&  !IGtQ[q, 0]
```

Rule 1078

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
 (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqrt[d + e*x + f
*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + B*c*x)/((a + c*x^2)*Sqrt[d + e*x
 + f*x^2]), x], x] /; FreeQ[{a, c, d, e, f, A, B, C}, x] && NeQ[e^2 - 4*d*f
```

```
, 0]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{x^3\left(a+bx+cx^2\right)^{3/2}}{d-fx^2}\,dx=\int\left(-\frac{x\left(a+bx+cx^2\right)^{3/2}}{f}+\frac{dx\left(a+bx+cx^2\right)^{3/2}}{f\left(d-fx^2\right)}\right)dx$$

$$=-\frac{\int x\left(a+bx+cx^2\right)^{3/2}\,dx}{f}+\frac{d\int\frac{x\left(a+bx+cx^2\right)^{3/2}}{d-fx^2}\,dx}{f}$$

$$=-\frac{d\left(a+bx+cx^2\right)^{3/2}}{3f^2}-\frac{\left(a+bx+cx^2\right)^{5/2}}{5cf}+\frac{d\int\frac{\sqrt{a+bx+cx^2}\left(\frac{3bd}{2}+3(cd+af)x+\frac{3}{2}bfx^2\right)}{d-fx^2}\,dx}{3f^2}+\ b$$

$$=-\frac{d\left(8c^2d+b^2f+8acf+2bcfx\right)\sqrt{a+bx+cx^2}}{8cf^3}-\frac{d\left(a+bx+cx^2\right)^{3/2}}{3f^2}+\frac{b(b+2cx)}{}$$

$$=-\frac{3b\left(b^2-4ac\right)(b+2cx)\sqrt{a+bx+cx^2}}{128c^3f}-\frac{d\left(8c^2d+b^2f+8acf+2bcfx\right)\sqrt{a+bx+}}{8cf^3}$$

$$=-\frac{3b\left(b^2-4ac\right)(b+2cx)\sqrt{a+bx+cx^2}}{128c^3f}-\frac{d\left(8c^2d+b^2f+8acf+2bcfx\right)\sqrt{a+bx+}}{8cf^3}$$

$$=-\frac{3b\left(b^2-4ac\right)(b+2cx)\sqrt{a+bx+cx^2}}{128c^3f}-\frac{d\left(8c^2d+b^2f+8acf+2bcfx\right)\sqrt{a+bx+}}{8cf^3}$$

$$=-\frac{3b\left(b^2-4ac\right)(b+2cx)\sqrt{a+bx+cx^2}}{128c^3f}-\frac{d\left(8c^2d+b^2f+8acf+2bcfx\right)\sqrt{a+bx+}}{8cf^3}$$

Mathematica [A] time = 1.49, size = 447, normalized size = 0.89

$$-\frac{\sqrt{f}\sqrt{a+x(b+cx)}\left(24c^2f\left(16a^2f+7abfx+b^2\left(10d+fx^2\right)\right)-30b^2cf^2(10a+bx)+16c^3f\left(160ad+48afx^2+70bdx+33bfx^3\right)+45b^4f^2+128c^4\left(15d^2+5dfx^2+3f^2\right.\right.}{c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*x + c*x^2)^(3/2))/(d - f*x^2),x]
```

```
[Out] (b*(-384*c^4*d^2 - 192*a*c^3*d*f + 3*b^4*f^2 - 24*a*b^2*c*f^2 + 16*c^2*f*(b
^2*d + 3*a^2*f))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/(2
56*c^(7/2)*f^3) + (-((Sqrt[f]*Sqrt[a + x*(b + c*x)]*(45*b^4*f^2 - 30*b^2*c*
f^2*(10*a + b*x) + 16*c^3*f*(160*a*d + 70*b*d*x + 48*a*f*x^2 + 33*b*f*x^3)
+ 128*c^4*(15*d^2 + 5*d*f*x^2 + 3*f^2*x^4) + 24*c^2*f*(16*a^2*f + 7*a*b*f*x
 + b^2*(10*d + f*x^2))))/c^3) + 960*d*(c*d - b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)
*ArcTanh[(-(b*Sqrt[d]) + 2*a*Sqrt[f] - 2*c*Sqrt[d]*x + b*Sqrt[f]*x)/(2*Sqrt
```

```
[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])] - 960*d*(c*d + b*Sq
rt[d]*Sqrt[f] + a*f)^(3/2)*ArcTanh[(-2*(a*Sqrt[f] + c*Sqrt[d]*x) - b*(Sqrt[
d] + Sqrt[f]*x))/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x
)])])/(1920*f^(7/2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [B] time = 0.03, size = 4884, normalized size = 9.75

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x)
```

```
[Out] -1/5*(c*x^2+b*x+a)^(5/2)/c/f+1/16/f*b^2/c^2*(c*x^2+b*x+a)^(3/2)-3/128/f*b^4
/c^3*(c*x^2+b*x+a)^(1/2)+3/256/f*b^5/c^(7/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+
b*x+a)^(1/2))-1/2/f^2*d*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*
f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*a-1/2/f^3*d^2*((x-(d*f)^(1/2
)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/
f)^(1/2)*c-1/2/f^2*d*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^
(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*a-1/2/f^3*d^2*((x+(d*f)^(1/2)/f
)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^
(1/2)*c+1/2/f^4*d^3/((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d+(d*f)^
(1/2)*b)/f+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+2*((a*f+c*d+(d*f)^(1/2
)*b)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f
)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))/(x-(d*f)^(1/2)/f))*c^2-1/8/f^2*d*((x+
(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)
^(1/2)*b)/f)^(1/2)*x*b+5/8/f^3*d*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*
c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*b*(d*f)^(1/2)-1/16/
f^2*d/c*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a
*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*b^2+1/32/f^2*d/c^(3/2)*ln(((x+(d*f)^(1/2)/f)
*c+1/2*(b*f-2*(d*f)^(1/2)*c)/f)/c^(1/2)+((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)
^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))*b^3-3/4/f^3
*d^2*ln(((x+(d*f)^(1/2)/f)*c+1/2*(b*f-2*(d*f)^(1/2)*c)/f)/c^(1/2)+((x+(d*f)
^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2
)*b)/f)^(1/2))*c^(1/2)*b+1/2/f^4*d^2*ln(((x+(d*f)^(1/2)/f)*c+1/2*(b*f-2*(d*
f)^(1/2)*c)/f)/c^(1/2)+((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f
)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))*c^(3/2)*(d*f)^(1/2)+1/2/f^3*
d^2/((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d-(d*f)^(1/2)*b)/f+(b*f-
2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+2*((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*((
x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*
f)^(1/2)*b)/f)^(1/2))/(x+(d*f)^(1/2)/f))*b^2+1/2/f^2*d/((a*f+c*d-(d*f)^(1/2
```

```
)*b)/f)^(1/2)*ln((2*(a*f+c*d-(d*f)^(1/2)*b)/f+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f
)^(1/2)/f)/f+2*((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*((x+(d*f)^(1/2)/f)^2*c+(b*
f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))/(x
+(d*f)^(1/2)/f))*a^2+1/2/f^4*d^3/((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a
*f+c*d-(d*f)^(1/2)*b)/f+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+2*((a*f+c
*d-(d*f)^(1/2)*b)/f)^(1/2)*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+
(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))/(x+(d*f)^(1/2)/f))*c^2+1
/8/f*b/c*x*(c*x^2+b*x+a)^(3/2)-3/64/f*b^3/c^2*(c*x^2+b*x+a)^(1/2)*x+3/32/f*
b^2/c^2*(c*x^2+b*x+a)^(1/2)*a+3/16/f*b/c^(3/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^
2+b*x+a)^(1/2))*a^2-3/32/f*b^3/c^(5/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)
^(1/2))*a-1/4/f^3*d*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(
1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*x*c*(d*f)^(1/2)-3/4/f^3*d*ln(((x
-(d*f)^(1/2)/f)*c+1/2*(b*f+2*(d*f)^(1/2)*c)/f)/c^(1/2)+((x-(d*f)^(1/2)/f)^2
*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/
2))*c^(1/2)*(d*f)^(1/2)*a-3/8/f^2*d/c^(1/2)*ln(((x-(d*f)^(1/2)/f)*c+1/2*(b*
f+2*(d*f)^(1/2)*c)/f)/c^(1/2)+((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*
(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))*a*b-3/16/f^3*d*ln(((x
-(d*f)^(1/2)/f)*c+1/2*(b*f+2*(d*f)^(1/2)*c)/f)/c^(1/2)+((x-(d*f)^(1/2)/f)^2
*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/
2))/c^(1/2)*b^2*(d*f)^(1/2)+1/f^3*d^2/((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*ln(
(2*(a*f+c*d+(d*f)^(1/2)*b)/f+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+2*((
a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c
)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))/(x-(d*f)^(1/2)/f))*
a*c+3/16/f^3*d*ln(((x+(d*f)^(1/2)/f)*c+1/2*(b*f-2*(d*f)^(1/2)*c)/f)/c^(1/2)
+((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-
(d*f)^(1/2)*b)/f)^(1/2))/c^(1/2)*b^2*(d*f)^(1/2)+1/f^3*d^2/((a*f+c*d-(d*f)^
(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d-(d*f)^(1/2)*b)/f+(b*f-2*(d*f)^(1/2)*c)*(x+
(d*f)^(1/2)/f)/f+2*((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*((x+(d*f)^(1/2)/f)^2*c
+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)
)/(x+(d*f)^(1/2)/f))*a*c+3/16/f*b/c*(c*x^2+b*x+a)^(1/2)*x*a+1/4/f^3*d*((x+(
d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^
(1/2)*b)/f)^(1/2)*x*c*(d*f)^(1/2)+3/4/f^3*d*ln(((x+(d*f)^(1/2)/f)*c+1/2*(b*
f-2*(d*f)^(1/2)*c)/f)/c^(1/2)+((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*
(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))*c^(1/2)*(d*f)^(1/2)*a
-3/8/f^2*d/c^(1/2)*ln(((x+(d*f)^(1/2)/f)*c+1/2*(b*f-2*(d*f)^(1/2)*c)/f)/c^(
1/2)+((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+
c*d-(d*f)^(1/2)*b)/f)^(1/2))*a*b+1/2/f^3*d^2/((a*f+c*d+(d*f)^(1/2)*b)/f)^(1
/2)*ln((2*(a*f+c*d+(d*f)^(1/2)*b)/f+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)
/f+2*((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^
(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))/(x-(d*f)^(1/
2)/f))*b^2+1/2/f^2*d/((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d+(d*f)
^(1/2)*b)/f+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+2*((a*f+c*d+(d*f)^(1/
2)*b)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/
f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))/(x-(d*f)^(1/2)/f))*a^2+1/32/f^2*d/c^
(3/2)*ln(((x-(d*f)^(1/2)/f)*c+1/2*(b*f+2*(d*f)^(1/2)*c)/f)/c^(1/2)+((x-(d*f
)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/
2)*b)/f)^(1/2))*b^3-3/4/f^3*d^2*ln(((x-(d*f)^(1/2)/f)*c+1/2*(b*f+2*(d*f)^(1
/2)*c)/f)/c^(1/2)+((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/
2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))*c^(1/2)*b-1/2/f^4*d^2*ln(((x-(d*f
)^(1/2)/f)*c+1/2*(b*f+2*(d*f)^(1/2)*c)/f)/c^(1/2)+((x-(d*f)^(1/2)/f)^2*c+(b
*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))*c
^(3/2)*(d*f)^(1/2)-1/16/f^2*d/c*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c
)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*b^2-1/6/f^2*d*((x-(d
*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(
1/2)*b)/f)^(3/2)-1/6/f^2*d*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+
(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(3/2)+1/f^3*d/((a*f+c*d+(d*f)^(
1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d+(d*f)^(1/2)*b)/f+(b*f+2*(d*f)^(1/2)*c)*(x-(
d*f)^(1/2)/f)/f+2*((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+
(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))
```

```
/(x-(d*f)^(1/2)/f))*b*(d*f)^(1/2)*a+1/f^4*d^2/((a*f+c*d+(d*f)^(1/2)*b)/f)^(
1/2)*ln((2*(a*f+c*d+(d*f)^(1/2)*b)/f+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f
)/f+2*((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)
^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))/(x-(d*f)^(1
/2)/f))*b*(d*f)^(1/2)*c-1/f^3*d/((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*
f+c*d-(d*f)^(1/2)*b)/f+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+2*((a*f+c*
d-(d*f)^(1/2)*b)/f)^(1/2)*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(
d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))/(x+(d*f)^(1/2)/f))*b*(d*f
)^(1/2)*a-1/f^4*d^2/((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d-(d*f)^
(1/2)*b)/f+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+2*((a*f+c*d-(d*f)^(1/2
)*b)/f)^(1/2)*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f
)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))/(x+(d*f)^(1/2)/f))*b*(d*f)^(1/2)*c-1/
8/f^2*d*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a
*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*x*b-5/8/f^3*d*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*
(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*b*(d*f)
^(1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(((c*sqrt(4*d*f))/(2*f^2)>0)', see `
assume?` for more details)Is ((c*sqrt(4*d*f))/(2*f^2)    +b/(2*f))    ^2
 -(c*((b*sqrt(4*d*f))                    /(2*f)                +(c*d)/f+a))
     /f^2 positive, negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\,{\left(c\,x^2+b\,x+a\right)}^{3/2}}{d-f\,x^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*x + c*x^2)^(3/2))/(d - f*x^2),x)
```

```
[Out] int((x^3*(a + b*x + c*x^2)^(3/2))/(d - f*x^2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c*x**2+b*x+a)**(3/2)/(-f*x**2+d),x)
```

```
[Out] Timed out
```

3.85 $\int \frac{x^2(a+bx+cx^2)^{3/2}}{d-fx^2} dx$

Optimal. Leaf size=417

$$-\frac{\left(48c^2f\left(a^2f+b^2d\right)-24ab^2cf^2+192ac^3df+3b^4f^2+128c^4d^2\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{128c^{5/2}f^3}-\frac{\sqrt{a+bx+cx^2}\left(2cx\left(1\right.\right.}{}$$

```
[Out] -1/8*(2*c*x+b)*(c*x^2+b*x+a)^(3/2)/c/f-1/128*(128*c^4*d^2+192*a*c^3*d*f+3*b
^4*f^2-24*a*b^2*c*f^2+48*c^2*f*(a^2*f+b^2*d))*arctanh(1/2*(2*c*x+b)/c^(1/2)
/(c*x^2+b*x+a)^(1/2))/c^(5/2)/f^3+1/2*arctanh(1/2*(b*d^(1/2)-2*a*f^(1/2)+x*
(2*c*d^(1/2)-b*f^(1/2)))/(c*x^2+b*x+a)^(1/2)/(c*d+a*f-b*d^(1/2)*f^(1/2))^(1
/2))*d^(1/2)*(c*d+a*f-b*d^(1/2)*f^(1/2))^(3/2)/f^3+1/2*arctanh(1/2*(b*d^(1/
2)+2*a*f^(1/2)+x*(2*c*d^(1/2)+b*f^(1/2)))/(c*x^2+b*x+a)^(1/2)/(c*d+a*f+b*d^
(1/2)*f^(1/2))^(1/2))*d^(1/2)*(c*d+a*f+b*d^(1/2)*f^(1/2))^(3/2)/f^3-1/64*(b
*(12*a*c*f-3*b^2*f+80*c^2*d)+2*c*(12*a*c*f-3*b^2*f+16*c^2*d)*x)*(c*x^2+b*x+
a)^(1/2)/c^2/f^2
```

Rubi [A] time = 1.02, antiderivative size = 417, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1071, 1070, 1078, 621, 206, 1033, 724}

$$-\frac{\left(48c^2f\left(a^2f+b^2d\right)-24ab^2cf^2+192ac^3df+3b^4f^2+128c^4d^2\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{128c^{5/2}f^3}-\frac{\sqrt{a+bx+cx^2}\left(2cx\left(1\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*x + c*x^2)^(3/2))/(d - f*x^2),x]
```

```
[Out] -((b*(80*c^2*d - 3*b^2*f + 12*a*c*f) + 2*c*(16*c^2*d - 3*b^2*f + 12*a*c*f)*
x)*Sqrt[a + b*x + c*x^2])/(64*c^2*f^2) - ((b + 2*c*x)*(a + b*x + c*x^2)^(3/
2))/(8*c*f) - ((128*c^4*d^2 + 192*a*c^3*d*f + 3*b^4*f^2 - 24*a*b^2*c*f^2 +
48*c^2*f*(b^2*d + a^2*f))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x
^2])])/(128*c^(5/2)*f^3) + (Sqrt[d]*(c*d - b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)*A
rcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d
- b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*f^3) + (Sqrt[d]*(c*d
 + b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*c*S
qrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x
+ c*x^2])])/(2*f^3)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
```

```
d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1033

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q
), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q
), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f
, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]
```

Rule 1070

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((B*c*f*(2*p + 2*q + 3)
+ C*(b*f*p) + 2*c*C*f*(p + q + 1)*x)*(a + b*x + c*x^2)^p*(d + f*x^2)^(q + 1
))/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3)), x] - Dist[1/(2*c*f^2*(p + q + 1)*
(2*p + 2*q + 3)), Int[(a + b*x + c*x^2)^(p - 1)*(d + f*x^2)^q*Simp[p*(b*d)*
(C*(-(b*f))*(q + 1) - c*(-(B*f))*(2*p + 2*q + 3)) + (p + q + 1)*(b^2*C*d*f*
p + a*c*(C*(2*d*f) + f*(-2*A*f)*(2*p + 2*q + 3))) + (2*p*(c*d - a*f)*(C*(-(
b*f))*(q + 1) - c*(-(B*f))*(2*p + 2*q + 3)) + (p + q + 1)*(-(b*c*(C*(-4*d*f
)*(2*p + q + 2) + f*(2*C*d + 2*A*f)*(2*p + 2*q + 3)))))*x + (p*(-(b*f))*(C*
(-(b*f))*(q + 1) - c*(-(B*f))*(2*p + 2*q + 3)) + (p + q + 1)*(C*f^2*p*(b^2
- 4*a*c) - c^2*(C*(-4*d*f)*(2*p + q + 2) + f*(2*C*d + 2*A*f)*(2*p + 2*q + 3
))))*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, A, B, C, q}, x] && NeQ[b^2 -
4*a*c, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0] && NeQ[2*p + 2*q + 3, 0] &&  !I
GtQ[p, 0] &&  !IGtQ[q, 0]
```

Rule 1071

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (C_.)*(x_)^2)*((d_) +
(f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((C*(b*f*p) + 2*c*C*f*(p + q + 1)*x)*
(a + b*x + c*x^2)^p*(d + f*x^2)^(q + 1))/(2*c*f^2*(p + q + 1)*(2*p + 2*q +
3)), x] - Dist[1/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3)), Int[(a + b*x + c*x^
2)^(p - 1)*(d + f*x^2)^q*Simp[p*(b*d)*(C*(-(b*f))*(q + 1)) + (p + q + 1)*(b
^2*C*d*f*p + a*c*(C*(2*d*f) + f*(-2*A*f)*(2*p + 2*q + 3))) + (2*p*(c*d - a*
f)*(C*(-(b*f))*(q + 1)) + (p + q + 1)*(-(b*c*(C*(-4*d*f)*(2*p + q + 2) + f*
(2*C*d + 2*A*f)*(2*p + 2*q + 3)))))*x + (p*(-(b*f))*(C*(-(b*f))*(q + 1)) +
(p + q + 1)*(C*f^2*p*(b^2 - 4*a*c) - c^2*(C*(-4*d*f)*(2*p + q + 2) + f*(2*C
*d + 2*A*f)*(2*p + 2*q + 3))))*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, A,
C, q}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0] && NeQ[2
*p + 2*q + 3, 0] &&  !IGtQ[p, 0] &&  !IGtQ[q, 0]
```

Rule 1078

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
 (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqrt[d + e*x + f
*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + B*c*x)/((a + c*x^2)*Sqrt[d + e*x
 + f*x^2]), x], x] /; FreeQ[{a, c, d, e, f, A, B, C}, x] && NeQ[e^2 - 4*d*f
, 0]
```

Rubi steps

$$\int \frac{x^2 \left(a+bx+cx^2\right)^{3/2}}{d-fx^2}\,dx = -\frac{(b+2cx)\left(a+bx+cx^2\right)^{3/2}}{8cf} - \frac{\int \frac{\sqrt{a+bx+cx^2}\left(-\frac{3}{4}\left(3b^2+4ac\right)df-12bcdfx-\frac{3}{4}f\left(16c^2d-3\left(b^2-4ac\right)f\right)x^2}{d-fx^2}}{12cf^2}$$

$$= -\frac{\left(b\left(80c^2d-3b^2f+12acf\right)+2c\left(16c^2d-3b^2f+12acf\right)x\right)\sqrt{a+bx+cx^2}}{64c^2f^2} - (b+2$$

$$= -\frac{\left(b\left(80c^2d-3b^2f+12acf\right)+2c\left(16c^2d-3b^2f+12acf\right)x\right)\sqrt{a+bx+cx^2}}{64c^2f^2} - (b+2$$

$$= -\frac{\left(b\left(80c^2d-3b^2f+12acf\right)+2c\left(16c^2d-3b^2f+12acf\right)x\right)\sqrt{a+bx+cx^2}}{64c^2f^2} - (b+2$$

$$= -\frac{\left(b\left(80c^2d-3b^2f+12acf\right)+2c\left(16c^2d-3b^2f+12acf\right)x\right)\sqrt{a+bx+cx^2}}{64c^2f^2} - (b+2$$

$$= -\frac{\left(b\left(80c^2d-3b^2f+12acf\right)+2c\left(16c^2d-3b^2f+12acf\right)x\right)\sqrt{a+bx+cx^2}}{64c^2f^2} - (b+2$$

Mathematica [A] time = 1.02, size = 395, normalized size = 0.95

$$-\left(\left(48c^2f\left(a^2f+b^2d\right)-24ab^2cf^2+192ac^3df+3b^4f^2+128c^4d^2\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)\right)-2\sqrt{c}\left(f\sqrt{a+x(b+\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*x + c*x^2)^(3/2))/(d - f*x^2),x]
```

```
[Out] (-((128*c^4*d^2 + 192*a*c^3*d*f + 3*b^4*f^2 - 24*a*b^2*c*f^2 + 48*c^2*f*(b^
2*d + a^2*f))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])]) - 2*S
qrt[c]*(f*Sqrt[a + x*(b + c*x)]*(-3*b^3*f + 2*b^2*c*f*x + 8*c^2*x*(4*c*d +
5*a*f + 2*c*f*x^2) + 4*b*c*(20*c*d + 5*a*f + 6*c*f*x^2)) + 32*c^2*Sqrt[d]*(
c*d - b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)*ArcTanh[(-(b*Sqrt[d]) + 2*a*Sqrt[f] -
2*c*Sqrt[d]*x + b*Sqrt[f]*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a
+ x*(b + c*x)])] + 32*c^2*Sqrt[d]*(c*d + b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)*Arc
Tanh[(-2*(a*Sqrt[f] + c*Sqrt[d]*x) - b*(Sqrt[d] + Sqrt[f]*x))/(2*Sqrt[c*d +
 b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])]))/(128*c^(5/2)*f^3)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [B] time = 0.02, size = 4900, normalized size = 11.75

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x)
```

```
[Out] -3/8/f/c^(1/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a^2-3/128/f/c^(5
/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*b^4-1/6*d/(d*f)^(1/2)/f*((x
-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f
)^(1/2)*b)/f)^(3/2)-5/8*d/f^2*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*
(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*b-1/2*d^2/f^3*ln(((x-(
d*f)^(1/2)/f)*c+1/2*(b*f+2*(d*f)^(1/2)*c)/f)/c^(1/2)+((x-(d*f)^(1/2)/f)^2*c
+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)
)*c^(3/2)+1/6*d/(d*f)^(1/2)/f*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*
(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(3/2)-5/8*d/f^2*((x+(d*f)^(1
/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b
)/f)^(1/2)*b-1/2*d^2/f^3*ln(((x+(d*f)^(1/2)/f)*c+1/2*(b*f-2*(d*f)^(1/2)*c)/
f)/c^(1/2)+((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f
+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))*c^(3/2)-1/8/f/c*(c*x^2+b*x+a)^(3/2)*b-3/
8/f*(c*x^2+b*x+a)^(1/2)*x*a+3/64/f/c^2*(c*x^2+b*x+a)^(1/2)*b^3-d^2/(d*f)^(1
/2)/f^2/((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d-(d*f)^(1/2)*b)/f+(
b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+2*((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2
)*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d
-(d*f)^(1/2)*b)/f)^(1/2))/(x+(d*f)^(1/2)/f))*a*c+3/8*d/(d*f)^(1/2)/f/c^(1/2
)*ln(((x+(d*f)^(1/2)/f)*c+1/2*(b*f-2*(d*f)^(1/2)*c)/f)/c^(1/2)+((x+(d*f)^(1
/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b
)/f)^(1/2))*a*b-3/8*d/(d*f)^(1/2)/f/c^(1/2)*ln(((x-(d*f)^(1/2)/f)*c+1/2*(b*
f+2*(d*f)^(1/2)*c)/f)/c^(1/2)+((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*
(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))*a*b+1/2*d/(d*f)^(1/2)
/f/((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d+(d*f)^(1/2)*b)/f+(b*f+2
*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+2*((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*((x
-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f
)^(1/2)*b)/f)^(1/2))/(x-(d*f)^(1/2)/f))*a^2+1/2*d^3/(d*f)^(1/2)/f^3/((a*f+c
*d+(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d+(d*f)^(1/2)*b)/f+(b*f+2*(d*f)^(1/
2)*c)*(x-(d*f)^(1/2)/f)/f+2*((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*((x-(d*f)^(1/
2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)
/f)^(1/2))/(x-(d*f)^(1/2)/f))*c^2-1/8*d/(d*f)^(1/2)/f*((x-(d*f)^(1/2)/f)^2*
c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2
)*x*b-1/16*d/(d*f)^(1/2)/f/c*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(
x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*b^2+1/32*d/(d*f)^(1/2)/
f/c^(3/2)*ln(((x-(d*f)^(1/2)/f)*c+1/2*(b*f+2*(d*f)^(1/2)*c)/f)/c^(1/2)+((x-
(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)
^(1/2)*b)/f)^(1/2))*b^3-3/4*d^2/(d*f)^(1/2)/f^2*ln(((x-(d*f)^(1/2)/f)*c+1/2
*(b*f+2*(d*f)^(1/2)*c)/f)/c^(1/2)+((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)
*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))*c^(1/2)*b+1/2*d^2
/(d*f)^(1/2)/f^2/((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d+(d*f)^(1/
2)*b)/f+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+2*((a*f+c*d+(d*f)^(1/2)*b
)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f
+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))/(x-(d*f)^(1/2)/f))*b^2+d/f^2/((a*f+c*d+(
d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d+(d*f)^(1/2)*b)/f+(b*f+2*(d*f)^(1/2)*c
)*(x-(d*f)^(1/2)/f)/f+2*((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*((x-(d*f)^(1/2)/f
)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^
(1/2))/(x-(d*f)^(1/2)/f))*b*a+d^2/f^3/((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*ln(
(2*(a*f+c*d+(d*f)^(1/2)*b)/f+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+2*((
```

```
a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c
)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))/(x-(d*f)^(1/2)/f))*
b*c+d^2/f^3/((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d-(d*f)^(1/2)*b)
/f+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+2*((a*f+c*d-(d*f)^(1/2)*b)/f)^
(1/2)*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f
+c*d-(d*f)^(1/2)*b)/f)^(1/2))/(x+(d*f)^(1/2)/f))*b*c-1/2*d/(d*f)^(1/2)/f/((
a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d-(d*f)^(1/2)*b)/f+(b*f-2*(d*f
)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+2*((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*((x+(d*f
)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/
2)*b)/f)^(1/2))/(x+(d*f)^(1/2)/f))*a^2-1/2*d^3/(d*f)^(1/2)/f^3/((a*f+c*d-(d
*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d-(d*f)^(1/2)*b)/f+(b*f-2*(d*f)^(1/2)*c)
*(x+(d*f)^(1/2)/f)/f+2*((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*((x+(d*f)^(1/2)/f)
^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(
1/2))/(x+(d*f)^(1/2)/f))*c^2+1/8*d/(d*f)^(1/2)/f*((x+(d*f)^(1/2)/f)^2*c+(b*
f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*x*b
+1/16*d/(d*f)^(1/2)/f/c*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*
f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*b^2-1/32*d/(d*f)^(1/2)/f/c^(
3/2)*ln(((x+(d*f)^(1/2)/f)*c+1/2*(b*f-2*(d*f)^(1/2)*c)/f)/c^(1/2)+((x+(d*f)
^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2
)*b)/f)^(1/2))*b^3+3/4*d^2/(d*f)^(1/2)/f^2*ln(((x+(d*f)^(1/2)/f)*c+1/2*(b*f
-2*(d*f)^(1/2)*c)/f)/c^(1/2)+((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(
x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))*c^(1/2)*b-1/2*d^2/(d*f
)^(1/2)/f^2/((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d-(d*f)^(1/2)*b)
/f+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+2*((a*f+c*d-(d*f)^(1/2)*b)/f)^
(1/2)*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f
+c*d-(d*f)^(1/2)*b)/f)^(1/2))/(x+(d*f)^(1/2)/f))*b^2+d/f^2/((a*f+c*d-(d*f)^
(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d-(d*f)^(1/2)*b)/f+(b*f-2*(d*f)^(1/2)*c)*(x+
(d*f)^(1/2)/f)/f+2*((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*((x+(d*f)^(1/2)/f)^2*c
+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)
)/(x+(d*f)^(1/2)/f))*b*a+d^2/(d*f)^(1/2)/f^2/((a*f+c*d+(d*f)^(1/2)*b)/f)^(1
/2)*ln((2*(a*f+c*d+(d*f)^(1/2)*b)/f+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)
/f+2*((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^
(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))/(x-(d*f)^(1/
2)/f))*a*c-1/4/f*x*(c*x^2+b*x+a)^(3/2)-3/16*d/f^2*ln(((x+(d*f)^(1/2)/f)*c+1
/2*(b*f-2*(d*f)^(1/2)*c)/f)/c^(1/2)+((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/
2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))/c^(1/2)*b^2+1/2
*d/(d*f)^(1/2)/f*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2
)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*a+1/2*d^2/(d*f)^(1/2)/f^2*((x+(d*f)
^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2
)*b)/f)^(1/2)*c-1/4*d/f^2*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(
d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*x*c-3/4*d/f^2*ln(((x-(d*f)
^(1/2)/f)*c+1/2*(b*f+2*(d*f)^(1/2)*c)/f)/c^(1/2)+((x-(d*f)^(1/2)/f)^2*c+(b*
f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))*c^
(1/2)*a-3/16*d/f^2*ln(((x-(d*f)^(1/2)/f)*c+1/2*(b*f+2*(d*f)^(1/2)*c)/f)/c^(
1/2)+((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+
c*d+(d*f)^(1/2)*b)/f)^(1/2))/c^(1/2)*b^2-1/2*d/(d*f)^(1/2)/f*((x-(d*f)^(1/2
)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/
f)^(1/2)*a-1/2*d^2/(d*f)^(1/2)/f^2*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2
)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*c+3/32/f/c*(c*x^2
+b*x+a)^(1/2)*x*b^2-3/16/f/c*(c*x^2+b*x+a)^(1/2)*b*a+3/16/f/c^(3/2)*ln((c*x
+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*b^2*a-1/4*d/f^2*((x+(d*f)^(1/2)/f)^2*c
+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)
*x*c-3/4*d/f^2*ln(((x+(d*f)^(1/2)/f)*c+1/2*(b*f-2*(d*f)^(1/2)*c)/f)/c^(1/2)
+((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-
(d*f)^(1/2)*b)/f)^(1/2))*c^(1/2)*a
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(((c*sqrt(4*d*f))/(2*f^2)>0)', see `
assume?` for more details)Is ((c*sqrt(4*d*f))/(2*f^2)    +b/(2*f))    ^2
 -(c*((b*sqrt(4*d*f))                      /(2*f)                 +(c*d)/f+a))
     /f^2 positive, negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\left(c\,x^2+b\,x+a\right)^{3/2}}{d-f\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*x + c*x^2)^(3/2))/(d - f*x^2),x)
```

```
[Out] int((x^2*(a + b*x + c*x^2)^(3/2))/(d - f*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{ax^2\sqrt{a+bx+cx^2}}{-d+fx^2}\,dx-\int \frac{bx^3\sqrt{a+bx+cx^2}}{-d+fx^2}\,dx-\int \frac{cx^4\sqrt{a+bx+cx^2}}{-d+fx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c*x**2+b*x+a)**(3/2)/(-f*x**2+d),x)
```

```
[Out] -Integral(a*x**2*sqrt(a + b*x + c*x**2)/(-d + f*x**2), x) - Integral(b*x**3
*sqrt(a + b*x + c*x**2)/(-d + f*x**2), x) - Integral(c*x**4*sqrt(a + b*x +
c*x**2)/(-d + f*x**2), x)
```

3.86 $\int \frac{x\left(a+bx+cx^2\right)^{3/2}}{d-fx^2}\,dx$

Optimal. Leaf size=349

$$-\frac{\sqrt{a+bx+cx^2}\left(8acf+b^2f+2bcfx+8c^2d\right)}{8cf^2}-\frac{b\left(12acf+b^2(-f)+24c^2d\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{16c^{3/2}f^2}-\frac{\left(af+b\left(-\right.\right.}{}$$

```
[Out] -1/3*(c*x^2+b*x+a)^(3/2)/f-1/16*b*(12*a*c*f-b^2*f+24*c^2*d)*arctanh(1/2*(2*
c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(3/2)/f^2-1/2*arctanh(1/2*(b*d^(1/2)-
2*a*f^(1/2)+x*(2*c*d^(1/2)-b*f^(1/2)))/(c*x^2+b*x+a)^(1/2)/(c*d+a*f-b*d^(1/
2)*f^(1/2))^(1/2))*(c*d+a*f-b*d^(1/2)*f^(1/2))^(3/2)/f^(5/2)+1/2*arctanh(1/
2*(b*d^(1/2)+2*a*f^(1/2)+x*(2*c*d^(1/2)+b*f^(1/2)))/(c*x^2+b*x+a)^(1/2)/(c*
d+a*f+b*d^(1/2)*f^(1/2))^(1/2))*(c*d+a*f+b*d^(1/2)*f^(1/2))^(3/2)/f^(5/2)-1
/8*(2*b*c*f*x+8*a*c*f+b^2*f+8*c^2*d)*(c*x^2+b*x+a)^(1/2)/c/f^2
```

Rubi [A] time = 0.52, antiderivative size = 349, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {1021, 1070, 1078, 621, 206, 1033, 724}

$$-\frac{\sqrt{a+bx+cx^2}\left(8acf+b^2f+2bcfx+8c^2d\right)}{8cf^2}-\frac{b\left(12acf+b^2(-f)+24c^2d\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{16c^{3/2}f^2}-\frac{\left(af+b\left(-\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*x + c*x^2)^(3/2))/(d - f*x^2),x]
```

```
[Out] -((8*c^2*d + b^2*f + 8*a*c*f + 2*b*c*f*x)*Sqrt[a + b*x + c*x^2])/(8*c*f^2)
- (a + b*x + c*x^2)^(3/2)/(3*f) - (b*(24*c^2*d - b^2*f + 12*a*c*f)*ArcTanh[
(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(16*c^(3/2)*f^2) - ((c*d -
b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)*ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt
[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c
*x^2])])/(2*f^(5/2)) + ((c*d + b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)*ArcTanh[(b*Sq
rt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqrt[d]*
Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*f^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1021

```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (f
_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(h*(a + b*x + c*x^2)^p*(d + f*x^2)^(q +
 1))/(2*f*(p + q + 1)), x] - Dist[1/(2*f*(p + q + 1)), Int[(a + b*x + c*x^2
)^(p - 1)*(d + f*x^2)^q*Simp[h*p*(b*d) + a*(-2*g*f)*(p + q + 1) + (2*h*p*(c
*d - a*f) + b*(-2*g*f)*(p + q + 1))*x + (h*p*(-(b*f)) + c*(-2*g*f)*(p + q +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, g, h, q}, x] && NeQ[b^2 - 4*a
*c, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0]
```

Rule 1033

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q
), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q
), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f
, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]
```

Rule 1070

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((B*c*f*(2*p + 2*q + 3)
+ C*(b*f*p) + 2*c*C*f*(p + q + 1)*x)*(a + b*x + c*x^2)^p*(d + f*x^2)^(q + 1
))/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3)), x] - Dist[1/(2*c*f^2*(p + q + 1)*
(2*p + 2*q + 3)), Int[(a + b*x + c*x^2)^(p - 1)*(d + f*x^2)^q*Simp[p*(b*d)*
(C*(-(b*f))*(q + 1) - c*(-(B*f))*(2*p + 2*q + 3)) + (p + q + 1)*(b^2*C*d*f*
p + a*c*(C*(2*d*f) + f*(-2*A*f)*(2*p + 2*q + 3))) + (2*p*(c*d - a*f)*(C*(-(
b*f))*(q + 1) - c*(-(B*f))*(2*p + 2*q + 3)) + (p + q + 1)*(-(b*c*(C*(-4*d*f
)*(2*p + q + 2) + f*(2*C*d + 2*A*f)*(2*p + 2*q + 3)))))*x + (p*(-(b*f))*(C*
(-(b*f))*(q + 1) - c*(-(B*f))*(2*p + 2*q + 3)) + (p + q + 1)*(C*f^2*p*(b^2
- 4*a*c) - c^2*(C*(-4*d*f)*(2*p + q + 2) + f*(2*C*d + 2*A*f)*(2*p + 2*q + 3
))))*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, A, B, C, q}, x] && NeQ[b^2 -
4*a*c, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0] && NeQ[2*p + 2*q + 3, 0] &&  !I
GtQ[p, 0] &&  !IGtQ[q, 0]
```

Rule 1078

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
 (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqrt[d + e*x + f
*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + B*c*x)/((a + c*x^2)*Sqrt[d + e*x
 + f*x^2]), x], x] /; FreeQ[{a, c, d, e, f, A, B, C}, x] && NeQ[e^2 - 4*d*f
, 0]
```

Rubi steps

$$\int \frac{x\left(a+bx+cx^2\right)^{3/2}}{d-fx^2}\,dx = -\frac{\left(a+bx+cx^2\right)^{3/2}}{3f} + \frac{\int \frac{\sqrt{a+bx+cx^2}\left(\frac{3bd}{2}+3(cd+af)x+\frac{3}{2}bfx^2\right)}{d-fx^2}\,dx}{3f}$$

$$= -\frac{\left(8c^2d+b^2f+8acf+2bcfx\right)\sqrt{a+bx+cx^2}}{8cf^2} - \frac{\left(a+bx+cx^2\right)^{3/2}}{3f} - \frac{\int \frac{-\frac{3}{8}bdf\left(8c^2d+b^2f\right.}{}}{}$$

$$= -\frac{\left(8c^2d+b^2f+8acf+2bcfx\right)\sqrt{a+bx+cx^2}}{8cf^2} - \frac{\left(a+bx+cx^2\right)^{3/2}}{3f} + \frac{\int \frac{\frac{3}{8}bdf^2\left(24c^2d-b^2\right.}{}}{}$$

$$= -\frac{\left(8c^2d+b^2f+8acf+2bcfx\right)\sqrt{a+bx+cx^2}}{8cf^2} - \frac{\left(a+bx+cx^2\right)^{3/2}}{3f} + \frac{\left(cd-b\sqrt{d}\sqrt{f}\right.}{}$$

$$= -\frac{\left(8c^2d+b^2f+8acf+2bcfx\right)\sqrt{a+bx+cx^2}}{8cf^2} - \frac{\left(a+bx+cx^2\right)^{3/2}}{3f} - \frac{b\left(24c^2d-b^2f\right.}{}$$

$$= -\frac{\left(8c^2d+b^2f+8acf+2bcfx\right)\sqrt{a+bx+cx^2}}{8cf^2} - \frac{\left(a+bx+cx^2\right)^{3/2}}{3f} - \frac{b\left(24c^2d-b^2f\right.}{}$$

Mathematica [A] time = 0.80, size = 330, normalized size = 0.95

$$\frac{b\left(-12acf+b^2f-24c^2d\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)}{16c^{3/2}f^2} - \frac{\sqrt{f}\sqrt{a+x(b+cx)}\left(2cf(16a+7bx)+3b^2f+8c^2\left(3d+fx^2\right)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*x + c*x^2)^(3/2))/(d - f*x^2),x]
```

```
[Out] (b*(-24*c^2*d + b^2*f - 12*a*c*f)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x
*(b + c*x)])])/(16*c^(3/2)*f^2) - (Sqrt[f]*Sqrt[a + x*(b + c*x)]*(3*b^2*f +
 2*c*f*(16*a + 7*b*x) + 8*c^2*(3*d + f*x^2)) - 12*c*(c*d - b*Sqrt[d]*Sqrt[f
] + a*f)^(3/2)*ArcTanh[(-(b*Sqrt[d]) + 2*a*Sqrt[f] - 2*c*Sqrt[d]*x + b*Sqrt
[f]*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])] + 12*
c*(c*d + b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)*ArcTanh[(-2*(a*Sqrt[f] + c*Sqrt[d]*
x) - b*(Sqrt[d] + Sqrt[f]*x))/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a
 + x*(b + c*x)])]))/(24*c*f^(5/2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [B] time = 0.02, size = 4567, normalized size = 13.09

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x)
```

```
[Out] -1/6/f*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*
f+c*d+(d*f)^(1/2)*b)/f)^(3/2)-1/6/f*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/
2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(3/2)-1/2/f^2*((x-(d*f
)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/
2)*b)/f)^(1/2)*c*d+1/2/f/((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d+(
d*f)^(1/2)*b)/f+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+2*((a*f+c*d+(d*f)
^(1/2)*b)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1
/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))/(x-(d*f)^(1/2)/f))*a^2-1/8/f*((x
-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f
)^(1/2)*b)/f)^(1/2)*x*b-5/8/f^2*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c
)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*b*(d*f)^(1/2)-1/16/f
/c*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*
d+(d*f)^(1/2)*b)/f)^(1/2)*b^2+1/32/f/c^(3/2)*ln(((x-(d*f)^(1/2)/f)*c+1/2*(b
*f+2*(d*f)^(1/2)*c)/f)/c^(1/2)+((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)
*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))*b^3-1/8/f*((x+(d*f)^
(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)
*b)/f)^(1/2)*x*b+5/8/f^2*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d
*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*b*(d*f)^(1/2)-1/16/f/c*((x+
(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)
^(1/2)*b)/f)^(1/2)*b^2+1/32/f/c^(3/2)*ln(((x+(d*f)^(1/2)/f)*c+1/2*(b*f-2*(d
*f)^(1/2)*c)/f)/c^(1/2)+((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*
f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))*b^3-1/2/f^2*((x+(d*f)^(1/2)
/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f
)^(1/2)*c*d+1/2/f/((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d-(d*f)^(1
/2)*b)/f+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+2*((a*f+c*d-(d*f)^(1/2)*
b)/f)^(1/2)*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/
f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))/(x+(d*f)^(1/2)/f))*a^2+1/f^2/((a*f+c*d+
(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d+(d*f)^(1/2)*b)/f+(b*f+2*(d*f)^(1/2)*
c)*(x-(d*f)^(1/2)/f)/f+2*((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*((x-(d*f)^(1/2)/
f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)
^(1/2))/(x-(d*f)^(1/2)/f))*(d*f)^(1/2)*b*a+1/f^2/((a*f+c*d+(d*f)^(1/2)*b)/f
)^(1/2)*ln((2*(a*f+c*d+(d*f)^(1/2)*b)/f+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2
)/f)/f+2*((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d
*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))/(x-(d*f)
^(1/2)/f))*a*c*d-1/f^2/((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d-(d*
f)^(1/2)*b)/f+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+2*((a*f+c*d-(d*f)^(
1/2)*b)/f)^(1/2)*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2
)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))/(x+(d*f)^(1/2)/f))*(d*f)^(1/2)*b*a
+1/f^2/((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d-(d*f)^(1/2)*b)/f+(b
*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+2*((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)
*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-
(d*f)^(1/2)*b)/f)^(1/2))/(x+(d*f)^(1/2)/f))*a*c*d-1/2/f*((x-(d*f)^(1/2)/f)^
2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1
/2)*a-1/2/f*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/
f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*a-3/8/f/c^(1/2)*ln(((x-(d*f)^(1/2)/f)*c+
1/2*(b*f+2*(d*f)^(1/2)*c)/f)/c^(1/2)+((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1
/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))*a*b-3/4/f^2*ln
(((x-(d*f)^(1/2)/f)*c+1/2*(b*f+2*(d*f)^(1/2)*c)/f)/c^(1/2)+((x-(d*f)^(1/2)/
```

```
f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)
^(1/2))*c^(1/2)*(d*f)^(1/2)*a-3/16/f^2*ln(((x-(d*f)^(1/2)/f)*c+1/2*(b*f+2*(
d*f)^(1/2)*c)/f)/c^(1/2)+((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d
*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))/c^(1/2)*b^2*(d*f)^(1/2)-3/
4/f^2*ln(((x-(d*f)^(1/2)/f)*c+1/2*(b*f+2*(d*f)^(1/2)*c)/f)/c^(1/2)+((x-(d*f
)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/
2)*b)/f)^(1/2))*c^(1/2)*d*b-1/2/f^3*ln(((x-(d*f)^(1/2)/f)*c+1/2*(b*f+2*(d*f
)^(1/2)*c)/f)/c^(1/2)+((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)
^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))*c^(3/2)*(d*f)^(1/2)*d+1/2/f^2
/((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d+(d*f)^(1/2)*b)/f+(b*f+2*(
d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+2*((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*((x-(
d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^
(1/2)*b)/f)^(1/2))/(x-(d*f)^(1/2)/f))*d*b^2+1/2/f^3/((a*f+c*d+(d*f)^(1/2)*b
)/f)^(1/2)*ln((2*(a*f+c*d+(d*f)^(1/2)*b)/f+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(
1/2)/f)/f+2*((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(b*f+2
*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))/(x-(d
*f)^(1/2)/f))*c^2*d^2+1/4/f^2*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*
(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*x*(d*f)^(1/2)*c+3/4/f^
2*ln(((x+(d*f)^(1/2)/f)*c+1/2*(b*f-2*(d*f)^(1/2)*c)/f)/c^(1/2)+((x+(d*f)^(1
/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b
)/f)^(1/2))*c^(1/2)*(d*f)^(1/2)*a-3/8/f/c^(1/2)*ln(((x+(d*f)^(1/2)/f)*c+1/2
*(b*f-2*(d*f)^(1/2)*c)/f)/c^(1/2)+((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)
*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))*a*b+3/16/f^2*ln((
(x+(d*f)^(1/2)/f)*c+1/2*(b*f-2*(d*f)^(1/2)*c)/f)/c^(1/2)+((x+(d*f)^(1/2)/f)
^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(
1/2))/c^(1/2)*b^2*(d*f)^(1/2)-3/4/f^2*ln(((x+(d*f)^(1/2)/f)*c+1/2*(b*f-2*(d
*f)^(1/2)*c)/f)/c^(1/2)+((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*
f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))*c^(1/2)*d*b+1/2/f^3*ln(((x+
(d*f)^(1/2)/f)*c+1/2*(b*f-2*(d*f)^(1/2)*c)/f)/c^(1/2)+((x+(d*f)^(1/2)/f)^2*
c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2
))*c^(3/2)*(d*f)^(1/2)*d+1/2/f^2/((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a
*f+c*d-(d*f)^(1/2)*b)/f+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+2*((a*f+c
*d-(d*f)^(1/2)*b)/f)^(1/2)*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+
(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))/(x+(d*f)^(1/2)/f))*d*b^2
+1/2/f^3/((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d-(d*f)^(1/2)*b)/f+
(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+2*((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/
2)*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*
d-(d*f)^(1/2)*b)/f)^(1/2))/(x+(d*f)^(1/2)/f))*c^2*d^2-1/4/f^2*((x-(d*f)^(1/
2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)
/f)^(1/2)*x*(d*f)^(1/2)*c+1/f^3/((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*
f+c*d+(d*f)^(1/2)*b)/f+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+2*((a*f+c*
d+(d*f)^(1/2)*b)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(
d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))/(x-(d*f)^(1/2)/f))*(d*f)^
(1/2)*b*c*d-1/f^3/((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d-(d*f)^(1
/2)*b)/f+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+2*((a*f+c*d-(d*f)^(1/2)*
b)/f)^(1/2)*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/
f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))/(x+(d*f)^(1/2)/f))*(d*f)^(1/2)*b*c*d
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(((c*sqrt(4*d*f))/(2*f^2)>0)', see `
assume?` for more details)Is ((c*sqrt(4*d*f))/(2*f^2)    +b/(2*f))    ^2
```

```
 -(c*((b*sqrt(4*d*f))                    /(2*f)                       +(c*d)/f+a))
     /f^2 positive, negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\left(c\,x^2+b\,x+a\right)^{3/2}}{d-f\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*x + c*x^2)^(3/2))/(d - f*x^2),x)
```

```
[Out] int((x*(a + b*x + c*x^2)^(3/2))/(d - f*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{ax\sqrt{a+bx+cx^2}}{-d+fx^2}\,dx-\int \frac{bx^2\sqrt{a+bx+cx^2}}{-d+fx^2}\,dx-\int \frac{cx^3\sqrt{a+bx+cx^2}}{-d+fx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x**2+b*x+a)**(3/2)/(-f*x**2+d),x)
```

```
[Out] -Integral(a*x*sqrt(a + b*x + c*x**2)/(-d + f*x**2), x) - Integral(b*x**2*sq
rt(a + b*x + c*x**2)/(-d + f*x**2), x) - Integral(c*x**3*sqrt(a + b*x + c*x
**2)/(-d + f*x**2), x)
```

3.87 $\int \frac{(a+bx+cx^2)^{3/2}}{d-fx^2}\,dx$

Optimal. Leaf size=315

$$-\frac{\left(12acf+3b^2f+8c^2d\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8\sqrt{c}f^2}+\frac{\left(af+b\left(-\sqrt{d}\right)\sqrt{f}+cd\right)^{3/2}\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+c}}\right)}{2\sqrt{d}f^2}$$

```
[Out] -1/8*(12*a*c*f+3*b^2*f+8*c^2*d)*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)
^(1/2))/f^2/c^(1/2)+1/2*arctanh(1/2*(b*d^(1/2)-2*a*f^(1/2)+x*(2*c*d^(1/2)-b
*f^(1/2)))/(c*x^2+b*x+a)^(1/2)/(c*d+a*f-b*d^(1/2)*f^(1/2))^(1/2))*(c*d+a*f-
b*d^(1/2)*f^(1/2))^(3/2)/f^2/d^(1/2)+1/2*arctanh(1/2*(b*d^(1/2)+2*a*f^(1/2)
+x*(2*c*d^(1/2)+b*f^(1/2)))/(c*x^2+b*x+a)^(1/2)/(c*d+a*f+b*d^(1/2)*f^(1/2))
^(1/2))*(c*d+a*f+b*d^(1/2)*f^(1/2))^(3/2)/f^2/d^(1/2)-1/4*(2*c*x+5*b)*(c*x^
2+b*x+a)^(1/2)/f
```

Rubi [A] time = 0.52, antiderivative size = 315, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {978, 1078, 621, 206, 1033, 724}

$$-\frac{\left(12acf+3b^2f+8c^2d\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8\sqrt{c}f^2}+\frac{\left(af+b\left(-\sqrt{d}\right)\sqrt{f}+cd\right)^{3/2}\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+c}}\right)}{2\sqrt{d}f^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^(3/2)/(d - f*x^2),x]
```

```
[Out] -((5*b + 2*c*x)*Sqrt[a + b*x + c*x^2])/(4*f) - ((8*c^2*d + 3*b^2*f + 12*a*c
*f)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(8*Sqrt[c]*f^2)
 + ((c*d - b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)*ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f]
+ (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[
a + b*x + c*x^2])])/(2*Sqrt[d]*f^2) + ((c*d + b*Sqrt[d]*Sqrt[f] + a*f)^(3/2
)*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c
*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[d]*f^2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 978

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (f_.)*(x_)^2)^(q_), x
_Symbol] :> Simp[((b*(3*p + 2*q) + 2*c*(p + q)*x)*(a + b*x + c*x^2)^(p - 1)
*(d + f*x^2)^(q + 1))/(2*f*(p + q)*(2*p + 2*q + 1)), x] - Dist[1/(2*f*(p +
q)*(2*p + 2*q + 1)), Int[(a + b*x + c*x^2)^(p - 2)*(d + f*x^2)^q*Simp[b^2*d
*(p - 1)*(2*p + q) - (p + q)*(b^2*d*(1 - p) - 2*a*(c*d - a*f*(2*p + 2*q + 1
))) - (2*b*(c*d - a*f)*(1 - p)*(2*p + q) - 2*(p + q)*b*(2*c*d*(2*p + q) - (
c*d + a*f)*(2*p + 2*q + 1)))*x + (b^2*f*p*(1 - p) + 2*c*(p + q)*(c*d*(2*p -
 1) - a*f*(4*p + 2*q - 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, q}, x]
 && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 1] && NeQ[p + q, 0] && NeQ[2*p + 2*q + 1,
 0] &&  !IGtQ[p, 0] &&  !IGtQ[q, 0]
```

Rule 1033

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q
), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q
), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f
, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]
```

Rule 1078

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
 (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqrt[d + e*x + f
*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + B*c*x)/((a + c*x^2)*Sqrt[d + e*x
 + f*x^2]), x], x] /; FreeQ[{a, c, d, e, f, A, B, C}, x] && NeQ[e^2 - 4*d*f
, 0]
```

Rubi steps

$$\int \frac{\left(a+bx+cx^2\right)^{3/2}}{d-fx^2}\,dx = -\frac{(5b+2cx)\sqrt{a+bx+cx^2}}{4f}+\frac{\int \frac{\frac{1}{4}\left(5b^2d+4a(cd+2af)\right)+4b(cd+af)x+\frac{1}{4}\left(8c^2d+3b^2f+12acf\right)x^2}{\sqrt{a+bx+cx^2}\left(d-fx^2\right)}\,dx}{2f}$$

$$= -\frac{(5b+2cx)\sqrt{a+bx+cx^2}}{4f}-\frac{\int \frac{-\frac{1}{4}d\left(8c^2d+3b^2f+12acf\right)-\frac{1}{4}f\left(5b^2d+4a(cd+2af)\right)-4bf(cd+af)x}{\sqrt{a+bx+cx^2}\left(d-fx^2\right)}\,dx}{2f^2}$$

$$= -\frac{(5b+2cx)\sqrt{a+bx+cx^2}}{4f}-\frac{\left(cd-b\sqrt{d}\sqrt{f}+af\right)^2\int \frac{1}{\left(-\sqrt{d}\sqrt{f}-fx\right)\sqrt{a+bx+cx^2}}\,dx}{2\sqrt{d}\,f^{3/2}}+\frac{(c\ldots}{\ldots}$$

$$= -\frac{(5b+2cx)\sqrt{a+bx+cx^2}}{4f}-\frac{\left(8c^2d+3b^2f+12acf\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8\sqrt{c}\,f^2}+\frac{(cd\ldots}{\ldots}$$

$$= -\frac{(5b+2cx)\sqrt{a+bx+cx^2}}{4f}-\frac{\left(8c^2d+3b^2f+12acf\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8\sqrt{c}\,f^2}+\frac{(cd\ldots}{\ldots}$$

Mathematica [A] time = 0.81, size = 298, normalized size = 0.95

$$-\frac{\frac{\left(12acf+3b^2f+8c^2d\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)}{\sqrt{c}}+\frac{4\left(af+b\left(-\sqrt{d}\right)\sqrt{f}+cd\right)^{3/2}\tanh^{-1}\left(\frac{2a\sqrt{f}-b\sqrt{d}+b\sqrt{f}x-2c\sqrt{d}x}{2\sqrt{a+x(b+cx)}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{\sqrt{d}}+\frac{4\left(af+b\sqrt{d}\sqrt{f}+cd\right)^{3/2}\ldots}{\ldots}}{8f^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^(3/2)/(d - f*x^2),x]
```

```
[Out] -1/8*(2*f*(5*b + 2*c*x)*Sqrt[a + x*(b + c*x)] + ((8*c^2*d + 3*b^2*f + 12*a*
c*f)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/Sqrt[c] + (4*(
c*d - b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)*ArcTanh[(-(b*Sqrt[d]) + 2*a*Sqrt[f] -
2*c*Sqrt[d]*x + b*Sqrt[f]*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a
+ x*(b + c*x)])])/Sqrt[d] + (4*(c*d + b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)*ArcTan
h[(-2*(a*Sqrt[f] + c*Sqrt[d]*x) - b*(Sqrt[d] + Sqrt[f]*x))/(2*Sqrt[c*d + b*
Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])])/Sqrt[d])/f^2
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [B] time = 0.02, size = 4574, normalized size = 14.52

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x)
```

```
[Out] -1/6/(d*f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2
)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(3/2)+1/6/(d*f)^(1/2)*((x+(d*f)^(1/2)/f)^
2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(3
/2)-1/2/f^2*ln(((x+(d*f)^(1/2)/f)*c+1/2*(b*f-2*(d*f)^(1/2)*c)/f)/c^(1/2)+((
x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*
f)^(1/2)*b)/f)^(1/2))*c^(3/2)*d-1/2/(d*f)^(1/2)/((a*f+c*d-(d*f)^(1/2)*b)/f)
^(1/2)*ln((2*(a*f+c*d-(d*f)^(1/2)*b)/f+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)
/f)/f+2*((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*
f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))/(x+(d*f)^
(1/2)/f))*a^2+1/32/(d*f)^(1/2)/c^(3/2)*ln(((x-(d*f)^(1/2)/f)*c+1/2*(b*f+2*(
d*f)^(1/2)*c)/f)/c^(1/2)+((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d
*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))*b^3-1/2/f^2*ln(((x-(d*f)^(
1/2)/f)*c+1/2*(b*f+2*(d*f)^(1/2)*c)/f)/c^(1/2)+((x-(d*f)^(1/2)/f)^2*c+(b*f+
2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))*c^(3
/2)*d+1/2/(d*f)^(1/2)/((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d+(d*f
)^(1/2)*b)/f+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+2*((a*f+c*d+(d*f)^(1
/2)*b)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)
/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))/(x-(d*f)^(1/2)/f))*a^2-1/4/f*((x+(d
*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(
1/2)*b)/f)^(1/2)*x*c+1/8/(d*f)^(1/2)*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1
/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*x*b+1/16/(d*f)^
(1/2)/c*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a
*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*b^2-3/4/f*ln(((x+(d*f)^(1/2)/f)*c+1/2*(b*f-2
*(d*f)^(1/2)*c)/f)/c^(1/2)+((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+
```

```
(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))*c^(1/2)*a-3/16/f*ln(((x+
(d*f)^(1/2)/f)*c+1/2*(b*f-2*(d*f)^(1/2)*c)/f)/c^(1/2)+((x+(d*f)^(1/2)/f)^2*
c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2
))/c^(1/2)*b^2-1/32/(d*f)^(1/2)/c^(3/2)*ln(((x+(d*f)^(1/2)/f)*c+1/2*(b*f-2*
(d*f)^(1/2)*c)/f)/c^(1/2)+((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(
d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))*b^3-1/4/f*((x-(d*f)^(1/2)
/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f
)^(1/2)*x*c-1/8/(d*f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x
-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*x*b-1/16/(d*f)^(1/2)/c*(
(x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d
*f)^(1/2)*b)/f)^(1/2)*b^2-3/4/f*ln(((x-(d*f)^(1/2)/f)*c+1/2*(b*f+2*(d*f)^(1
/2)*c)/f)/c^(1/2)+((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/
2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))*c^(1/2)*a-3/16/f*ln(((x-(d*f)^(1/
2)/f)*c+1/2*(b*f+2*(d*f)^(1/2)*c)/f)/c^(1/2)+((x-(d*f)^(1/2)/f)^2*c+(b*f+2*
(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))/c^(1/2
)*b^2+1/(d*f)^(1/2)/f/((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d+(d*f
)^(1/2)*b)/f+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+2*((a*f+c*d+(d*f)^(1
/2)*b)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)
/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))/(x-(d*f)^(1/2)/f))*a*c*d+1/f^2/((a*
f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d+(d*f)^(1/2)*b)/f+(b*f+2*(d*f)^
(1/2)*c)*(x-(d*f)^(1/2)/f)/f+2*((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*((x-(d*f)^
(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)
*b)/f)^(1/2))/(x-(d*f)^(1/2)/f))*b*c*d+1/2/(d*f)^(1/2)/f^2/((a*f+c*d+(d*f)^
(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d+(d*f)^(1/2)*b)/f+(b*f+2*(d*f)^(1/2)*c)*(x-
(d*f)^(1/2)/f)/f+2*((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c
+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)
)/(x-(d*f)^(1/2)/f))*c^2*d^2-3/4/(d*f)^(1/2)/f*ln(((x-(d*f)^(1/2)/f)*c+1/2*
(b*f+2*(d*f)^(1/2)*c)/f)/c^(1/2)+((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*
c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))*c^(1/2)*d*b+1/2/(d
*f)^(1/2)/f/((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d+(d*f)^(1/2)*b)
/f+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+2*((a*f+c*d+(d*f)^(1/2)*b)/f)^
(1/2)*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f
+c*d+(d*f)^(1/2)*b)/f)^(1/2))/(x-(d*f)^(1/2)/f))*d*b^2+3/4/(d*f)^(1/2)/f*ln
(((x+(d*f)^(1/2)/f)*c+1/2*(b*f-2*(d*f)^(1/2)*c)/f)/c^(1/2)+((x+(d*f)^(1/2)/
f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)
^(1/2))*c^(1/2)*d*b-1/2/(d*f)^(1/2)/f/((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*ln(
(2*(a*f+c*d-(d*f)^(1/2)*b)/f+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+2*((
a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c
)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))/(x+(d*f)^(1/2)/f))*
d*b^2+1/f^2/((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d-(d*f)^(1/2)*b)
/f+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+2*((a*f+c*d-(d*f)^(1/2)*b)/f)^
(1/2)*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f
+c*d-(d*f)^(1/2)*b)/f)^(1/2))/(x+(d*f)^(1/2)/f))*b*c*d-1/2/(d*f)^(1/2)/f^2/
((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d-(d*f)^(1/2)*b)/f+(b*f-2*(d
*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+2*((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*((x+(d
*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(
1/2)*b)/f)^(1/2))/(x+(d*f)^(1/2)/f))*c^2*d^2-5/8/f*((x-(d*f)^(1/2)/f)^2*c+(
b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*b
-1/2/(d*f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2
)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*a-5/8/f*((x+(d*f)^(1/2)/f)^2*c+(b*f
-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*b+1/
2/(d*f)^(1/2)*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f
)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*a+3/8/(d*f)^(1/2)/c^(1/2)*ln(((x+(d*f)
^(1/2)/f)*c+1/2*(b*f-2*(d*f)^(1/2)*c)/f)/c^(1/2)+((x+(d*f)^(1/2)/f)^2*c+(b*
f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))*a*
b+1/2/(d*f)^(1/2)/f*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(
1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*c*d+1/f/((a*f+c*d-(d*f)^(1/2)*b)
/f)^(1/2)*ln((2*(a*f+c*d-(d*f)^(1/2)*b)/f+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1
/2)/f)/f+2*((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*
```

```
(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))/(x+(d*
f)^(1/2)/f))*b*a-3/8/(d*f)^(1/2)/c^(1/2)*ln(((x-(d*f)^(1/2)/f)*c+1/2*(b*f+2
*(d*f)^(1/2)*c)/f)/c^(1/2)+((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-
(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))*a*b-1/2/(d*f)^(1/2)/f*((
x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*
f)^(1/2)*b)/f)^(1/2)*c*d+1/f/((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c
*d+(d*f)^(1/2)*b)/f+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+2*((a*f+c*d+(
d*f)^(1/2)*b)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f
)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))/(x-(d*f)^(1/2)/f))*b*a-1/(d*
f)^(1/2)/f/((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d-(d*f)^(1/2)*b)/
f+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+2*((a*f+c*d-(d*f)^(1/2)*b)/f)^(
1/2)*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+
c*d-(d*f)^(1/2)*b)/f)^(1/2))/(x+(d*f)^(1/2)/f))*a*c*d
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(((c*sqrt(4*d*f))/(2*f^2)>0)', see `
assume?` for more details)Is ((c*sqrt(4*d*f))/(2*f^2)    +b/(2*f))    ^2
 -(c*((b*sqrt(4*d*f))                 /(2*f)                +(c*d)/f+a))
     /f^2 positive, negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,x^2+b\,x+a\right)^{3/2}}{d-f\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)^(3/2)/(d - f*x^2),x)
```

```
[Out] int((a + b*x + c*x^2)^(3/2)/(d - f*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{a\sqrt{a+bx+cx^2}}{-d+fx^2}\,dx-\int \frac{bx\sqrt{a+bx+cx^2}}{-d+fx^2}\,dx-\int \frac{cx^2\sqrt{a+bx+cx^2}}{-d+fx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(3/2)/(-f*x**2+d),x)
```

```
[Out] -Integral(a*sqrt(a + b*x + c*x**2)/(-d + f*x**2), x) - Integral(b*x*sqrt(a
+ b*x + c*x**2)/(-d + f*x**2), x) - Integral(c*x**2*sqrt(a + b*x + c*x**2)/
(-d + f*x**2), x)
```

3.88 $\int \frac{\left(a+bx+cx^2\right)^{3/2}}{x\left(d-fx^2\right)}\,dx$

Optimal. Leaf size=469

$$-\frac{a^{3/2}\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{d}-\frac{b\left(b^2-12ac\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{16c^{3/2}d}-\frac{\sqrt{a+bx+cx^2}\,\left(8acf+b^2f+2bcfx+8\right.}{8cdf}$$

```
[Out] -a^(3/2)*arctanh(1/2*(b*x+2*a)/a^(1/2)/(c*x^2+b*x+a)^(1/2))/d-1/16*b*(-12*a
*c+b^2)*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(3/2)/d-1/16*b
*(12*a*c*f-b^2*f+24*c^2*d)*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2
))/c^(3/2)/d/f-1/2*arctanh(1/2*(b*d^(1/2)-2*a*f^(1/2)+x*(2*c*d^(1/2)-b*f^(1
/2)))/(c*x^2+b*x+a)^(1/2)/(c*d+a*f-b*d^(1/2)*f^(1/2))^(1/2))*(c*d+a*f-b*d^(
1/2)*f^(1/2))^(3/2)/d/f^(3/2)+1/2*arctanh(1/2*(b*d^(1/2)+2*a*f^(1/2)+x*(2*c
*d^(1/2)+b*f^(1/2)))/(c*x^2+b*x+a)^(1/2)/(c*d+a*f+b*d^(1/2)*f^(1/2))^(1/2))
*(c*d+a*f+b*d^(1/2)*f^(1/2))^(3/2)/d/f^(3/2)+1/8*(2*b*c*x+8*a*c+b^2)*(c*x^2
+b*x+a)^(1/2)/c/d-1/8*(2*b*c*f*x+8*a*c*f+b^2*f+8*c^2*d)*(c*x^2+b*x+a)^(1/2)
/c/d/f
```

Rubi [A] time = 1.27, antiderivative size = 469, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 11, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.393, Rules used = {6725, 734, 814, 843, 621, 206, 724, 1021, 1070, 1078, 1033}

$$-\frac{a^{3/2}\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{d}-\frac{\sqrt{a+bx+cx^2}\,\left(8acf+b^2f+2bcfx+8c^2d\right)}{8cdf}-\frac{b\left(12acf+b^2(-f)+24c^2d\right)\tanh}{16c^{3/2}df}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^(3/2)/(x*(d - f*x^2)),x]
```

```
[Out] ((b^2 + 8*a*c + 2*b*c*x)*Sqrt[a + b*x + c*x^2])/(8*c*d) - ((8*c^2*d + b^2*f
 + 8*a*c*f + 2*b*c*f*x)*Sqrt[a + b*x + c*x^2])/(8*c*d*f) - (a^(3/2)*ArcTanh
[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/d - (b*(b^2 - 12*a*c)*ArcT
anh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(16*c^(3/2)*d) - (b*(24
*c^2*d - b^2*f + 12*a*c*f)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*
x^2])])/(16*c^(3/2)*d*f) - ((c*d - b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)*ArcTanh[(
b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt
[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*d*f^(3/2)) + ((c*d + b*Sqrt[
d]*Sqrt[f] + a*f)^(3/2)*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b
*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])]
)/(2*d*f^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1021

```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (f
_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(h*(a + b*x + c*x^2)^p*(d + f*x^2)^(q +
 1))/(2*f*(p + q + 1)), x] - Dist[1/(2*f*(p + q + 1)), Int[(a + b*x + c*x^2
)^(p - 1)*(d + f*x^2)^q*Simp[h*p*(b*d) + a*(-2*g*f)*(p + q + 1) + (2*h*p*(c
*d - a*f) + b*(-2*g*f)*(p + q + 1))*x + (h*p*(-(b*f)) + c*(-2*g*f)*(p + q +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, g, h, q}, x] && NeQ[b^2 - 4*a
*c, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0]
```

Rule 1033

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q
), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q
), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f
, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]
```

Rule 1070

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
```

```
)^2)*((d_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((B*c*f*(2*p + 2*q + 3)
+ C*(b*f*p) + 2*c*C*f*(p + q + 1)*x)*(a + b*x + c*x^2)^p*(d + f*x^2)^(q + 1
))/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3)), x] - Dist[1/(2*c*f^2*(p + q + 1)*
(2*p + 2*q + 3)), Int[(a + b*x + c*x^2)^(p - 1)*(d + f*x^2)^q*Simp[p*(b*d)*
(C*(-(b*f))*(q + 1) - c*(-(B*f))*(2*p + 2*q + 3)) + (p + q + 1)*(b^2*C*d*f*
p + a*c*(C*(2*d*f) + f*(-2*A*f)*(2*p + 2*q + 3))) + (2*p*(c*d - a*f)*(C*(-(
b*f))*(q + 1) - c*(-(B*f))*(2*p + 2*q + 3)) + (p + q + 1)*(-(b*c*(C*(-4*d*f
)*(2*p + q + 2) + f*(2*C*d + 2*A*f)*(2*p + 2*q + 3)))))*x + (p*(-(b*f))*(C*
(-(b*f))*(q + 1) - c*(-(B*f))*(2*p + 2*q + 3)) + (p + q + 1)*(C*f^2*p*(b^2
- 4*a*c) - c^2*(C*(-4*d*f)*(2*p + q + 2) + f*(2*C*d + 2*A*f)*(2*p + 2*q + 3
))))*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, A, B, C, q}, x] && NeQ[b^2 -
4*a*c, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0] && NeQ[2*p + 2*q + 3, 0] &&  !I
GtQ[p, 0] &&  !IGtQ[q, 0]
```

Rule 1078

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
 (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqrt[d + e*x + f
*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + B*c*x)/((a + c*x^2)*Sqrt[d + e*x
 + f*x^2]), x], x] /; FreeQ[{a, c, d, e, f, A, B, C}, x] && NeQ[e^2 - 4*d*f
, 0]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{\left(a+bx+cx^2\right)^{3/2}}{x\left(d-fx^2\right)}dx = \int \left(\frac{\left(a+bx+cx^2\right)^{3/2}}{dx} - \frac{fx\left(a+bx+cx^2\right)^{3/2}}{d\left(-d+fx^2\right)}\right)dx$$

$$= \frac{\int \frac{\left(a+bx+cx^2\right)^{3/2}}{x}dx}{d} - \frac{f\int \frac{x\left(a+bx+cx^2\right)^{3/2}}{-d+fx^2}dx}{d}$$

$$= \frac{\int \frac{\sqrt{a+bx+cx^2}\left(-\frac{3bd}{2}-3(cd+af)x-\frac{3}{2}bfx^2\right)}{-d+fx^2}dx}{3d} - \frac{\int \frac{(-2a-bx)\sqrt{a+bx+cx^2}}{x}dx}{2d}$$

$$= \frac{\left(b^2+8ac+2bcx\right)\sqrt{a+bx+cx^2}}{8cd} - \frac{\left(8c^2d+b^2f+8acf+2bcfx\right)\sqrt{a+bx+cx^2}}{8cdf} + \frac{\int}{}$$

$$= \frac{\left(b^2+8ac+2bcx\right)\sqrt{a+bx+cx^2}}{8cd} - \frac{\left(8c^2d+b^2f+8acf+2bcfx\right)\sqrt{a+bx+cx^2}}{8cdf} + \frac{a^2}{}$$

$$= \frac{\left(b^2+8ac+2bcx\right)\sqrt{a+bx+cx^2}}{8cd} - \frac{\left(8c^2d+b^2f+8acf+2bcfx\right)\sqrt{a+bx+cx^2}}{8cdf} - \frac{(2}{}$$

$$= \frac{\left(b^2+8ac+2bcx\right)\sqrt{a+bx+cx^2}}{8cd} - \frac{\left(8c^2d+b^2f+8acf+2bcfx\right)\sqrt{a+bx+cx^2}}{8cdf} - \frac{a^{3/}}{}$$

$$= \frac{\left(b^2+8ac+2bcx\right)\sqrt{a+bx+cx^2}}{8cd} - \frac{\left(8c^2d+b^2f+8acf+2bcfx\right)\sqrt{a+bx+cx^2}}{8cdf} - \frac{a^{3/}}{}$$

Mathematica [A] time = 0.50, size = 755, normalized size = 1.61

$$-\frac{2a^{3/2}f^{3/2}\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+x(b+cx)}}\right)+2cd\sqrt{f}\sqrt{a+x(b+cx)}-af\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}\tanh^{-1}\left(\frac{2a\sqrt{f}-b\sqrt{d}-}{2\sqrt{a+x(b+cx)}\sqrt{a}}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^(3/2)/(x*(d - f*x^2)),x]
```

```
[Out] -1/2*(2*c*d*Sqrt[f]*Sqrt[a + x*(b + c*x)] + 2*a^(3/2)*f^(3/2)*ArcTanh[(2*a
+ b*x)/(2*Sqrt[a]*Sqrt[a + x*(b + c*x)])] + 3*b*Sqrt[c]*d*Sqrt[f]*ArcTanh[(
b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])] - c*d*Sqrt[c*d - b*Sqrt[d]*Sq
rt[f] + a*f]*ArcTanh[(-(b*Sqrt[d]) + 2*a*Sqrt[f] - 2*c*Sqrt[d]*x + b*Sqrt[f
]*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])] + b*Sqr
t[d]*Sqrt[f]*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*ArcTanh[(-(b*Sqrt[d]) + 2*
a*Sqrt[f] - 2*c*Sqrt[d]*x + b*Sqrt[f]*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] +
a*f]*Sqrt[a + x*(b + c*x)])] - a*f*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*ArcT
anh[(-(b*Sqrt[d]) + 2*a*Sqrt[f] - 2*c*Sqrt[d]*x + b*Sqrt[f]*x)/(2*Sqrt[c*d
- b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])] + c*d*Sqrt[c*d + b*Sqrt[
d]*Sqrt[f] + a*f]*ArcTanh[(-2*(a*Sqrt[f] + c*Sqrt[d]*x) - b*(Sqrt[d] + Sqrt
[f]*x))/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])] + b*
Sqrt[d]*Sqrt[f]*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*ArcTanh[(-2*(a*Sqrt[f]
+ c*Sqrt[d]*x) - b*(Sqrt[d] + Sqrt[f]*x))/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] +
 a*f]*Sqrt[a + x*(b + c*x)])] + a*f*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Arc
Tanh[(-2*(a*Sqrt[f] + c*Sqrt[d]*x) - b*(Sqrt[d] + Sqrt[f]*x))/(2*Sqrt[c*d +
 b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])])/(d*f^(3/2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/x/(-f*x^2+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/x/(-f*x^2+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:index.cc index_m operator + Error: Bad Argum
ent Value
```

maple [B] time = 0.02, size = 4765, normalized size = 10.16

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(3/2)/x/(-f*x^2+d),x)
```

```
[Out] 1/3/d*(c*x^2+b*x+a)^(3/2)-1/6/d*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c
)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(3/2)-1/6/d*((x+(d*f)^(1/2
)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/
f)^(3/2)+1/4/d*b*(c*x^2+b*x+a)^(1/2)*x+1/8/d/c*(c*x^2+b*x+a)^(1/2)*b^2-1/16
/d/c^(3/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*b^3-1/8/d*((x-(d*f)^
(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)
*b)/f)^(1/2)*x*b-1/16/d/c*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(
d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*b^2-3/4/f*ln(((x-(d*f)^(1/
2)/f)*c+1/2*(b*f+2*(d*f)^(1/2)*c)/f)/c^(1/2)+((x-(d*f)^(1/2)/f)^2*c+(b*f+2*
(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))*c^(1/2
)*b-1/2/f^2*ln(((x-(d*f)^(1/2)/f)*c+1/2*(b*f+2*(d*f)^(1/2)*c)/f)/c^(1/2)+((
x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*
f)^(1/2)*b)/f)^(1/2))*c^(3/2)*(d*f)^(1/2)+1/2/f/((a*f+c*d+(d*f)^(1/2)*b)/f)
^(1/2)*ln((2*(a*f+c*d+(d*f)^(1/2)*b)/f+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)
/f)/f+2*((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*
f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))/(x-(d*f)^
(1/2)/f))*b^2+1/2/d/((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d+(d*f)^
(1/2)*b)/f+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+2*((a*f+c*d+(d*f)^(1/2
)*b)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f
)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))/(x-(d*f)^(1/2)/f))*a^2+1/2/d/((a*f+c*
d-(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d-(d*f)^(1/2)*b)/f+(b*f-2*(d*f)^(1/2
)*c)*(x+(d*f)^(1/2)/f)/f+2*((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*((x+(d*f)^(1/2
)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/
f)^(1/2))/(x+(d*f)^(1/2)/f))*a^2-1/8/d*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^
(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*x*b-1/16/d/c*
((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(
d*f)^(1/2)*b)/f)^(1/2)*b^2+1/32/d/c^(3/2)*ln(((x+(d*f)^(1/2)/f)*c+1/2*(b*f-
2*(d*f)^(1/2)*c)/f)/c^(1/2)+((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x
+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))*b^3-3/4/f*ln(((x+(d*f)^
(1/2)/f)*c+1/2*(b*f-2*(d*f)^(1/2)*c)/f)/c^(1/2)+((x+(d*f)^(1/2)/f)^2*c+(b*f
-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))*c^(
1/2)*b+1/2/f^2*ln(((x+(d*f)^(1/2)/f)*c+1/2*(b*f-2*(d*f)^(1/2)*c)/f)/c^(1/2)
```

```
+((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-
(d*f)^(1/2)*b)/f)^(1/2))*c^(3/2)*(d*f)^(1/2)+1/2/f/((a*f+c*d-(d*f)^(1/2)*b)
/f)^(1/2)*ln((2*(a*f+c*d-(d*f)^(1/2)*b)/f+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1
/2)/f)/f+2*((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*
(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))/(x+(d*
f)^(1/2)/f))*b^2+1/32/d/c^(3/2)*ln(((x-(d*f)^(1/2)/f)*c+1/2*(b*f+2*(d*f)^(1
/2)*c)/f)/c^(1/2)+((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/
2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))*b^3-1/4/d/f*((x-(d*f)^(1/2)/f)^2*
c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2
)*x*(d*f)^(1/2)*c-3/4/d/f*ln(((x-(d*f)^(1/2)/f)*c+1/2*(b*f+2*(d*f)^(1/2)*c)
/f)/c^(1/2)+((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/
f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))*c^(1/2)*(d*f)^(1/2)*a-3/16/d/f*ln(((x-(
d*f)^(1/2)/f)*c+1/2*(b*f+2*(d*f)^(1/2)*c)/f)/c^(1/2)+((x-(d*f)^(1/2)/f)^2*c
+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)
)/c^(1/2)*b^2*(d*f)^(1/2)+1/f^2/((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*
f+c*d+(d*f)^(1/2)*b)/f+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+2*((a*f+c*
d+(d*f)^(1/2)*b)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(
d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))/(x-(d*f)^(1/2)/f))*(d*f)^
(1/2)*b*c+3/16/d/f*ln(((x+(d*f)^(1/2)/f)*c+1/2*(b*f-2*(d*f)^(1/2)*c)/f)/c^(
1/2)+((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+
c*d-(d*f)^(1/2)*b)/f)^(1/2))/c^(1/2)*b^2*(d*f)^(1/2)-1/f^2/((a*f+c*d-(d*f)^
(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d-(d*f)^(1/2)*b)/f+(b*f-2*(d*f)^(1/2)*c)*(x+
(d*f)^(1/2)/f)/f+2*((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*((x+(d*f)^(1/2)/f)^2*c
+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)
)/(x+(d*f)^(1/2)/f))*(d*f)^(1/2)*b*c+1/4/d/f*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*
(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*x*(d*f)
^(1/2)*c+3/4/d/f*ln(((x+(d*f)^(1/2)/f)*c+1/2*(b*f-2*(d*f)^(1/2)*c)/f)/c^(1/
2)+((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*
d-(d*f)^(1/2)*b)/f)^(1/2))*c^(1/2)*(d*f)^(1/2)*a+1/f/((a*f+c*d+(d*f)^(1/2)*
b)/f)^(1/2)*ln((2*(a*f+c*d+(d*f)^(1/2)*b)/f+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^
(1/2)/f)/f+2*((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(b*f+
2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))/(x-(
d*f)^(1/2)/f))*a*c+1/2*d/f^2/((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c
*d+(d*f)^(1/2)*b)/f+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+2*((a*f+c*d+(
d*f)^(1/2)*b)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f
)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))/(x-(d*f)^(1/2)/f))*c^2+3/4/d
*b/c^(1/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a+1/f/((a*f+c*d-(d*f
)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d-(d*f)^(1/2)*b)/f+(b*f-2*(d*f)^(1/2)*c)*(
x+(d*f)^(1/2)/f)/f+2*((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*((x+(d*f)^(1/2)/f)^2
*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/
2))/(x+(d*f)^(1/2)/f))*a*c+1/2*d/f^2/((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*ln((
2*(a*f+c*d-(d*f)^(1/2)*b)/f+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+2*((a
*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)
*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))/(x+(d*f)^(1/2)/f))*c
^2+5/8/d/f*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f
+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*b*(d*f)^(1/2)-3/8/d/c^(1/2)*ln(((x+(d*f)^
(1/2)/f)*c+1/2*(b*f-2*(d*f)^(1/2)*c)/f)/c^(1/2)+((x+(d*f)^(1/2)/f)^2*c+(b*f
-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))*a*b
+1/d/f/((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d+(d*f)^(1/2)*b)/f+(b
*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+2*((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)
*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+
(d*f)^(1/2)*b)/f)^(1/2))/(x-(d*f)^(1/2)/f))*(d*f)^(1/2)*b*a-1/d/f/((a*f+c*d
-(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d-(d*f)^(1/2)*b)/f+(b*f-2*(d*f)^(1/2)
*c)*(x+(d*f)^(1/2)/f)/f+2*((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*((x+(d*f)^(1/2)
/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f
)^(1/2))/(x+(d*f)^(1/2)/f))*(d*f)^(1/2)*b*a-1/2/d*((x+(d*f)^(1/2)/f)^2*c+(b
*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*a-
1/2/f*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f
+c*d-(d*f)^(1/2)*b)/f)^(1/2)*c+1/d*a*(c*x^2+b*x+a)^(1/2)-1/d*a^(3/2)*ln((b*
```

```
x+2*a+2*(c*x^2+b*x+a)^(1/2)*a^(1/2))/x)-1/2/d*((x-(d*f)^(1/2)/f)^2*c+(b*f+2
*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*a-1/2/
f*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d
+(d*f)^(1/2)*b)/f)^(1/2)*c-5/8/d/f*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2
)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*b*(d*f)^(1/2)-3/8
/d/c^(1/2)*ln(((x-(d*f)^(1/2)/f)*c+1/2*(b*f+2*(d*f)^(1/2)*c)/f)/c^(1/2)+((x
-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f
)^(1/2)*b)/f)^(1/2))*a*b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(cx^2 + bx + a\right)^{\frac{3}{2}}}{\left(fx^2 - d\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/x/(-f*x^2+d),x, algorithm="maxima")
[Out] -integrate((c*x^2 + b*x + a)^(3/2)/((f*x^2 - d)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,x^2 + b\,x + a\right)^{3/2}}{x\,\left(d - f\,x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)^(3/2)/(x*(d - f*x^2)),x)
[Out] int((a + b*x + c*x^2)^(3/2)/(x*(d - f*x^2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{a\sqrt{a + bx + cx^2}}{-dx + fx^3}\,dx - \int \frac{bx\sqrt{a + bx + cx^2}}{-dx + fx^3}\,dx - \int \frac{cx^2\sqrt{a + bx + cx^2}}{-dx + fx^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(3/2)/x/(-f*x**2+d),x)
[Out] -Integral(a*sqrt(a + b*x + c*x**2)/(-d*x + f*x**3), x) - Integral(b*x*sqrt(
a + b*x + c*x**2)/(-d*x + f*x**3), x) - Integral(c*x**2*sqrt(a + b*x + c*x*
*2)/(-d*x + f*x**3), x)
```

3.89 $\int \frac{\left(a+bx+cx^2\right)^{3/2}}{x^2\left(d-fx^2\right)}\,dx$

Optimal. Leaf size=463

$$-\frac{\left(12acf+3b^2f+8c^2d\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8\sqrt{c}\,df}+\frac{3\left(4ac+b^2\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8\sqrt{c}\,d}+\frac{\left(af+b\left(-\sqrt{d}\right)\sqrt{f}+cd\right)}{}$$

```
[Out] -(c*x^2+b*x+a)^(3/2)/d/x-3/2*b*arctanh(1/2*(b*x+2*a)/a^(1/2)/(c*x^2+b*x+a)^
(1/2))*a^(1/2)/d+3/8*(4*a*c+b^2)*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a
)^(1/2))/d/c^(1/2)-1/8*(12*a*c*f+3*b^2*f+8*c^2*d)*arctanh(1/2*(2*c*x+b)/c^(
1/2)/(c*x^2+b*x+a)^(1/2))/d/f/c^(1/2)+1/2*arctanh(1/2*(b*d^(1/2)-2*a*f^(1/2
)+x*(2*c*d^(1/2)-b*f^(1/2)))/(c*x^2+b*x+a)^(1/2)/(c*d+a*f-b*d^(1/2)*f^(1/2)
)^(1/2))*(c*d+a*f-b*d^(1/2)*f^(1/2))^(3/2)/d^(3/2)/f+1/2*arctanh(1/2*(b*d^(
1/2)+2*a*f^(1/2)+x*(2*c*d^(1/2)+b*f^(1/2)))/(c*x^2+b*x+a)^(1/2)/(c*d+a*f+b*
d^(1/2)*f^(1/2))^(1/2))*(c*d+a*f+b*d^(1/2)*f^(1/2))^(3/2)/d^(3/2)/f+3/4*(2*
c*x+3*b)*(c*x^2+b*x+a)^(1/2)/d-1/4*(2*c*x+5*b)*(c*x^2+b*x+a)^(1/2)/d
```

Rubi [A] time = 1.20, antiderivative size = 463, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {6725, 732, 814, 843, 621, 206, 724, 978, 1078, 1033}

$$-\frac{\left(12acf+3b^2f+8c^2d\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8\sqrt{c}\,df}+\frac{3\left(4ac+b^2\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8\sqrt{c}\,d}+\frac{\left(af+b\left(-\sqrt{d}\right)\sqrt{f}+cd\right)}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^(3/2)/(x^2*(d - f*x^2)),x]
```

```
[Out] (3*(3*b + 2*c*x)*Sqrt[a + b*x + c*x^2])/(4*d) - ((5*b + 2*c*x)*Sqrt[a + b*x
 + c*x^2])/(4*d) - (a + b*x + c*x^2)^(3/2)/(d*x) - (3*Sqrt[a]*b*ArcTanh[(2*
a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/(2*d) + (3*(b^2 + 4*a*c)*ArcTa
nh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(8*Sqrt[c]*d) - ((8*c^2*
d + 3*b^2*f + 12*a*c*f)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2
])])/(8*Sqrt[c]*d*f) + ((c*d - b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)*ArcTanh[(b*Sq
rt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt[d]*
Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*d^(3/2)*f) + ((c*d + b*Sqrt[d]*S
qrt[f] + a*f)^(3/2)*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqr
t[f])*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2
*d^(3/2)*f)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
```

```
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 1)), x] - Di
st[p/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && (IntegerQ[p]
 || LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a,
 b, c, d, e, m, p, x]
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 978

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (f_.)*(x_)^2)^(q_), x
_Symbol] :> Simp[((b*(3*p + 2*q) + 2*c*(p + q)*x)*(a + b*x + c*x^2)^(p - 1)
*(d + f*x^2)^(q + 1))/(2*f*(p + q)*(2*p + 2*q + 1)), x] - Dist[1/(2*f*(p +
q)*(2*p + 2*q + 1)), Int[(a + b*x + c*x^2)^(p - 2)*(d + f*x^2)^q*Simp[b^2*d
*(p - 1)*(2*p + q) - (p + q)*(b^2*d*(1 - p) - 2*a*(c*d - a*f*(2*p + 2*q + 1
))) - (2*b*(c*d - a*f)*(1 - p)*(2*p + q) - 2*(p + q)*b*(2*c*d*(2*p + q) - (
c*d + a*f)*(2*p + 2*q + 1)))*x + (b^2*f*p*(1 - p) + 2*c*(p + q)*(c*d*(2*p -
 1) - a*f*(4*p + 2*q - 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, q}, x]
 && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 1] && NeQ[p + q, 0] && NeQ[2*p + 2*q + 1,
 0] &&  !IGtQ[p, 0] &&  !IGtQ[q, 0]
```

Rule 1033

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q
), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q
), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f
, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]
```

Rule 1078

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
 (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqrt[d + e*x + f
*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + B*c*x)/((a + c*x^2)*Sqrt[d + e*x
 + f*x^2]), x], x] /; FreeQ[{a, c, d, e, f, A, B, C}, x] && NeQ[e^2 - 4*d*f
, 0]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{\left(a+bx+cx^2\right)^{3/2}}{x^2\left(d-fx^2\right)}\,dx = \int \left(\frac{\left(a+bx+cx^2\right)^{3/2}}{dx^2}+\frac{f\left(a+bx+cx^2\right)^{3/2}}{d\left(d-fx^2\right)}\right)dx$$

$$= \frac{\int \frac{\left(a+bx+cx^2\right)^{3/2}}{x^2}\,dx}{d}+\frac{f\int \frac{\left(a+bx+cx^2\right)^{3/2}}{d-fx^2}\,dx}{d}$$

$$= -\frac{(5b+2cx)\sqrt{a+bx+cx^2}}{4d}-\frac{\left(a+bx+cx^2\right)^{3/2}}{dx}+\frac{\int \frac{\frac{1}{4}\left(5b^2d+4a(cd+2af)\right)+4b(cd+af)x+\frac{1}{4}\left(8c^2d+3\right.}{\sqrt{a+bx+cx^2}\left(d-fx^2\right)}}{2d}$$

$$= \frac{3(3b+2cx)\sqrt{a+bx+cx^2}}{4d}-\frac{(5b+2cx)\sqrt{a+bx+cx^2}}{4d}-\frac{\left(a+bx+cx^2\right)^{3/2}}{dx}-\frac{3\int \frac{-4abc}{x\sqrt{a}}}{}$$

$$= \frac{3(3b+2cx)\sqrt{a+bx+cx^2}}{4d}-\frac{(5b+2cx)\sqrt{a+bx+cx^2}}{4d}-\frac{\left(a+bx+cx^2\right)^{3/2}}{dx}+\frac{(3ab)\int}{}$$

$$= \frac{3(3b+2cx)\sqrt{a+bx+cx^2}}{4d}-\frac{(5b+2cx)\sqrt{a+bx+cx^2}}{4d}-\frac{\left(a+bx+cx^2\right)^{3/2}}{dx}-\frac{\left(8c^2d+3\right.}{}$$

$$= \frac{3(3b+2cx)\sqrt{a+bx+cx^2}}{4d}-\frac{(5b+2cx)\sqrt{a+bx+cx^2}}{4d}-\frac{\left(a+bx+cx^2\right)^{3/2}}{dx}-\frac{3\sqrt{a}\,b\tanh}{}$$

Mathematica [A] time = 0.64, size = 765, normalized size = 1.65

$$-\frac{2c^{3/2}d^{3/2}x\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)+2a\sqrt{d}f\sqrt{a+x(b+cx)}+cdx\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}\tanh^{-1}\left(\frac{2a\sqrt{f}-b\sqrt{}}{2\sqrt{a+x(b+cx)}}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^(3/2)/(x^2*(d - f*x^2)),x]
```

```
[Out] -1/2*(2*a*Sqrt[d]*f*Sqrt[a + x*(b + c*x)] + 3*Sqrt[a]*b*Sqrt[d]*f*x*ArcTanh
[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + x*(b + c*x)])] + 2*c^(3/2)*d^(3/2)*x*ArcTa
nh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])] + c*d*Sqrt[c*d - b*Sqrt[d
]*Sqrt[f] + a*f]*x*ArcTanh[(-(b*Sqrt[d]) + 2*a*Sqrt[f] - 2*c*Sqrt[d]*x + b*
Sqrt[f]*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])] -
 b*Sqrt[d]*Sqrt[f]*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*x*ArcTanh[(-(b*Sqrt[
d]) + 2*a*Sqrt[f] - 2*c*Sqrt[d]*x + b*Sqrt[f]*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sq
```

```
rt[f] + a*f]*Sqrt[a + x*(b + c*x)])] + a*f*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a
*f]*x*ArcTanh[(-(b*Sqrt[d]) + 2*a*Sqrt[f] - 2*c*Sqrt[d]*x + b*Sqrt[f]*x)/(2
*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])] + c*d*Sqrt[c*d
 + b*Sqrt[d]*Sqrt[f] + a*f]*x*ArcTanh[(-2*(a*Sqrt[f] + c*Sqrt[d]*x) - b*(Sq
rt[d] + Sqrt[f]*x))/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b +
c*x)])] + b*Sqrt[d]*Sqrt[f]*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*x*ArcTanh[(
-2*(a*Sqrt[f] + c*Sqrt[d]*x) - b*(Sqrt[d] + Sqrt[f]*x))/(2*Sqrt[c*d + b*Sqr
t[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])] + a*f*Sqrt[c*d + b*Sqrt[d]*Sqrt
[f] + a*f]*x*ArcTanh[(-2*(a*Sqrt[f] + c*Sqrt[d]*x) - b*(Sqrt[d] + Sqrt[f]*x
))/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])])/(d^(3/2)
*f*x)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/x^2/(-f*x^2+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/x^2/(-f*x^2+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [B] time = 0.02, size = 4799, normalized size = 10.37

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(3/2)/x^2/(-f*x^2+d),x)
```

```
[Out] -1/6*f/d/(d*f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^
(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(3/2)-1/4/d*((x-(d*f)^(1/2)/f)^2*c+(b
*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*x*
c-3/4/d*ln(((x-(d*f)^(1/2)/f)*c+1/2*(b*f+2*(d*f)^(1/2)*c)/f)/c^(1/2)+((x-(d
*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(
1/2)*b)/f)^(1/2))*c^(1/2)*a-3/16/d*ln(((x-(d*f)^(1/2)/f)*c+1/2*(b*f+2*(d*f)
^(1/2)*c)/f)/c^(1/2)+((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^
(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))/c^(1/2)*b^2-3/4/(d*f)^(1/2)*ln
(((x-(d*f)^(1/2)/f)*c+1/2*(b*f+2*(d*f)^(1/2)*c)/f)/c^(1/2)+((x-(d*f)^(1/2)/
f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)
^(1/2))*c^(1/2)*b+1/2/(d*f)^(1/2)/((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(
a*f+c*d+(d*f)^(1/2)*b)/f+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+2*((a*f+
c*d+(d*f)^(1/2)*b)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x
-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))/(x-(d*f)^(1/2)/f))*b^2+
3/8/d*b^2/c^(1/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+1/d*b/a*(c*x^
2+b*x+a)^(3/2)+1/6*f/d/(d*f)^(1/2)*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2
)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(3/2)-1/4/d*((x+(d*f)^(
1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*
b)/f)^(1/2)*x*c-3/4/d*ln(((x+(d*f)^(1/2)/f)*c+1/2*(b*f-2*(d*f)^(1/2)*c)/f)/
c^(1/2)+((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a
*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))*c^(1/2)*a-3/16/d*ln(((x+(d*f)^(1/2)/f)*c+1/
```

```
2*(b*f-2*(d*f)^(1/2)*c)/f)/c^(1/2)+((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2
)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))/c^(1/2)*b^2+3/4/
(d*f)^(1/2)*ln(((x+(d*f)^(1/2)/f)*c+1/2*(b*f-2*(d*f)^(1/2)*c)/f)/c^(1/2)+((
x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*
f)^(1/2)*b)/f)^(1/2))*c^(1/2)*b-1/2/(d*f)^(1/2)/((a*f+c*d-(d*f)^(1/2)*b)/f)
^(1/2)*ln((2*(a*f+c*d-(d*f)^(1/2)*b)/f+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)
/f)/f+2*((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*
f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))/(x+(d*f)^
(1/2)/f))*b^2-1/8*f/d/(d*f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)
*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*x*b-1/16*f/d/(d*f)
^(1/2)/c*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(
a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*b^2+1/32*f/d/(d*f)^(1/2)/c^(3/2)*ln(((x-(d*
f)^(1/2)/f)*c+1/2*(b*f+2*(d*f)^(1/2)*c)/f)/c^(1/2)+((x-(d*f)^(1/2)/f)^2*c+(
b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))*
b^3+1/2*f/d/(d*f)^(1/2)/((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d+(d
*f)^(1/2)*b)/f+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+2*((a*f+c*d+(d*f)^
(1/2)*b)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/
2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))/(x-(d*f)^(1/2)/f))*a^2+1/2/f*d/(d
*f)^(1/2)/((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d+(d*f)^(1/2)*b)/f
+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+2*((a*f+c*d+(d*f)^(1/2)*b)/f)^(1
/2)*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c
*d+(d*f)^(1/2)*b)/f)^(1/2))/(x-(d*f)^(1/2)/f))*c^2+1/8*f/d/(d*f)^(1/2)*((x+
(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)
^(1/2)*b)/f)^(1/2)*x*b+1/16*f/d/(d*f)^(1/2)/c*((x+(d*f)^(1/2)/f)^2*c+(b*f-2
*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*b^2-1/
32*f/d/(d*f)^(1/2)/c^(3/2)*ln(((x+(d*f)^(1/2)/f)*c+1/2*(b*f-2*(d*f)^(1/2)*c
)/f)/c^(1/2)+((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)
/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))*b^3-1/2*f/d/(d*f)^(1/2)/((a*f+c*d-(d*f
)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d-(d*f)^(1/2)*b)/f+(b*f-2*(d*f)^(1/2)*c)*(
x+(d*f)^(1/2)/f)/f+2*((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*((x+(d*f)^(1/2)/f)^2
*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/
2))/(x+(d*f)^(1/2)/f))*a^2-1/2/f*d/(d*f)^(1/2)/((a*f+c*d-(d*f)^(1/2)*b)/f)^
(1/2)*ln((2*(a*f+c*d-(d*f)^(1/2)*b)/f+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/
f)/f+2*((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f
)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))/(x+(d*f)^(
1/2)/f))*c^2+1/d/((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d+(d*f)^(1/
2)*b)/f+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+2*((a*f+c*d+(d*f)^(1/2)*b
)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f
+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))/(x-(d*f)^(1/2)/f))*b*a+1/f/((a*f+c*d+(d*
f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d+(d*f)^(1/2)*b)/f+(b*f+2*(d*f)^(1/2)*c)*
(x-(d*f)^(1/2)/f)/f+2*((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*((x-(d*f)^(1/2)/f)^
2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1
/2))/(x-(d*f)^(1/2)/f))*b*c+1/2*f/d/(d*f)^(1/2)*((x+(d*f)^(1/2)/f)^2*c+(b*f
-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*a+1/
d/((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d-(d*f)^(1/2)*b)/f+(b*f-2*
(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+2*((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*((x+
(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)
^(1/2)*b)/f)^(1/2))/(x+(d*f)^(1/2)/f))*b*a+1/f/((a*f+c*d-(d*f)^(1/2)*b)/f)^
(1/2)*ln((2*(a*f+c*d-(d*f)^(1/2)*b)/f+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/
f)/f+2*((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f
)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))/(x+(d*f)^(
1/2)/f))*b*c-1/(d*f)^(1/2)/((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d
-(d*f)^(1/2)*b)/f+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+2*((a*f+c*d-(d*
f)^(1/2)*b)/f)^(1/2)*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^
(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))/(x+(d*f)^(1/2)/f))*a*c+1/d*c/a
*(c*x^2+b*x+a)^(3/2)*x-5/8/d*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(
x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*b-1/2/(d*f)^(1/2)*((x-(
d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^
(1/2)*b)/f)^(1/2)*c-1/2/f*ln(((x-(d*f)^(1/2)/f)*c+1/2*(b*f+2*(d*f)^(1/2)*c)
```

```
/f)/c^(1/2)+((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/
f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))*c^(3/2)+9/4/d*b*(c*x^2+b*x+a)^(1/2)-5/8
/d*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*
d-(d*f)^(1/2)*b)/f)^(1/2)*b+1/2/(d*f)^(1/2)*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(
d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*c-1/2/f*
ln(((x+(d*f)^(1/2)/f)*c+1/2*(b*f-2*(d*f)^(1/2)*c)/f)/c^(1/2)+((x+(d*f)^(1/2
)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/
f)^(1/2))*c^(3/2)+3/2/d*c*(c*x^2+b*x+a)^(1/2)*x+3/2/d*c^(1/2)*a*ln((c*x+1/2
*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-1/d/a/x*(c*x^2+b*x+a)^(5/2)+3/8*f/d/(d*f)^
(1/2)/c^(1/2)*ln(((x+(d*f)^(1/2)/f)*c+1/2*(b*f-2*(d*f)^(1/2)*c)/f)/c^(1/2)+
((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(
d*f)^(1/2)*b)/f)^(1/2))*a*b-3/8*f/d/(d*f)^(1/2)/c^(1/2)*ln(((x-(d*f)^(1/2)/
f)*c+1/2*(b*f+2*(d*f)^(1/2)*c)/f)/c^(1/2)+((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*
f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))*a*b+1/(d*
f)^(1/2)/((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d+(d*f)^(1/2)*b)/f+
(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+2*((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/
2)*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*
d+(d*f)^(1/2)*b)/f)^(1/2))/(x-(d*f)^(1/2)/f))*a*c-1/2*f/d/(d*f)^(1/2)*((x-(
d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^
(1/2)*b)/f)^(1/2)*a-3/2/d*b*a^(1/2)*ln((b*x+2*a+2*(c*x^2+b*x+a)^(1/2)*a^(1/
2))/x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(cx^2+bx+a\right)^{\frac{3}{2}}}{\left(fx^2-d\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/x^2/(-f*x^2+d),x, algorithm="maxima")
```

```
[Out] -integrate((c*x^2 + b*x + a)^(3/2)/((f*x^2 - d)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,x^2+b\,x+a\right)^{3/2}}{x^2\,\left(d-f\,x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)^(3/2)/(x^2*(d - f*x^2)),x)
```

```
[Out] int((a + b*x + c*x^2)^(3/2)/(x^2*(d - f*x^2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{a\sqrt{a+bx+cx^2}}{-dx^2+fx^4}\,dx-\int \frac{bx\sqrt{a+bx+cx^2}}{-dx^2+fx^4}\,dx-\int \frac{cx^2\sqrt{a+bx+cx^2}}{-dx^2+fx^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(3/2)/x**2/(-f*x**2+d),x)
```

```
[Out] -Integral(a*sqrt(a + b*x + c*x**2)/(-d*x**2 + f*x**4), x) - Integral(b*x*sq
rt(a + b*x + c*x**2)/(-d*x**2 + f*x**4), x) - Integral(c*x**2*sqrt(a + b*x
+ c*x**2)/(-d*x**2 + f*x**4), x)
```

3.90 $\int \frac{(a+bx+cx^2)^{3/2}}{x^3(d-fx^2)}\,dx$

Optimal. Leaf size=614

$$-\frac{a^{3/2}f\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{d^2}-\frac{bf\left(b^2-12ac\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{16c^{3/2}d^2}-\frac{\sqrt{a+bx+cx^2}\left(8acf+b^2f+2bcfx+8c\right.}{8cd^2}$$

```
[Out] -1/2*(c*x^2+b*x+a)^(3/2)/d/x^2-a^(3/2)*f*arctanh(1/2*(b*x+2*a)/a^(1/2)/(c*x
^2+b*x+a)^(1/2))/d^2-1/16*b*(-12*a*c+b^2)*f*arctanh(1/2*(2*c*x+b)/c^(1/2)/(
c*x^2+b*x+a)^(1/2))/c^(3/2)/d^2-1/16*b*(12*a*c*f-b^2*f+24*c^2*d)*arctanh(1/
2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(3/2)/d^2-3/8*(4*a*c+b^2)*arctan
h(1/2*(b*x+2*a)/a^(1/2)/(c*x^2+b*x+a)^(1/2))/d/a^(1/2)+3/2*b*arctanh(1/2*(2
*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))*c^(1/2)/d-1/2*arctanh(1/2*(b*d^(1/2)-2
*a*f^(1/2)+x*(2*c*d^(1/2)-b*f^(1/2)))/(c*x^2+b*x+a)^(1/2)/(c*d+a*f-b*d^(1/2
)*f^(1/2))^(1/2))*(c*d+a*f-b*d^(1/2)*f^(1/2))^(3/2)/d^2/f^(1/2)+1/2*arctanh
(1/2*(b*d^(1/2)+2*a*f^(1/2)+x*(2*c*d^(1/2)+b*f^(1/2)))/(c*x^2+b*x+a)^(1/2)/
(c*d+a*f+b*d^(1/2)*f^(1/2))^(1/2))*(c*d+a*f+b*d^(1/2)*f^(1/2))^(3/2)/d^2/f^
(1/2)-3/4*(-2*c*x+b)*(c*x^2+b*x+a)^(1/2)/d/x+1/8*f*(2*b*c*x+8*a*c+b^2)*(c*x
^2+b*x+a)^(1/2)/c/d^2-1/8*(2*b*c*f*x+8*a*c*f+b^2*f+8*c^2*d)*(c*x^2+b*x+a)^(
1/2)/c/d^2
```

Rubi [A] time = 1.44, antiderivative size = 614, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 13, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.464, Rules used = {6725, 732, 812, 843, 621, 206, 724, 734, 814, 1021, 1070, 1078, 1033}

$$-\frac{a^{3/2}f\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{d^2}-\frac{\sqrt{a+bx+cx^2}\left(8acf+b^2f+2bcfx+8c^2d\right)}{8cd^2}-\frac{bf\left(b^2-12ac\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+c}\right.}{16c^{3/2}d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^(3/2)/(x^3*(d - f*x^2)),x]
```

```
[Out] (-3*(b - 2*c*x)*Sqrt[a + b*x + c*x^2])/(4*d*x) + (f*(b^2 + 8*a*c + 2*b*c*x)
*Sqrt[a + b*x + c*x^2])/(8*c*d^2) - ((8*c^2*d + b^2*f + 8*a*c*f + 2*b*c*f*x
)*Sqrt[a + b*x + c*x^2])/(8*c*d^2) - (a + b*x + c*x^2)^(3/2)/(2*d*x^2) - (3
*(b^2 + 4*a*c)*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/(8*S
qrt[a]*d) - (a^(3/2)*f*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2]
)])/d^2 + (3*b*Sqrt[c]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2]
)])/(2*d) - (b*(b^2 - 12*a*c)*f*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x
 + c*x^2])])/(16*c^(3/2)*d^2) - (b*(24*c^2*d - b^2*f + 12*a*c*f)*ArcTanh[(b
 + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(16*c^(3/2)*d^2) - ((c*d - b*
Sqrt[d]*Sqrt[f] + a*f)^(3/2)*ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d
] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x
^2])])/(2*d^2*Sqrt[f]) + ((c*d + b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)*ArcTanh[(b*
Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqrt[d
]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*d^2*Sqrt[f])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 1)), x] - Di
st[p/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && (IntegerQ[p]
 || LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a,
 b, c, d, e, m, p, x]
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 812

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2)
- d*g*(2*p + 1) + e*g*(m + 1)*x)*(a + b*x + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p
 + 2)), x] + Dist[p/(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m +
 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && RationalQ[p] && p > 0 && (LtQ[m, -1] || Eq
Q[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p
 + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1021

```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (f
_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(h*(a + b*x + c*x^2)^p*(d + f*x^2)^(q +
 1))/(2*f*(p + q + 1)), x] - Dist[1/(2*f*(p + q + 1)), Int[(a + b*x + c*x^2
)^(p - 1)*(d + f*x^2)^q*Simp[h*p*(b*d) + a*(-2*g*f)*(p + q + 1) + (2*h*p*(c
*d - a*f) + b*(-2*g*f)*(p + q + 1))*x + (h*p*(-(b*f)) + c*(-2*g*f)*(p + q +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, g, h, q}, x] && NeQ[b^2 - 4*a
*c, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0]
```

Rule 1033

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q
), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q
), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f
, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]
```

Rule 1070

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((B*c*f*(2*p + 2*q + 3)
+ C*(b*f*p) + 2*c*C*f*(p + q + 1)*x)*(a + b*x + c*x^2)^p*(d + f*x^2)^(q + 1
))/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3)), x] - Dist[1/(2*c*f^2*(p + q + 1)*
(2*p + 2*q + 3)), Int[(a + b*x + c*x^2)^(p - 1)*(d + f*x^2)^q*Simp[p*(b*d)*
(C*(-(b*f))*(q + 1) - c*(-(B*f))*(2*p + 2*q + 3)) + (p + q + 1)*(b^2*C*d*f*
p + a*c*(C*(2*d*f) + f*(-2*A*f)*(2*p + 2*q + 3))) + (2*p*(c*d - a*f)*(C*(-(
b*f))*(q + 1) - c*(-(B*f))*(2*p + 2*q + 3)) + (p + q + 1)*(-(b*c*(C*(-4*d*f
)*(2*p + q + 2) + f*(2*C*d + 2*A*f)*(2*p + 2*q + 3)))))*x + (p*(-(b*f))*(C*
(-(b*f))*(q + 1) - c*(-(B*f))*(2*p + 2*q + 3)) + (p + q + 1)*(C*f^2*p*(b^2
- 4*a*c) - c^2*(C*(-4*d*f)*(2*p + q + 2) + f*(2*C*d + 2*A*f)*(2*p + 2*q + 3
))))*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, A, B, C, q}, x] && NeQ[b^2 -
4*a*c, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0] && NeQ[2*p + 2*q + 3, 0] &&  !I
GtQ[p, 0] &&  !IGtQ[q, 0]
```

Rule 1078

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
 (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqrt[d + e*x + f
*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + B*c*x)/((a + c*x^2)*Sqrt[d + e*x
 + f*x^2]), x], x] /; FreeQ[{a, c, d, e, f, A, B, C}, x] && NeQ[e^2 - 4*d*f
, 0]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{\left(a+bx+cx^2\right)^{3/2}}{x^3\left(d-fx^2\right)}dx = \int \left(\frac{\left(a+bx+cx^2\right)^{3/2}}{dx^3}+\frac{f\left(a+bx+cx^2\right)^{3/2}}{d^2x}+\frac{f^2x\left(a+bx+cx^2\right)^{3/2}}{d^2\left(d-fx^2\right)}\right)dx$$

$$=\frac{\int \frac{\left(a+bx+cx^2\right)^{3/2}}{x^3}dx}{d}+\frac{f\int \frac{\left(a+bx+cx^2\right)^{3/2}}{x}dx}{d^2}+\frac{f^2\int \frac{x\left(a+bx+cx^2\right)^{3/2}}{d-fx^2}dx}{d^2}$$

$$=-\frac{\left(a+bx+cx^2\right)^{3/2}}{2dx^2}+\frac{3\int \frac{(b+2cx)\sqrt{a+bx+cx^2}}{x^2}dx}{4d}+\frac{f\int \frac{\sqrt{a+bx+cx^2}\left(\frac{3bd}{2}+3(cd+af)x+\frac{3}{2}bfx^2\right)}{d-fx^2}dx}{3d^2}$$

$$=-\frac{3(b-2cx)\sqrt{a+bx+cx^2}}{4dx}+\frac{f\left(b^2+8ac+2bcx\right)\sqrt{a+bx+cx^2}}{8cd^2}-\frac{\left(8c^2d+b^2f+8\right.}{}$$

$$=-\frac{3(b-2cx)\sqrt{a+bx+cx^2}}{4dx}+\frac{f\left(b^2+8ac+2bcx\right)\sqrt{a+bx+cx^2}}{8cd^2}-\frac{\left(8c^2d+b^2f+8\right.}{}$$

$$=-\frac{3(b-2cx)\sqrt{a+bx+cx^2}}{4dx}+\frac{f\left(b^2+8ac+2bcx\right)\sqrt{a+bx+cx^2}}{8cd^2}-\frac{\left(8c^2d+b^2f+8\right.}{}$$

$$=-\frac{3(b-2cx)\sqrt{a+bx+cx^2}}{4dx}+\frac{f\left(b^2+8ac+2bcx\right)\sqrt{a+bx+cx^2}}{8cd^2}-\frac{\left(8c^2d+b^2f+8\right.}{}$$

$$=-\frac{3(b-2cx)\sqrt{a+bx+cx^2}}{4dx}+\frac{f\left(b^2+8ac+2bcx\right)\sqrt{a+bx+cx^2}}{8cd^2}-\frac{\left(8c^2d+b^2f+8\right.}{}$$

Mathematica [A] time = 0.97, size = 303, normalized size = 0.49

$$-\frac{\frac{\left(4a(2af+3cd)+3b^2d\right)\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+x(b+cx)}}\right)}{\sqrt{a}}-\frac{4\left(af+b\left(-\sqrt{d}\right)\sqrt{f}+cd\right)^{3/2}\tanh^{-1}\left(\frac{2a\sqrt{f}-b\sqrt{d}+b\sqrt{f}x-2c\sqrt{d}x}{2\sqrt{a+x(b+cx)}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{\sqrt{f}}+\frac{4\left(af+b\sqrt{d}\sqrt{f}+cd\right)^{3/2}}{}}{8d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^(3/2)/(x^3*(d - f*x^2)),x]
```

```
[Out] -1/8*((2*d*(2*a + 5*b*x)*Sqrt[a + x*(b + c*x)])/x^2 + ((3*b^2*d + 4*a*(3*c*
d + 2*a*f))*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + x*(b + c*x)])])/Sqrt[a]
 - (4*(c*d - b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)*ArcTanh[(-(b*Sqrt[d]) + 2*a*Sqr
t[f] - 2*c*Sqrt[d]*x + b*Sqrt[f]*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*
Sqrt[a + x*(b + c*x)])])/Sqrt[f] + (4*(c*d + b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)
*ArcTanh[(-2*(a*Sqrt[f] + c*Sqrt[d]*x) - b*(Sqrt[d] + Sqrt[f]*x))/(2*Sqrt[c
*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])])/Sqrt[f])/d^2
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/x^3/(-f*x^2+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/x^3/(-f*x^2+d),x, algorithm="giac")
```

```
[Out] sage2
```

maple [B] time = 0.02, size = 5056, normalized size = 8.23

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(3/2)/x^3/(-f*x^2+d),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(cx^2 + bx + a\right)^{\frac{3}{2}}}{\left(fx^2 - d\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/x^3/(-f*x^2+d),x, algorithm="maxima")
```

```
[Out] -integrate((c*x^2 + b*x + a)^(3/2)/((f*x^2 - d)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,x^2 + b\,x + a\right)^{3/2}}{x^3\,\left(d - f\,x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)^(3/2)/(x^3*(d - f*x^2)),x)
```

```
[Out] int((a + b*x + c*x^2)^(3/2)/(x^3*(d - f*x^2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{a\sqrt{a + bx + cx^2}}{-dx^3 + fx^5}\,dx - \int \frac{bx\sqrt{a + bx + cx^2}}{-dx^3 + fx^5}\,dx - \int \frac{cx^2\sqrt{a + bx + cx^2}}{-dx^3 + fx^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(3/2)/x**3/(-f*x**2+d),x)
```

```
[Out] -Integral(a*sqrt(a + b*x + c*x**2)/(-d*x**3 + f*x**5), x) - Integral(b*x*sq
rt(a + b*x + c*x**2)/(-d*x**3 + f*x**5), x) - Integral(c*x**2*sqrt(a + b*x
+ c*x**2)/(-d*x**3 + f*x**5), x)
```

3.91 $\int \frac{\left(a+bx+cx^2\right)^{3/2}}{1-x^2}\,dx$

Optimal. Leaf size=189

$$-\frac{\left(12ac+3b^2+8c^2\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8\sqrt{c}}-\frac{1}{4}(5b+2cx)\sqrt{a+bx+cx^2}-\frac{1}{2}(a-b+c)^{3/2}\tanh^{-1}\left(\frac{2a+x(b-}{2\sqrt{a-b+c}\sqrt{a}}\right.$$

```
[Out] -1/2*(a-b+c)^(3/2)*arctanh(1/2*(2*a-b+(b-2*c)*x)/(a-b+c)^(1/2)/(c*x^2+b*x+a
)^(1/2))+1/2*(a+b+c)^(3/2)*arctanh(1/2*(2*a+b+(b+2*c)*x)/(a+b+c)^(1/2)/(c*x
^2+b*x+a)^(1/2))-1/8*(12*a*c+3*b^2+8*c^2)*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*
x^2+b*x+a)^(1/2))/c^(1/2)-1/4*(2*c*x+5*b)*(c*x^2+b*x+a)^(1/2)
```

Rubi [A] time = 0.28, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {978, 1078, 621, 206, 1033, 724}

$$-\frac{\left(12ac+3b^2+8c^2\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8\sqrt{c}}-\frac{1}{4}(5b+2cx)\sqrt{a+bx+cx^2}-\frac{1}{2}(a-b+c)^{3/2}\tanh^{-1}\left(\frac{2a+x(b-}{2\sqrt{a-b+c}\sqrt{a}}\right.$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^(3/2)/(1 - x^2),x]
```

```
[Out] -((5*b + 2*c*x)*Sqrt[a + b*x + c*x^2])/4 - ((a - b + c)^(3/2)*ArcTanh[(2*a
- b + (b - 2*c)*x)/(2*Sqrt[a - b + c]*Sqrt[a + b*x + c*x^2])])/2 - ((3*b^2
+ 12*a*c + 8*c^2)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(
8*Sqrt[c]) + ((a + b + c)^(3/2)*ArcTanh[(2*a + b + (b + 2*c)*x)/(2*Sqrt[a +
 b + c]*Sqrt[a + b*x + c*x^2])])/2
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 978

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (f_.)*(x_)^2)^(q_), x
_Symbol] :> Simp[((b*(3*p + 2*q) + 2*c*(p + q)*x)*(a + b*x + c*x^2)^(p - 1)
*(d + f*x^2)^(q + 1))/(2*f*(p + q)*(2*p + 2*q + 1)), x] - Dist[1/(2*f*(p +
q)*(2*p + 2*q + 1)), Int[(a + b*x + c*x^2)^(p - 2)*(d + f*x^2)^q*Simp[b^2*d
*(p - 1)*(2*p + q) - (p + q)*(b^2*d*(1 - p) - 2*a*(c*d - a*f*(2*p + 2*q + 1
))) - (2*b*(c*d - a*f)*(1 - p)*(2*p + q) - 2*(p + q)*b*(2*c*d*(2*p + q) - (
c*d + a*f)*(2*p + 2*q + 1)))*x + (b^2*f*p*(1 - p) + 2*c*(p + q)*(c*d*(2*p -
```

```
1) - a*f*(4*p + 2*q - 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, q}, x]
&& NeQ[b^2 - 4*a*c, 0] && GtQ[p, 1] && NeQ[p + q, 0] && NeQ[2*p + 2*q + 1,
0] && !IGtQ[p, 0] && !IGtQ[q, 0]
```

Rule 1033

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q
), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q
), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f
, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]
```

Rule 1078

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
 (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqrt[d + e*x + f
*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + B*c*x)/((a + c*x^2)*Sqrt[d + e*x
 + f*x^2]), x], x] /; FreeQ[{a, c, d, e, f, A, B, C}, x] && NeQ[e^2 - 4*d*f
, 0]
```

Rubi steps

$$\int \frac{\left(a+bx+cx^2\right)^{3/2}}{1-x^2}\,dx = -\frac{1}{4}(5b+2cx)\sqrt{a+bx+cx^2} + \frac{1}{2}\int \frac{\frac{1}{4}\left(8a^2+5b^2+4ac\right)+4b(a+c)x+\frac{1}{4}\left(3b^2+12ac\right.}{\left(1-x^2\right)\sqrt{a+bx+cx^2}}$$

$$= -\frac{1}{4}(5b+2cx)\sqrt{a+bx+cx^2} - \frac{1}{2}\int \frac{\frac{1}{4}\left(-8a^2-5b^2-4ac\right)+\frac{1}{4}\left(-3b^2-12ac-8c^2\right)-4b}{\left(1-x^2\right)\sqrt{a+bx+cx^2}}$$

$$= -\frac{1}{4}(5b+2cx)\sqrt{a+bx+cx^2} - \frac{1}{2}(a-b+c)^2\int \frac{1}{(-1-x)\sqrt{a+bx+cx^2}}\,dx + \frac{1}{2}(a+b+$$

$$= -\frac{1}{4}(5b+2cx)\sqrt{a+bx+cx^2} - \frac{\left(3b^2+12ac+8c^2\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8\sqrt{c}} + (a-b+c)$$

$$= -\frac{1}{4}(5b+2cx)\sqrt{a+bx+cx^2} - \frac{1}{2}(a-b+c)^{3/2}\tanh^{-1}\left(\frac{2a-b+(b-2c)x}{2\sqrt{a-b+c}\sqrt{a+bx+cx^2}}\right) -$$

Mathematica [A] time = 0.58, size = 181, normalized size = 0.96

$$\frac{1}{8}\left(-\frac{\left(4c(3a+2c)+3b^2\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)}{\sqrt{c}} - 2(5b+2cx)\sqrt{a+x(b+cx)} - 4(a-b+c)^{3/2}\tanh^{-1}\left(\frac{2a+}{2\sqrt{a-b}}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^(3/2)/(1 - x^2),x]
```

```
[Out] (-2*(5*b + 2*c*x)*Sqrt[a + x*(b + c*x)] - 4*(a - b + c)^(3/2)*ArcTanh[(2*a
+ b*(-1 + x) - 2*c*x)/(2*Sqrt[a - b + c]*Sqrt[a + x*(b + c*x)])] - ((3*b^2
+ 4*c*(3*a + 2*c))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/
Sqrt[c] + 4*(a + b + c)^(3/2)*ArcTanh[(2*a + b + b*x + 2*c*x)/(2*Sqrt[a + b
 + c]*Sqrt[a + x*(b + c*x)])])/8
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(-x^2+1),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(-x^2+1),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [B] time = 0.02, size = 1346, normalized size = 7.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(3/2)/(-x^2+1),x)
```

```
[Out] -1/2*c^(3/2)*ln((1/2*b-c+c*(x+1))/c^(1/2)+((x+1)^2*c+(b-2*c)*(x+1)+a-b+c)^(
1/2))-1/2*c^(3/2)*ln((1/2*b+c+c*(x-1))/c^(1/2)+((x-1)^2*c+(b+2*c)*(x-1)+a+b
+c)^(1/2))+1/2*a*((x+1)^2*c+(b-2*c)*(x+1)+a-b+c)^(1/2)+1/2*c*((x+1)^2*c+(b-
2*c)*(x+1)+a-b+c)^(1/2)-5/8*((x+1)^2*c+(b-2*c)*(x+1)+a-b+c)^(1/2)*b-1/2*a*(
(x-1)^2*c+(b+2*c)*(x-1)+a+b+c)^(1/2)-1/2*c*((x-1)^2*c+(b+2*c)*(x-1)+a+b+c)^
(1/2)-5/8*((x-1)^2*c+(b+2*c)*(x-1)+a+b+c)^(1/2)*b+3/8*b/c^(1/2)*ln((1/2*b-c
+c*(x+1))/c^(1/2)+((x+1)^2*c+(b-2*c)*(x+1)+a-b+c)^(1/2))*a-3/8*b/c^(1/2)*ln
((1/2*b+c+c*(x-1))/c^(1/2)+((x-1)^2*c+(b+2*c)*(x-1)+a+b+c)^(1/2))*a-1/2*a*(
a-b+c)^(1/2)*ln((2*a-2*b+2*c+(b-2*c)*(x+1)+2*(a-b+c)^(1/2)*((x+1)^2*c+(b-2*
c)*(x+1)+a-b+c)^(1/2))/(x+1))+3/4*b*ln((1/2*b-c+c*(x+1))/c^(1/2)+((x+1)^2*c
+(b-2*c)*(x+1)+a-b+c)^(1/2))*c^(1/2)+1/2*b*(a-b+c)^(1/2)*ln((2*a-2*b+2*c+(b
-2*c)*(x+1)+2*(a-b+c)^(1/2)*((x+1)^2*c+(b-2*c)*(x+1)+a-b+c)^(1/2))/(x+1))-1
/2*c*(a-b+c)^(1/2)*ln((2*a-2*b+2*c+(b-2*c)*(x+1)+2*(a-b+c)^(1/2)*((x+1)^2*c
+(b-2*c)*(x+1)+a-b+c)^(1/2))/(x+1))+1/8*b*((x+1)^2*c+(b-2*c)*(x+1)+a-b+c)^(
1/2)*x+1/16/c*((x+1)^2*c+(b-2*c)*(x+1)+a-b+c)^(1/2)*b^2-1/32/c^(3/2)*ln((1/
2*b-c+c*(x+1))/c^(1/2)+((x+1)^2*c+(b-2*c)*(x+1)+a-b+c)^(1/2))*b^3-1/4*c*((x
+1)^2*c+(b-2*c)*(x+1)+a-b+c)^(1/2)*x+1/2*a*(a+b+c)^(1/2)*ln((2*a+2*b+2*c+(b
+2*c)*(x-1)+2*(a+b+c)^(1/2)*((x-1)^2*c+(b+2*c)*(x-1)+a+b+c)^(1/2))/(x-1))-3
/4*b*ln((1/2*b+c+c*(x-1))/c^(1/2)+((x-1)^2*c+(b+2*c)*(x-1)+a+b+c)^(1/2))*c^
(1/2)+1/2*b*(a+b+c)^(1/2)*ln((2*a+2*b+2*c+(b+2*c)*(x-1)+2*(a+b+c)^(1/2)*((x
-1)^2*c+(b+2*c)*(x-1)+a+b+c)^(1/2))/(x-1))+1/2*c*(a+b+c)^(1/2)*ln((2*a+2*b+
2*c+(b+2*c)*(x-1)+2*(a+b+c)^(1/2)*((x-1)^2*c+(b+2*c)*(x-1)+a+b+c)^(1/2))/(x
-1))-1/8*b*((x-1)^2*c+(b+2*c)*(x-1)+a+b+c)^(1/2)*x-1/16/c*((x-1)^2*c+(b+2*c
)*(x-1)+a+b+c)^(1/2)*b^2+1/32/c^(3/2)*ln((1/2*b+c+c*(x-1))/c^(1/2)+((x-1)^2
*c+(b+2*c)*(x-1)+a+b+c)^(1/2))*b^3-1/4*c*((x-1)^2*c+(b+2*c)*(x-1)+a+b+c)^(1
/2)*x-3/16/c^(1/2)*ln((1/2*b-c+c*(x+1))/c^(1/2)+((x+1)^2*c+(b-2*c)*(x+1)+a-
b+c)^(1/2))*b^2-3/4*c^(1/2)*ln((1/2*b-c+c*(x+1))/c^(1/2)+((x+1)^2*c+(b-2*c)
*(x+1)+a-b+c)^(1/2))*a-3/4*c^(1/2)*ln((1/2*b+c+c*(x-1))/c^(1/2)+((x-1)^2*c+
(b+2*c)*(x-1)+a+b+c)^(1/2))*a-3/16/c^(1/2)*ln((1/2*b+c+c*(x-1))/c^(1/2)+((x
-1)^2*c+(b+2*c)*(x-1)+a+b+c)^(1/2))*b^2+1/6*((x+1)^2*c+(b-2*c)*(x+1)+a-b+c)
^(3/2)-1/6*((x-1)^2*c+(b+2*c)*(x-1)+a+b+c)^(3/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(-x^2+1),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{\left(c\,x^2+b\,x+a\right)^{3/2}}{x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a + b*x + c*x^2)^(3/2)/(x^2 - 1),x)
```

```
[Out] -int((a + b*x + c*x^2)^(3/2)/(x^2 - 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{a\sqrt{a+bx+cx^2}}{x^2-1}\,dx-\int \frac{bx\sqrt{a+bx+cx^2}}{x^2-1}\,dx-\int \frac{cx^2\sqrt{a+bx+cx^2}}{x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(3/2)/(-x**2+1),x)
```

```
[Out] -Integral(a*sqrt(a + b*x + c*x**2)/(x**2 - 1), x) - Integral(b*x*sqrt(a + b
*x + c*x**2)/(x**2 - 1), x) - Integral(c*x**2*sqrt(a + b*x + c*x**2)/(x**2
- 1), x)
```

3.92 $\int \frac{\sqrt{-1-x+x^2}}{1-x^2} \, dx$

Optimal. Leaf size=75

$$-\frac{1}{2}\tan^{-1}\left(\frac{3-x}{2\sqrt{x^2-x-1}}\right)+\tanh^{-1}\left(\frac{1-2x}{2\sqrt{x^2-x-1}}\right)+\frac{1}{2}\tanh^{-1}\left(\frac{3x+1}{2\sqrt{x^2-x-1}}\right)$$

```
[Out] -1/2*arctan(1/2*(3-x)/(x^2-x-1)^(1/2))+arctanh(1/2*(1-2*x)/(x^2-x-1)^(1/2))
+1/2*arctanh(1/2*(1+3*x)/(x^2-x-1)^(1/2))
```

Rubi [A] time = 0.06, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {990, 621, 206, 1033, 724, 204}

$$-\frac{1}{2}\tan^{-1}\left(\frac{3-x}{2\sqrt{x^2-x-1}}\right)+\tanh^{-1}\left(\frac{1-2x}{2\sqrt{x^2-x-1}}\right)+\frac{1}{2}\tanh^{-1}\left(\frac{3x+1}{2\sqrt{x^2-x-1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-1 - x + x^2]/(1 - x^2),x]
```

```
[Out] -ArcTan[(3 - x)/(2*Sqrt[-1 - x + x^2])]/2 + ArcTanh[(1 - 2*x)/(2*Sqrt[-1 -
x + x^2])] + ArcTanh[(1 + 3*x)/(2*Sqrt[-1 - x + x^2])]/2
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 990

```
Int[Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2]/((d_) + (f_.)*(x_)^2), x_Symbol]
 :> Dist[c/f, Int[1/Sqrt[a + b*x + c*x^2], x], x] - Dist[1/f, Int[(c*d - a*
f - b*f*x)/(Sqrt[a + b*x + c*x^2]*(d + f*x^2)), x], x] /; FreeQ[{a, b, c, d
, f}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1033

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q
```

```
), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q
), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f
, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]
```

Rubi steps

$$\int \frac{\sqrt{-1-x+x^2}}{1-x^2}\,dx = -\int \frac{1}{\sqrt{-1-x+x^2}}\,dx - \int \frac{x}{\left(1-x^2\right)\sqrt{-1-x+x^2}}\,dx$$
$$= -\left(\frac{1}{2}\int \frac{1}{(-1-x)\sqrt{-1-x+x^2}}\,dx\right) - \frac{1}{2}\int \frac{1}{(1-x)\sqrt{-1-x+x^2}}\,dx - 2\,\text{Subst}\left(\int \frac{1}{4-x^2}\,d\right.$$
$$= \tanh^{-1}\left(\frac{1-2x}{2\sqrt{-1-x+x^2}}\right) + \text{Subst}\left(\int \frac{1}{-4-x^2}\,dx, x, \frac{3-x}{\sqrt{-1-x+x^2}}\right) + \text{Subst}\left(\int \frac{1}{4-x^2}\,d\right.$$
$$= -\frac{1}{2}\tan^{-1}\left(\frac{3-x}{2\sqrt{-1-x+x^2}}\right) + \tanh^{-1}\left(\frac{1-2x}{2\sqrt{-1-x+x^2}}\right) + \frac{1}{2}\tanh^{-1}\left(\frac{1+3x}{2\sqrt{-1-x+x^2}}\right)$$

Mathematica [A] time = 0.04, size = 75, normalized size = 1.00

$$-\frac{1}{2}\tan^{-1}\left(\frac{3-x}{2\sqrt{x^2-x-1}}\right) + \tanh^{-1}\left(\frac{1-2x}{2\sqrt{x^2-x-1}}\right) + \frac{1}{2}\tanh^{-1}\left(\frac{3x+1}{2\sqrt{x^2-x-1}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-1 - x + x^2]/(1 - x^2),x]
[Out] -1/2*ArcTan[(3 - x)/(2*Sqrt[-1 - x + x^2])] + ArcTanh[(1 - 2*x)/(2*Sqrt[-1
- x + x^2])] + ArcTanh[(1 + 3*x)/(2*Sqrt[-1 - x + x^2])]/2
```

fricas [A] time = 1.39, size = 70, normalized size = 0.93

$$\arctan\left(-x+\sqrt{x^2-x-1}+1\right)-\frac{1}{2}\log\left(-x+\sqrt{x^2-x-1}\right)+\frac{1}{2}\log\left(-x+\sqrt{x^2-x-1}-2\right)+\log\left(-2\,x+2\,\sqrt{x^2-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-x-1)^(1/2)/(-x^2+1),x, algorithm="fricas")
[Out] arctan(-x + sqrt(x^2 - x - 1) + 1) - 1/2*log(-x + sqrt(x^2 - x - 1)) + 1/2*
log(-x + sqrt(x^2 - x - 1) - 2) + log(-2*x + 2*sqrt(x^2 - x - 1) + 1)
```

giac [A] time = 0.20, size = 73, normalized size = 0.97

$$\arctan\left(-x+\sqrt{x^2-x-1}+1\right)-\frac{1}{2}\log\left(\left|-x+\sqrt{x^2-x-1}\right|\right)+\frac{1}{2}\log\left(\left|-x+\sqrt{x^2-x-1}-2\right|\right)+\log\left(\left|-2\,x+2\,\sqrt{x\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-x-1)^(1/2)/(-x^2+1),x, algorithm="giac")
[Out] arctan(-x + sqrt(x^2 - x - 1) + 1) - 1/2*log(abs(-x + sqrt(x^2 - x - 1))) +
 1/2*log(abs(-x + sqrt(x^2 - x - 1) - 2)) + log(abs(-2*x + 2*sqrt(x^2 - x -
 1) + 1))
```

maple [A] time = 0.02, size = 102, normalized size = 1.36

$$-\frac{\operatorname{arctanh}\left(\frac{-3x-1}{2\sqrt{-3x+(x+1)^2-2}}\right)}{2}+\frac{\arctan\left(\frac{x-3}{2\sqrt{x+(x-1)^2-2}}\right)}{2}-\frac{3\ln\left(x-\frac{1}{2}+\sqrt{-3x+(x+1)^2-2}\right)}{4}-\frac{\ln\left(x-\frac{1}{2}+\sqrt{x+(x-\right.}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-x-1)^(1/2)/(-x^2+1),x)
```

```
[Out] -1/2*((x-1)^2+x-2)^(1/2)-1/4*ln(-1/2+x+((x-1)^2+x-2)^(1/2))+1/2*arctan(1/2*
(-3+x)/((x-1)^2+x-2)^(1/2))+1/2*((x+1)^2-3*x-2)^(1/2)-3/4*ln(-1/2+x+((x+1)^
2-3*x-2)^(1/2))-1/2*arctanh(1/2*(-1-3*x)/((x+1)^2-3*x-2)^(1/2))
```

maxima [A] time = 0.97, size = 83, normalized size = 1.11

$$\frac{1}{2}\arcsin\left(\frac{2\sqrt{5}x}{5|2x-2|}-\frac{6\sqrt{5}}{5|2x-2|}\right)-\log\left(x+\sqrt{x^2-x-1}-\frac{1}{2}\right)-\frac{1}{2}\log\left(\frac{2\sqrt{x^2-x-1}}{|2x+2|}+\frac{2}{|2x+2|}-\frac{3}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-x-1)^(1/2)/(-x^2+1),x, algorithm="maxima")
```

```
[Out] 1/2*arcsin(2/5*sqrt(5)*x/abs(2*x - 2) - 6/5*sqrt(5)/abs(2*x - 2)) - log(x +
 sqrt(x^2 - x - 1) - 1/2) - 1/2*log(2*sqrt(x^2 - x - 1)/abs(2*x + 2) + 2/ab
s(2*x + 2) - 3/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{\sqrt{x^2-x-1}}{x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2 - x - 1)^(1/2)/(x^2 - 1),x)
```

```
[Out] -int((x^2 - x - 1)^(1/2)/(x^2 - 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt{x^2-x-1}}{x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-x-1)**(1/2)/(-x**2+1),x)
```

```
[Out] -Integral(sqrt(x**2 - x - 1)/(x**2 - 1), x)
```

3.93 $\int \frac{(x+x^2)^{3/2}}{1+x^2} dx$

Optimal. Leaf size=130

$$\frac{1}{4}\sqrt{x^2+x}(2x+5)+\sqrt{1+\sqrt{2}}\tan^{-1}\left(\frac{-x+\sqrt{2}+1}{\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{x^2+x}}\right)-\sqrt{\sqrt{2}-1}\tanh^{-1}\left(\frac{-x-\sqrt{2}+1}{\sqrt{2\left(\sqrt{2}-1\right)}\sqrt{x^2+x}}\right)-\frac{5}{4}\tanh$$

```
[Out] -5/4*arctanh(x/(x^2+x)^(1/2))+1/4*(5+2*x)*(x^2+x)^(1/2)-arctanh((1-x-2^(1/2
))/(x^2+x)^(1/2)/(-2+2*2^(1/2))^(1/2))*(2^(1/2)-1)^(1/2)+arctan((1-x+2^(1/2
))/(x^2+x)^(1/2)/(2+2*2^(1/2))^(1/2))*(1+2^(1/2))^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.529, Rules used = {978, 1078, 620, 206, 12, 1036, 1030, 207, 203}

$$\frac{1}{4}\sqrt{x^2+x}(2x+5)+\sqrt{1+\sqrt{2}}\tan^{-1}\left(\frac{-x+\sqrt{2}+1}{\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{x^2+x}}\right)-\sqrt{\sqrt{2}-1}\tanh^{-1}\left(\frac{-x-\sqrt{2}+1}{\sqrt{2\left(\sqrt{2}-1\right)}\sqrt{x^2+x}}\right)-\frac{5}{4}\tanh$$

Antiderivative was successfully verified.

```
[In] Int[(x + x^2)^(3/2)/(1 + x^2),x]
```

```
[Out] ((5 + 2*x)*Sqrt[x + x^2])/4 + Sqrt[1 + Sqrt[2]]*ArcTan[(1 + Sqrt[2] - x)/(S
qrt[2*(1 + Sqrt[2])]*Sqrt[x + x^2])] - Sqrt[-1 + Sqrt[2]]*ArcTanh[(1 - Sqrt
[2] - x)/(Sqrt[2*(-1 + Sqrt[2])]*Sqrt[x + x^2])] - (5*ArcTanh[x/Sqrt[x + x^
2]])/4
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 978

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (f_.)*(x_)^2)^(q_), x
_Symbol] :> Simp[((b*(3*p + 2*q) + 2*c*(p + q)*x)*(a + b*x + c*x^2)^(p - 1)
*(d + f*x^2)^(q + 1))/(2*f*(p + q)*(2*p + 2*q + 1)), x] - Dist[1/(2*f*(p +
q)*(2*p + 2*q + 1)), Int[(a + b*x + c*x^2)^(p - 2)*(d + f*x^2)^q*Simp[b^2*d
*(p - 1)*(2*p + q) - (p + q)*(b^2*d*(1 - p) - 2*a*(c*d - a*f*(2*p + 2*q + 1
))) - (2*b*(c*d - a*f)*(1 - p)*(2*p + q) - 2*(p + q)*b*(2*c*d*(2*p + q) - (
c*d + a*f)*(2*p + 2*q + 1)))*x + (b^2*f*p*(1 - p) + 2*c*(p + q)*(c*d*(2*p -
 1) - a*f*(4*p + 2*q - 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, q}, x]
 && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 1] && NeQ[p + q, 0] && NeQ[2*p + 2*q + 1,
 0] &&  !IGtQ[p, 0] &&  !IGtQ[q, 0]
```

Rule 1030

```
Int[((g_) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f
_.)*(x_)^2]), x_Symbol] :> Dist[-2*a*g*h, Subst[Int[1/Simp[2*a^2*g*h*c + a*
e*x^2, x], x], x, Simp[a*h - g*c*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[
{a, c, d, e, f, g, h}, x] && EqQ[a*h^2*e + 2*g*h*(c*d - a*f) - g^2*c*e, 0]
```

Rule 1036

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 + a*c*e^2, 2]}, Dist
[1/(2*q), Int[Simp[-(a*h*e) - g*(c*d - a*f - q) + (h*(c*d - a*f + q) - g*c*
e)*x, x]/((a + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Si
mp[-(a*h*e) - g*(c*d - a*f + q) + (h*(c*d - a*f - q) - g*c*e)*x, x]/((a + c
*x^2)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f, g, h}, x] &&
 NeQ[e^2 - 4*d*f, 0] && NegQ[-(a*c)]
```

Rule 1078

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
 (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqrt[d + e*x + f
*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + B*c*x)/((a + c*x^2)*Sqrt[d + e*x
 + f*x^2]), x], x] /; FreeQ[{a, c, d, e, f, A, B, C}, x] && NeQ[e^2 - 4*d*f
, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(x+x^2\right)^{3/2}}{1+x^2}\,dx &= \frac{1}{4}(5+2x)\sqrt{x+x^2}-\frac{1}{2}\int \frac{\frac{5}{4}+4x+\frac{5x^2}{4}}{\left(1+x^2\right)\sqrt{x+x^2}}\,dx\\
&= \frac{1}{4}(5+2x)\sqrt{x+x^2}-\frac{1}{2}\int \frac{4x}{\left(1+x^2\right)\sqrt{x+x^2}}\,dx-\frac{5}{8}\int \frac{1}{\sqrt{x+x^2}}\,dx\\
&= \frac{1}{4}(5+2x)\sqrt{x+x^2}-\frac{5}{4}\operatorname{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\frac{x}{\sqrt{x+x^2}}\right)-2\int \frac{x}{\left(1+x^2\right)\sqrt{x+x^2}}\,dx\\
&= \frac{1}{4}(5+2x)\sqrt{x+x^2}-\frac{5}{4}\tanh^{-1}\left(\frac{x}{\sqrt{x+x^2}}\right)+\frac{\int \frac{-1+\left(-1-\sqrt{2}\right)x}{\left(1+x^2\right)\sqrt{x+x^2}}\,dx}{\sqrt{2}}-\frac{\int \frac{-1+\left(-1+\sqrt{2}\right)x}{\left(1+x^2\right)\sqrt{x+x^2}}\,dx}{\sqrt{2}}\\
&= \frac{1}{4}(5+2x)\sqrt{x+x^2}-\frac{5}{4}\tanh^{-1}\left(\frac{x}{\sqrt{x+x^2}}\right)+\left(-2+\sqrt{2}\right)\operatorname{Subst}\left(\int \frac{1}{2\left(1-\sqrt{2}\right)+x^2}\,dx,x,\right.\\
&= \frac{1}{4}(5+2x)\sqrt{x+x^2}+\sqrt{1+\sqrt{2}}\,\tan^{-1}\left(\frac{1+\sqrt{2}-x}{\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{x+x^2}}\right)-\sqrt{-1+\sqrt{2}}\,\tanh^{-1}\left(\frac{}{\sqrt{2}}\right.
\end{aligned}
$$

Mathematica [C] time = 0.16, size = 120, normalized size = 0.92

$$\frac{\sqrt{x}\sqrt{x+1}\left(2\sqrt{x+1}x^{3/2}+5\sqrt{x+1}\sqrt{x}+4(-1+i)^{3/2}\tan^{-1}\left(\sqrt{-1+i}\sqrt{\frac{x}{x+1}}\right)-5\sinh^{-1}\left(\sqrt{x}\right)+4(1+i)^{3/2}\tanh$$
$$4\sqrt{x(x+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x + x^2)^(3/2)/(1 + x^2),x]
```

```
[Out] (Sqrt[x]*Sqrt[1 + x]*(5*Sqrt[x]*Sqrt[1 + x] + 2*x^(3/2)*Sqrt[1 + x] - 5*Arc
Sinh[Sqrt[x]] + 4*(-1 + I)^(3/2)*ArcTan[Sqrt[-1 + I]*Sqrt[x/(1 + x)]] + 4*(
1 + I)^(3/2)*ArcTanh[Sqrt[1 + I]*Sqrt[x/(1 + x)]]))/(4*Sqrt[x*(1 + x)])
```

fricas [B] time = 1.28, size = 777, normalized size = 5.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x)^(3/2)/(x^2+1),x, algorithm="fricas")
```

```
[Out] -1/8*8^(1/4)*sqrt(2*sqrt(2) + 4)*(sqrt(2) - 2)*log(8*x^2 - 8*sqrt(x^2 + x)*
x + 2*(8^(1/4)*sqrt(x^2 + x)*(sqrt(2) - 1) - 8^(1/4)*(sqrt(2)*x - x - 1))*s
qrt(2*sqrt(2) + 4) + 4*x + 4*sqrt(2) + 4) + 1/8*8^(1/4)*sqrt(2*sqrt(2) + 4)
*(sqrt(2) - 2)*log(8*x^2 - 8*sqrt(x^2 + x)*x - 2*(8^(1/4)*sqrt(x^2 + x)*(sq
rt(2) - 1) - 8^(1/4)*(sqrt(2)*x - x - 1))*sqrt(2*sqrt(2) + 4) + 4*x + 4*sqr
t(2) + 4) + 1/2*8^(1/4)*sqrt(2)*sqrt(2*sqrt(2) + 4)*arctan(1/7*sqrt(2)*(sqr
t(2)*(5*x + 1) + 6*x + 4) + 1/112*sqrt(8*x^2 - 8*sqrt(x^2 + x)*x - 2*(8^(1/
4)*sqrt(x^2 + x)*(sqrt(2) - 1) - 8^(1/4)*(sqrt(2)*x - x - 1))*sqrt(2*sqrt(2
) + 4) + 4*x + 4*sqrt(2) + 4)*(8*sqrt(2)*(5*sqrt(2) + 6) + (8^(3/4)*(5*sqrt
(2) + 6) + 8*8^(1/4)*(2*sqrt(2) + 1))*sqrt(2*sqrt(2) + 4) + 64*sqrt(2) + 32
) - 1/7*sqrt(x^2 + x)*(sqrt(2)*(5*sqrt(2) + 6) + 8*sqrt(2) + 4) + 1/7*sqrt(
2)*(8*x + 3) + 1/56*(8^(3/4)*(sqrt(2)*(5*x + 1) + 6*x + 4) - sqrt(x^2 + x)*
(8^(3/4)*(5*sqrt(2) + 6) + 8*8^(1/4)*(2*sqrt(2) + 1)) + 8*8^(1/4)*(sqrt(2)*
(2*x - 1) + x + 3))*sqrt(2*sqrt(2) + 4) + 4/7*x + 5/7) + 1/2*8^(1/4)*sqrt(2
)*sqrt(2*sqrt(2) + 4)*arctan(-1/7*sqrt(2)*(sqrt(2)*(5*x + 1) + 6*x + 4) - 1
/112*sqrt(8*x^2 - 8*sqrt(x^2 + x)*x + 2*(8^(1/4)*sqrt(x^2 + x)*(sqrt(2) - 1
) - 8^(1/4)*(sqrt(2)*x - x - 1))*sqrt(2*sqrt(2) + 4) + 4*x + 4*sqrt(2) + 4)
*(8*sqrt(2)*(5*sqrt(2) + 6) - (8^(3/4)*(5*sqrt(2) + 6) + 8*8^(1/4)*(2*sqrt(
2) + 1))*sqrt(2*sqrt(2) + 4) + 64*sqrt(2) + 32) + 1/7*sqrt(x^2 + x)*(sqrt(2
)*(5*sqrt(2) + 6) + 8*sqrt(2) + 4) - 1/7*sqrt(2)*(8*x + 3) + 1/56*(8^(3/4)*
(sqrt(2)*(5*x + 1) + 6*x + 4) - sqrt(x^2 + x)*(8^(3/4)*(5*sqrt(2) + 6) + 8*
8^(1/4)*(2*sqrt(2) + 1)) + 8*8^(1/4)*(sqrt(2)*(2*x - 1) + x + 3))*sqrt(2*sq
rt(2) + 4) - 4/7*x - 5/7) + 1/4*sqrt(x^2 + x)*(2*x + 5) + 5/8*log(-2*x + 2*
sqrt(x^2 + x) - 1)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x)^(3/2)/(x^2+1),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to divide, perhaps due to rounding er
ror%%%{%%{poly1[293782586339316530197997184853348485491135961433723696543O1
45666147448671085140650508776826583251953125OO,-181983623107889624631149356
157406719337484171310120143823570373289503707930369037595766180522699542236
3281250,-457037569568074726029024220476354238878560215202443954711967761282
5585897607153309996504109115565063476562500,5657088028436513895060368238114
```

045087528706221186372034279204932374221645044901876954909719232468137744140
6250000]:[1,0,-31106,-99112,-65203241]%%},[6]%%%}+%%%{%%{poly1[563214846686
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2571922564697265625O,-69671463056492042794032844569028505467609156013361581
339753892610088915797181572983287609014002602233886718 75,-18501639301691189
171876668611027555220095054439393285921280605772669698963742122921653330793
61994541992187500000,21154356016599020416908584996741185285458255144337770O
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99112,-65203241]%%},[5]%%%}+%%%{%%{poly1[249752783489288169448537286816821 8
934326063649034465040023204984927006741616526424834382657324058227539062 50,
102899415255489529312024941498712410711762889441826371792182609371037315453
946796356559154403034765625000000 0,-829533110278394819601873642487583961356
974446292618272798853390412960478561827576658458766459696930368041992187 5,-
621314982897707824179256504166985600883550135798254936235154038672746633126
7769426651845176824134578207397460937 5]:[1,0,-31106,-99112,-65203241]%%},[4
]%%%}+%%%{%%{poly1[134061679438690600698329033958374395472346927695250329 21
595687410910143475305657725850348520828167724609375 0,144822363547001571041 1
105400521262660890267467474254168325112744678537043998787547620879624916147
4060058593750,-450743946228316845834503722080596923970970618377104928698371
490711344578730358390494165951866069947869873046875 0,-501259088879939665231
933655971301952587189466470689047006275328975844492780545187141383181367765
715754302978515625]:[1,0,-31106,-99112,-65203241]%%},[3]%%%}+%%%{%%{poly1[-
381381386714161407348746082633738466949446345681605193953880120440269250489
6705133556440560506922912597656 25,-928090484018458467281328667589231013642 6
540844620787163618570825609266807260813310410092811870469665527343 75,125754
762806389252665809919975988058534449424198247881435471309616603663322695610
14277652901709172789611816406250,16358037436900106479732840612571178737540 6
383526961222766588716228204232812664410486840411838682451928710937500 00]:[1
,0,-31106,-99112,-65203241]%%},[2]%%%}+%%%{%%{poly1[-282916504651425899151 8
864770923875171856122508474820224931553878066663155128213074011486795035416
25976562500,-110475220241836404058124193839468468131104619301709288761928 26
646859750070859837003681504660562347747802734375,926224105736204220013902 83
280266603413021660935063007466600699398333831523463749530142976259339105900
26855468750,379944404413115369887516401121805461986384147356195938788669206
085547964391733575642579455121057385054046630859375]:[1,0,-31106,-99112,-65
203241]%%},[1]%%%}+%%%{%%{poly1[145744257028158078272793741263638276606104 7
294766508516836262066352309092494446076038057333731054382324218 75,451515992
466773368261871056920647955432794275385491983650499399141514582936757420365
836369871082510375976562 5,-494995282225192497001838548818146861751126606116
946395167844591941183044764975183478723518773095487619018554687 5,-118464124
267461539474328024609355477174113903628048164370055623880000124338618560214
7216185055836801604309082031 25]:[1,0,-31106,-99112,-65203241]%%},[0]%%%} /
%%%{%%{poly1[-44900316662769750,27813501156752721637 5,6985142263227090007 50
,1028664866672275019250]:[1,0,-31106,-99112,-65203241]%%},[1]%%%}+%%%{%%{[2
929989585103994875,368557731538442832750,907666859350191995250,-45807860415
265619064625]:[1,0,-31106,-99112,-65203241]%%},[0]%%%} Error: Bad Argument
Value

maple [B] time = 0.14, size = 789, normalized size = 6.07

$$\frac{\sqrt{x^2+x}\,x}{2}-\frac{5\ln\left(x+\frac{1}{2}+\sqrt{x^2+x}\right)}{8}+\frac{5\sqrt{x^2+x}}{4}+\frac{\sqrt{\frac{4\left(x-\sqrt{2}-1\right)^2}{\left(-x+1-\sqrt{2}\right)^2}-\frac{3\sqrt{2}\left(x-\sqrt{2}-1\right)^2}{\left(-x+1-\sqrt{2}\right)^2}+4+3\sqrt{2}}\;\sqrt{2}\left(-4\sqrt{2}\ \mathrm{arctan}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+x)^(3/2)/(x^2+1),x)
```

```
[Out] 1/2*x*(x^2+x)^(1/2)+5/4*(x^2+x)^(1/2)-5/8*ln(1/2+x+(x^2+x)^(1/2))+1/2*(4*(-
2^(1/2)-1+x)^2/(1-x-2^(1/2))^2-3*2^(1/2)*(-2^(1/2)-1+x)^2/(1-x-2^(1/2))^2+4
+3*2^(1/2))^(1/2)*2^(1/2)*((-2+2*2^(1/2))^(1/2)*arctan(1/2*(-2+2*2^(1/2))^(
1/2)*((3*2^(1/2)-4)*(-(-2^(1/2)-1+x)^2/(1-x-2^(1/2))^2+12*2^(1/2)+17))^(1/2
)*(24*(-2^(1/2)-1+x)^2/(1-x-2^(1/2))^2+17*2^(1/2)*(-2^(1/2)-1+x)^2/(1-x-2^(
1/2))^2-2^(1/2))*(-2^(1/2)-1+x)/(1-x-2^(1/2))*(3*2^(1/2)-4)/((-2^(1/2)-1+x)
^4/(1-x-2^(1/2))^4-34*(-2^(1/2)-1+x)^2/(1-x-2^(1/2))^2+1))*(1+2^(1/2))^(1/2
)*2^(1/2)-2*(-2+2*2^(1/2))^(1/2)*arctan(1/2*(-2+2*2^(1/2))^(1/2)*((3*2^(1/2
)-4)*(-(-2^(1/2)-1+x)^2/(1-x-2^(1/2))^2+12*2^(1/2)+17))^(1/2)*(24*(-2^(1/2)
-1+x)^2/(1-x-2^(1/2))^2+17*2^(1/2)*(-2^(1/2)-1+x)^2/(1-x-2^(1/2))^2-2^(1/2)
)*(-2^(1/2)-1+x)/(1-x-2^(1/2))*(3*2^(1/2)-4)/((-2^(1/2)-1+x)^4/(1-x-2^(1/2)
)^4-34*(-2^(1/2)-1+x)^2/(1-x-2^(1/2))^2+1))*(1+2^(1/2))^(1/2)-4*arctanh(1/2
*(4*(-2^(1/2)-1+x)^2/(1-x-2^(1/2))^2-3*2^(1/2)*(-2^(1/2)-1+x)^2/(1-x-2^(1/2
))^2+4+3*2^(1/2))^(1/2)/(1+2^(1/2))^(1/2))*2^(1/2)+6*arctanh(1/2*(4*(-2^(1/
2)-1+x)^2/(1-x-2^(1/2))^2-3*2^(1/2)*(-2^(1/2)-1+x)^2/(1-x-2^(1/2))^2+4+3*2^
(1/2))^(1/2)/(1+2^(1/2))^(1/2)))/(-(3*2^(1/2)*(-2^(1/2)-1+x)^2/(1-x-2^(1/2)
)^2-4*(-2^(1/2)-1+x)^2/(1-x-2^(1/2))^2-3*2^(1/2)-4)/((-2^(1/2)-1+x)/(1-x-2^
(1/2))+1)^2)^(1/2)/((-2^(1/2)-1+x)/(1-x-2^(1/2))+1)/(3*2^(1/2)-4)/(1+2^(1/2
))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^2+x\right)^{\frac{3}{2}}}{x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x)^(3/2)/(x^2+1),x, algorithm="maxima")
```

```
[Out] integrate((x^2 + x)^(3/2)/(x^2 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(x^2+x\right)^{3/2}}{x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + x^2)^(3/2)/(x^2 + 1),x)
```

```
[Out] int((x + x^2)^(3/2)/(x^2 + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x\left(x+1\right)\right)^{\frac{3}{2}}}{x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+x)**(3/2)/(x**2+1),x)
```

```
[Out] Integral((x*(x + 1))**(3/2)/(x**2 + 1), x)
```

3.94 $\int \frac{x^4}{\sqrt{a+bx+cx^2}\left(d-fx^2\right)}\,dx$

Optimal. Leaf size=369

$$-\frac{\left(3b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{5/2}f}+\frac{3b\sqrt{a+bx+cx^2}}{4c^2f}+\frac{d^{3/2}\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2f^2\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}+\frac{d^{3/2}\,\mathrm{tanh}\cdots}{2\cdots}$$

```
[Out] -1/8*(-4*a*c+3*b^2)*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(5
/2)/f-d*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/f^2/c^(1/2)+3/4*
b*(c*x^2+b*x+a)^(1/2)/c^2/f-1/2*x*(c*x^2+b*x+a)^(1/2)/c/f+1/2*d^(3/2)*arcta
nh(1/2*(b*d^(1/2)-2*a*f^(1/2)+x*(2*c*d^(1/2)-b*f^(1/2)))/(c*x^2+b*x+a)^(1/2
)/(c*d+a*f-b*d^(1/2)*f^(1/2))^(1/2))/f^2/(c*d+a*f-b*d^(1/2)*f^(1/2))^(1/2)+
1/2*d^(3/2)*arctanh(1/2*(b*d^(1/2)+2*a*f^(1/2)+x*(2*c*d^(1/2)+b*f^(1/2)))/(
c*x^2+b*x+a)^(1/2)/(c*d+a*f+b*d^(1/2)*f^(1/2))^(1/2))/f^2/(c*d+a*f+b*d^(1/2
)*f^(1/2))^(1/2)
```

Rubi [A] time = 0.81, antiderivative size = 369, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6725, 621, 206, 742, 640, 984, 724}

$$-\frac{\left(3b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{5/2}f}+\frac{3b\sqrt{a+bx+cx^2}}{4c^2f}+\frac{d^{3/2}\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2f^2\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}+\frac{d^{3/2}\,\mathrm{tanh}\cdots}{2\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(Sqrt[a + b*x + c*x^2]*(d - f*x^2)),x]
```

```
[Out] (3*b*Sqrt[a + b*x + c*x^2])/(4*c^2*f) - (x*Sqrt[a + b*x + c*x^2])/(2*c*f) -
 (d*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(Sqrt[c]*f^2) -
 ((3*b^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(
8*c^(5/2)*f) + (d^(3/2)*ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b
*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])]
)/(2*f^2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]) + (d^(3/2)*ArcTanh[(b*Sqrt[d]
 + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[
f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*f^2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*
f])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
```

```
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 742

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
raticQ[a, b, c, d, e, m, p, x]
```

Rule 984

```
Int[1/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Sy
mbol] :> Dist[1/2, Int[1/((a - Rt[-(a*c), 2]*x)*Sqrt[d + e*x + f*x^2]), x],
 x] + Dist[1/2, Int[1/((a + Rt[-(a*c), 2]*x)*Sqrt[d + e*x + f*x^2]), x], x]
 /; FreeQ[{a, c, d, e, f}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{x^4}{\sqrt{a+bx+cx^2}\left(d-fx^2\right)}\,dx = \int \left(-\frac{d}{f^2\sqrt{a+bx+cx^2}} - \frac{x^2}{f\sqrt{a+bx+cx^2}} + \frac{d^2}{f^2\sqrt{a+bx+cx^2}\left(d-fx^2\right)}\right) dx$$

$$= -\frac{d\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{f^2} + \frac{d^2\int \frac{1}{\sqrt{a+bx+cx^2}\left(d-fx^2\right)}\,dx}{f^2} - \frac{\int \frac{x^2}{\sqrt{a+bx+cx^2}}\,dx}{f}$$

$$= -\frac{x\sqrt{a+bx+cx^2}}{2cf} - \frac{(2d)\,\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{f^2} + \frac{d^2\int \frac{1}{\left(d-\sqrt{d}\,\sqrt{f}\,x\right)\sqrt{a+\cdots}}}{2f^2}$$

$$= \frac{3b\sqrt{a+bx+cx^2}}{4c^2f} - \frac{x\sqrt{a+bx+cx^2}}{2cf} - \frac{d\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}\,f^2} - \frac{d^2\,\text{Subst}\left(\int \cdots\right.}{\cdots}$$

$$= \frac{3b\sqrt{a+bx+cx^2}}{4c^2f} - \frac{x\sqrt{a+bx+cx^2}}{2cf} - \frac{d\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}\,f^2} + \frac{d^{3/2}\tanh^{-1}\cdots}{2f^2\cdots}$$

$$= \frac{3b\sqrt{a+bx+cx^2}}{4c^2f} - \frac{x\sqrt{a+bx+cx^2}}{2cf} - \frac{d\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}\,f^2} - \frac{\left(3b^2-4ac\right)\cdots}{\cdots}$$

Mathematica [A] time = 1.71, size = 300, normalized size = 0.81

$$-\frac{\frac{\left(-4acf+3b^2f+8c^2d\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)}{c^{5/2}}-\frac{2f(2cx-3b)\sqrt{a+x(b+cx)}}{c^2}+\frac{4d^{3/2}\tanh^{-1}\left(\frac{2a\sqrt{f}+b\sqrt{d}+b\sqrt{f}x+2c\sqrt{d}x}{2\sqrt{a+x(b+cx)}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}+\frac{4d^{3/2}\tanh^{-1}\left(\frac{-\ldots}{2\sqrt{\ldots}}\right)}{\sqrt{af\ldots}}}{8f^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(Sqrt[a + b*x + c*x^2]*(d - f*x^2)),x]
```

```
[Out] ((-2*f*(-3*b + 2*c*x)*Sqrt[a + x*(b + c*x)])/c^2 - ((8*c^2*d + 3*b^2*f - 4*
a*c*f)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/c^(5/2) + (4
*d^(3/2)*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + 2*c*Sqrt[d]*x + b*Sqrt[f]*x)/(2
*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])])/Sqrt[c*d + b*
Sqrt[d]*Sqrt[f] + a*f] + (4*d^(3/2)*ArcTanh[(-2*a*Sqrt[f] + 2*c*Sqrt[d]*x +
 b*(Sqrt[d] - Sqrt[f]*x))/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x
*(b + c*x)])])/Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f])/(8*f^2)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [A] time = 0.03, size = 516, normalized size = 1.40

$$-\frac{d^2\ln\left(\frac{\frac{2af+2cd-2\sqrt{df}\,b}{f}+\frac{\left(bf-2\sqrt{df}\,c\right)\left(x+\frac{\sqrt{df}}{f}\right)}{f}+2\sqrt{\frac{af+cd-\sqrt{df}\,b}{f}}\sqrt{\left(x+\frac{\sqrt{df}}{f}\right)^2c+\frac{\left(bf-2\sqrt{df}\,c\right)\left(x+\frac{\sqrt{df}}{f}\right)}{f}+\frac{af+cd-\sqrt{df}\,b}{f}}}{x+\frac{\sqrt{df}}{f}}\right)}{2\sqrt{df}\,\sqrt{\frac{af+cd-\sqrt{df}\,b}{f}}\,f^2}+\frac{d^2\ln\left(\frac{\frac{2af+2cd+2\sqrt{d\ldots}}{f}\ldots}{\ldots}\right)}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x)
```

```
[Out] -1/2*x*(c*x^2+b*x+a)^(1/2)/c/f+3/4*b*(c*x^2+b*x+a)^(1/2)/c^2/f-3/8/f*b^2/c^
(5/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+1/2/f*a/c^(3/2)*ln((c*x+1
/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-1/f^2*d*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*
x+a)^(1/2))/c^(1/2)-1/2/f^2*d^2/(d*f)^(1/2)/((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/
2)*ln((2*(a*f+c*d-(d*f)^(1/2)*b)/f+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/
f+2*((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(
1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))/(x+(d*f)^(1/2
)/f))+1/2/f^2*d^2/(d*f)^(1/2)/((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+
```

```
c*d+(d*f)^(1/2)*b)/f+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+2*((a*f+c*d+
(d*f)^(1/2)*b)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*
f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))/(x-(d*f)^(1/2)/f))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(((c*sqrt(4*d*f))/(2*f^2)>0)', see `
assume?` for more details)Is ((c*sqrt(4*d*f))/(2*f^2)    +b/(2*f))    ^2
 -(c*((b*sqrt(4*d*f))                  /(2*f)                 +(c*d)/f+a))
    /f^2 zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4}{\left(d-f\,x^2\right)\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((d - f*x^2)*(a + b*x + c*x^2)^(1/2)),x)
```

```
[Out] int(x^4/((d - f*x^2)*(a + b*x + c*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^4}{-d\sqrt{a+bx+cx^2}+fx^2\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(c*x**2+b*x+a)**(1/2)/(-f*x**2+d),x)
```

```
[Out] -Integral(x**4/(-d*sqrt(a + b*x + c*x**2) + f*x**2*sqrt(a + b*x + c*x**2)),
 x)
```

3.95 $\int \frac{x^3}{\sqrt{a+bx+cx^2}\left(d-fx^2\right)}\,dx$

Optimal. Leaf size=287

$$\frac{b\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2c^{3/2}f}-\frac{d\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2f^{3/2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}+\frac{d\tanh^{-1}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2f^{3/2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}-\sqrt{a}$$

```
[Out] 1/2*b*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(3/2)/f-(c*x^2+b
*x+a)^(1/2)/c/f-1/2*d*arctanh(1/2*(b*d^(1/2)-2*a*f^(1/2)+x*(2*c*d^(1/2)-b*f
^(1/2)))/(c*x^2+b*x+a)^(1/2)/(c*d+a*f-b*d^(1/2)*f^(1/2))^(1/2))/f^(3/2)/(c*
d+a*f-b*d^(1/2)*f^(1/2))^(1/2)+1/2*d*arctanh(1/2*(b*d^(1/2)+2*a*f^(1/2)+x*(
2*c*d^(1/2)+b*f^(1/2)))/(c*x^2+b*x+a)^(1/2)/(c*d+a*f+b*d^(1/2)*f^(1/2))^(1/
2))/f^(3/2)/(c*d+a*f+b*d^(1/2)*f^(1/2))^(1/2)
```

Rubi [A] time = 0.63, antiderivative size = 287, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {6725, 640, 621, 206, 1033, 724}

$$\frac{b\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2c^{3/2}f}-\frac{d\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2f^{3/2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}+\frac{d\tanh^{-1}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2f^{3/2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}-\sqrt{a}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(Sqrt[a + b*x + c*x^2]*(d - f*x^2)),x]
```

```
[Out] -(Sqrt[a + b*x + c*x^2]/(c*f)) + (b*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a +
 b*x + c*x^2])])/(2*c^(3/2)*f) - (d*ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c
*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*
x + c*x^2])])/(2*f^(3/2)*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]) + (d*ArcTanh[
(b*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqr
t[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*f^(3/2)*Sqrt[c*d + b*Sqrt[d
]*Sqrt[f] + a*f])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1033

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q
), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q
), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f
, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{x^3}{\sqrt{a+bx+cx^2}\left(d-fx^2\right)}\,dx = \int\left(-\frac{x}{f\sqrt{a+bx+cx^2}}+\frac{dx}{f\sqrt{a+bx+cx^2}\left(d-fx^2\right)}\right)dx$$

$$= -\frac{\int \frac{x}{\sqrt{a+bx+cx^2}}\,dx}{f}+\frac{d\int \frac{x}{\sqrt{a+bx+cx^2}\left(d-fx^2\right)}\,dx}{f}$$

$$= -\frac{\sqrt{a+bx+cx^2}}{cf}+\frac{b\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{2cf}+\frac{d\int \frac{1}{\left(-\sqrt{d}\sqrt{f}-fx\right)\sqrt{a+bx+cx^2}}\,dx}{2f}+\frac{d\int \frac{}{\left(\sqrt{d}\right.}$$

$$= -\frac{\sqrt{a+bx+cx^2}}{cf}+\frac{b\,\mathrm{Subst}\left(\int \frac{1}{4c-x^2}\,dx,x,\frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{cf}-\frac{d\,\mathrm{Subst}\left(\int \frac{1}{4cdf-4b\sqrt{d}\,f^3}\right.}{}$$

$$= -\frac{\sqrt{a+bx+cx^2}}{cf}+\frac{b\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2c^{3/2}f}-\frac{d\tanh^{-1}\left(\frac{b\sqrt{d}-2a\sqrt{f}+\left(2c\sqrt{d}-b\sqrt{f}\right.}{2\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx}}\right.}{2f^{3/2}\sqrt{cd-b\sqrt{d}\sqrt{f}+af}}$$

Mathematica [A] time = 1.16, size = 325, normalized size = 1.13

$$\frac{\frac{b\sqrt{f}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)}{c^{3/2}}+\frac{d\tanh^{-1}\left(\frac{2a\sqrt{f}+b\sqrt{d}+b\sqrt{f}x+2c\sqrt{d}x}{2\sqrt{a+x(b+cx)}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}-\frac{d\tanh^{-1}\left(\frac{-2a\sqrt{f}+b\left(\sqrt{d}-\sqrt{f}x\right)+2c\sqrt{d}x}{2\sqrt{a+x(b+cx)}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}-\frac{2\sqrt{f}x^2}{\sqrt{a+x(b+cx)}}-\frac{2b\sqrt{}}{c\sqrt{a+x(}}}{2f^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(Sqrt[a + b*x + c*x^2]*(d - f*x^2)),x]
```

```
[Out] ((-2*a*Sqrt[f])/(c*Sqrt[a + x*(b + c*x)]) - (2*b*Sqrt[f]*x)/(c*Sqrt[a + x*(
b + c*x)]) - (2*Sqrt[f]*x^2)/Sqrt[a + x*(b + c*x)] + (b*Sqrt[f]*ArcTanh[(b
+ 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/c^(3/2) + (d*ArcTanh[(b*Sqrt[d
] + 2*a*Sqrt[f] + 2*c*Sqrt[d]*x + b*Sqrt[f]*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt
[f] + a*f]*Sqrt[a + x*(b + c*x)])])/Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f] - (
d*ArcTanh[(-2*a*Sqrt[f] + 2*c*Sqrt[d]*x + b*(Sqrt[d] - Sqrt[f]*x))/(2*Sqrt[
```

```
c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])])/Sqrt[c*d - b*Sqrt[d
]*Sqrt[f] + a*f])/(2*f^(3/2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [A] time = 0.02, size = 410, normalized size = 1.43

$$\frac{d\ln\left(\frac{\frac{2af+2cd-2\sqrt{df}\,b}{f}+\frac{\left(bf-2\sqrt{df}\,c\right)\left(x+\frac{\sqrt{df}}{f}\right)}{f}+2\sqrt{\frac{af+cd-\sqrt{df}\,b}{f}}\sqrt{\left(x+\frac{\sqrt{df}}{f}\right)^{2}c+\frac{\left(bf-2\sqrt{df}\,c\right)\left(x+\frac{\sqrt{df}}{f}\right)}{f}+\frac{af+cd-\sqrt{df}\,b}{f}}}{x+\frac{\sqrt{df}}{f}}\right)}{2\sqrt{\frac{af+cd-\sqrt{df}\,b}{f}}\,f^{2}}+\frac{d\ln\left(\frac{\frac{2af+2cd+2\sqrt{df}\,b}{f}+\cdots}{\cdots}\right)}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x)
```

```
[Out] -(c*x^2+b*x+a)^(1/2)/c/f+1/2/f*b/c^(3/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+
a)^(1/2))+1/2/f^2*d/((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d-(d*f)^
(1/2)*b)/f+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+2*((a*f+c*d-(d*f)^(1/2
)*b)/f)^(1/2)*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f
)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))/(x+(d*f)^(1/2)/f))+1/2/f^2*d/((a*f+c*
d+(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d+(d*f)^(1/2)*b)/f+(b*f+2*(d*f)^(1/2
)*c)*(x-(d*f)^(1/2)/f)/f+2*((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*((x-(d*f)^(1/2
)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/
f)^(1/2))/(x-(d*f)^(1/2)/f))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(((c*sqrt(4*d*f))/(2*f^2)>0)', see `
assume?` for more details)Is ((c*sqrt(4*d*f))/(2*f^2)    +b/(2*f))    ^2
 -(c*((b*sqrt(4*d*f))                  /(2*f)              +(c*d)/f+a))
    /f^2 zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3}{\left(d - f\,x^2\right)\,\sqrt{c\,x^2 + b\,x + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((d - f*x^2)*(a + b*x + c*x^2)^(1/2)),x)
```

```
[Out] int(x^3/((d - f*x^2)*(a + b*x + c*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^3}{-d\sqrt{a + bx + cx^2} + fx^2\sqrt{a + bx + cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(c*x**2+b*x+a)**(1/2)/(-f*x**2+d),x)
```

```
[Out] -Integral(x**3/(-d*sqrt(a + b*x + c*x**2) + f*x**2*sqrt(a + b*x + c*x**2)),
 x)
```

3.96 $\int \frac{x^2}{\sqrt{a+bx+cx^2}\left(d-fx^2\right)} dx$

Optimal. Leaf size=266

$$\frac{\sqrt{d}\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2f\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}+\frac{\sqrt{d}\tanh^{-1}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2f\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}-\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}f}$$

```
[Out] -arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/f/c^(1/2)+1/2*arctanh(1
/2*(b*d^(1/2)-2*a*f^(1/2)+x*(2*c*d^(1/2)-b*f^(1/2)))/(c*x^2+b*x+a)^(1/2)/(c
*d+a*f-b*d^(1/2)*f^(1/2))^(1/2))*d^(1/2)/f/(c*d+a*f-b*d^(1/2)*f^(1/2))^(1/2
)+1/2*arctanh(1/2*(b*d^(1/2)+2*a*f^(1/2)+x*(2*c*d^(1/2)+b*f^(1/2)))/(c*x^2+
b*x+a)^(1/2)/(c*d+a*f+b*d^(1/2)*f^(1/2))^(1/2))*d^(1/2)/f/(c*d+a*f+b*d^(1/2
)*f^(1/2))^(1/2)
```

Rubi [A] time = 0.22, antiderivative size = 266, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {1079, 621, 206, 984, 724}

$$\frac{\sqrt{d}\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2f\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}+\frac{\sqrt{d}\tanh^{-1}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2f\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}-\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}f}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(Sqrt[a + b*x + c*x^2]*(d - f*x^2)),x]
```

```
[Out] -(ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])]/(Sqrt[c]*f)) + (Sq
rt[d]*ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sq
rt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*f*Sqrt[c*d -
b*Sqrt[d]*Sqrt[f] + a*f]) + (Sqrt[d]*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*
c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b
*x + c*x^2])])/(2*f*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 984

```
Int[1/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Sy
mbol] :> Dist[1/2, Int[1/((a - Rt[-(a*c), 2]*x)*Sqrt[d + e*x + f*x^2]), x],
```

```
x] + Dist[1/2, Int[1/((a + Rt[-(a*c), 2]*x)*Sqrt[d + e*x + f*x^2]), x], x]
/; FreeQ[{a, c, d, e, f}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]
```

Rule 1079

```
Int[((A_.) + (C_.)*(x_)^2)/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) +
 (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqrt[d + e*x + f*x^2], x], x]
 + Dist[(A*c - a*C)/c, Int[1/((a + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /;
 FreeQ[{a, c, d, e, f, A, C}, x] && NeQ[e^2 - 4*d*f, 0]
```

Rubi steps

$$\int \frac{x^2}{\sqrt{a+bx+cx^2}\left(d-fx^2\right)}\,dx = -\frac{\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{f}+\frac{d\int \frac{1}{\sqrt{a+bx+cx^2}\left(d-fx^2\right)}\,dx}{f}$$

$$= -\frac{2\,\mathrm{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{f}+\frac{d\int \frac{1}{\left(d-\sqrt{d}\sqrt{f}x\right)\sqrt{a+bx+cx^2}}\,dx}{2f}+\frac{d\int \frac{1}{\left(d+\sqrt{d}\right.}\cdots$$

$$= -\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}f}-\frac{d\,\mathrm{Subst}\left(\int \frac{1}{4cd^2-4bd^{3/2}\sqrt{f}+4adf-x^2}\,dx, x, \frac{-bd+2a\sqrt{d}\sqrt{f}-\cdots}{\sqrt{a+bx\cdots}}\right.}{f}\cdots$$

$$= -\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}f}+\frac{\sqrt{d}\tanh^{-1}\left(\frac{b\sqrt{d}-2a\sqrt{f}+\left(2c\sqrt{d}-b\sqrt{f}\right)x}{2\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2f\sqrt{cd-b\sqrt{d}\sqrt{f}+af}}+\frac{\sqrt{d}\tanh\cdots}{2\cdots}$$

Mathematica [A] time = 0.43, size = 250, normalized size = 0.94

$$\frac{\sqrt{d}\left(\frac{\tanh^{-1}\left(\frac{-2a\sqrt{f}+b\sqrt{d}-b\sqrt{f}x+2c\sqrt{d}x}{2\sqrt{a+x(b+cx)}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}+\frac{\tanh^{-1}\left(\frac{2a\sqrt{f}+b\sqrt{d}+b\sqrt{f}x+2c\sqrt{d}x}{2\sqrt{a+x(b+cx)}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)-\frac{2\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)}{\sqrt{c}}}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(Sqrt[a + b*x + c*x^2]*(d - f*x^2)),x]
```

```
[Out] ((-2*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/Sqrt[c] + Sqrt
[d]*(ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + 2*c*Sqrt[d]*x - b*Sqrt[f]*x)/(2*Sqr
t[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])]/Sqrt[c*d - b*Sqrt[
d]*Sqrt[f] + a*f] + ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + 2*c*Sqrt[d]*x + b*Sq
rt[f]*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])]/Sqr
t[c*d + b*Sqrt[d]*Sqrt[f] + a*f]))/(2*f)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [A] time = 0.02, size = 399, normalized size = 1.50

$$-\frac{d\ln\left(\frac{\frac{2af+2cd-2\sqrt{df}\,b}{f}+\frac{\left(bf-2\sqrt{df}\,c\right)\left(x+\frac{\sqrt{df}}{f}\right)}{f}+2\sqrt{\frac{af+cd-\sqrt{df}\,b}{f}}\sqrt{\left(x+\frac{\sqrt{df}}{f}\right)^2 c+\frac{\left(bf-2\sqrt{df}\,c\right)\left(x+\frac{\sqrt{df}}{f}\right)}{f}+\frac{af+cd-\sqrt{df}\,b}{f}}}{x+\frac{\sqrt{df}}{f}}\right)}{2\sqrt{df}\,\sqrt{\frac{af+cd-\sqrt{df}\,b}{f}}\,f}+\frac{d\ln\left(\frac{2af+2cd+2\sqrt{df}\,b}{f}\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x)
```

```
[Out] -1/f*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2)-1/2*d/(d*f)^(1/2)/
f/((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d-(d*f)^(1/2)*b)/f+(b*f-2*
(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+2*((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*((x+
(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)
^(1/2)*b)/f)^(1/2))/(x+(d*f)^(1/2)/f))+1/2*d/(d*f)^(1/2)/f/((a*f+c*d+(d*f)^
(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d+(d*f)^(1/2)*b)/f+(b*f+2*(d*f)^(1/2)*c)*(x-
(d*f)^(1/2)/f)/f+2*((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c
+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)
)/(x-(d*f)^(1/2)/f))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(((c*sqrt(4*d*f))/(2*f^2)>0)', see `
assume?` for more details)Is ((c*sqrt(4*d*f))/(2*f^2)    +b/(2*f))    ^2
 -(c*((b*sqrt(4*d*f))                  /(2*f)                  +(c*d)/f+a))
     /f^2 zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2}{\left(d-f\,x^2\right)\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((d - f*x^2)*(a + b*x + c*x^2)^(1/2)),x)
```

```
[Out] int(x^2/((d - f*x^2)*(a + b*x + c*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^2}{-d\sqrt{a+bx+cx^2}+fx^2\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(c*x**2+b*x+a)**(1/2)/(-f*x**2+d),x)
```

```
[Out] -Integral(x**2/(-d*sqrt(a + b*x + c*x**2) + f*x**2*sqrt(a + b*x + c*x**2)),
 x)
```

3.97 $\int \frac{x}{\sqrt{a+bx+cx^2}\left(d-fx^2\right)}\,dx$

Optimal. Leaf size=220

$$\frac{\tanh^{-1}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2\sqrt{f}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}-\frac{\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2\sqrt{f}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}$$

```
[Out] -1/2*arctanh(1/2*(b*d^(1/2)-2*a*f^(1/2)+x*(2*c*d^(1/2)-b*f^(1/2)))/(c*x^2+b
*x+a)^(1/2)/(c*d+a*f-b*d^(1/2)*f^(1/2))^(1/2))/f^(1/2)/(c*d+a*f-b*d^(1/2)*f
^(1/2))^(1/2)+1/2*arctanh(1/2*(b*d^(1/2)+2*a*f^(1/2)+x*(2*c*d^(1/2)+b*f^(1/
2)))/(c*x^2+b*x+a)^(1/2)/(c*d+a*f+b*d^(1/2)*f^(1/2))^(1/2))/f^(1/2)/(c*d+a*
f+b*d^(1/2)*f^(1/2))^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1033, 724, 206}

$$\frac{\tanh^{-1}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2\sqrt{f}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}-\frac{\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2\sqrt{f}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(Sqrt[a + b*x + c*x^2]*(d - f*x^2)),x]
```

```
[Out] -ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*
d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])]/(2*Sqrt[f]*Sqrt[c*d -
b*Sqrt[d]*Sqrt[f] + a*f]) + ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d]
 + b*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^
2])]/(2*Sqrt[f]*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1033

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q
), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q
), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f
, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]
```

Rubi steps

$$\int \frac{x}{\sqrt{a+bx+cx^2}\left(d-fx^2\right)}dx = \frac{1}{2}\int \frac{1}{\left(-\sqrt{d}\sqrt{f}-fx\right)\sqrt{a+bx+cx^2}}dx + \frac{1}{2}\int \frac{1}{\left(\sqrt{d}\sqrt{f}-fx\right)\sqrt{a+bx+cx^2}}d$$

$$= -\text{Subst}\left(\int \frac{1}{4cdf-4b\sqrt{d}f^{3/2}+4af^2-x^2}dx, x, \frac{b\sqrt{d}\sqrt{f}-2af-\left(-2c\sqrt{d}\sqrt{f}\right.}{\sqrt{a+bx+cx^2}}\right.$$

$$= -\frac{\tanh^{-1}\left(\frac{b\sqrt{d}\sqrt{f}-2af-\left(-2c\sqrt{d}\sqrt{f}+bf\right)x}{2\sqrt{f}\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{f}\sqrt{cd-b\sqrt{d}\sqrt{f}+af}} - \frac{\tanh^{-1}\left(\frac{-b\sqrt{d}\sqrt{f}-2af-\left(2c\sqrt{d}\sqrt{f}+bf\right)x}{2\sqrt{f}\sqrt{cd+b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{f}\sqrt{cd+b\sqrt{d}\sqrt{f}+af}}$$

Mathematica [A] time = 0.17, size = 211, normalized size = 0.96

$$-\frac{\frac{\tanh^{-1}\left(\frac{-2a\sqrt{f}+b\left(\sqrt{d}-\sqrt{f}x\right)+2c\sqrt{d}x}{2\sqrt{a+x(b+cx)}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}+\frac{\tanh^{-1}\left(\frac{-2\left(a\sqrt{f}+c\sqrt{d}x\right)-b\left(\sqrt{d}+\sqrt{f}x\right)}{2\sqrt{a+x(b+cx)}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}}{2\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(Sqrt[a + b*x + c*x^2]*(d - f*x^2)),x]
```

```
[Out] -1/2*(ArcTanh[(-2*a*Sqrt[f] + 2*c*Sqrt[d]*x + b*(Sqrt[d] - Sqrt[f]*x))/(2*S
qrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])]/Sqrt[c*d - b*Sqr
t[d]*Sqrt[f] + a*f] + ArcTanh[(-2*(a*Sqrt[f] + c*Sqrt[d]*x) - b*(Sqrt[d] +
Sqrt[f]*x))/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])]/
Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f])/Sqrt[f]
```

fricas [B] time = 2.40, size = 2753, normalized size = 12.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt((c*d + a*f + (c^2*d^2*f + a^2*f^3 - (b^2 - 2*a*c)*d*f^2)*sqrt(b^2*
d/(c^4*d^4*f + a^4*f^5 - 2*(b^2*c^2 - 2*a*c^3)*d^3*f^2 + (b^4 - 4*a*b^2*c +
 6*a^2*c^2)*d^2*f^3 - 2*(a^2*b^2 - 2*a^3*c)*d*f^4)))/(c^2*d^2*f + a^2*f^3 -
 (b^2 - 2*a*c)*d*f^2))*log((2*b*c*d*x + b^2*d + 2*(b^2*d*f - (c^3*d^3*f + a
^3*f^4 - (b^2*c - 3*a*c^2)*d^2*f^2 - (a*b^2 - 3*a^2*c)*d*f^3)*sqrt(b^2*d/(c
^4*d^4*f + a^4*f^5 - 2*(b^2*c^2 - 2*a*c^3)*d^3*f^2 + (b^4 - 4*a*b^2*c + 6*a
^2*c^2)*d^2*f^3 - 2*(a^2*b^2 - 2*a^3*c)*d*f^4)))*sqrt(c*x^2 + b*x + a)*sqrt
((c*d + a*f + (c^2*d^2*f + a^2*f^3 - (b^2 - 2*a*c)*d*f^2)*sqrt(b^2*d/(c^4*d
^4*f + a^4*f^5 - 2*(b^2*c^2 - 2*a*c^3)*d^3*f^2 + (b^4 - 4*a*b^2*c + 6*a^2*c
^2)*d^2*f^3 - 2*(a^2*b^2 - 2*a^3*c)*d*f^4)))/(c^2*d^2*f + a^2*f^3 - (b^2 -
2*a*c)*d*f^2)) - (2*a*c^2*d^2*f + 2*a^3*f^3 - 2*(a*b^2 - 2*a^2*c)*d*f^2 + (
b*c^2*d^2*f + a^2*b*f^3 - (b^3 - 2*a*b*c)*d*f^2)*x)*sqrt(b^2*d/(c^4*d^4*f +
 a^4*f^5 - 2*(b^2*c^2 - 2*a*c^3)*d^3*f^2 + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^
2*f^3 - 2*(a^2*b^2 - 2*a^3*c)*d*f^4)))/x) - 1/4*sqrt((c*d + a*f + (c^2*d^2*
f + a^2*f^3 - (b^2 - 2*a*c)*d*f^2)*sqrt(b^2*d/(c^4*d^4*f + a^4*f^5 - 2*(b^2
*c^2 - 2*a*c^3)*d^3*f^2 + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^2*f^3 - 2*(a^2*b^
2 - 2*a^3*c)*d*f^4)))/(c^2*d^2*f + a^2*f^3 - (b^2 - 2*a*c)*d*f^2))*log((2*b
*c*d*x + b^2*d - 2*(b^2*d*f - (c^3*d^3*f + a^3*f^4 - (b^2*c - 3*a*c^2)*d^2*
f^2 - (a*b^2 - 3*a^2*c)*d*f^3)*sqrt(b^2*d/(c^4*d^4*f + a^4*f^5 - 2*(b^2*c^2
 - 2*a*c^3)*d^3*f^2 + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^2*f^3 - 2*(a^2*b^2 -
2*a^3*c)*d*f^4)))*sqrt(c*x^2 + b*x + a)*sqrt((c*d + a*f + (c^2*d^2*f + a^2*
```

```
f^3 - (b^2 - 2*a*c)*d*f^2)*sqrt(b^2*d/(c^4*d^4*f + a^4*f^5 - 2*(b^2*c^2 - 2
*a*c^3)*d^3*f^2 + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^2*f^3 - 2*(a^2*b^2 - 2*a^
3*c)*d*f^4)))/(c^2*d^2*f + a^2*f^3 - (b^2 - 2*a*c)*d*f^2)) - (2*a*c^2*d^2*f
 + 2*a^3*f^3 - 2*(a*b^2 - 2*a^2*c)*d*f^2 + (b*c^2*d^2*f + a^2*b*f^3 - (b^3
- 2*a*b*c)*d*f^2)*x)*sqrt(b^2*d/(c^4*d^4*f + a^4*f^5 - 2*(b^2*c^2 - 2*a*c^3
)*d^3*f^2 + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^2*f^3 - 2*(a^2*b^2 - 2*a^3*c)*d
*f^4)))/x) + 1/4*sqrt((c*d + a*f - (c^2*d^2*f + a^2*f^3 - (b^2 - 2*a*c)*d*f
^2)*sqrt(b^2*d/(c^4*d^4*f + a^4*f^5 - 2*(b^2*c^2 - 2*a*c^3)*d^3*f^2 + (b^4
- 4*a*b^2*c + 6*a^2*c^2)*d^2*f^3 - 2*(a^2*b^2 - 2*a^3*c)*d*f^4)))/(c^2*d^2*
f + a^2*f^3 - (b^2 - 2*a*c)*d*f^2))*log((2*b*c*d*x + b^2*d + 2*(b^2*d*f + (
c^3*d^3*f + a^3*f^4 - (b^2*c - 3*a*c^2)*d^2*f^2 - (a*b^2 - 3*a^2*c)*d*f^3)*
sqrt(b^2*d/(c^4*d^4*f + a^4*f^5 - 2*(b^2*c^2 - 2*a*c^3)*d^3*f^2 + (b^4 - 4*
a*b^2*c + 6*a^2*c^2)*d^2*f^3 - 2*(a^2*b^2 - 2*a^3*c)*d*f^4)))*sqrt(c*x^2 +
b*x + a)*sqrt((c*d + a*f - (c^2*d^2*f + a^2*f^3 - (b^2 - 2*a*c)*d*f^2)*sqrt
(b^2*d/(c^4*d^4*f + a^4*f^5 - 2*(b^2*c^2 - 2*a*c^3)*d^3*f^2 + (b^4 - 4*a*b^
2*c + 6*a^2*c^2)*d^2*f^3 - 2*(a^2*b^2 - 2*a^3*c)*d*f^4)))/(c^2*d^2*f + a^2*
f^3 - (b^2 - 2*a*c)*d*f^2)) + (2*a*c^2*d^2*f + 2*a^3*f^3 - 2*(a*b^2 - 2*a^2
*c)*d*f^2 + (b*c^2*d^2*f + a^2*b*f^3 - (b^3 - 2*a*b*c)*d*f^2)*x)*sqrt(b^2*d
/(c^4*d^4*f + a^4*f^5 - 2*(b^2*c^2 - 2*a*c^3)*d^3*f^2 + (b^4 - 4*a*b^2*c +
6*a^2*c^2)*d^2*f^3 - 2*(a^2*b^2 - 2*a^3*c)*d*f^4)))/x) - 1/4*sqrt((c*d + a*
f - (c^2*d^2*f + a^2*f^3 - (b^2 - 2*a*c)*d*f^2)*sqrt(b^2*d/(c^4*d^4*f + a^4
*f^5 - 2*(b^2*c^2 - 2*a*c^3)*d^3*f^2 + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^2*f^
3 - 2*(a^2*b^2 - 2*a^3*c)*d*f^4)))/(c^2*d^2*f + a^2*f^3 - (b^2 - 2*a*c)*d*f
^2))*log((2*b*c*d*x + b^2*d - 2*(b^2*d*f + (c^3*d^3*f + a^3*f^4 - (b^2*c -
3*a*c^2)*d^2*f^2 - (a*b^2 - 3*a^2*c)*d*f^3)*sqrt(b^2*d/(c^4*d^4*f + a^4*f^5
 - 2*(b^2*c^2 - 2*a*c^3)*d^3*f^2 + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^2*f^3 -
2*(a^2*b^2 - 2*a^3*c)*d*f^4)))*sqrt(c*x^2 + b*x + a)*sqrt((c*d + a*f - (c^2
*d^2*f + a^2*f^3 - (b^2 - 2*a*c)*d*f^2)*sqrt(b^2*d/(c^4*d^4*f + a^4*f^5 - 2
*(b^2*c^2 - 2*a*c^3)*d^3*f^2 + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^2*f^3 - 2*(a
^2*b^2 - 2*a^3*c)*d*f^4)))/(c^2*d^2*f + a^2*f^3 - (b^2 - 2*a*c)*d*f^2)) + (
2*a*c^2*d^2*f + 2*a^3*f^3 - 2*(a*b^2 - 2*a^2*c)*d*f^2 + (b*c^2*d^2*f + a^2*
b*f^3 - (b^3 - 2*a*b*c)*d*f^2)*x)*sqrt(b^2*d/(c^4*d^4*f + a^4*f^5 - 2*(b^2*
c^2 - 2*a*c^3)*d^3*f^2 + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^2*f^3 - 2*(a^2*b^2
 - 2*a^3*c)*d*f^4)))/x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument ValueEvaluation time: 0.52sym2pol
y/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argum
ent Value
```

maple [B] time = 0.02, size = 354, normalized size = 1.61

$$\frac{\ln\left(\frac{\frac{2af+2cd-2\sqrt{df}\,b}{f}+\frac{\left(bf-2\sqrt{df}\,c\right)\left(x+\frac{\sqrt{df}}{f}\right)}{f}+2\sqrt{\frac{af+cd-\sqrt{df}\,b}{f}}\sqrt{\left(x+\frac{\sqrt{df}}{f}\right)^{2}c+\frac{\left(bf-2\sqrt{df}\,c\right)\left(x+\frac{\sqrt{df}}{f}\right)}{f}+\frac{af+cd-\sqrt{df}\,b}{f}}}{x+\frac{\sqrt{df}}{f}}\right)}{2\sqrt{\frac{af+cd-\sqrt{df}\,b}{f}}\,f}+\ln\left(\frac{\frac{2af+2cd+2\sqrt{df}\,b}{f}+\frac{(bf+2\cdots}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x)
```

```
[Out] 1/2/f/((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d-(d*f)^(1/2)*b)/f+(b*
f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+2*((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*
((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(
d*f)^(1/2)*b)/f)^(1/2))/(x+(d*f)^(1/2)/f))+1/2/f/((a*f+c*d+(d*f)^(1/2)*b)/f
)^(1/2)*ln((2*(a*f+c*d+(d*f)^(1/2)*b)/f+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2
)/f)/f+2*((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d
*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))/(x-(d*f)
^(1/2)/f))
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(((c*sqrt(4*d*f))/(2*f^2)>0)', see `
assume?` for more details)Is ((c*sqrt(4*d*f))/(2*f^2)    +b/(2*f))    ^2
 -(c*((b*sqrt(4*d*f))                 /(2*f)                  +(c*d)/f+a))
     /f^2 zero or nonzero?
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x}{\left(d - f\,x^2\right)\,\sqrt{c\,x^2 + b\,x + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((d - f*x^2)*(a + b*x + c*x^2)^(1/2)),x)
```

```
[Out] int(x/((d - f*x^2)*(a + b*x + c*x^2)^(1/2)), x)
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{-d\sqrt{a + bx + cx^2} + fx^2\sqrt{a + bx + cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x**2+b*x+a)**(1/2)/(-f*x**2+d),x)
```

```
[Out] -Integral(x/(-d*sqrt(a + b*x + c*x**2) + f*x**2*sqrt(a + b*x + c*x**2)), x)
```

3.98 $\int \frac{1}{\sqrt{a+bx+cx^2}\left(d-fx^2\right)}\,dx$

Optimal. Leaf size=220

$$\frac{\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2\sqrt{d}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}+\frac{\tanh^{-1}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2\sqrt{d}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}$$

```
[Out] 1/2*arctanh(1/2*(b*d^(1/2)-2*a*f^(1/2)+x*(2*c*d^(1/2)-b*f^(1/2)))/(c*x^2+b*
x+a)^(1/2)/(c*d+a*f-b*d^(1/2)*f^(1/2))^(1/2))/d^(1/2)/(c*d+a*f-b*d^(1/2)*f^
(1/2))^(1/2)+1/2*arctanh(1/2*(b*d^(1/2)+2*a*f^(1/2)+x*(2*c*d^(1/2)+b*f^(1/2
)))/(c*x^2+b*x+a)^(1/2)/(c*d+a*f+b*d^(1/2)*f^(1/2))^(1/2))/d^(1/2)/(c*d+a*f
+b*d^(1/2)*f^(1/2))^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {984, 724, 206}

$$\frac{\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2\sqrt{d}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}+\frac{\tanh^{-1}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2\sqrt{d}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b*x + c*x^2]*(d - f*x^2)),x]
```

```
[Out] ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d
 - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])]/(2*Sqrt[d]*Sqrt[c*d - b
*Sqrt[d]*Sqrt[f] + a*f]) + ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d]
+ b*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2
])]/(2*Sqrt[d]*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 984

```
Int[1/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Sy
mbol] :> Dist[1/2, Int[1/((a - Rt[-(a*c), 2]*x)*Sqrt[d + e*x + f*x^2]), x],
 x] + Dist[1/2, Int[1/((a + Rt[-(a*c), 2]*x)*Sqrt[d + e*x + f*x^2]), x], x]
 /; FreeQ[{a, c, d, e, f}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+bx+cx^2}\left(d-fx^2\right)}dx = \frac{1}{2}\int \frac{1}{\left(d-\sqrt{d}\sqrt{f}x\right)\sqrt{a+bx+cx^2}}dx + \frac{1}{2}\int \frac{1}{\left(d+\sqrt{d}\sqrt{f}x\right)\sqrt{a+bx+cx^2}}dx$$

$$= -\text{Subst}\left(\int \frac{1}{4cd^2-4bd^{3/2}\sqrt{f}+4adf-x^2}dx, x, \frac{-bd+2a\sqrt{d}\sqrt{f}-\left(2cd-b\sqrt{d}\sqrt{f}\right)x}{\sqrt{a+bx+cx^2}}\right)$$

$$= -\frac{\tanh^{-1}\left(\frac{-bd+2a\sqrt{d}\sqrt{f}-\left(2cd-b\sqrt{d}\sqrt{f}\right)x}{2\sqrt{d}\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{d}\sqrt{cd-b\sqrt{d}\sqrt{f}+af}} - \frac{\tanh^{-1}\left(\frac{-bd-2a\sqrt{d}\sqrt{f}-\left(2cd+b\sqrt{d}\sqrt{f}\right)x}{2\sqrt{d}\sqrt{cd+b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{d}\sqrt{cd+b\sqrt{d}\sqrt{f}+af}}$$

Mathematica [A] time = 0.10, size = 209, normalized size = 0.95

$$\frac{\frac{\tanh^{-1}\left(\frac{2a\sqrt{f}+b\sqrt{d}+b\sqrt{f}x+2c\sqrt{d}x}{2\sqrt{a+x(b+cx)}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}+\frac{\tanh^{-1}\left(\frac{-2a\sqrt{f}+b\left(\sqrt{d}-\sqrt{f}x\right)+2c\sqrt{d}x}{2\sqrt{a+x(b+cx)}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}}{2\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b*x + c*x^2]*(d - f*x^2)),x]
```

```
[Out] (ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + 2*c*Sqrt[d]*x + b*Sqrt[f]*x)/(2*Sqrt[c*
d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])]/Sqrt[c*d + b*Sqrt[d]*S
qrt[f] + a*f] + ArcTanh[(-2*a*Sqrt[f] + 2*c*Sqrt[d]*x + b*(Sqrt[d] - Sqrt[f
]*x))/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])]/Sqrt[c
*d - b*Sqrt[d]*Sqrt[f] + a*f])/(2*Sqrt[d])
```

fricas [B] time = 2.15, size = 2641, normalized size = 12.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt((c*d + a*f + (c^2*d^3 + a^2*d*f^2 - (b^2 - 2*a*c)*d^2*f)*sqrt(b^2*
f/(c^4*d^5 + a^4*d*f^4 - 2*(b^2*c^2 - 2*a*c^3)*d^4*f + (b^4 - 4*a*b^2*c + 6
*a^2*c^2)*d^3*f^2 - 2*(a^2*b^2 - 2*a^3*c)*d^2*f^3)))/(c^2*d^3 + a^2*d*f^2 -
 (b^2 - 2*a*c)*d^2*f))*log((2*b*c*x + b^2 + 2*(b*c*d + a*b*f - (b*c^2*d^3 +
 a^2*b*d*f^2 - (b^3 - 2*a*b*c)*d^2*f)*sqrt(b^2*f/(c^4*d^5 + a^4*d*f^4 - 2*(
b^2*c^2 - 2*a*c^3)*d^4*f + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3*f^2 - 2*(a^2*b
^2 - 2*a^3*c)*d^2*f^3)))*sqrt(c*x^2 + b*x + a)*sqrt((c*d + a*f + (c^2*d^3 +
 a^2*d*f^2 - (b^2 - 2*a*c)*d^2*f)*sqrt(b^2*f/(c^4*d^5 + a^4*d*f^4 - 2*(b^2*
c^2 - 2*a*c^3)*d^4*f + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3*f^2 - 2*(a^2*b^2 -
 2*a^3*c)*d^2*f^3)))/(c^2*d^3 + a^2*d*f^2 - (b^2 - 2*a*c)*d^2*f)) - (2*a*c^
2*d^2 + 2*a^3*f^2 - 2*(a*b^2 - 2*a^2*c)*d*f + (b*c^2*d^2 + a^2*b*f^2 - (b^3
 - 2*a*b*c)*d*f)*x)*sqrt(b^2*f/(c^4*d^5 + a^4*d*f^4 - 2*(b^2*c^2 - 2*a*c^3)
*d^4*f + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3*f^2 - 2*(a^2*b^2 - 2*a^3*c)*d^2*
f^3)))/x) - 1/4*sqrt((c*d + a*f + (c^2*d^3 + a^2*d*f^2 - (b^2 - 2*a*c)*d^2*
f)*sqrt(b^2*f/(c^4*d^5 + a^4*d*f^4 - 2*(b^2*c^2 - 2*a*c^3)*d^4*f + (b^4 - 4
*a*b^2*c + 6*a^2*c^2)*d^3*f^2 - 2*(a^2*b^2 - 2*a^3*c)*d^2*f^3)))/(c^2*d^3 +
 a^2*d*f^2 - (b^2 - 2*a*c)*d^2*f))*log((2*b*c*x + b^2 - 2*(b*c*d + a*b*f -
(b*c^2*d^3 + a^2*b*d*f^2 - (b^3 - 2*a*b*c)*d^2*f)*sqrt(b^2*f/(c^4*d^5 + a^4
*d*f^4 - 2*(b^2*c^2 - 2*a*c^3)*d^4*f + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3*f^
2 - 2*(a^2*b^2 - 2*a^3*c)*d^2*f^3)))*sqrt(c*x^2 + b*x + a)*sqrt((c*d + a*f
+ (c^2*d^3 + a^2*d*f^2 - (b^2 - 2*a*c)*d^2*f)*sqrt(b^2*f/(c^4*d^5 + a^4*d*f
```

```
^4 - 2*(b^2*c^2 - 2*a*c^3)*d^4*f + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3*f^2 -
2*(a^2*b^2 - 2*a^3*c)*d^2*f^3)))/(c^2*d^3 + a^2*d*f^2 - (b^2 - 2*a*c)*d^2*f
)) - (2*a*c^2*d^2 + 2*a^3*f^2 - 2*(a*b^2 - 2*a^2*c)*d*f + (b*c^2*d^2 + a^2*
b*f^2 - (b^3 - 2*a*b*c)*d*f)*x)*sqrt(b^2*f/(c^4*d^5 + a^4*d*f^4 - 2*(b^2*c^
2 - 2*a*c^3)*d^4*f + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3*f^2 - 2*(a^2*b^2 - 2
*a^3*c)*d^2*f^3)))/x) + 1/4*sqrt((c*d + a*f - (c^2*d^3 + a^2*d*f^2 - (b^2 -
 2*a*c)*d^2*f)*sqrt(b^2*f/(c^4*d^5 + a^4*d*f^4 - 2*(b^2*c^2 - 2*a*c^3)*d^4*
f + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3*f^2 - 2*(a^2*b^2 - 2*a^3*c)*d^2*f^3))
)/(c^2*d^3 + a^2*d*f^2 - (b^2 - 2*a*c)*d^2*f))*log((2*b*c*x + b^2 + 2*(b*c*
d + a*b*f + (b*c^2*d^3 + a^2*b*d*f^2 - (b^3 - 2*a*b*c)*d^2*f)*sqrt(b^2*f/(c
^4*d^5 + a^4*d*f^4 - 2*(b^2*c^2 - 2*a*c^3)*d^4*f + (b^4 - 4*a*b^2*c + 6*a^2
*c^2)*d^3*f^2 - 2*(a^2*b^2 - 2*a^3*c)*d^2*f^3)))*sqrt(c*x^2 + b*x + a)*sqrt
((c*d + a*f - (c^2*d^3 + a^2*d*f^2 - (b^2 - 2*a*c)*d^2*f)*sqrt(b^2*f/(c^4*d
^5 + a^4*d*f^4 - 2*(b^2*c^2 - 2*a*c^3)*d^4*f + (b^4 - 4*a*b^2*c + 6*a^2*c^2
)*d^3*f^2 - 2*(a^2*b^2 - 2*a^3*c)*d^2*f^3)))/(c^2*d^3 + a^2*d*f^2 - (b^2 -
2*a*c)*d^2*f)) + (2*a*c^2*d^2 + 2*a^3*f^2 - 2*(a*b^2 - 2*a^2*c)*d*f + (b*c^
2*d^2 + a^2*b*f^2 - (b^3 - 2*a*b*c)*d*f)*x)*sqrt(b^2*f/(c^4*d^5 + a^4*d*f^4
 - 2*(b^2*c^2 - 2*a*c^3)*d^4*f + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3*f^2 - 2*
(a^2*b^2 - 2*a^3*c)*d^2*f^3)))/x) - 1/4*sqrt((c*d + a*f - (c^2*d^3 + a^2*d*
f^2 - (b^2 - 2*a*c)*d^2*f)*sqrt(b^2*f/(c^4*d^5 + a^4*d*f^4 - 2*(b^2*c^2 - 2
*a*c^3)*d^4*f + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3*f^2 - 2*(a^2*b^2 - 2*a^3*
c)*d^2*f^3)))/(c^2*d^3 + a^2*d*f^2 - (b^2 - 2*a*c)*d^2*f))*log((2*b*c*x + b
^2 - 2*(b*c*d + a*b*f + (b*c^2*d^3 + a^2*b*d*f^2 - (b^3 - 2*a*b*c)*d^2*f)*s
qrt(b^2*f/(c^4*d^5 + a^4*d*f^4 - 2*(b^2*c^2 - 2*a*c^3)*d^4*f + (b^4 - 4*a*b
^2*c + 6*a^2*c^2)*d^3*f^2 - 2*(a^2*b^2 - 2*a^3*c)*d^2*f^3)))*sqrt(c*x^2 + b
*x + a)*sqrt((c*d + a*f - (c^2*d^3 + a^2*d*f^2 - (b^2 - 2*a*c)*d^2*f)*sqrt(
b^2*f/(c^4*d^5 + a^4*d*f^4 - 2*(b^2*c^2 - 2*a*c^3)*d^4*f + (b^4 - 4*a*b^2*c
 + 6*a^2*c^2)*d^3*f^2 - 2*(a^2*b^2 - 2*a^3*c)*d^2*f^3)))/(c^2*d^3 + a^2*d*f
^2 - (b^2 - 2*a*c)*d^2*f)) + (2*a*c^2*d^2 + 2*a^3*f^2 - 2*(a*b^2 - 2*a^2*c)
*d*f + (b*c^2*d^2 + a^2*b*f^2 - (b^3 - 2*a*b*c)*d*f)*x)*sqrt(b^2*f/(c^4*d^5
 + a^4*d*f^4 - 2*(b^2*c^2 - 2*a*c^3)*d^4*f + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*
d^3*f^2 - 2*(a^2*b^2 - 2*a^3*c)*d^2*f^3)))/x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument ValueEvaluation time: 0.58sym2pol
y/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argum
ent Value
```

maple [B] time = 0.02, size = 358, normalized size = 1.63

$$-\frac{\ln\left(\frac{\frac{2af+2cd-2\sqrt{df}\,b}{f}+\frac{\left(bf-2\sqrt{df}\,c\right)\left(x+\frac{\sqrt{df}}{f}\right)}{f}+2\sqrt{\frac{af+cd-\sqrt{df}\,b}{f}}\sqrt{\left(x+\frac{\sqrt{df}}{f}\right)^{2}c+\frac{\left(bf-2\sqrt{df}\,c\right)\left(x+\frac{\sqrt{df}}{f}\right)}{f}+\frac{af+cd-\sqrt{df}\,b}{f}}}{x+\frac{\sqrt{df}}{f}}\right)}{2\sqrt{df}\,\sqrt{\frac{af+cd-\sqrt{df}\,b}{f}}}+\frac{\ln\left(\frac{\frac{2af+2cd+2\sqrt{df}\,b}{f}+\frac{(bf \cdots}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x)
```

```
[Out] -1/2/(d*f)^(1/2)/((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d-(d*f)^(1/
2)*b)/f+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+2*((a*f+c*d-(d*f)^(1/2)*b
)/f)^(1/2)*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f
+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))/(x+(d*f)^(1/2)/f))+1/2/(d*f)^(1/2)/((a*f
+c*d+(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d+(d*f)^(1/2)*b)/f+(b*f+2*(d*f)^(
1/2)*c)*(x-(d*f)^(1/2)/f)/f+2*((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*((x-(d*f)^(
1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*
b)/f)^(1/2))/(x-(d*f)^(1/2)/f))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(((c*sqrt(4*d*f))/(2*f^2)>0)', see `
assume?` for more details)Is ((c*sqrt(4*d*f))/(2*f^2)    +b/(2*f))    ^2
 -(c*((b*sqrt(4*d*f))                    /(2*f)                 +(c*d)/f+a))
     /f^2 positive, negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(d-f\,x^2\right)\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d - f*x^2)*(a + b*x + c*x^2)^(1/2)),x)
```

```
[Out] int(1/((d - f*x^2)*(a + b*x + c*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{-d\sqrt{a+bx+cx^2}+fx^2\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**2+b*x+a)**(1/2)/(-f*x**2+d),x)
```

```
[Out] -Integral(1/(-d*sqrt(a + b*x + c*x**2) + f*x**2*sqrt(a + b*x + c*x**2)), x)
```

3.99 $\int \frac{1}{x\sqrt{a+bx+cx^2}\left(d-fx^2\right)}\,dx$

Optimal. Leaf size=267

$$-\frac{\sqrt{f}\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2d\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}+\frac{\sqrt{f}\tanh^{-1}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2d\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}-\frac{\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a}\,d}$$

```
[Out] -arctanh(1/2*(b*x+2*a)/a^(1/2)/(c*x^2+b*x+a)^(1/2))/d/a^(1/2)-1/2*arctanh(1
/2*(b*d^(1/2)-2*a*f^(1/2)+x*(2*c*d^(1/2)-b*f^(1/2)))/(c*x^2+b*x+a)^(1/2)/(c
*d+a*f-b*d^(1/2)*f^(1/2))^(1/2))*f^(1/2)/d/(c*d+a*f-b*d^(1/2)*f^(1/2))^(1/2
)+1/2*arctanh(1/2*(b*d^(1/2)+2*a*f^(1/2)+x*(2*c*d^(1/2)+b*f^(1/2)))/(c*x^2+
b*x+a)^(1/2)/(c*d+a*f+b*d^(1/2)*f^(1/2))^(1/2))*f^(1/2)/d/(c*d+a*f+b*d^(1/2
)*f^(1/2))^(1/2)
```

Rubi [A] time = 0.66, antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6725, 724, 206, 1033}

$$-\frac{\sqrt{f}\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2d\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}+\frac{\sqrt{f}\tanh^{-1}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2d\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}-\frac{\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[a + b*x + c*x^2]*(d - f*x^2)),x]
```

```
[Out] -(ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])]/(Sqrt[a]*d)) - (Sq
rt[f]*ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sq
rt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])]])/(2*d*Sqrt[c*d -
b*Sqrt[d]*Sqrt[f] + a*f]) + (Sqrt[f]*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*
c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b
*x + c*x^2])]])/(2*d*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1033

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q
), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q
), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f
, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{1}{x\sqrt{a+bx+cx^2}\left(d-fx^2\right)}dx = \int \left(\frac{1}{dx\sqrt{a+bx+cx^2}} - \frac{fx}{d\sqrt{a+bx+cx^2}\left(-d+fx^2\right)}\right)dx$$

$$= \frac{\int \frac{1}{x\sqrt{a+bx+cx^2}}dx}{d} - \frac{f\int \frac{x}{\sqrt{a+bx+cx^2}\left(-d+fx^2\right)}dx}{d}$$

$$= -\frac{2\,\text{Subst}\left(\int \frac{1}{4a-x^2}dx, x, \frac{2a+bx}{\sqrt{a+bx+cx^2}}\right)}{d} - \frac{f\int \frac{1}{\left(-\sqrt{d}\sqrt{f}+fx\right)\sqrt{a+bx+cx^2}}dx}{2d} - \frac{f\int \frac{}{(\sqrt{d}}$$

$$= -\frac{\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a}\,d} + \frac{f\,\text{Subst}\left(\int \frac{1}{4cdf-4b\sqrt{d}f^{3/2}+4af^2-x^2}dx, x, \frac{-b\sqrt{d}\sqrt{f}+2a}{\sqrt{a+}}\right.}{d}$$

$$= -\frac{\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a}\,d} - \frac{\sqrt{f}\tanh^{-1}\left(\frac{b\sqrt{d}-2a\sqrt{f}+\left(2c\sqrt{d}-b\sqrt{f}\right)x}{2\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2d\sqrt{cd-b\sqrt{d}\sqrt{f}+af}} + \frac{\sqrt{f}\tan}{}$$

Mathematica [A] time = 0.44, size = 252, normalized size = 0.94

$$\frac{\sqrt{f}\left(\frac{\tanh^{-1}\left(\frac{2a\sqrt{f}+b\sqrt{d}+b\sqrt{f}x+2c\sqrt{d}x}{2\sqrt{a+x(b+cx)}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{\sqrt{af+b\sqrt{d}\sqrt{f}+cd}} - \frac{\tanh^{-1}\left(\frac{-2a\sqrt{f}+b\sqrt{d}-b\sqrt{f}x+2c\sqrt{d}x}{2\sqrt{a+x(b+cx)}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right) - \frac{2\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+x(b+cx)}}\right)}{\sqrt{a}}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[a + b*x + c*x^2]*(d - f*x^2)),x]
```

```
[Out] ((-2*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + x*(b + c*x)])])/Sqrt[a] + Sqrt
[f]*(-(ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + 2*c*Sqrt[d]*x - b*Sqrt[f]*x)/(2*S
qrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])]/Sqrt[c*d - b*Sqr
t[d]*Sqrt[f] + a*f]) + ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + 2*c*Sqrt[d]*x + b
*Sqrt[f]*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])]/
Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]))/(2*d)
```

fricas [B] time = 149.60, size = 5995, normalized size = 22.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="fricas")
```

```
[Out] [1/4*(a*d*sqrt((c*d*f + a*f^2 + (c^2*d^4 + a^2*d^2*f^2 - (b^2 - 2*a*c)*d^3*
f)*sqrt(b^2*f^3/(c^4*d^7 + a^4*d^3*f^4 - 2*(b^2*c^2 - 2*a*c^3)*d^6*f + (b^4
 - 4*a*b^2*c + 6*a^2*c^2)*d^5*f^2 - 2*(a^2*b^2 - 2*a^3*c)*d^4*f^3)))/(c^2*d
^4 + a^2*d^2*f^2 - (b^2 - 2*a*c)*d^3*f))*log((2*b*c*f^2*x + b^2*f^2 + 2*(b^
2*d*f^2 - (c^3*d^5 + a^3*d^2*f^3 - (b^2*c - 3*a*c^2)*d^4*f - (a*b^2 - 3*a^2
*c)*d^3*f^2)*sqrt(b^2*f^3/(c^4*d^7 + a^4*d^3*f^4 - 2*(b^2*c^2 - 2*a*c^3)*d^
6*f + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^5*f^2 - 2*(a^2*b^2 - 2*a^3*c)*d^4*f^3
```

```
)))*sqrt(c*x^2 + b*x + a)*sqrt((c*d*f + a*f^2 + (c^2*d^4 + a^2*d^2*f^2 - (b
^2 - 2*a*c)*d^3*f)*sqrt(b^2*f^3/(c^4*d^7 + a^4*d^3*f^4 - 2*(b^2*c^2 - 2*a*c
^3)*d^6*f + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^5*f^2 - 2*(a^2*b^2 - 2*a^3*c)*d
^4*f^3)))/(c^2*d^4 + a^2*d^2*f^2 - (b^2 - 2*a*c)*d^3*f)) - (2*a*c^2*d^3*f +
 2*a^3*d*f^3 - 2*(a*b^2 - 2*a^2*c)*d^2*f^2 + (b*c^2*d^3*f + a^2*b*d*f^3 - (
b^3 - 2*a*b*c)*d^2*f^2)*x)*sqrt(b^2*f^3/(c^4*d^7 + a^4*d^3*f^4 - 2*(b^2*c^2
 - 2*a*c^3)*d^6*f + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^5*f^2 - 2*(a^2*b^2 - 2*
a^3*c)*d^4*f^3)))/x) - a*d*sqrt((c*d*f + a*f^2 + (c^2*d^4 + a^2*d^2*f^2 - (
b^2 - 2*a*c)*d^3*f)*sqrt(b^2*f^3/(c^4*d^7 + a^4*d^3*f^4 - 2*(b^2*c^2 - 2*a*
c^3)*d^6*f + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^5*f^2 - 2*(a^2*b^2 - 2*a^3*c)*
d^4*f^3)))/(c^2*d^4 + a^2*d^2*f^2 - (b^2 - 2*a*c)*d^3*f))*log((2*b*c*f^2*x
+ b^2*f^2 - 2*(b^2*d*f^2 - (c^3*d^5 + a^3*d^2*f^3 - (b^2*c - 3*a*c^2)*d^4*f
 - (a*b^2 - 3*a^2*c)*d^3*f^2)*sqrt(b^2*f^3/(c^4*d^7 + a^4*d^3*f^4 - 2*(b^2*
c^2 - 2*a*c^3)*d^6*f + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^5*f^2 - 2*(a^2*b^2 -
 2*a^3*c)*d^4*f^3)))*sqrt(c*x^2 + b*x + a)*sqrt((c*d*f + a*f^2 + (c^2*d^4 +
 a^2*d^2*f^2 - (b^2 - 2*a*c)*d^3*f)*sqrt(b^2*f^3/(c^4*d^7 + a^4*d^3*f^4 - 2
*(b^2*c^2 - 2*a*c^3)*d^6*f + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^5*f^2 - 2*(a^2
*b^2 - 2*a^3*c)*d^4*f^3)))/(c^2*d^4 + a^2*d^2*f^2 - (b^2 - 2*a*c)*d^3*f)) -
 (2*a*c^2*d^3*f + 2*a^3*d*f^3 - 2*(a*b^2 - 2*a^2*c)*d^2*f^2 + (b*c^2*d^3*f
+ a^2*b*d*f^3 - (b^3 - 2*a*b*c)*d^2*f^2)*x)*sqrt(b^2*f^3/(c^4*d^7 + a^4*d^3
*f^4 - 2*(b^2*c^2 - 2*a*c^3)*d^6*f + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^5*f^2
- 2*(a^2*b^2 - 2*a^3*c)*d^4*f^3)))/x) + a*d*sqrt((c*d*f + a*f^2 - (c^2*d^4
+ a^2*d^2*f^2 - (b^2 - 2*a*c)*d^3*f)*sqrt(b^2*f^3/(c^4*d^7 + a^4*d^3*f^4 -
2*(b^2*c^2 - 2*a*c^3)*d^6*f + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^5*f^2 - 2*(a^
2*b^2 - 2*a^3*c)*d^4*f^3)))/(c^2*d^4 + a^2*d^2*f^2 - (b^2 - 2*a*c)*d^3*f))*
log((2*b*c*f^2*x + b^2*f^2 + 2*(b^2*d*f^2 + (c^3*d^5 + a^3*d^2*f^3 - (b^2*c
 - 3*a*c^2)*d^4*f - (a*b^2 - 3*a^2*c)*d^3*f^2)*sqrt(b^2*f^3/(c^4*d^7 + a^4*
d^3*f^4 - 2*(b^2*c^2 - 2*a*c^3)*d^6*f + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^5*f
^2 - 2*(a^2*b^2 - 2*a^3*c)*d^4*f^3)))*sqrt(c*x^2 + b*x + a)*sqrt((c*d*f + a
*f^2 - (c^2*d^4 + a^2*d^2*f^2 - (b^2 - 2*a*c)*d^3*f)*sqrt(b^2*f^3/(c^4*d^7
+ a^4*d^3*f^4 - 2*(b^2*c^2 - 2*a*c^3)*d^6*f + (b^4 - 4*a*b^2*c + 6*a^2*c^2)
*d^5*f^2 - 2*(a^2*b^2 - 2*a^3*c)*d^4*f^3)))/(c^2*d^4 + a^2*d^2*f^2 - (b^2 -
 2*a*c)*d^3*f)) + (2*a*c^2*d^3*f + 2*a^3*d*f^3 - 2*(a*b^2 - 2*a^2*c)*d^2*f^
2 + (b*c^2*d^3*f + a^2*b*d*f^3 - (b^3 - 2*a*b*c)*d^2*f^2)*x)*sqrt(b^2*f^3/(
c^4*d^7 + a^4*d^3*f^4 - 2*(b^2*c^2 - 2*a*c^3)*d^6*f + (b^4 - 4*a*b^2*c + 6*
a^2*c^2)*d^5*f^2 - 2*(a^2*b^2 - 2*a^3*c)*d^4*f^3)))/x) - a*d*sqrt((c*d*f +
a*f^2 - (c^2*d^4 + a^2*d^2*f^2 - (b^2 - 2*a*c)*d^3*f)*sqrt(b^2*f^3/(c^4*d^7
 + a^4*d^3*f^4 - 2*(b^2*c^2 - 2*a*c^3)*d^6*f + (b^4 - 4*a*b^2*c + 6*a^2*c^2
)*d^5*f^2 - 2*(a^2*b^2 - 2*a^3*c)*d^4*f^3)))/(c^2*d^4 + a^2*d^2*f^2 - (b^2
- 2*a*c)*d^3*f))*log((2*b*c*f^2*x + b^2*f^2 - 2*(b^2*d*f^2 + (c^3*d^5 + a^3
*d^2*f^3 - (b^2*c - 3*a*c^2)*d^4*f - (a*b^2 - 3*a^2*c)*d^3*f^2)*sqrt(b^2*f^
3/(c^4*d^7 + a^4*d^3*f^4 - 2*(b^2*c^2 - 2*a*c^3)*d^6*f + (b^4 - 4*a*b^2*c +
 6*a^2*c^2)*d^5*f^2 - 2*(a^2*b^2 - 2*a^3*c)*d^4*f^3)))*sqrt(c*x^2 + b*x + a
)*sqrt((c*d*f + a*f^2 - (c^2*d^4 + a^2*d^2*f^2 - (b^2 - 2*a*c)*d^3*f)*sqrt(
b^2*f^3/(c^4*d^7 + a^4*d^3*f^4 - 2*(b^2*c^2 - 2*a*c^3)*d^6*f + (b^4 - 4*a*b
^2*c + 6*a^2*c^2)*d^5*f^2 - 2*(a^2*b^2 - 2*a^3*c)*d^4*f^3)))/(c^2*d^4 + a^2
*d^2*f^2 - (b^2 - 2*a*c)*d^3*f)) + (2*a*c^2*d^3*f + 2*a^3*d*f^3 - 2*(a*b^2
- 2*a^2*c)*d^2*f^2 + (b*c^2*d^3*f + a^2*b*d*f^3 - (b^3 - 2*a*b*c)*d^2*f^2)*
x)*sqrt(b^2*f^3/(c^4*d^7 + a^4*d^3*f^4 - 2*(b^2*c^2 - 2*a*c^3)*d^6*f + (b^4
 - 4*a*b^2*c + 6*a^2*c^2)*d^5*f^2 - 2*(a^2*b^2 - 2*a^3*c)*d^4*f^3)))/x) + 2
*sqrt(a)*log(-(8*a*b*x + (b^2 + 4*a*c)*x^2 - 4*sqrt(c*x^2 + b*x + a)*(b*x +
 2*a)*sqrt(a) + 8*a^2)/x^2))/(a*d), 1/4*(a*d*sqrt((c*d*f + a*f^2 + (c^2*d^4
 + a^2*d^2*f^2 - (b^2 - 2*a*c)*d^3*f)*sqrt(b^2*f^3/(c^4*d^7 + a^4*d^3*f^4 -
 2*(b^2*c^2 - 2*a*c^3)*d^6*f + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^5*f^2 - 2*(a
^2*b^2 - 2*a^3*c)*d^4*f^3)))/(c^2*d^4 + a^2*d^2*f^2 - (b^2 - 2*a*c)*d^3*f))
*log((2*b*c*f^2*x + b^2*f^2 + 2*(b^2*d*f^2 - (c^3*d^5 + a^3*d^2*f^3 - (b^2*
c - 3*a*c^2)*d^4*f - (a*b^2 - 3*a^2*c)*d^3*f^2)*sqrt(b^2*f^3/(c^4*d^7 + a^4
*d^3*f^4 - 2*(b^2*c^2 - 2*a*c^3)*d^6*f + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^5*
f^2 - 2*(a^2*b^2 - 2*a^3*c)*d^4*f^3)))*sqrt(c*x^2 + b*x + a)*sqrt((c*d*f +
```

```
a*f^2 + (c^2*d^4 + a^2*d^2*f^2 - (b^2 - 2*a*c)*d^3*f)*sqrt(b^2*f^3/(c^4*d^7
 + a^4*d^3*f^4 - 2*(b^2*c^2 - 2*a*c^3)*d^6*f + (b^4 - 4*a*b^2*c + 6*a^2*c^2
)*d^5*f^2 - 2*(a^2*b^2 - 2*a^3*c)*d^4*f^3)))/(c^2*d^4 + a^2*d^2*f^2 - (b^2
- 2*a*c)*d^3*f)) - (2*a*c^2*d^3*f + 2*a^3*d*f^3 - 2*(a*b^2 - 2*a^2*c)*d^2*f
^2 + (b*c^2*d^3*f + a^2*b*d*f^3 - (b^3 - 2*a*b*c)*d^2*f^2)*x)*sqrt(b^2*f^3/
(c^4*d^7 + a^4*d^3*f^4 - 2*(b^2*c^2 - 2*a*c^3)*d^6*f + (b^4 - 4*a*b^2*c + 6
*a^2*c^2)*d^5*f^2 - 2*(a^2*b^2 - 2*a^3*c)*d^4*f^3)))/x) - a*d*sqrt((c*d*f +
 a*f^2 + (c^2*d^4 + a^2*d^2*f^2 - (b^2 - 2*a*c)*d^3*f)*sqrt(b^2*f^3/(c^4*d^
7 + a^4*d^3*f^4 - 2*(b^2*c^2 - 2*a*c^3)*d^6*f + (b^4 - 4*a*b^2*c + 6*a^2*c^
2)*d^5*f^2 - 2*(a^2*b^2 - 2*a^3*c)*d^4*f^3)))/(c^2*d^4 + a^2*d^2*f^2 - (b^2
 - 2*a*c)*d^3*f))*log((2*b*c*f^2*x + b^2*f^2 - 2*(b^2*d*f^2 - (c^3*d^5 + a^
3*d^2*f^3 - (b^2*c - 3*a*c^2)*d^4*f - (a*b^2 - 3*a^2*c)*d^3*f^2)*sqrt(b^2*f
^3/(c^4*d^7 + a^4*d^3*f^4 - 2*(b^2*c^2 - 2*a*c^3)*d^6*f + (b^4 - 4*a*b^2*c
+ 6*a^2*c^2)*d^5*f^2 - 2*(a^2*b^2 - 2*a^3*c)*d^4*f^3)))*sqrt(c*x^2 + b*x +
a)*sqrt((c*d*f + a*f^2 + (c^2*d^4 + a^2*d^2*f^2 - (b^2 - 2*a*c)*d^3*f)*sqrt
(b^2*f^3/(c^4*d^7 + a^4*d^3*f^4 - 2*(b^2*c^2 - 2*a*c^3)*d^6*f + (b^4 - 4*a*
b^2*c + 6*a^2*c^2)*d^5*f^2 - 2*(a^2*b^2 - 2*a^3*c)*d^4*f^3)))/(c^2*d^4 + a^
2*d^2*f^2 - (b^2 - 2*a*c)*d^3*f)) - (2*a*c^2*d^3*f + 2*a^3*d*f^3 - 2*(a*b^2
 - 2*a^2*c)*d^2*f^2 + (b*c^2*d^3*f + a^2*b*d*f^3 - (b^3 - 2*a*b*c)*d^2*f^2)
*x)*sqrt(b^2*f^3/(c^4*d^7 + a^4*d^3*f^4 - 2*(b^2*c^2 - 2*a*c^3)*d^6*f + (b^
4 - 4*a*b^2*c + 6*a^2*c^2)*d^5*f^2 - 2*(a^2*b^2 - 2*a^3*c)*d^4*f^3)))/x) +
a*d*sqrt((c*d*f + a*f^2 - (c^2*d^4 + a^2*d^2*f^2 - (b^2 - 2*a*c)*d^3*f)*sqr
t(b^2*f^3/(c^4*d^7 + a^4*d^3*f^4 - 2*(b^2*c^2 - 2*a*c^3)*d^6*f + (b^4 - 4*a
*b^2*c + 6*a^2*c^2)*d^5*f^2 - 2*(a^2*b^2 - 2*a^3*c)*d^4*f^3)))/(c^2*d^4 + a
^2*d^2*f^2 - (b^2 - 2*a*c)*d^3*f))*log((2*b*c*f^2*x + b^2*f^2 + 2*(b^2*d*f^
2 + (c^3*d^5 + a^3*d^2*f^3 - (b^2*c - 3*a*c^2)*d^4*f - (a*b^2 - 3*a^2*c)*d^
3*f^2)*sqrt(b^2*f^3/(c^4*d^7 + a^4*d^3*f^4 - 2*(b^2*c^2 - 2*a*c^3)*d^6*f +
(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^5*f^2 - 2*(a^2*b^2 - 2*a^3*c)*d^4*f^3)))*sq
rt(c*x^2 + b*x + a)*sqrt((c*d*f + a*f^2 - (c^2*d^4 + a^2*d^2*f^2 - (b^2 - 2
*a*c)*d^3*f)*sqrt(b^2*f^3/(c^4*d^7 + a^4*d^3*f^4 - 2*(b^2*c^2 - 2*a*c^3)*d^
6*f + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^5*f^2 - 2*(a^2*b^2 - 2*a^3*c)*d^4*f^3
)))/(c^2*d^4 + a^2*d^2*f^2 - (b^2 - 2*a*c)*d^3*f)) + (2*a*c^2*d^3*f + 2*a^3
*d*f^3 - 2*(a*b^2 - 2*a^2*c)*d^2*f^2 + (b*c^2*d^3*f + a^2*b*d*f^3 - (b^3 -
2*a*b*c)*d^2*f^2)*x)*sqrt(b^2*f^3/(c^4*d^7 + a^4*d^3*f^4 - 2*(b^2*c^2 - 2*a
*c^3)*d^6*f + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^5*f^2 - 2*(a^2*b^2 - 2*a^3*c)
*d^4*f^3)))/x) - a*d*sqrt((c*d*f + a*f^2 - (c^2*d^4 + a^2*d^2*f^2 - (b^2 -
2*a*c)*d^3*f)*sqrt(b^2*f^3/(c^4*d^7 + a^4*d^3*f^4 - 2*(b^2*c^2 - 2*a*c^3)*d
^6*f + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^5*f^2 - 2*(a^2*b^2 - 2*a^3*c)*d^4*f^
3)))/(c^2*d^4 + a^2*d^2*f^2 - (b^2 - 2*a*c)*d^3*f))*log((2*b*c*f^2*x + b^2*
f^2 - 2*(b^2*d*f^2 + (c^3*d^5 + a^3*d^2*f^3 - (b^2*c - 3*a*c^2)*d^4*f - (a*
b^2 - 3*a^2*c)*d^3*f^2)*sqrt(b^2*f^3/(c^4*d^7 + a^4*d^3*f^4 - 2*(b^2*c^2 -
2*a*c^3)*d^6*f + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^5*f^2 - 2*(a^2*b^2 - 2*a^3
*c)*d^4*f^3)))*sqrt(c*x^2 + b*x + a)*sqrt((c*d*f + a*f^2 - (c^2*d^4 + a^2*d
^2*f^2 - (b^2 - 2*a*c)*d^3*f)*sqrt(b^2*f^3/(c^4*d^7 + a^4*d^3*f^4 - 2*(b^2*
c^2 - 2*a*c^3)*d^6*f + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^5*f^2 - 2*(a^2*b^2 -
 2*a^3*c)*d^4*f^3)))/(c^2*d^4 + a^2*d^2*f^2 - (b^2 - 2*a*c)*d^3*f)) + (2*a*
c^2*d^3*f + 2*a^3*d*f^3 - 2*(a*b^2 - 2*a^2*c)*d^2*f^2 + (b*c^2*d^3*f + a^2*
b*d*f^3 - (b^3 - 2*a*b*c)*d^2*f^2)*x)*sqrt(b^2*f^3/(c^4*d^7 + a^4*d^3*f^4 -
 2*(b^2*c^2 - 2*a*c^3)*d^6*f + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^5*f^2 - 2*(a
^2*b^2 - 2*a^3*c)*d^4*f^3)))/x) + 4*sqrt(-a)*arctan(1/2*sqrt(c*x^2 + b*x +
a)*(b*x + 2*a)*sqrt(-a)/(a*c*x^2 + a*b*x + a^2)))/(a*d)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="giac")
```

```
[Out] sage2
```

maple [A] time = 0.02, size = 391, normalized size = 1.46

$$\frac{\ln\left(\frac{\frac{2af+2cd-2\sqrt{df}\,b}{f}+\frac{\left(bf-2\sqrt{df}\,c\right)\left(x+\frac{\sqrt{df}}{f}\right)}{f}+2\sqrt{\frac{af+cd-\sqrt{df}\,b}{f}}\sqrt{\left(x+\frac{\sqrt{df}}{f}\right)^2c+\frac{\left(bf-2\sqrt{df}\,c\right)\left(x+\frac{\sqrt{df}}{f}\right)}{f}+\frac{af+cd-\sqrt{df}\,b}{f}}}{x+\frac{\sqrt{df}}{f}}\right)}{2\sqrt{\frac{af+cd-\sqrt{df}\,b}{f}}\,d}+\frac{\ln\left(\frac{\frac{2af+2cd+2\sqrt{df}\,b}{f}+\frac{\left(bf+2 \cdots\right.}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x)
```

```
[Out] 1/2/d/((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d-(d*f)^(1/2)*b)/f+(b*
f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+2*((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*
((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(
d*f)^(1/2)*b)/f)^(1/2))/(x+(d*f)^(1/2)/f))+1/2/d/((a*f+c*d+(d*f)^(1/2)*b)/f
)^(1/2)*ln((2*(a*f+c*d+(d*f)^(1/2)*b)/f+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2
)/f)/f+2*((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d
*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))/(x-(d*f)
^(1/2)/f))-1/d/a^(1/2)*ln((b*x+2*a+2*(c*x^2+b*x+a)^(1/2)*a^(1/2))/x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{\sqrt{cx^2+bx+a}\left(fx^2-d\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="maxima")
```

```
[Out] -integrate(1/(sqrt(c*x^2 + b*x + a)*(f*x^2 - d)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x\,\left(d-f\,x^2\right)\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(d - f*x^2)*(a + b*x + c*x^2)^(1/2)),x)
```

```
[Out] int(1/(x*(d - f*x^2)*(a + b*x + c*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{-dx\sqrt{a+bx+cx^2}+fx^3\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x**2+b*x+a)**(1/2)/(-f*x**2+d),x)
```

```
[Out] -Integral(1/(-d*x*sqrt(a + b*x + c*x**2) + f*x**3*sqrt(a + b*x + c*x**2)),
x)
```

3.100 $\int \frac{1}{x^2\sqrt{a+bx+cx^2}\left(d-fx^2\right)}\,dx$

Optimal. Leaf size=291

$$\frac{b\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2a^{3/2}d}+\frac{f\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2d^{3/2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}+\frac{f\tanh^{-1}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2d^{3/2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}-\sqrt{a+\cdots}$$

```
[Out] 1/2*b*arctanh(1/2*(b*x+2*a)/a^(1/2)/(c*x^2+b*x+a)^(1/2))/a^(3/2)/d-(c*x^2+b
*x+a)^(1/2)/a/d/x+1/2*f*arctanh(1/2*(b*d^(1/2)-2*a*f^(1/2)+x*(2*c*d^(1/2)-b
*f^(1/2)))/(c*x^2+b*x+a)^(1/2)/(c*d+a*f-b*d^(1/2)*f^(1/2))^(1/2))/d^(3/2)/(
c*d+a*f-b*d^(1/2)*f^(1/2))^(1/2)+1/2*f*arctanh(1/2*(b*d^(1/2)+2*a*f^(1/2)+x
*(2*c*d^(1/2)+b*f^(1/2)))/(c*x^2+b*x+a)^(1/2)/(c*d+a*f+b*d^(1/2)*f^(1/2))^(
1/2))/d^(3/2)/(c*d+a*f+b*d^(1/2)*f^(1/2))^(1/2)
```

Rubi [A] time = 0.66, antiderivative size = 291, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {6725, 730, 724, 206, 984}

$$\frac{b\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2a^{3/2}d}+\frac{f\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2d^{3/2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}+\frac{f\tanh^{-1}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2d^{3/2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}-\sqrt{a+\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[a + b*x + c*x^2]*(d - f*x^2)),x]
```

```
[Out] -(Sqrt[a + b*x + c*x^2]/(a*d*x)) + (b*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a
 + b*x + c*x^2])])/(2*a^(3/2)*d) + (f*ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2
*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a +
b*x + c*x^2])])/(2*d^(3/2)*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]) + (f*ArcTan
h[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d + b*S
qrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*d^(3/2)*Sqrt[c*d + b*Sqrt
[d]*Sqrt[f] + a*f])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 730

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 - b*d*e + a*e^2)), x] + Dist[(2*c*d - b*e)/(2*(c*d^2 - b*d*e + a*e^2)),
 Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e,
 m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2
*c*d - b*e, 0] && EqQ[m + 2*p + 3, 0]
```

Rule 984

```
Int[1/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Sy
mbol] :> Dist[1/2, Int[1/((a - Rt[-(a*c), 2]*x)*Sqrt[d + e*x + f*x^2]), x],
 x] + Dist[1/2, Int[1/((a + Rt[-(a*c), 2]*x)*Sqrt[d + e*x + f*x^2]), x], x]
 /; FreeQ[{a, c, d, e, f}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{1}{x^2\sqrt{a+bx+cx^2}\left(d-fx^2\right)}\,dx = \int \left(\frac{1}{dx^2\sqrt{a+bx+cx^2}} + \frac{f}{d\sqrt{a+bx+cx^2}\left(d-fx^2\right)}\right)dx$$

$$= \frac{\int \frac{1}{x^2\sqrt{a+bx+cx^2}}\,dx}{d} + \frac{f\int \frac{1}{\sqrt{a+bx+cx^2}\left(d-fx^2\right)}\,dx}{d}$$

$$= -\frac{\sqrt{a+bx+cx^2}}{adx} - \frac{b\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx}{2ad} + \frac{f\int \frac{1}{\left(d-\sqrt{d}\sqrt{f}x\right)\sqrt{a+bx+cx^2}}\,dx}{2d} + \frac{f\int \ldots}{\ldots}$$

$$= -\frac{\sqrt{a+bx+cx^2}}{adx} + \frac{b\,\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+bx}{\sqrt{a+bx+cx^2}}\right)}{ad} - \frac{f\,\text{Subst}\left(\int \frac{\ldots}{4cd^2-4\ldots}\right)}{\ldots}$$

$$= -\frac{\sqrt{a+bx+cx^2}}{adx} + \frac{b\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2a^{3/2}d} + \frac{f\tanh^{-1}\left(\frac{b\sqrt{d}-2a\sqrt{f}+(2c\ldots}{2\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\ldots}\right)}{2d^{3/2}\sqrt{cd-b\sqrt{d}\sqrt{f}\ldots}}$$

Mathematica [A] time = 1.03, size = 325, normalized size = 1.12

$$\frac{\frac{b\sqrt{d}\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+x(b+cx)}}\right)}{a^{3/2}} + \frac{f\tanh^{-1}\left(\frac{2a\sqrt{f}+b\sqrt{d}+b\sqrt{f}x+2c\sqrt{d}x}{2\sqrt{a+x(b+cx)}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{\sqrt{af+b\sqrt{d}\sqrt{f}+cd}} + \frac{f\tanh^{-1}\left(\frac{-2a\sqrt{f}+b\left(\sqrt{d}-\sqrt{f}x\right)+2c\sqrt{d}x}{2\sqrt{a+x(b+cx)}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}} - \frac{2b\sqrt{d}}{a\sqrt{a+x(b+cx)}} - \frac{\ldots}{a\sqrt{\ldots}}}{2d^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Sqrt[a + b*x + c*x^2]*(d - f*x^2)),x]
```

```
[Out] ((-2*b*Sqrt[d])/(a*Sqrt[a + x*(b + c*x)]) - (2*Sqrt[d])/(x*Sqrt[a + x*(b +
c*x)]) - (2*c*Sqrt[d]*x)/(a*Sqrt[a + x*(b + c*x)]) + (b*Sqrt[d]*ArcTanh[(2*
a + b*x)/(2*Sqrt[a]*Sqrt[a + x*(b + c*x)])])/a^(3/2) + (f*ArcTanh[(b*Sqrt[d
] + 2*a*Sqrt[f] + 2*c*Sqrt[d]*x + b*Sqrt[f]*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt
[f] + a*f]*Sqrt[a + x*(b + c*x)])])/Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f] + (
f*ArcTanh[(-2*a*Sqrt[f] + 2*c*Sqrt[d]*x + b*(Sqrt[d] - Sqrt[f]*x))/(2*Sqrt[
c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])])/Sqrt[c*d - b*Sqrt[d
]*Sqrt[f] + a*f])/(2*d^(3/2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument ValueEvaluation time: 1.11sym2pol
y/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argum
ent Value
```

maple [A] time = 0.02, size = 427, normalized size = 1.47

$$-\frac{f \ln\left(\frac{\frac{2af+2cd-2\sqrt{df}\,b}{f}+\frac{\left(bf-2\sqrt{df}\,c\right)\left(x+\frac{\sqrt{df}}{f}\right)}{f}+2\sqrt{\frac{af+cd-\sqrt{df}\,b}{f}}\,\sqrt{\left(x+\frac{\sqrt{df}}{f}\right)^{2} c+\frac{\left(bf-2\sqrt{df}\,c\right)\left(x+\frac{\sqrt{df}}{f}\right)}{f}+\frac{af+cd-\sqrt{df}\,b}{f}}}{x+\frac{\sqrt{df}}{f}}\right)}{2\sqrt{df}\,\sqrt{\frac{af+cd-\sqrt{df}\,b}{f}}\,d}+\frac{f \ln\left(\frac{\frac{2af+2cd+2\sqrt{df}\,b}{f}+\cdots}{\cdots}\right)}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x)
```

```
[Out] -1/2*f/d/(d*f)^(1/2)/((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d-(d*f)
^(1/2)*b)/f+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+2*((a*f+c*d-(d*f)^(1/
2)*b)/f)^(1/2)*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/
f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))/(x+(d*f)^(1/2)/f))-(c*x^2+b*x+a)^(1/
2)/a/d/x+1/2/d*b/a^(3/2)*ln((b*x+2*a+2*(c*x^2+b*x+a)^(1/2)*a^(1/2))/x)+1/2*
f/d/(d*f)^(1/2)/((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d+(d*f)^(1/2
)*b)/f+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+2*((a*f+c*d+(d*f)^(1/2)*b)
/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+
(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))/(x-(d*f)^(1/2)/f))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{\sqrt{cx^2+bx+a}\left(fx^2-d\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="maxima")
```

```
[Out] -integrate(1/(sqrt(c*x^2 + b*x + a)*(f*x^2 - d)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^2\,\left(d-f\,x^2\right)\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(d - f*x^2)*(a + b*x + c*x^2)^(1/2)),x)
```

```
[Out] int(1/(x^2*(d - f*x^2)*(a + b*x + c*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{-dx^2\sqrt{a+bx+cx^2}+fx^4\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(c*x**2+b*x+a)**(1/2)/(-f*x**2+d),x)
```

```
[Out] -Integral(1/(-d*x**2*sqrt(a + b*x + c*x**2) + f*x**4*sqrt(a + b*x + c*x**2)
), x)
```

3.101 $\int \frac{1}{x^3\sqrt{a+bx+cx^2}\left(d-fx^2\right)}\,dx$

Optimal. Leaf size=376

$$-\frac{\left(3b^2-4ac\right)\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{8a^{5/2}d}+\frac{3b\sqrt{a+bx+cx^2}}{4a^2dx}-\frac{f^{3/2}\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2d^2\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}+\frac{f^{3/2}\tanh^{-1}}{2d^2}$$

```
[Out] -1/8*(-4*a*c+3*b^2)*arctanh(1/2*(b*x+2*a)/a^(1/2)/(c*x^2+b*x+a)^(1/2))/a^(5
/2)/d-f*arctanh(1/2*(b*x+2*a)/a^(1/2)/(c*x^2+b*x+a)^(1/2))/d^2/a^(1/2)-1/2*
(c*x^2+b*x+a)^(1/2)/a/d/x^2+3/4*b*(c*x^2+b*x+a)^(1/2)/a^2/d/x-1/2*f^(3/2)*a
rctanh(1/2*(b*d^(1/2)-2*a*f^(1/2)+x*(2*c*d^(1/2)-b*f^(1/2)))/(c*x^2+b*x+a)^
(1/2)/(c*d+a*f-b*d^(1/2)*f^(1/2))^(1/2))/d^2/(c*d+a*f-b*d^(1/2)*f^(1/2))^(1
/2)+1/2*f^(3/2)*arctanh(1/2*(b*d^(1/2)+2*a*f^(1/2)+x*(2*c*d^(1/2)+b*f^(1/2)
))/(c*x^2+b*x+a)^(1/2)/(c*d+a*f+b*d^(1/2)*f^(1/2))^(1/2))/d^2/(c*d+a*f+b*d^
(1/2)*f^(1/2))^(1/2)
```

Rubi [A] time = 0.73, antiderivative size = 376, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {6725, 744, 806, 724, 206, 1033}

$$-\frac{\left(3b^2-4ac\right)\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{8a^{5/2}d}+\frac{3b\sqrt{a+bx+cx^2}}{4a^2dx}-\frac{f^{3/2}\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2d^2\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}+\frac{f^{3/2}\tanh^{-1}}{2d^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*Sqrt[a + b*x + c*x^2]*(d - f*x^2)),x]
```

```
[Out] -Sqrt[a + b*x + c*x^2]/(2*a*d*x^2) + (3*b*Sqrt[a + b*x + c*x^2])/(4*a^2*d*x
) - ((3*b^2 - 4*a*c)*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])]
)/(8*a^(5/2)*d) - (f*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])]
)/(Sqrt[a]*d^2) - (f^(3/2)*ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d]
- b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2
]])/(2*d^2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]) + (f^(3/2)*ArcTanh[(b*Sqrt
[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sq
rt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*d^2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] +
 a*f])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 744

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(
```

```
d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x,
x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && Ne
Q[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumS
implerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 1033

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q
), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q
), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f
, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{1}{x^3\sqrt{a+bx+cx^2}\left(d-fx^2\right)}\,dx = \int \left(\frac{1}{dx^3\sqrt{a+bx+cx^2}}+\frac{f}{d^2x\sqrt{a+bx+cx^2}}+\frac{f^2x}{d^2\sqrt{a+bx+cx^2}\left(d-fx^2\right)}\right.$$

$$=\frac{\int \frac{1}{x^3\sqrt{a+bx+cx^2}}\,dx}{d}+\frac{f\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx}{d^2}+\frac{f^2\int \frac{x}{\sqrt{a+bx+cx^2}\left(d-fx^2\right)}\,dx}{d^2}$$

$$=-\frac{\sqrt{a+bx+cx^2}}{2adx^2}-\frac{\int \frac{\frac{3b}{2}+cx}{x^2\sqrt{a+bx+cx^2}}\,dx}{2ad}-\frac{(2f)\,\mathrm{Subst}\left(\int \frac{1}{4a-x^2}\,dx,x,\frac{2a+bx}{\sqrt{a+bx+cx}}\right.}{d^2}$$

$$=-\frac{\sqrt{a+bx+cx^2}}{2adx^2}+\frac{3b\sqrt{a+bx+cx^2}}{4a^2dx}-\frac{f\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a}\,d^2}+\frac{\left(3b^2\right.}{}$$

$$=-\frac{\sqrt{a+bx+cx^2}}{2adx^2}+\frac{3b\sqrt{a+bx+cx^2}}{4a^2dx}-\frac{f\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a}\,d^2}-\frac{f^{3/2}\,\mathrm{t}}{}$$

$$=-\frac{\sqrt{a+bx+cx^2}}{2adx^2}+\frac{3b\sqrt{a+bx+cx^2}}{4a^2dx}-\frac{\left(3b^2-4ac\right)\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right.}{8a^{5/2}d}$$

Mathematica [A] time = 2.01, size = 314, normalized size = 0.84

$$\frac{2\sqrt{a}\left(\frac{2a^2f^{3/2}\tanh^{-1}\left(\frac{2a\sqrt{f}+b\sqrt{d}+b\sqrt{f}x+2c\sqrt{d}x}{2\sqrt{a+x(b+cx)}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}-\frac{2a^2f^{3/2}\tanh^{-1}\left(\frac{-2a\sqrt{f}+b\left(\sqrt{d}-\sqrt{f}x\right)+2c\sqrt{d}x}{2\sqrt{a+x(b+cx)}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}-\frac{d(2a-3bx)\sqrt{a+x(b+cx)}}{x^2}\right)+\left(4a(cd\ \cdots\right.}{8a^{5/2}d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*Sqrt[a + b*x + c*x^2]*(d - f*x^2)),x]
```

```
[Out] ((-3*b^2*d + 4*a*(c*d - 2*a*f))*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + x*(
b + c*x)])] + 2*Sqrt[a]*(-((d*(2*a - 3*b*x)*Sqrt[a + x*(b + c*x)])/x^2) + (
2*a^2*f^(3/2)*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + 2*c*Sqrt[d]*x + b*Sqrt[f]*
x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])]])/Sqrt[c*d
 + b*Sqrt[d]*Sqrt[f] + a*f] - (2*a^2*f^(3/2)*ArcTanh[(-2*a*Sqrt[f] + 2*c*Sq
rt[d]*x + b*(Sqrt[d] - Sqrt[f]*x))/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*S
qrt[a + x*(b + c*x)])])/Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]))/(8*a^(5/2)*d^
2)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument ValueEvaluation time: 1.43sym2pol
y/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argum
ent Value
```

maple [A] time = 0.02, size = 519, normalized size = 1.38

$$\frac{f\ln\left(\frac{\frac{2af+2cd-2\sqrt{df}\,b}{f}+\frac{\left(bf-2\sqrt{df}\,c\right)\left(x+\frac{\sqrt{df}}{f}\right)}{f}+2\sqrt{\frac{af+cd-\sqrt{df}\,b}{f}}\sqrt{\left(x+\frac{\sqrt{df}}{f}\right)^2c+\frac{\left(bf-2\sqrt{df}\,c\right)\left(x+\frac{\sqrt{df}}{f}\right)}{f}+\frac{af+cd-\sqrt{df}\,b}{f}}}{x+\frac{\sqrt{df}}{f}}\right)}{2\sqrt{\frac{af+cd-\sqrt{df}\,b}{f}}\,d^2}+\frac{f\ln\left(\frac{\frac{2af+2cd+2\sqrt{df}\,b}{f}+\frac{(bf+\ \cdots}{\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x)
```

```
[Out] 1/2*f/d^2/((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d-(d*f)^(1/2)*b)/f
+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+2*((a*f+c*d-(d*f)^(1/2)*b)/f)^(1
/2)*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c
```

```
*d-(d*f)^(1/2)*b)/f)^(1/2))/(x+(d*f)^(1/2)/f))+1/2*f/d^2/((a*f+c*d+(d*f)^(1
/2)*b)/f)^(1/2)*ln((2*(a*f+c*d+(d*f)^(1/2)*b)/f+(b*f+2*(d*f)^(1/2)*c)*(x-(d
*f)^(1/2)/f)/f+2*((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(
b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))/
(x-(d*f)^(1/2)/f))-1/2*(c*x^2+b*x+a)^(1/2)/a/d/x^2+3/4*b*(c*x^2+b*x+a)^(1/2
)/a^2/d/x-3/8/d*b^2/a^(5/2)*ln((b*x+2*a+2*(c*x^2+b*x+a)^(1/2)*a^(1/2))/x)+1
/2/d*c/a^(3/2)*ln((b*x+2*a+2*(c*x^2+b*x+a)^(1/2)*a^(1/2))/x)-f/d^2/a^(1/2)*
ln((b*x+2*a+2*(c*x^2+b*x+a)^(1/2)*a^(1/2))/x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{\sqrt{cx^2+bx+a}\left(fx^2-d\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="maxima")
[Out] -integrate(1/(sqrt(c*x^2 + b*x + a)*(f*x^2 - d)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^3\,\left(d-f\,x^2\right)\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(d - f*x^2)*(a + b*x + c*x^2)^(1/2)),x)
[Out] int(1/(x^3*(d - f*x^2)*(a + b*x + c*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{-dx^3\sqrt{a+bx+cx^2}+fx^5\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(c*x**2+b*x+a)**(1/2)/(-f*x**2+d),x)
[Out] -Integral(1/(-d*x**3*sqrt(a + b*x + c*x**2) + f*x**5*sqrt(a + b*x + c*x**2)
), x)
```

3.102 $\int \frac{x^4}{\left(a+bx+cx^2\right)^{3/2}\left(d-fx^2\right)}\,dx$

Optimal. Leaf size=466

$$-\frac{2d^2\left(b\left(b^2f-c(3af+cd)\right)-cx\left(2acf+b^2(-f)+2c^2d\right)\right)}{f^2\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left(b^2df-(af+cd)^2\right)}+\frac{2d(b+2cx)}{f^2\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}+\frac{2b\sqrt{a+bx+cx^2}}{cf\left(b^2-4ac\right)}-\frac{}{f(}$$

```
[Out] -arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(3/2)/f+1/2*d^(3/2)*a
rctanh(1/2*(b*d^(1/2)-2*a*f^(1/2)+x*(2*c*d^(1/2)-b*f^(1/2)))/(c*x^2+b*x+a)^
(1/2)/(c*d+a*f-b*d^(1/2)*f^(1/2))^(1/2))/f/(c*d+a*f-b*d^(1/2)*f^(1/2))^(3/2
)+1/2*d^(3/2)*arctanh(1/2*(b*d^(1/2)+2*a*f^(1/2)+x*(2*c*d^(1/2)+b*f^(1/2)))
/(c*x^2+b*x+a)^(1/2)/(c*d+a*f+b*d^(1/2)*f^(1/2))^(1/2))/f/(c*d+a*f+b*d^(1/2
)*f^(1/2))^(3/2)-2*x*(b*x+2*a)/(-4*a*c+b^2)/f/(c*x^2+b*x+a)^(1/2)+2*d*(2*c*
x+b)/(-4*a*c+b^2)/f^2/(c*x^2+b*x+a)^(1/2)-2*d^2*(b*(b^2*f-c*(3*a*f+c*d))-c*
(2*a*c*f-b^2*f+2*c^2*d)*x)/(-4*a*c+b^2)/f^2/(b^2*d*f-(a*f+c*d)^2)/(c*x^2+b*
x+a)^(1/2)+2*b*(c*x^2+b*x+a)^(1/2)/c/(-4*a*c+b^2)/f
```

Rubi [A] time = 1.35, antiderivative size = 466, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.321, Rules used = {6725, 613, 738, 640, 621, 206, 975, 1033, 724}

$$-\frac{2d^2\left(b\left(b^2f-c(3af+cd)\right)-cx\left(2acf+b^2(-f)+2c^2d\right)\right)}{f^2\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left(b^2df-(af+cd)^2\right)}+\frac{2d(b+2cx)}{f^2\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}+\frac{2b\sqrt{a+bx+cx^2}}{cf\left(b^2-4ac\right)}-\frac{}{f(}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((a + b*x + c*x^2)^(3/2)*(d - f*x^2)),x]
```

```
[Out] (-2*x*(2*a + b*x))/((b^2 - 4*a*c)*f*Sqrt[a + b*x + c*x^2]) + (2*d*(b + 2*c*
x))/((b^2 - 4*a*c)*f^2*Sqrt[a + b*x + c*x^2]) - (2*d^2*(b*(b^2*f - c*(c*d +
 3*a*f)) - c*(2*c^2*d - b^2*f + 2*a*c*f)*x))/((b^2 - 4*a*c)*f^2*(b^2*d*f -
(c*d + a*f)^2)*Sqrt[a + b*x + c*x^2]) + (2*b*Sqrt[a + b*x + c*x^2])/(c*(b^2
 - 4*a*c)*f) - ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])]/(c^(3
/2)*f) + (d^(3/2)*ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[
f])*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*f
*(c*d - b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)) + (d^(3/2)*ArcTanh[(b*Sqrt[d] + 2*a
*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a
*f]*Sqrt[a + b*x + c*x^2])])/(2*f*(c*d + b*Sqrt[d]*Sqrt[f] + a*f)^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 613

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[(-2*(b +
2*c*x))/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
```

```
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 738

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
+ c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*
c)), Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*c
*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p +
 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 1] &&
IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 975

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (f_.)*(x_)^2)^(q_), x
_Symbol] :> Simp[((b^3*f + b*c*(c*d - 3*a*f) + c*(2*c^2*d + b^2*f - c*(2*a*
f))*x)*(a + b*x + c*x^2)^(p + 1)*(d + f*x^2)^(q + 1))/((b^2 - 4*a*c)*(b^2*d
*f + (c*d - a*f)^2)*(p + 1)), x] - Dist[1/((b^2 - 4*a*c)*(b^2*d*f + (c*d -
a*f)^2)*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + f*x^2)^q*Simp[2*c*(b^2
*d*f + (c*d - a*f)^2)*(p + 1) - (2*c^2*d + b^2*f - c*(2*a*f))*(a*f*(p + 1)
- c*d*(p + 2)) + (2*f*(b^3*f + b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d +
b^2*f - c*(2*a*f))*(b*f*(p + 1)))*x + c*f*(2*c^2*d + b^2*f - c*(2*a*f))*(2*
p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, q}, x] && NeQ[b^2 - 4
*a*c, 0] && LtQ[p, -1] && NeQ[b^2*d*f + (c*d - a*f)^2, 0] &&  !( !IntegerQ[
p] && ILtQ[q, -1]) &&  !IGtQ[q, 0]
```

Rule 1033

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q
), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q
), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f
, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{x^4}{\left(a+bx+cx^2\right)^{3/2}\left(d-fx^2\right)}dx = \int \left(-\frac{d}{f^2\left(a+bx+cx^2\right)^{3/2}} - \frac{x^2}{f\left(a+bx+cx^2\right)^{3/2}} + \frac{d^2}{f^2\left(a+bx+cx^2\right)^{3/2}\left(d-f\right.}\right.$$

$$= -\frac{d\int \frac{1}{\left(a+bx+cx^2\right)^{3/2}}dx}{f^2} + \frac{d^2\int \frac{1}{\left(a+bx+cx^2\right)^{3/2}\left(d-fx^2\right)}dx}{f^2} - \frac{\int \frac{x^2}{\left(a+bx+cx^2\right)^{3/2}}dx}{f}$$

$$= -\frac{2x(2a+bx)}{\left(b^2-4ac\right)f\sqrt{a+bx+cx^2}} + \frac{2d(b+2cx)}{\left(b^2-4ac\right)f^2\sqrt{a+bx+cx^2}} - \frac{2d^2\left(b\left(b^2f-\right.\right.}{\left(b^2-4ac\right)f}$$

$$= -\frac{2x(2a+bx)}{\left(b^2-4ac\right)f\sqrt{a+bx+cx^2}} + \frac{2d(b+2cx)}{\left(b^2-4ac\right)f^2\sqrt{a+bx+cx^2}} - \frac{2d^2\left(b\left(b^2f-\right.\right.}{\left(b^2-4ac\right)f}$$

$$= -\frac{2x(2a+bx)}{\left(b^2-4ac\right)f\sqrt{a+bx+cx^2}} + \frac{2d(b+2cx)}{\left(b^2-4ac\right)f^2\sqrt{a+bx+cx^2}} - \frac{2d^2\left(b\left(b^2f-\right.\right.}{\left(b^2-4ac\right)f}$$

$$= -\frac{2x(2a+bx)}{\left(b^2-4ac\right)f\sqrt{a+bx+cx^2}} + \frac{2d(b+2cx)}{\left(b^2-4ac\right)f^2\sqrt{a+bx+cx^2}} - \frac{2d^2\left(b\left(b^2f-\right.\right.}{\left(b^2-4ac\right)f}$$

Mathematica [A] time = 1.41, size = 562, normalized size = 1.21

$$\frac{\frac{f\left(a\left(b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)-2\sqrt{c}\left(a(b-2cx)+bcx^2\right)\sqrt{a+x(b+cx)}\right)}{ac^{3/2}\left(4ac-b^2\right)} + \frac{d^{3/2}f\left(\frac{\left(b^2-4ac\right)\left(af+b\sqrt{d}\sqrt{f}+cd\right)\tanh^{-1}\left(\frac{-2a\sqrt{f}+b\left(\sqrt{d}-\sqrt{f}x\right)+2c\sqrt{d}x}{2\sqrt{a+x(b+cx)}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}+\right.}{2\left(b^2-4ac\right)\left((af+c\right.}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/((a + b*x + c*x^2)^(3/2)*(d - f*x^2)),x]
```

```
[Out] ((2*d*(b + 2*c*x))/((b^2 - 4*a*c)*Sqrt[a + x*(b + c*x)]) - (2*f*x^3*(b^2 -
2*a*c + b*c*x))/(a*(b^2 - 4*a*c)*Sqrt[a + x*(b + c*x)]) - (2*d^2*(b^3*f - b
*c*(c*d + 3*a*f) + b^2*c*f*x - 2*c^2*(c*d + a*f)*x))/((b^2 - 4*a*c)*(b^2*d*
f - (c*d + a*f)^2)*Sqrt[a + x*(b + c*x)]) + (f*(-2*Sqrt[c]*(b*c*x^2 + a*(b
- 2*c*x))*Sqrt[a + x*(b + c*x)] + a*(b^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*Sq
rt[c]*Sqrt[a + x*(b + c*x)])]))/(a*c^(3/2)*(-b^2 + 4*a*c)) + (d^(3/2)*f*(((
b^2 - 4*a*c)*(c*d + b*Sqrt[d]*Sqrt[f] + a*f)*ArcTanh[(-2*a*Sqrt[f] + 2*c*Sq
rt[d]*x + b*(Sqrt[d] - Sqrt[f]*x))/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*S
qrt[a + x*(b + c*x)])]])/Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f] + ((-b^2 + 4*a*
c)*(c*d - b*Sqrt[d]*Sqrt[f] + a*f)*ArcTanh[(-2*(a*Sqrt[f] + c*Sqrt[d]*x) -
b*(Sqrt[d] + Sqrt[f]*x))/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*
(b + c*x)])]])/Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]))/(2*(b^2 - 4*a*c)*(-(b^2
*d*f) + (c*d + a*f)^2)))/f^2
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [B] time = 0.02, size = 1648, normalized size = 3.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x)
```

```
[Out] 1/f*x/c/(c*x^2+b*x+a)^(1/2)-1/2/f*b/c^2/(c*x^2+b*x+a)^(1/2)-1/f*b^2/c/(4*a*
c-b^2)/(c*x^2+b*x+a)^(1/2)*x-1/2/f*b^3/c^2/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)-
1/f/c^(3/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-4/f^2*d/(4*a*c-b^2)
/(c*x^2+b*x+a)^(1/2)*c*x-2/f^2*d/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*b+1/2/f*d^
2/(d*f)^(1/2)/(a*f+c*d-(d*f)^(1/2)*b)/((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(
1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)+2/f^2*d^2/(a*f
+c*d-(d*f)^(1/2)*b)/(4*a*c-b^2)/((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c
)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*x*c^2-1/f*d^2/(d*f)^
(1/2)/(a*f+c*d-(d*f)^(1/2)*b)/(4*a*c-b^2)/((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*
f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*x*b*c+1/f^
2*d^2/(a*f+c*d-(d*f)^(1/2)*b)/(4*a*c-b^2)/((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*
f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*b*c-1/2/f*
d^2/(d*f)^(1/2)/(a*f+c*d-(d*f)^(1/2)*b)/(4*a*c-b^2)/((x+(d*f)^(1/2)/f)^2*c+
(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*
b^2-1/2/f*d^2/(d*f)^(1/2)/(a*f+c*d-(d*f)^(1/2)*b)/((a*f+c*d-(d*f)^(1/2)*b)/
f)^(1/2)*ln((2*(a*f+c*d-(d*f)^(1/2)*b)/f+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/
2)/f)/f+2*((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(
d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))/(x+(d*f
)^(1/2)/f))-1/2/f*d^2/(d*f)^(1/2)/(a*f+c*d+(d*f)^(1/2)*b)/((x-(d*f)^(1/2)/f
)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^
(1/2)+2/f^2*d^2/(a*f+c*d+(d*f)^(1/2)*b)/(4*a*c-b^2)/((x-(d*f)^(1/2)/f)^2*c+
(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*
x*c^2+1/f*d^2/(d*f)^(1/2)/(a*f+c*d+(d*f)^(1/2)*b)/(4*a*c-b^2)/((x-(d*f)^(1/
2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)
/f)^(1/2)*x*b*c+1/f^2*d^2/(a*f+c*d+(d*f)^(1/2)*b)/(4*a*c-b^2)/((x-(d*f)^(1/
2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)
/f)^(1/2)*b*c+1/2/f*d^2/(d*f)^(1/2)/(a*f+c*d+(d*f)^(1/2)*b)/(4*a*c-b^2)/((x
-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f
)^(1/2)*b)/f)^(1/2)*b^2+1/2/f*d^2/(d*f)^(1/2)/(a*f+c*d+(d*f)^(1/2)*b)/((a*f
+c*d+(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d+(d*f)^(1/2)*b)/f+(b*f+2*(d*f)^(
1/2)*c)*(x-(d*f)^(1/2)/f)/f+2*((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*((x-(d*f)^(
1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*
b)/f)^(1/2))/(x-(d*f)^(1/2)/f))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(((c*sqrt(4*d*f))/(2*f^2)>0)', see `
assume?` for more details)Is ((c*sqrt(4*d*f))/(2*f^2)    +b/(2*f))    ^2
 -(c*((b*sqrt(4*d*f))                 /(2*f)                  +(c*d)/f+a))
     /f^2 zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4}{\left(d-f\,x^2\right)\,\left(c\,x^2+b\,x+a\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((d - f*x^2)*(a + b*x + c*x^2)^(3/2)),x)
```

```
[Out] int(x^4/((d - f*x^2)*(a + b*x + c*x^2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^4}{-ad\sqrt{a+bx+cx^2}+afx^2\sqrt{a+bx+cx^2}-bdx\sqrt{a+bx+cx^2}+bfx^3\sqrt{a+bx+cx^2}-cdx^2\sqrt{a+bx+cx^2}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(c*x**2+b*x+a)**(3/2)/(-f*x**2+d),x)
```

```
[Out] -Integral(x**4/(-a*d*sqrt(a + b*x + c*x**2) + a*f*x**2*sqrt(a + b*x + c*x**
2) - b*d*x*sqrt(a + b*x + c*x**2) + b*f*x**3*sqrt(a + b*x + c*x**2) - c*d*x
**2*sqrt(a + b*x + c*x**2) + c*f*x**4*sqrt(a + b*x + c*x**2)), x)
```

3.103 $\int \frac{x^3}{\left(a+bx+cx^2\right)^{3/2}\left(d-fx^2\right)} \, dx$

Optimal. Leaf size=341

$$-\frac{2d\left(a\left(2acf+b^2(-f)+2c^2d\right)+bcx(cd-af)\right)}{f\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left(b^2df-(af+cd)^2\right)}-\frac{2(2a+bx)}{f\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}-\frac{d\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+\dots}}\right)}{2\sqrt{f}\left(af+b\left(-\sqrt{d}\right)\sqrt{f}+\dots\right)}$$

```
[Out] -1/2*d*arctanh(1/2*(b*d^(1/2)-2*a*f^(1/2)+x*(2*c*d^(1/2)-b*f^(1/2)))/(c*x^2
+b*x+a)^(1/2)/(c*d+a*f-b*d^(1/2)*f^(1/2))^(1/2))/f^(1/2)/(c*d+a*f-b*d^(1/2)
*f^(1/2))^(3/2)+1/2*d*arctanh(1/2*(b*d^(1/2)+2*a*f^(1/2)+x*(2*c*d^(1/2)+b*f
^(1/2)))/(c*x^2+b*x+a)^(1/2)/(c*d+a*f+b*d^(1/2)*f^(1/2))^(1/2))/f^(1/2)/(c*
d+a*f+b*d^(1/2)*f^(1/2))^(3/2)-2*(b*x+2*a)/(-4*a*c+b^2)/f/(c*x^2+b*x+a)^(1/
2)-2*d*(a*(2*a*c*f-b^2*f+2*c^2*d)+b*c*(-a*f+c*d)*x)/(-4*a*c+b^2)/f/(b^2*d*f
-(a*f+c*d)^2)/(c*x^2+b*x+a)^(1/2)
```

Rubi [A] time = 1.04, antiderivative size = 341, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {6725, 636, 1018, 1033, 724, 206}

$$-\frac{2d\left(a\left(2acf+b^2(-f)+2c^2d\right)+bcx(cd-af)\right)}{f\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left(b^2df-(af+cd)^2\right)}-\frac{2(2a+bx)}{f\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}-\frac{d\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+\dots}}\right)}{2\sqrt{f}\left(af+b\left(-\sqrt{d}\right)\sqrt{f}+\dots\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((a + b*x + c*x^2)^(3/2)*(d - f*x^2)),x]
```

```
[Out] (-2*(2*a + b*x))/((b^2 - 4*a*c)*f*Sqrt[a + b*x + c*x^2]) - (2*d*(a*(2*c^2*d
 - b^2*f + 2*a*c*f) + b*c*(c*d - a*f)*x))/((b^2 - 4*a*c)*f*(b^2*d*f - (c*d
+ a*f)^2)*Sqrt[a + b*x + c*x^2]) - (d*ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2
*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a +
b*x + c*x^2])])/(2*Sqrt[f]*(c*d - b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)) + (d*ArcT
anh[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d + b
*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[f]*(c*d + b*Sqrt[d
]*Sqrt[f] + a*f)^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 636

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[(-2*(b*d - 2*a*e + (2*c*d - b*e)*x))/((b^2 - 4*a*c)*Sqrt[a + b*x
 + c*x^2]), x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b
^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1018

```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (f
_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((a + b*x + c*x^2)^(p + 1)*(d + f*x^2)^
(q + 1)*((g*c)*(-(b*(c*d + a*f))) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(2*a*f
)) + c*(g*(2*c^2*d + b^2*f - c*(2*a*f)) - h*(b*c*d + a*b*f))*x))/((b^2 - 4*
a*c)*(b^2*d*f + (c*d - a*f)^2)*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*(b^2*d*
f + (c*d - a*f)^2)*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + f*x^2)^q*Si
mp[(b*h - 2*g*c)*((c*d - a*f)^2 - (b*d)*(-(b*f)))*(p + 1) + (b^2*(g*f) - b*
(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(a*f*(p + 1) - c*d*(p + 2)) - (2*f*(
(g*c)*(-(b*(c*d + a*f))) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(2*a*f)))*(p +
q + 2) - (b^2*(g*f) - b*(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(b*f*(p + 1)
))*x - c*f*(b^2*(g*f) - b*(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(2*p + 2*q
 + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, g, h, q}, x] && NeQ[b^2 - 4*
a*c, 0] && LtQ[p, -1] && NeQ[b^2*d*f + (c*d - a*f)^2, 0] &&  !( !IntegerQ[p
] && ILtQ[q, -1])
```

Rule 1033

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q
), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q
), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f
, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{x^3}{\left(a+bx+cx^2\right)^{3/2}\left(d-fx^2\right)}\,dx = \int \left(-\frac{x}{f\left(a+bx+cx^2\right)^{3/2}}+\frac{dx}{f\left(a+bx+cx^2\right)^{3/2}\left(d-fx^2\right)}\right)dx$$

$$= -\frac{\int \frac{x}{\left(a+bx+cx^2\right)^{3/2}}\,dx}{f}+\frac{d\int \frac{x}{\left(a+bx+cx^2\right)^{3/2}\left(d-fx^2\right)}\,dx}{f}$$

$$= -\frac{2(2a+bx)}{\left(b^2-4ac\right)f\sqrt{a+bx+cx^2}}-\frac{2d\left(a\left(2c^2d-b^2f+2acf\right)+bc(cd-af)x\right)}{\left(b^2-4ac\right)f\left(b^2df-(cd+af)^2\right)\sqrt{a+bx+c}}$$

$$= -\frac{2(2a+bx)}{\left(b^2-4ac\right)f\sqrt{a+bx+cx^2}}-\frac{2d\left(a\left(2c^2d-b^2f+2acf\right)+bc(cd-af)x\right)}{\left(b^2-4ac\right)f\left(b^2df-(cd+af)^2\right)\sqrt{a+bx+c}}$$

$$= -\frac{2(2a+bx)}{\left(b^2-4ac\right)f\sqrt{a+bx+cx^2}}-\frac{2d\left(a\left(2c^2d-b^2f+2acf\right)+bc(cd-af)x\right)}{\left(b^2-4ac\right)f\left(b^2df-(cd+af)^2\right)\sqrt{a+bx+c}}$$

$$= -\frac{2(2a+bx)}{\left(b^2-4ac\right)f\sqrt{a+bx+cx^2}}-\frac{2d\left(a\left(2c^2d-b^2f+2acf\right)+bc(cd-af)x\right)}{\left(b^2-4ac\right)f\left(b^2df-(cd+af)^2\right)\sqrt{a+bx+c}}$$

Mathematica [A] time = 1.20, size = 414, normalized size = 1.21

$$\frac{1}{2}\left(\frac{8a^3f+4a^2(bfx+2cd)-4abd(b-3cx)-4b^3dx}{(b^2-4ac)\sqrt{a+x(b+cx)}\left(f\left(b^2d-a^2f\right)-2acdf-c^2d^2\right)}-\frac{d\log\left(\sqrt{d}\sqrt{f}-fx\right)}{\sqrt{f}\left(af+b\sqrt{d}\sqrt{f}+cd\right)^{3/2}}-\frac{d\log\left(\sqrt{d}\right.}{\sqrt{f}\left(af+b\left(-\right.\right.}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((a + b*x + c*x^2)^(3/2)*(d - f*x^2)),x]
```

```
[Out] ((8*a^3*f - 4*b^3*d*x - 4*a*b*d*(b - 3*c*x) + 4*a^2*(2*c*d + b*f*x))/((b^2
- 4*a*c)*(-(c^2*d^2) - 2*a*c*d*f + f*(b^2*d - a^2*f))*Sqrt[a + x*(b + c*x)]
) - (d*Log[Sqrt[d]*Sqrt[f] - f*x])/(Sqrt[f]*(c*d + b*Sqrt[d]*Sqrt[f] + a*f)
^(3/2)) - (d*Log[Sqrt[d]*Sqrt[f] + f*x])/(Sqrt[f]*(c*d - b*Sqrt[d]*Sqrt[f]
+ a*f)^(3/2)) + (d*Log[Sqrt[d]*(-(b*Sqrt[d]) + 2*a*Sqrt[f] - 2*c*Sqrt[d]*x
+ b*Sqrt[f]*x + 2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)]
)])/(Sqrt[f]*(c*d - b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)) + (d*Log[Sqrt[d]*(b*(Sq
rt[d] + Sqrt[f]*x) + 2*(a*Sqrt[f] + c*Sqrt[d]*x + Sqrt[c*d + b*Sqrt[d]*Sqrt
[f] + a*f]*Sqrt[a + x*(b + c*x)]))])/(Sqrt[f]*(c*d + b*Sqrt[d]*Sqrt[f] + a*
f)^(3/2)))/2
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Valueint()  Bad Argument Type
```

maple [B] time = 0.02, size = 1480, normalized size = 4.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x)
```

```
[Out] 1/f/c/(c*x^2+b*x+a)^(1/2)+2/f*b/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x+1/f*b^2/c
/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)-1/2/f*d/(a*f+c*d-(d*f)^(1/2)*b)/((x+(d*f)^
(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)
*b)/f)^(1/2)-2/f^2*d/(a*f+c*d-(d*f)^(1/2)*b)/(4*a*c-b^2)/((x+(d*f)^(1/2)/f)
^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(
1/2)*(d*f)^(1/2)*x*c^2+1/f*d/(a*f+c*d-(d*f)^(1/2)*b)/(4*a*c-b^2)/((x+(d*f)^
(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)
*b)/f)^(1/2)*x*b*c-1/f^2*d/(a*f+c*d-(d*f)^(1/2)*b)/(4*a*c-b^2)/((x+(d*f)^(1
/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b
)/f)^(1/2)*(d*f)^(1/2)*b*c+1/2/f*d/(a*f+c*d-(d*f)^(1/2)*b)/(4*a*c-b^2)/((x+
```

```
(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)
^(1/2)*b)/f)^(1/2)*b^2+1/2/f*d/(a*f+c*d-(d*f)^(1/2)*b)/((a*f+c*d-(d*f)^(1/2
)*b)/f)^(1/2)*ln((2*(a*f+c*d-(d*f)^(1/2)*b)/f+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f
)^(1/2)/f)/f+2*((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*((x+(d*f)^(1/2)/f)^2*c+(b*
f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))/(x
+(d*f)^(1/2)/f))-1/2/f*d/(a*f+c*d+(d*f)^(1/2)*b)/((x-(d*f)^(1/2)/f)^2*c+(b*
f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)+2/f
^2*d/(a*f+c*d+(d*f)^(1/2)*b)/(4*a*c-b^2)/((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f
)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*(d*f)^(1/2)
*x*c^2+1/f*d/(a*f+c*d+(d*f)^(1/2)*b)/(4*a*c-b^2)/((x-(d*f)^(1/2)/f)^2*c+(b*
f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*x*b
*c+1/f^2*d/(a*f+c*d+(d*f)^(1/2)*b)/(4*a*c-b^2)/((x-(d*f)^(1/2)/f)^2*c+(b*f+
2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*(d*f)
^(1/2)*b*c+1/2/f*d/(a*f+c*d+(d*f)^(1/2)*b)/(4*a*c-b^2)/((x-(d*f)^(1/2)/f)^2
*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/
2)*b^2+1/2/f*d/(a*f+c*d+(d*f)^(1/2)*b)/((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*ln
((2*(a*f+c*d+(d*f)^(1/2)*b)/f+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+2*(
(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*
c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))/(x-(d*f)^(1/2)/f))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(((c*sqrt(4*d*f))/(2*f^2)>0)', see `
assume?` for more details)Is ((c*sqrt(4*d*f))/(2*f^2)    +b/(2*f))    ^2
 -(c*((b*sqrt(4*d*f))                  /(2*f)                  +(c*d)/f+a))
     /f^2 zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3}{\left(d-f\,x^2\right)\,{\left(c\,x^2+b\,x+a\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((d - f*x^2)*(a + b*x + c*x^2)^(3/2)),x)
```

```
[Out] int(x^3/((d - f*x^2)*(a + b*x + c*x^2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^3}{-ad\sqrt{a+bx+cx^2}+afx^2\sqrt{a+bx+cx^2}-bdx\sqrt{a+bx+cx^2}+bfx^3\sqrt{a+bx+cx^2}-cdx^2\sqrt{a+bx+cx^2}+\dots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(c*x**2+b*x+a)**(3/2)/(-f*x**2+d),x)
```

```
[Out] -Integral(x**3/(-a*d*sqrt(a + b*x + c*x**2) + a*f*x**2*sqrt(a + b*x + c*x**
2) - b*d*x*sqrt(a + b*x + c*x**2) + b*f*x**3*sqrt(a + b*x + c*x**2) - c*d*x
**2*sqrt(a + b*x + c*x**2) + c*f*x**4*sqrt(a + b*x + c*x**2)), x)
```

3.104 $\int \frac{x^2}{\left(a+bx+cx^2\right)^{3/2}\left(d-fx^2\right)}\,dx$

Optimal. Leaf size=297

$$\frac{2\left(cx\left(b^2d-2a(af+cd)\right)+ab(cd-af)\right)}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left(b^2df-(af+cd)^2\right)}+\frac{\sqrt{d}\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2\left(af+b\left(-\sqrt{d}\right)\sqrt{f}+cd\right)^{3/2}}+\frac{\sqrt{d}\tanh^{-1}\left(\frac{2a\sqrt{f}}{2\sqrt{a+b}}\right.}{2\left(af+b\sqrt{\right.}}$$

```
[Out] 1/2*arctanh(1/2*(b*d^(1/2)-2*a*f^(1/2)+x*(2*c*d^(1/2)-b*f^(1/2)))/(c*x^2+b*
x+a)^(1/2)/(c*d+a*f-b*d^(1/2)*f^(1/2))^(1/2))*d^(1/2)/(c*d+a*f-b*d^(1/2)*f^
(1/2))^(3/2)+1/2*arctanh(1/2*(b*d^(1/2)+2*a*f^(1/2)+x*(2*c*d^(1/2)+b*f^(1/2
)))/(c*x^2+b*x+a)^(1/2)/(c*d+a*f+b*d^(1/2)*f^(1/2))^(1/2))*d^(1/2)/(c*d+a*f
+b*d^(1/2)*f^(1/2))^(3/2)+2*(a*b*(-a*f+c*d)+c*(b^2*d-2*a*(a*f+c*d))*x)/(-4*
a*c+b^2)/(b^2*d*f-(a*f+c*d)^2)/(c*x^2+b*x+a)^(1/2)
```

Rubi [A] time = 0.45, antiderivative size = 297, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1065, 1033, 724, 206}

$$\frac{2\left(cx\left(b^2d-2a(af+cd)\right)+ab(cd-af)\right)}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left(b^2df-(af+cd)^2\right)}+\frac{\sqrt{d}\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2\left(af+b\left(-\sqrt{d}\right)\sqrt{f}+cd\right)^{3/2}}+\frac{\sqrt{d}\tanh^{-1}\left(\frac{2a\sqrt{f}}{2\sqrt{a+b}}\right.}{2\left(af+b\sqrt{\right.}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((a + b*x + c*x^2)^(3/2)*(d - f*x^2)),x]
```

```
[Out] (2*(a*b*(c*d - a*f) + c*(b^2*d - 2*a*(c*d + a*f))*x))/((b^2 - 4*a*c)*(b^2*d
*f - (c*d + a*f)^2)*Sqrt[a + b*x + c*x^2]) + (Sqrt[d]*ArcTanh[(b*Sqrt[d] -
2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f]
+ a*f]*Sqrt[a + b*x + c*x^2])])/(2*(c*d - b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)) +
 (Sqrt[d]*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(
2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*(c*d + b*
Sqrt[d]*Sqrt[f] + a*f)^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1033

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q
), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q
), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f
, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]
```

Rule 1065

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (C_.)*(x_)^2)*((d_) +
(f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((a + b*x + c*x^2)^(p + 1)*(d + f*x^2
)^(q + 1)*((A*c - a*C)*(-(b*(c*d + a*f))) + (A*b)*(2*c^2*d + b^2*f - c*(2*a
*f)) + c*(A*(2*c^2*d + b^2*f - c*(2*a*f)) + C*(b^2*d - 2*a*(c*d - a*f)))*x)
)/((b^2 - 4*a*c)*(b^2*d*f + (c*d - a*f)^2)*(p + 1)), x] + Dist[1/((b^2 - 4*
a*c)*(b^2*d*f + (c*d - a*f)^2)*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d +
 f*x^2)^q*Simp[(-2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d)*(-(b*f)))*(p + 1) +
(b^2*(C*d + A*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - a*C*f)))*(a*f*(p + 1) -
c*d*(p + 2)) - (2*f*((A*c - a*C)*(-(b*(c*d + a*f))) + (A*b)*(2*c^2*d + b^2*
f - c*(2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) + 2*(A*c*(c*d - a*f) - a*(c*
C*d - a*C*f)))*(b*f*(p + 1)))*x - c*f*(b^2*(C*d + A*f) + 2*(A*c*(c*d - a*f)
 - a*(c*C*d - a*C*f)))*(2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d
, f, A, C, q}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[b^2*d*f + (c*
d - a*f)^2, 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[q, 0]
```

Rubi steps

$$\int \frac{x^2}{\left(a+bx+cx^2\right)^{3/2}\left(d-fx^2\right)}dx = \frac{2\left(ab(cd-af)+c\left(b^2d-2a(cd+af)\right)x\right)}{\left(b^2-4ac\right)\left(b^2df-(cd+af)^2\right)\sqrt{a+bx+cx^2}} + \frac{2\int \frac{-\frac{1}{2}\left(b^2-4ac\right)d(cd+af)+\frac{1}{2}b\left(b^2-4 \cdots\right.}{\sqrt{a+bx+cx^2}\left(d-fx^2\right)}}{\left(b^2-4ac\right)\left(b^2df-(cd+\cdots\right.}$$

$$= \frac{2\left(ab(cd-af)+c\left(b^2d-2a(cd+af)\right)x\right)}{\left(b^2-4ac\right)\left(b^2df-(cd+af)^2\right)\sqrt{a+bx+cx^2}} - \frac{\left(\sqrt{d}\sqrt{f}\right)\int \frac{1}{\left(-\sqrt{d}\sqrt{f}-fx\right)\sqrt{a+b\cdots}}}{2\left(cd-b\sqrt{d}\sqrt{f}+af\cdots\right.}$$

$$= \frac{2\left(ab(cd-af)+c\left(b^2d-2a(cd+af)\right)x\right)}{\left(b^2-4ac\right)\left(b^2df-(cd+af)^2\right)\sqrt{a+bx+cx^2}} + \frac{\left(\sqrt{d}\sqrt{f}\right)\text{Subst}\left(\int \frac{\cdots}{4cdf-4b\sqrt{\cdots}}\right.}{\cdots}$$

$$= \frac{2\left(ab(cd-af)+c\left(b^2d-2a(cd+af)\right)x\right)}{\left(b^2-4ac\right)\left(b^2df-(cd+af)^2\right)\sqrt{a+bx+cx^2}} + \frac{\sqrt{d}\tanh^{-1}\left(\frac{b\sqrt{d}-2a\sqrt{f}+(2c\cdots}{2\sqrt{cd-b\sqrt{d}\sqrt{f}+af\cdots}}\right.}{2\left(cd-b\sqrt{d}\sqrt{f}+a\cdots\right.}$$

Mathematica [A] time = 0.40, size = 352, normalized size = 1.19

$$\frac{2\left(\frac{a^2f(b+2cx)+acd(2cx-b)-b^2cdx}{\sqrt{a+x(b+cx)}}+\frac{\sqrt{d}\left(b^2-4ac\right)\left(af+b\sqrt{d}\sqrt{f}+cd\right)\tanh^{-1}\left(\frac{-2a\sqrt{f}+b\left(\sqrt{d}-\sqrt{f}x\right)+2c\sqrt{d}x}{2\sqrt{a+x(b+cx)}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{4\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}+\frac{\sqrt{d}\left(4ac-b^2\right)\left(af+b\left(-\sqrt{d}\right)\sqrt{f}+cd\right)\cdots}{4\sqrt{af+\cdots}}\right)}{\left(b^2-4ac\right)\left((af+cd)^2-b^2df\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((a + b*x + c*x^2)^(3/2)*(d - f*x^2)),x]
```

```
[Out] (2*((-(b^2*c*d*x) + a*c*d*(-b + 2*c*x) + a^2*f*(b + 2*c*x))/Sqrt[a + x*(b +
 c*x)] + ((b^2 - 4*a*c)*Sqrt[d]*(c*d + b*Sqrt[d]*Sqrt[f] + a*f)*ArcTanh[(-2
*a*Sqrt[f] + 2*c*Sqrt[d]*x + b*(Sqrt[d] - Sqrt[f]*x))/(2*Sqrt[c*d - b*Sqrt[
d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])])/(4*Sqrt[c*d - b*Sqrt[d]*Sqrt[f]
+ a*f]) + ((-b^2 + 4*a*c)*Sqrt[d]*(c*d - b*Sqrt[d]*Sqrt[f] + a*f)*ArcTanh[(
-2*(a*Sqrt[f] + c*Sqrt[d]*x) - b*(Sqrt[d] + Sqrt[f]*x))/(2*Sqrt[c*d + b*Sqr
t[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])])/(4*Sqrt[c*d + b*Sqrt[d]*Sqrt[f
] + a*f]))/((b^2 - 4*a*c)*(-(b^2*d*f) + (c*d + a*f)^2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Valueint()  Bad Argument Type
```

maple [B] time = 0.02, size = 1427, normalized size = 4.80

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x)
```

```
[Out] -2/f*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)+1/2*d/(d*f)^(1/2)/(a*f+c*d-(
d*f)^(1/2)*b)/((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f
)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)+2*d/f/(a*f+c*d-(d*f)^(1/2)*b)/(4*a*c-b
^2)/((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c
*d-(d*f)^(1/2)*b)/f)^(1/2)*x*c^2-d/(d*f)^(1/2)/(a*f+c*d-(d*f)^(1/2)*b)/(4*a
*c-b^2)/((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a
*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*x*b*c+d/f/(a*f+c*d-(d*f)^(1/2)*b)/(4*a*c-b^2
)/((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d
-(d*f)^(1/2)*b)/f)^(1/2)*b*c-1/2*d/(d*f)^(1/2)/(a*f+c*d-(d*f)^(1/2)*b)/(4*a
*c-b^2)/((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a
*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*b^2-1/2*d/(d*f)^(1/2)/(a*f+c*d-(d*f)^(1/2)*b
)/((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d-(d*f)^(1/2)*b)/f+(b*f-2*
(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+2*((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*((x+
(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)
^(1/2)*b)/f)^(1/2))/(x+(d*f)^(1/2)/f))-1/2*d/(d*f)^(1/2)/(a*f+c*d+(d*f)^(1/
2)*b)/((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f
+c*d+(d*f)^(1/2)*b)/f)^(1/2)+2*d/f/(a*f+c*d+(d*f)^(1/2)*b)/(4*a*c-b^2)/((x-
(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)
^(1/2)*b)/f)^(1/2)*x*c^2+d/(d*f)^(1/2)/(a*f+c*d+(d*f)^(1/2)*b)/(4*a*c-b^2)/
((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(
d*f)^(1/2)*b)/f)^(1/2)*x*b*c+d/f/(a*f+c*d+(d*f)^(1/2)*b)/(4*a*c-b^2)/((x-(d
*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(
1/2)*b)/f)^(1/2)*b*c+1/2*d/(d*f)^(1/2)/(a*f+c*d+(d*f)^(1/2)*b)/(4*a*c-b^2)/
((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(
d*f)^(1/2)*b)/f)^(1/2)*b^2+1/2*d/(d*f)^(1/2)/(a*f+c*d+(d*f)^(1/2)*b)/((a*f+
c*d+(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d+(d*f)^(1/2)*b)/f+(b*f+2*(d*f)^(1
/2)*c)*(x-(d*f)^(1/2)/f)/f+2*((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*((x-(d*f)^(1
/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b
)/f)^(1/2))/(x-(d*f)^(1/2)/f))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(((c*sqrt(4*d*f))/(2*f^2)>0)', see `
assume?` for more details)Is ((c*sqrt(4*d*f))/(2*f^2)    +b/(2*f))    ^2
 -(c*((b*sqrt(4*d*f))                   /(2*f)                +(c*d)/f+a))
     /f^2 zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2}{\left(d-f\,x^2\right)\,{\left(c\,x^2+b\,x+a\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((d - f*x^2)*(a + b*x + c*x^2)^(3/2)),x)
```

```
[Out] int(x^2/((d - f*x^2)*(a + b*x + c*x^2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^2}{-ad\sqrt{a+bx+cx^2}+afx^2\sqrt{a+bx+cx^2}-bdx\sqrt{a+bx+cx^2}+bfx^3\sqrt{a+bx+cx^2}-cdx^2\sqrt{a+bx+cx^2}+\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(c*x**2+b*x+a)**(3/2)/(-f*x**2+d),x)
```

```
[Out] -Integral(x**2/(-a*d*sqrt(a + b*x + c*x**2) + a*f*x**2*sqrt(a + b*x + c*x**
2) - b*d*x*sqrt(a + b*x + c*x**2) + b*f*x**3*sqrt(a + b*x + c*x**2) - c*d*x
**2*sqrt(a + b*x + c*x**2) + c*f*x**4*sqrt(a + b*x + c*x**2)), x)
```

3.105 $\int \frac{x}{\left(a+bx+cx^2\right)^{3/2}\left(d-fx^2\right)}\,dx$

Optimal. Leaf size=299

$$-\frac{2\left(a\left(2acf+b^2(-f)+2c^2d\right)+bcx(cd-af)\right)}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left(b^2df-(af+cd)^2\right)}-\frac{\sqrt{f}\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2\left(af+b\left(-\sqrt{d}\right)\sqrt{f}+cd\right)^{3/2}}+\frac{\sqrt{f}\tanh^{-1}\left(\frac{2a\ldots}{2\sqrt{a+\ldots}}\right)}{2\left(af+b\ldots\right.}$$

```
[Out] -1/2*arctanh(1/2*(b*d^(1/2)-2*a*f^(1/2)+x*(2*c*d^(1/2)-b*f^(1/2)))/(c*x^2+b
*x+a)^(1/2)/(c*d+a*f-b*d^(1/2)*f^(1/2))^(1/2))*f^(1/2)/(c*d+a*f-b*d^(1/2)*f
^(1/2))^(3/2)+1/2*arctanh(1/2*(b*d^(1/2)+2*a*f^(1/2)+x*(2*c*d^(1/2)+b*f^(1/
2)))/(c*x^2+b*x+a)^(1/2)/(c*d+a*f+b*d^(1/2)*f^(1/2))^(1/2))*f^(1/2)/(c*d+a*
f+b*d^(1/2)*f^(1/2))^(3/2)-2*(a*(2*a*c*f-b^2*f+2*c^2*d)+b*c*(-a*f+c*d)*x)/(
-4*a*c+b^2)/(b^2*d*f-(a*f+c*d)^2)/(c*x^2+b*x+a)^(1/2)
```

Rubi [A] time = 0.40, antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1018, 1033, 724, 206}

$$-\frac{2\left(a\left(2acf+b^2(-f)+2c^2d\right)+bcx(cd-af)\right)}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left(b^2df-(af+cd)^2\right)}-\frac{\sqrt{f}\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2\left(af+b\left(-\sqrt{d}\right)\sqrt{f}+cd\right)^{3/2}}+\frac{\sqrt{f}\tanh^{-1}\left(\frac{2a\ldots}{2\sqrt{a+\ldots}}\right)}{2\left(af+b\ldots\right.}$$

Antiderivative was successfully verified.

```
[In] Int[x/((a + b*x + c*x^2)^(3/2)*(d - f*x^2)),x]
```

```
[Out] (-2*(a*(2*c^2*d - b^2*f + 2*a*c*f) + b*c*(c*d - a*f)*x))/((b^2 - 4*a*c)*(b^
2*d*f - (c*d + a*f)^2)*Sqrt[a + b*x + c*x^2]) - (Sqrt[f]*ArcTanh[(b*Sqrt[d]
 - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[
f] + a*f]*Sqrt[a + b*x + c*x^2])]])/(2*(c*d - b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)
) + (Sqrt[f]*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x
)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])]])/(2*(c*d +
 b*Sqrt[d]*Sqrt[f] + a*f)^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1018

```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (f
_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((a + b*x + c*x^2)^(p + 1)*(d + f*x^2)^
(q + 1)*((g*c)*(-(b*(c*d + a*f))) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(2*a*f
)) + c*(g*(2*c^2*d + b^2*f - c*(2*a*f)) - h*(b*c*d + a*b*f))*x))/((b^2 - 4*
a*c)*(b^2*d*f + (c*d - a*f)^2)*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*(b^2*d*
f + (c*d - a*f)^2)*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + f*x^2)^q*Si
mp[(b*h - 2*g*c)*((c*d - a*f)^2 - (b*d)*(-(b*f)))*(p + 1) + (b^2*(g*f) - b*
```

```
(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(a*f*(p + 1) - c*d*(p + 2)) - (2*f*(
(g*c)*(-(b*(c*d + a*f))) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(2*a*f)))*(p +
q + 2) - (b^2*(g*f) - b*(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(b*f*(p + 1)
))*x - c*f*(b^2*(g*f) - b*(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(2*p + 2*q
 + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, g, h, q}, x] && NeQ[b^2 - 4*
a*c, 0] && LtQ[p, -1] && NeQ[b^2*d*f + (c*d - a*f)^2, 0] &&  !( !IntegerQ[p
] && ILtQ[q, -1])
```

Rule 1033

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q
), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q
), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f
, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]
```

Rubi steps

$$\int \frac{x}{\left(a+bx+cx^2\right)^{3/2}\left(d-fx^2\right)}\,dx = -\frac{2\left(a\left(2c^2d-b^2f+2acf\right)+bc(cd-af)x\right)}{\left(b^2-4ac\right)\left(b^2df-(cd+af)^2\right)\sqrt{a+bx+cx^2}} + \frac{2\int \frac{\frac{1}{2}b\left(b^2-4ac\right)df-\frac{1}{2}\left(b^2-4ac\right)f(c}{\sqrt{a+bx+cx^2}\left(d-fx^2\right)}}{\left(b^2-4ac\right)\left(b^2df-(cd+}$$

$$= -\frac{2\left(a\left(2c^2d-b^2f+2acf\right)+bc(cd-af)x\right)}{\left(b^2-4ac\right)\left(b^2df-(cd+af)^2\right)\sqrt{a+bx+cx^2}} + \frac{f\int \frac{1}{\left(-\sqrt{d}\sqrt{f}-fx\right)\sqrt{a+bx+cx^2}}\,d}{2\left(cd-b\sqrt{d}\sqrt{f}+af\right)}$$

$$= -\frac{2\left(a\left(2c^2d-b^2f+2acf\right)+bc(cd-af)x\right)}{\left(b^2-4ac\right)\left(b^2df-(cd+af)^2\right)\sqrt{a+bx+cx^2}} - \frac{f\,\mathrm{Subst}\left(\int \frac{1}{4cdf-4b\sqrt{d}f^{3/2}+4}\right.}{cd}$$

$$= -\frac{2\left(a\left(2c^2d-b^2f+2acf\right)+bc(cd-af)x\right)}{\left(b^2-4ac\right)\left(b^2df-(cd+af)^2\right)\sqrt{a+bx+cx^2}} - \frac{\sqrt{f}\tanh^{-1}\left(\frac{b\sqrt{d}-2a\sqrt{f}+(2}{2\sqrt{cd-b\sqrt{d}\sqrt{f}+}}\right.}{2\left(cd-b\sqrt{d}\sqrt{f}+\right.}$$

Mathematica [A] time = 0.35, size = 356, normalized size = 1.19

$$\frac{2\left(\frac{2a^2cf+a\left(b^2(-f)-bcfx+2c^2d\right)+bc^2dx}{\sqrt{a+x(b+cx)}} - \frac{\sqrt{f}\left(b^2-4ac\right)\left(af+b\sqrt{d}\sqrt{f}+cd\right)\tanh^{-1}\left(\frac{-2a\sqrt{f}+b\left(\sqrt{d}-\sqrt{f}x\right)+2c\sqrt{d}x}{2\sqrt{a+x(b+cx)}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{4\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}} + \frac{\sqrt{f}\left(4ac-b^2\right)\left(af+b\left(-\sqrt{d}\right)\sqrt{\right.}}{4}\right.}{\left(b^2-4ac\right)\left((af+cd)^2-b^2df\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((a + b*x + c*x^2)^(3/2)*(d - f*x^2)),x]
```

```
[Out] (2*((2*a^2*c*f + b*c^2*d*x + a*(2*c^2*d - b^2*f - b*c*f*x))/Sqrt[a + x*(b +
 c*x)] - ((b^2 - 4*a*c)*Sqrt[f]*(c*d + b*Sqrt[d]*Sqrt[f] + a*f)*ArcTanh[(-2
*a*Sqrt[f] + 2*c*Sqrt[d]*x + b*(Sqrt[d] - Sqrt[f]*x))/(2*Sqrt[c*d - b*Sqrt[
d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])])/(4*Sqrt[c*d - b*Sqrt[d]*Sqrt[f]
+ a*f]) + ((-b^2 + 4*a*c)*Sqrt[f]*(c*d - b*Sqrt[d]*Sqrt[f] + a*f)*ArcTanh[(
-2*(a*Sqrt[f] + c*Sqrt[d]*x) - b*(Sqrt[d] + Sqrt[f]*x))/(2*Sqrt[c*d + b*Sqr
t[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])])/(4*Sqrt[c*d + b*Sqrt[d]*Sqrt[f
] + a*f])))/((b^2 - 4*a*c)*(-(b^2*d*f) + (c*d + a*f)^2))
```

fricas [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Valueint()  Bad Argument Type
```

maple [**B**] time = 0.02, size = 1360, normalized size = 4.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x)
```

```
[Out] -1/2/(a*f+c*d-(d*f)^(1/2)*b)/((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(
x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)-2/f/(a*f+c*d-(d*f)^(1/2
)*b)/(4*a*c-b^2)/((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2
)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*(d*f)^(1/2)*x*c^2+1/(a*f+c*d-(d*f)^
(1/2)*b)/(4*a*c-b^2)/((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^
(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*x*b*c-1/f/(a*f+c*d-(d*f)^(1/2)*
b)/(4*a*c-b^2)/((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/
f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*(d*f)^(1/2)*b*c+1/2/(a*f+c*d-(d*f)^(1
/2)*b)/(4*a*c-b^2)/((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1
/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*b^2+1/2/(a*f+c*d-(d*f)^(1/2)*b)/(
(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d-(d*f)^(1/2)*b)/f+(b*f-2*(d*
f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+2*((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*((x+(d*
f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1
/2)*b)/f)^(1/2))/(x+(d*f)^(1/2)/f))-1/2/(a*f+c*d+(d*f)^(1/2)*b)/((x-(d*f)^(
1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*
b)/f)^(1/2)+2/f/(a*f+c*d+(d*f)^(1/2)*b)/(4*a*c-b^2)/((x-(d*f)^(1/2)/f)^2*c+
(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*
(d*f)^(1/2)*x*c^2+1/(a*f+c*d+(d*f)^(1/2)*b)/(4*a*c-b^2)/((x-(d*f)^(1/2)/f)^
2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1
/2)*x*b*c+1/f/(a*f+c*d+(d*f)^(1/2)*b)/(4*a*c-b^2)/((x-(d*f)^(1/2)/f)^2*c+(b
*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*(d
*f)^(1/2)*b*c+1/2/(a*f+c*d+(d*f)^(1/2)*b)/(4*a*c-b^2)/((x-(d*f)^(1/2)/f)^2*
c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2
)*b^2+1/2/(a*f+c*d+(d*f)^(1/2)*b)/((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(
a*f+c*d+(d*f)^(1/2)*b)/f+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+2*((a*f+
c*d+(d*f)^(1/2)*b)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x
-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))/(x-(d*f)^(1/2)/f))
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(((c*sqrt(4*d*f))/(2*f^2)>0)', see `
assume?` for more details)Is ((c*sqrt(4*d*f))/(2*f^2)    +b/(2*f))    ^2
 -(c*((b*sqrt(4*d*f))                    /(2*f)                  +(c*d)/f+a))
     /f^2 zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x}{\left(d-f\,x^2\right)\,{\left(c\,x^2+b\,x+a\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((d - f*x^2)*(a + b*x + c*x^2)^(3/2)),x)
```

```
[Out] int(x/((d - f*x^2)*(a + b*x + c*x^2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{-ad\sqrt{a+bx+cx^2}+afx^2\sqrt{a+bx+cx^2}-bdx\sqrt{a+bx+cx^2}+bfx^3\sqrt{a+bx+cx^2}-cdx^2\sqrt{a+bx+cx^2}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x**2+b*x+a)**(3/2)/(-f*x**2+d),x)
```

```
[Out] -Integral(x/(-a*d*sqrt(a + b*x + c*x**2) + a*f*x**2*sqrt(a + b*x + c*x**2)
- b*d*x*sqrt(a + b*x + c*x**2) + b*f*x**3*sqrt(a + b*x + c*x**2) - c*d*x**2
*sqrt(a + b*x + c*x**2) + c*f*x**4*sqrt(a + b*x + c*x**2)), x)
```

3.106 $\int \frac{1}{\left(a+bx+cx^2\right)^{3/2}\left(d-fx^2\right)}\,dx$

Optimal. Leaf size=310

$$-\frac{2\left(b\left(b^2f-c(3af+cd)\right)-cx\left(2acf+b^2(-f)+2c^2d\right)\right)}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left(b^2df-(af+cd)^2\right)}+\frac{f\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2\sqrt{d}\left(af+b\left(-\sqrt{d}\right)\sqrt{f}+cd\right)^{3/2}}+\frac{f\tanh^{-1}\left(\cdots\right.}{2\sqrt{d}\,(\cdots}$$

```
[Out] 1/2*f*arctanh(1/2*(b*d^(1/2)-2*a*f^(1/2)+x*(2*c*d^(1/2)-b*f^(1/2)))/(c*x^2+
b*x+a)^(1/2)/(c*d+a*f-b*d^(1/2)*f^(1/2))^(1/2))/d^(1/2)/(c*d+a*f-b*d^(1/2)*
f^(1/2))^(3/2)+1/2*f*arctanh(1/2*(b*d^(1/2)+2*a*f^(1/2)+x*(2*c*d^(1/2)+b*f^
(1/2)))/(c*x^2+b*x+a)^(1/2)/(c*d+a*f+b*d^(1/2)*f^(1/2))^(1/2))/d^(1/2)/(c*d
+a*f+b*d^(1/2)*f^(1/2))^(3/2)-2*(b*(b^2*f-c*(3*a*f+c*d))-c*(2*a*c*f-b^2*f+2
*c^2*d)*x)/(-4*a*c+b^2)/(b^2*d*f-(a*f+c*d)^2)/(c*x^2+b*x+a)^(1/2)
```

Rubi [A] time = 0.41, antiderivative size = 310, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {975, 1033, 724, 206}

$$-\frac{2\left(b\left(b^2f-c(3af+cd)\right)-cx\left(2acf+b^2(-f)+2c^2d\right)\right)}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left(b^2df-(af+cd)^2\right)}+\frac{f\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2\sqrt{d}\left(af+b\left(-\sqrt{d}\right)\sqrt{f}+cd\right)^{3/2}}+\frac{f\tanh^{-1}\left(\cdots\right.}{2\sqrt{d}\,(\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x + c*x^2)^(3/2)*(d - f*x^2)),x]
```

```
[Out] (-2*(b*(b^2*f - c*(c*d + 3*a*f)) - c*(2*c^2*d - b^2*f + 2*a*c*f)*x))/((b^2
- 4*a*c)*(b^2*d*f - (c*d + a*f)^2)*Sqrt[a + b*x + c*x^2]) + (f*ArcTanh[(b*S
qrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt[d]
*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[d]*(c*d - b*Sqrt[d]*Sqrt[f
] + a*f)^(3/2)) + (f*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sq
rt[f])*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(
2*Sqrt[d]*(c*d + b*Sqrt[d]*Sqrt[f] + a*f)^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 975

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (f_.)*(x_)^2)^(q_), x
_Symbol] :> Simp[((b^3*f + b*c*(c*d - 3*a*f) + c*(2*c^2*d + b^2*f - c*(2*a*
f))*x)*(a + b*x + c*x^2)^(p + 1)*(d + f*x^2)^(q + 1))/((b^2 - 4*a*c)*(b^2*d
*f + (c*d - a*f)^2)*(p + 1)), x] - Dist[1/((b^2 - 4*a*c)*(b^2*d*f + (c*d -
a*f)^2)*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + f*x^2)^q*Simp[2*c*(b^2
*d*f + (c*d - a*f)^2)*(p + 1) - (2*c^2*d + b^2*f - c*(2*a*f))*(a*f*(p + 1)
- c*d*(p + 2)) + (2*f*(b^3*f + b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d +
```

```
b^2*f - c*(2*a*f))*(b*f*(p + 1)))*x + c*f*(2*c^2*d + b^2*f - c*(2*a*f))*(2*
p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, q}, x] && NeQ[b^2 - 4
*a*c, 0] && LtQ[p, -1] && NeQ[b^2*d*f + (c*d - a*f)^2, 0] &&  !( !IntegerQ[
p] && ILtQ[q, -1]) &&  !IGtQ[q, 0]
```

Rule 1033

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q
), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q
), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f
, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]
```

Rubi steps

$$\int \frac{1}{\left(a+bx+cx^2\right)^{3/2}\left(d-fx^2\right)}\,dx = -\frac{2\left(b\left(b^2f-c(cd+3af)\right)-c\left(2c^2d-b^2f+2acf\right)x\right)}{\left(b^2-4ac\right)\left(b^2df-(cd+af)^2\right)\sqrt{a+bx+cx^2}} - \frac{2\int \frac{\frac{1}{2}\left(b^2-4ac\right)f(cd+af)-}{\sqrt{a+bx+cx^2}\,(}}{\left(b^2-4ac\right)\left(b^2df-\right.}$$

$$= -\frac{2\left(b\left(b^2f-c(cd+3af)\right)-c\left(2c^2d-b^2f+2acf\right)x\right)}{\left(b^2-4ac\right)\left(b^2df-(cd+af)^2\right)\sqrt{a+bx+cx^2}} - \frac{f^{3/2}\int \frac{1}{\left(-\sqrt{d}\sqrt{f}-fx\right)\sqrt{a}}}{2\sqrt{d}\left(cd-b\sqrt{d}\sqrt{f}\right.}$$

$$= -\frac{2\left(b\left(b^2f-c(cd+3af)\right)-c\left(2c^2d-b^2f+2acf\right)x\right)}{\left(b^2-4ac\right)\left(b^2df-(cd+af)^2\right)\sqrt{a+bx+cx^2}} + \frac{f^{3/2}\,\text{Subst}\left(\int \frac{}{4cdf-4b}\right.}{}$$

$$= -\frac{2\left(b\left(b^2f-c(cd+3af)\right)-c\left(2c^2d-b^2f+2acf\right)x\right)}{\left(b^2-4ac\right)\left(b^2df-(cd+af)^2\right)\sqrt{a+bx+cx^2}} + \frac{f\tanh^{-1}\left(\frac{b\sqrt{d}-2a\sqrt{f}}{2\sqrt{cd-b\sqrt{d}}}\right.}{2\sqrt{d}\left(cd-b\sqrt{d}\right.}$$

Mathematica [A] time = 0.40, size = 360, normalized size = 1.16

$$\frac{2\left(\frac{f\left(b^2-4ac\right)\left(af+b\sqrt{d}\sqrt{f}+cd\right)\tanh^{-1}\left(\frac{-2a\sqrt{f}+b\left(\sqrt{d}-\sqrt{f}x\right)+2c\sqrt{d}x}{2\sqrt{a+x(b+cx)}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{4\sqrt{d}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}+\frac{f\left(4ac-b^2\right)\left(af+b\left(-\sqrt{d}\right)\sqrt{f}+cd\right)\tanh^{-1}\left(\frac{-2\left(a\sqrt{f}+c\sqrt{d}x\right)-b\left(\sqrt{d}+\sqrt{f}x\right)}{2\sqrt{a+x(b+cx)}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{4\sqrt{d}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right.}{\left(b^2-4ac\right)\left((af+cd)^2-b^2df\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x + c*x^2)^(3/2)*(d - f*x^2)),x]
```

```
[Out] (2*((b^3*f - b*c*(c*d + 3*a*f) + b^2*c*f*x - 2*c^2*(c*d + a*f)*x)/Sqrt[a +
x*(b + c*x)] + ((b^2 - 4*a*c)*f*(c*d + b*Sqrt[d]*Sqrt[f] + a*f)*ArcTanh[(-2
*a*Sqrt[f] + 2*c*Sqrt[d]*x + b*(Sqrt[d] - Sqrt[f]*x))/(2*Sqrt[c*d - b*Sqrt[
d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])])/(4*Sqrt[d]*Sqrt[c*d - b*Sqrt[d]*
Sqrt[f] + a*f]) + ((-b^2 + 4*a*c)*f*(c*d - b*Sqrt[d]*Sqrt[f] + a*f)*ArcTanh
[(-2*(a*Sqrt[f] + c*Sqrt[d]*x) - b*(Sqrt[d] + Sqrt[f]*x))/(2*Sqrt[c*d + b*S
qrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])])/(4*Sqrt[d]*Sqrt[c*d + b*Sqrt
[d]*Sqrt[f] + a*f])))/((b^2 - 4*a*c)*(-(b^2*d*f) + (c*d + a*f)^2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Valueint()  Bad Argument Type
```

maple [B] time = 0.02, size = 1376, normalized size = 4.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x)
```

```
[Out] 1/2/(d*f)^(1/2)/(a*f+c*d-(d*f)^(1/2)*b)*f/((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*
f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)+2/(a*f+c*d
-(d*f)^(1/2)*b)/(4*a*c-b^2)/((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x
+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*x*c^2-1/(d*f)^(1/2)/(a*f
+c*d-(d*f)^(1/2)*b)/(4*a*c-b^2)/((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c
)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*x*b*c*f+1/(a*f+c*d-(
d*f)^(1/2)*b)/(4*a*c-b^2)/((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(
d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*b*c-1/2/(d*f)^(1/2)/(a*f+c
*d-(d*f)^(1/2)*b)/(4*a*c-b^2)/((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*
(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*b^2*f-1/2/(d*f)^(1/2)/
(a*f+c*d-(d*f)^(1/2)*b)*f/((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d-
(d*f)^(1/2)*b)/f+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+2*((a*f+c*d-(d*f
)^(1/2)*b)/f)^(1/2)*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(
1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))/(x+(d*f)^(1/2)/f))-1/2/(d*f)^(1
/2)/(a*f+c*d+(d*f)^(1/2)*b)*f/((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*
(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)+2/(a*f+c*d+(d*f)^(1/2)
*b)/(4*a*c-b^2)/((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)
/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*x*c^2+1/(d*f)^(1/2)/(a*f+c*d+(d*f)^(
1/2)*b)/(4*a*c-b^2)/((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(
1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*x*b*c*f+1/(a*f+c*d+(d*f)^(1/2)*b
)/(4*a*c-b^2)/((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f
)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*b*c+1/2/(d*f)^(1/2)/(a*f+c*d+(d*f)^(1/
2)*b)/(4*a*c-b^2)/((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/
2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*b^2*f+1/2/(d*f)^(1/2)/(a*f+c*d+(d*
f)^(1/2)*b)*f/((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d+(d*f)^(1/2)*
b)/f+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+2*((a*f+c*d+(d*f)^(1/2)*b)/f
)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a
*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))/(x-(d*f)^(1/2)/f))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(((c*sqrt(4*d*f))/(2*f^2)>0)', see `
assume?` for more details)Is ((c*sqrt(4*d*f))/(2*f^2)    +b/(2*f))    ^2
 -(c*((b*sqrt(4*d*f))                    /(2*f)                  +(c*d)/f+a))
     /f^2 positive, negative or zero?
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(d-f\,x^2\right)\,{\left(c\,x^2+b\,x+a\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d - f*x^2)*(a + b*x + c*x^2)^(3/2)),x)
```

```
[Out] int(1/((d - f*x^2)*(a + b*x + c*x^2)^(3/2)), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{-ad\sqrt{a+bx+cx^2}+afx^2\sqrt{a+bx+cx^2}-bdx\sqrt{a+bx+cx^2}+bfx^3\sqrt{a+bx+cx^2}-cdx^2\sqrt{a+bx+cx^2}+\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**2+b*x+a)**(3/2)/(-f*x**2+d),x)
```

```
[Out] -Integral(1/(-a*d*sqrt(a + b*x + c*x**2) + a*f*x**2*sqrt(a + b*x + c*x**2)
- b*d*x*sqrt(a + b*x + c*x**2) + b*f*x**3*sqrt(a + b*x + c*x**2) - c*d*x**2
*sqrt(a + b*x + c*x**2) + c*f*x**4*sqrt(a + b*x + c*x**2)), x)
```

3.107 $\int \frac{1}{x\left(a+bx+cx^2\right)^{3/2}\left(d-fx^2\right)}\,dx$

Optimal. Leaf size=394

$$-\frac{\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{a^{3/2}d}-\frac{2f\left(a\left(2acf+b^2(-f)+2c^2d\right)+bcx(cd-af)\right)}{d\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left(b^2df-(af+cd)^2\right)}+\frac{2\left(-2ac+b^2+bcx\right)}{ad\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}-\frac{f^{3/2}}{}$$

```
[Out] -arctanh(1/2*(b*x+2*a)/a^(1/2)/(c*x^2+b*x+a)^(1/2))/a^(3/2)/d-1/2*f^(3/2)*a
rctanh(1/2*(b*d^(1/2)-2*a*f^(1/2)+x*(2*c*d^(1/2)-b*f^(1/2)))/(c*x^2+b*x+a)^
(1/2)/(c*d+a*f-b*d^(1/2)*f^(1/2))^(1/2))/d/(c*d+a*f-b*d^(1/2)*f^(1/2))^(3/2
)+1/2*f^(3/2)*arctanh(1/2*(b*d^(1/2)+2*a*f^(1/2)+x*(2*c*d^(1/2)+b*f^(1/2)))
/(c*x^2+b*x+a)^(1/2)/(c*d+a*f+b*d^(1/2)*f^(1/2))^(1/2))/d/(c*d+a*f+b*d^(1/2
)*f^(1/2))^(3/2)+2*(b*c*x-2*a*c+b^2)/a/(-4*a*c+b^2)/d/(c*x^2+b*x+a)^(1/2)-2
*f*(a*(2*a*c*f-b^2*f+2*c^2*d)+b*c*(-a*f+c*d)*x)/(-4*a*c+b^2)/d/(b^2*d*f-(a*
f+c*d)^2)/(c*x^2+b*x+a)^(1/2)
```

Rubi [A] time = 1.18, antiderivative size = 394, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6725, 740, 12, 724, 206, 1018, 1033}

$$-\frac{\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{a^{3/2}d}-\frac{2f\left(a\left(2acf+b^2(-f)+2c^2d\right)+bcx(cd-af)\right)}{d\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left(b^2df-(af+cd)^2\right)}+\frac{2\left(-2ac+b^2+bcx\right)}{ad\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}-\frac{f^{3/2}}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x + c*x^2)^(3/2)*(d - f*x^2)),x]
```

```
[Out] (2*(b^2 - 2*a*c + b*c*x))/(a*(b^2 - 4*a*c)*d*Sqrt[a + b*x + c*x^2]) - (2*f*
(a*(2*c^2*d - b^2*f + 2*a*c*f) + b*c*(c*d - a*f)*x))/((b^2 - 4*a*c)*d*(b^2*
d*f - (c*d + a*f)^2)*Sqrt[a + b*x + c*x^2]) - ArcTanh[(2*a + b*x)/(2*Sqrt[a
]*Sqrt[a + b*x + c*x^2])]/(a^(3/2)*d) - (f^(3/2)*ArcTanh[(b*Sqrt[d] - 2*a*S
qrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f
]*Sqrt[a + b*x + c*x^2])]])/(2*d*(c*d - b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)) + (f
^(3/2)*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*S
qrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])]])/(2*d*(c*d + b*S
qrt[d]*Sqrt[f] + a*f)^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 1018

```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (f
_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((a + b*x + c*x^2)^(p + 1)*(d + f*x^2)^
(q + 1)*((g*c)*(-(b*(c*d + a*f))) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(2*a*f
)) + c*(g*(2*c^2*d + b^2*f - c*(2*a*f)) - h*(b*c*d + a*b*f))*x))/((b^2 - 4*
a*c)*(b^2*d*f + (c*d - a*f)^2)*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*(b^2*d*
f + (c*d - a*f)^2)*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + f*x^2)^q*Si
mp[(b*h - 2*g*c)*((c*d - a*f)^2 - (b*d)*(-(b*f)))*(p + 1) + (b^2*(g*f) - b*
(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(a*f*(p + 1) - c*d*(p + 2)) - (2*f*(
(g*c)*(-(b*(c*d + a*f))) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(2*a*f)))*(p +
q + 2) - (b^2*(g*f) - b*(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(b*f*(p + 1)
))*x - c*f*(b^2*(g*f) - b*(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(2*p + 2*q
 + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, g, h, q}, x] && NeQ[b^2 - 4*
a*c, 0] && LtQ[p, -1] && NeQ[b^2*d*f + (c*d - a*f)^2, 0] &&  !( !IntegerQ[p
] && ILtQ[q, -1])
```

Rule 1033

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q
), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q
), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f
, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{1}{x\left(a+bx+cx^2\right)^{3/2}\left(d-fx^2\right)}\,dx = \int \left(\frac{1}{dx\left(a+bx+cx^2\right)^{3/2}} - \frac{fx}{d\left(a+bx+cx^2\right)^{3/2}\left(-d+fx^2\right)}\right) dx$$

$$= \frac{\int \frac{1}{x\left(a+bx+cx^2\right)^{3/2}}\,dx}{d} - \frac{f\int \frac{x}{\left(a+bx+cx^2\right)^{3/2}\left(-d+fx^2\right)}\,dx}{d}$$

$$= \frac{2\left(b^2-2ac+bcx\right)}{a\left(b^2-4ac\right)d\sqrt{a+bx+cx^2}} - \frac{2f\left(a\left(2c^2d-b^2f+2acf\right)+bc(cd-a\right.}{\left(b^2-4ac\right)d\left(b^2df-(cd+af)^2\right)\sqrt{a+bx}}$$

$$= \frac{2\left(b^2-2ac+bcx\right)}{a\left(b^2-4ac\right)d\sqrt{a+bx+cx^2}} - \frac{2f\left(a\left(2c^2d-b^2f+2acf\right)+bc(cd-a\right.}{\left(b^2-4ac\right)d\left(b^2df-(cd+af)^2\right)\sqrt{a+bx}}$$

$$= \frac{2\left(b^2-2ac+bcx\right)}{a\left(b^2-4ac\right)d\sqrt{a+bx+cx^2}} - \frac{2f\left(a\left(2c^2d-b^2f+2acf\right)+bc(cd-a\right.}{\left(b^2-4ac\right)d\left(b^2df-(cd+af)^2\right)\sqrt{a+bx}}$$

$$= \frac{2\left(b^2-2ac+bcx\right)}{a\left(b^2-4ac\right)d\sqrt{a+bx+cx^2}} - \frac{2f\left(a\left(2c^2d-b^2f+2acf\right)+bc(cd-a\right.}{\left(b^2-4ac\right)d\left(b^2df-(cd+af)^2\right)\sqrt{a+bx}}$$

Mathematica [A] time = 0.87, size = 436, normalized size = 1.11

$$\frac{-\frac{\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+x(b+cx)}}\right)}{a^{3/2}} - \frac{2f\left(a\left(2acf+b^2(-f)+2c^2d\right)+bcx(cd-af)\right)}{\left(b^2-4ac\right)\sqrt{a+x(b+cx)}\left(b^2df-(af+cd)^2\right)} + \frac{f^{3/2}\left(\left(af+b\left(-\sqrt{d}\right)\sqrt{f}+cd\right)^{3/2}\tanh^{-1}\left(\frac{2a\sqrt{f}+b\sqrt{d}+b\sqrt{f}x+2c\sqrt{d}x}{2\sqrt{a+x(b+cx)}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)+\left(\right.\right.}{2\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}\sqrt{af+b\sqrt{d}}}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x + c*x^2)^(3/2)*(d - f*x^2)),x]
```

```
[Out] ((2*(b^2 - 2*a*c + b*c*x))/(a*(b^2 - 4*a*c)*Sqrt[a + x*(b + c*x)]) - (2*f*(
a*(2*c^2*d - b^2*f + 2*a*c*f) + b*c*(c*d - a*f)*x))/((b^2 - 4*a*c)*(b^2*d*f
 - (c*d + a*f)^2)*Sqrt[a + x*(b + c*x)]) - ArcTanh[(2*a + b*x)/(2*Sqrt[a]*S
qrt[a + x*(b + c*x)])]/a^(3/2) + (f^(3/2)*((c*d + b*Sqrt[d]*Sqrt[f] + a*f)^
(3/2)*ArcTanh[(-(b*Sqrt[d]) + 2*a*Sqrt[f] - 2*c*Sqrt[d]*x + b*Sqrt[f]*x)/(2
*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])] + (c*d - b*Sqr
t[d]*Sqrt[f] + a*f)^(3/2)*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + 2*c*Sqrt[d]*x
+ b*Sqrt[f]*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)]
)]))/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] +
a*f]*(-(b^2*d*f) + (c*d + a*f)^2)))/d
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$sage_2$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="giac")
```

```
[Out] sage2
```

maple [B] time = 0.02, size = 1518, normalized size = 3.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x)
```

```
[Out] -1/2/d/(a*f+c*d-(d*f)^(1/2)*b)*f/((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*
c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)-2/d/(a*f+c*d-(d*f)^
(1/2)*b)/(4*a*c-b^2)/((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^
(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*(d*f)^(1/2)*x*c^2+1/d/(a*f+c*d-
(d*f)^(1/2)*b)/(4*a*c-b^2)/((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+
(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*x*b*c*f-1/d/(a*f+c*d-(d*f
)^(1/2)*b)/(4*a*c-b^2)/((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f
)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*(d*f)^(1/2)*b*c+1/2/d/(a*f+c*
d-(d*f)^(1/2)*b)/(4*a*c-b^2)/((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(
x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*b^2*f+1/2/d/(a*f+c*d-(d
*f)^(1/2)*b)*f/((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d-(d*f)^(1/2)
*b)/f+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+2*((a*f+c*d-(d*f)^(1/2)*b)/
f)^(1/2)*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(
a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))/(x+(d*f)^(1/2)/f))-1/2/d/(a*f+c*d+(d*f)^(1
/2)*b)*f/((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(
a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)+2/d/(a*f+c*d+(d*f)^(1/2)*b)/(4*a*c-b^2)/((x
-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f
)^(1/2)*b)/f)^(1/2)*(d*f)^(1/2)*x*c^2+1/d/(a*f+c*d+(d*f)^(1/2)*b)/(4*a*c-b^
2)/((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*
d+(d*f)^(1/2)*b)/f)^(1/2)*x*b*c*f+1/d/(a*f+c*d+(d*f)^(1/2)*b)/(4*a*c-b^2)/(
(x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d
*f)^(1/2)*b)/f)^(1/2)*(d*f)^(1/2)*b*c+1/2/d/(a*f+c*d+(d*f)^(1/2)*b)/(4*a*c-
b^2)/((x-(d*f)^(1/2)/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+
c*d+(d*f)^(1/2)*b)/f)^(1/2)*b^2*f+1/2/d/(a*f+c*d+(d*f)^(1/2)*b)*f/((a*f+c*d
+(d*f)^(1/2)*b)/f)^(1/2)*ln((2*(a*f+c*d+(d*f)^(1/2)*b)/f+(b*f+2*(d*f)^(1/2)
*c)*(x-(d*f)^(1/2)/f)/f+2*((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*((x-(d*f)^(1/2)
/f)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f
)^(1/2))/(x-(d*f)^(1/2)/f))+1/d/a/(c*x^2+b*x+a)^(1/2)-2/d*b/a/(4*a*c-b^2)/(
c*x^2+b*x+a)^(1/2)*c*x-1/d*b^2/a/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)-1/d/a^(3/2
)*ln((b*x+2*a+2*(c*x^2+b*x+a)^(1/2)*a^(1/2))/x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{\left(cx^2 + bx + a\right)^{\frac{3}{2}}\left(fx^2 - d\right)x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="maxima")
```

```
[Out] -integrate(1/((c*x^2 + b*x + a)^(3/2)*(f*x^2 - d)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x\,\left(d-f\,x^2\right)\,{\left(c\,x^2+b\,x+a\right)}^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(d - f*x^2)*(a + b*x + c*x^2)^(3/2)),x)
```

```
[Out] int(1/(x*(d - f*x^2)*(a + b*x + c*x^2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{-adx\sqrt{a+bx+cx^2}+afx^3\sqrt{a+bx+cx^2}-bdx^2\sqrt{a+bx+cx^2}+bfx^4\sqrt{a+bx+cx^2}-cdx^3\sqrt{a+bx+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x**2+b*x+a)**(3/2)/(-f*x**2+d),x)
```

```
[Out] -Integral(1/(-a*d*x*sqrt(a + b*x + c*x**2) + a*f*x**3*sqrt(a + b*x + c*x**2
) - b*d*x**2*sqrt(a + b*x + c*x**2) + b*f*x**4*sqrt(a + b*x + c*x**2) - c*d
*x**3*sqrt(a + b*x + c*x**2) + c*f*x**5*sqrt(a + b*x + c*x**2)), x)
```

3.108 $\int \frac{1}{x^2\left(a+bx+cx^2\right)^{3/2}\left(d-fx^2\right)}\,dx$

Optimal. Leaf size=454

$$\frac{3b\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2a^{5/2}d}-\frac{\left(3b^2-8ac\right)\sqrt{a+bx+cx^2}}{a^2dx\left(b^2-4ac\right)}-\frac{2f\left(b\left(b^2f-c(3af+cd)\right)-cx\left(2acf+b^2(-f)+2c^2d\right)\right)}{d\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left(b^2df-(af+cd)^2\right)}$$

```
[Out] 3/2*b*arctanh(1/2*(b*x+2*a)/a^(1/2)/(c*x^2+b*x+a)^(1/2))/a^(5/2)/d+1/2*f^2*
arctanh(1/2*(b*d^(1/2)-2*a*f^(1/2)+x*(2*c*d^(1/2)-b*f^(1/2)))/(c*x^2+b*x+a)
^(1/2)/(c*d+a*f-b*d^(1/2)*f^(1/2))^(1/2))/d^(3/2)/(c*d+a*f-b*d^(1/2)*f^(1/2
))^(3/2)+1/2*f^2*arctanh(1/2*(b*d^(1/2)+2*a*f^(1/2)+x*(2*c*d^(1/2)+b*f^(1/2
)))/(c*x^2+b*x+a)^(1/2)/(c*d+a*f+b*d^(1/2)*f^(1/2))^(1/2))/d^(3/2)/(c*d+a*f
+b*d^(1/2)*f^(1/2))^(3/2)+2*(b*c*x-2*a*c+b^2)/a/(-4*a*c+b^2)/d/x/(c*x^2+b*x
+a)^(1/2)-2*f*(b*(b^2*f-c*(3*a*f+c*d))-c*(2*a*c*f-b^2*f+2*c^2*d)*x)/(-4*a*c
+b^2)/d/(b^2*d*f-(a*f+c*d)^2)/(c*x^2+b*x+a)^(1/2)-(-8*a*c+3*b^2)*(c*x^2+b*x
+a)^(1/2)/a^2/(-4*a*c+b^2)/d/x
```

Rubi [A] time = 1.19, antiderivative size = 454, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6725, 740, 806, 724, 206, 975, 1033}

$$-\frac{\left(3b^2-8ac\right)\sqrt{a+bx+cx^2}}{a^2dx\left(b^2-4ac\right)}+\frac{3b\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2a^{5/2}d}-\frac{2f\left(b\left(b^2f-c(3af+cd)\right)-cx\left(2acf+b^2(-f)+2c^2d\right)\right)}{d\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left(b^2df-(af+cd)^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x + c*x^2)^(3/2)*(d - f*x^2)),x]
```

```
[Out] (2*(b^2 - 2*a*c + b*c*x))/(a*(b^2 - 4*a*c)*d*x*Sqrt[a + b*x + c*x^2]) - (2*
f*(b*(b^2*f - c*(c*d + 3*a*f)) - c*(2*c^2*d - b^2*f + 2*a*c*f)*x))/((b^2 -
4*a*c)*d*(b^2*d*f - (c*d + a*f)^2)*Sqrt[a + b*x + c*x^2]) - ((3*b^2 - 8*a*c
)*Sqrt[a + b*x + c*x^2])/(a^2*(b^2 - 4*a*c)*d*x) + (3*b*ArcTanh[(2*a + b*x)
/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/(2*a^(5/2)*d) + (f^2*ArcTanh[(b*Sqrt[d
] - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt
[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*d^(3/2)*(c*d - b*Sqrt[d]*Sqrt[f] + a
*f)^(3/2)) + (f^2*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[
f])*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*d
^(3/2)*(c*d + b*Sqrt[d]*Sqrt[f] + a*f)^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
```

```
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 975

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (f_.)*(x_)^2)^(q_), x
_Symbol] :> Simp[((b^3*f + b*c*(c*d - 3*a*f) + c*(2*c^2*d + b^2*f - c*(2*a*
f))*x)*(a + b*x + c*x^2)^(p + 1)*(d + f*x^2)^(q + 1))/((b^2 - 4*a*c)*(b^2*d
*f + (c*d - a*f)^2)*(p + 1)), x] - Dist[1/((b^2 - 4*a*c)*(b^2*d*f + (c*d -
a*f)^2)*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + f*x^2)^q*Simp[2*c*(b^2
*d*f + (c*d - a*f)^2)*(p + 1) - (2*c^2*d + b^2*f - c*(2*a*f))*(a*f*(p + 1)
- c*d*(p + 2)) + (2*f*(b^3*f + b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d +
b^2*f - c*(2*a*f))*(b*f*(p + 1)))*x + c*f*(2*c^2*d + b^2*f - c*(2*a*f))*(2*
p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, q}, x] && NeQ[b^2 - 4
*a*c, 0] && LtQ[p, -1] && NeQ[b^2*d*f + (c*d - a*f)^2, 0] &&  !( !IntegerQ[
p] && ILtQ[q, -1]) &&  !IGtQ[q, 0]
```

Rule 1033

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q
), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q
), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f
, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{1}{x^2\left(a+bx+cx^2\right)^{3/2}\left(d-fx^2\right)}\,dx = \int \left(\frac{1}{dx^2\left(a+bx+cx^2\right)^{3/2}} + \frac{f}{d\left(a+bx+cx^2\right)^{3/2}\left(d-fx^2\right)}\right)dx$$

$$= \frac{\int \frac{1}{x^2\left(a+bx+cx^2\right)^{3/2}}\,dx}{d} + \frac{f\int \frac{1}{\left(a+bx+cx^2\right)^{3/2}\left(d-fx^2\right)}\,dx}{d}$$

$$= \frac{2\left(b^2-2ac+bcx\right)}{a\left(b^2-4ac\right)dx\sqrt{a+bx+cx^2}} - \frac{2f\left(b\left(b^2f-c(cd+3af)\right)-c\left(2c^2d-b^2f\right.\right.}{\left(b^2-4ac\right)d\left(b^2df-(cd+af)^2\right)\sqrt{a+}}$$

$$= \frac{2\left(b^2-2ac+bcx\right)}{a\left(b^2-4ac\right)dx\sqrt{a+bx+cx^2}} - \frac{2f\left(b\left(b^2f-c(cd+3af)\right)-c\left(2c^2d-b^2f\right.\right.}{\left(b^2-4ac\right)d\left(b^2df-(cd+af)^2\right)\sqrt{a+}}$$

$$= \frac{2\left(b^2-2ac+bcx\right)}{a\left(b^2-4ac\right)dx\sqrt{a+bx+cx^2}} - \frac{2f\left(b\left(b^2f-c(cd+3af)\right)-c\left(2c^2d-b^2f\right.\right.}{\left(b^2-4ac\right)d\left(b^2df-(cd+af)^2\right)\sqrt{a+}}$$

$$= \frac{2\left(b^2-2ac+bcx\right)}{a\left(b^2-4ac\right)dx\sqrt{a+bx+cx^2}} - \frac{2f\left(b\left(b^2f-c(cd+3af)\right)-c\left(2c^2d-b^2f\right.\right.}{\left(b^2-4ac\right)d\left(b^2df-(cd+af)^2\right)\sqrt{a+}}$$

Mathematica [A] time = 1.19, size = 488, normalized size = 1.07

$$\frac{\frac{3b\left(b^2-4ac\right)\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+x(b+cx)}}\right)}{a^{5/2}} + \frac{2\left(8ac-3b^2\right)\sqrt{a+x(b+cx)}}{a^2x} - \frac{f^2\left(\frac{\left(b^2-4ac\right)\left(af+b\sqrt{d}\sqrt{f}+cd\right)\tanh^{-1}\left(\frac{2a\sqrt{f}-b\sqrt{d}+b\sqrt{f}x-2c\sqrt{d}x}{2\sqrt{a+x(b+cx)}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}} + \frac{\left(4ac-b^2\right)\left(af+\right.}{}\right.}{\sqrt{d}\left((af+cd)^2-b^2df\right)}}{2d\left(b^2-4ac\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x + c*x^2)^(3/2)*(d - f*x^2)),x]
```

```
[Out] ((4*(b^2 - 2*a*c + b*c*x))/(a*x*Sqrt[a + x*(b + c*x)]) - (4*f*(b^3*f - b*c*
(c*d + 3*a*f) + b^2*c*f*x - 2*c^2*(c*d + a*f)*x))/((b^2*d*f - (c*d + a*f)^2
)*Sqrt[a + x*(b + c*x)]) + (2*(-3*b^2 + 8*a*c)*Sqrt[a + x*(b + c*x)])/(a^2*
x) + (3*b*(b^2 - 4*a*c)*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + x*(b + c*x)
])])/a^(5/2) - (f^2*(((b^2 - 4*a*c)*(c*d + b*Sqrt[d]*Sqrt[f] + a*f)*ArcTanh
[(-(b*Sqrt[d]) + 2*a*Sqrt[f] - 2*c*Sqrt[d]*x + b*Sqrt[f]*x)/(2*Sqrt[c*d - b
*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])]))/Sqrt[c*d - b*Sqrt[d]*Sqrt[
f] + a*f] + ((-b^2 + 4*a*c)*(c*d - b*Sqrt[d]*Sqrt[f] + a*f)*ArcTanh[(b*Sqrt
[d] + 2*a*Sqrt[f] + 2*c*Sqrt[d]*x + b*Sqrt[f]*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sq
rt[f] + a*f]*Sqrt[a + x*(b + c*x)])]))/Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]))
/(Sqrt[d]*(-(b^2*d*f) + (c*d + a*f)^2)))/(2*(b^2 - 4*a*c)*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument ValueEvaluation time: 1.84int()
Bad Argument Type
```

maple [B] time = 0.02, size = 1656, normalized size = 3.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x)
```

```
[Out] 1/2*f^2/d/(d*f)^(1/2)/(a*f+c*d-(d*f)^(1/2)*b)/((x+(d*f)^(1/2)/f)^2*c+(b*f-2
*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)+2*f/d/
(a*f+c*d-(d*f)^(1/2)*b)/(4*a*c-b^2)/((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(1/
2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*x*c^2-f^2/d/(d*f
)^(1/2)/(a*f+c*d-(d*f)^(1/2)*b)/(4*a*c-b^2)/((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(
d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*x*b*c+f/
d/(a*f+c*d-(d*f)^(1/2)*b)/(4*a*c-b^2)/((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)^(
1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*b*c-1/2*f^2/d/
(d*f)^(1/2)/(a*f+c*d-(d*f)^(1/2)*b)/(4*a*c-b^2)/((x+(d*f)^(1/2)/f)^2*c+(b*f
-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*b^2-
1/2*f^2/d/(d*f)^(1/2)/(a*f+c*d-(d*f)^(1/2)*b)/((a*f+c*d-(d*f)^(1/2)*b)/f)^(
1/2)*ln((2*(a*f+c*d-(d*f)^(1/2)*b)/f+(b*f-2*(d*f)^(1/2)*c)*(x+(d*f)^(1/2)/f
)/f+2*((a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2)*((x+(d*f)^(1/2)/f)^2*c+(b*f-2*(d*f)
^(1/2)*c)*(x+(d*f)^(1/2)/f)/f+(a*f+c*d-(d*f)^(1/2)*b)/f)^(1/2))/(x+(d*f)^(1
/2)/f))-1/d/a/x/(c*x^2+b*x+a)^(1/2)-3/2/d*b/a^2/(c*x^2+b*x+a)^(1/2)+3/d*b^2
/a^2/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*c*x+3/2/d*b^3/a^2/(4*a*c-b^2)/(c*x^2+b
*x+a)^(1/2)+3/2/d*b/a^(5/2)*ln((b*x+2*a+2*(c*x^2+b*x+a)^(1/2)*a^(1/2))/x)-8
/d*c^2/a/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x-4/d*c/a/(4*a*c-b^2)/(c*x^2+b*x+a
)^(1/2)*b-1/2*f^2/d/(d*f)^(1/2)/(a*f+c*d+(d*f)^(1/2)*b)/((x-(d*f)^(1/2)/f)^
2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1
/2)+2*f/d/(a*f+c*d+(d*f)^(1/2)*b)/(4*a*c-b^2)/((x-(d*f)^(1/2)/f)^2*c+(b*f+2
*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*x*c^2+
f^2/d/(d*f)^(1/2)/(a*f+c*d+(d*f)^(1/2)*b)/(4*a*c-b^2)/((x-(d*f)^(1/2)/f)^2*
c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2
)*x*b*c+f/d/(a*f+c*d+(d*f)^(1/2)*b)/(4*a*c-b^2)/((x-(d*f)^(1/2)/f)^2*c+(b*f
+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*b*c+
1/2*f^2/d/(d*f)^(1/2)/(a*f+c*d+(d*f)^(1/2)*b)/(4*a*c-b^2)/((x-(d*f)^(1/2)/f
)^2*c+(b*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^
(1/2)*b^2+1/2*f^2/d/(d*f)^(1/2)/(a*f+c*d+(d*f)^(1/2)*b)/((a*f+c*d+(d*f)^(1/
2)*b)/f)^(1/2)*ln((2*(a*f+c*d+(d*f)^(1/2)*b)/f+(b*f+2*(d*f)^(1/2)*c)*(x-(d*
f)^(1/2)/f)/f+2*((a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(b
*f+2*(d*f)^(1/2)*c)*(x-(d*f)^(1/2)/f)/f+(a*f+c*d+(d*f)^(1/2)*b)/f)^(1/2))/(
x-(d*f)^(1/2)/f))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{\left(cx^2 + bx + a\right)^{\frac{3}{2}} \left(fx^2 - d\right) x^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="maxima")
```

```
[Out] -integrate(1/((c*x^2 + b*x + a)^(3/2)*(f*x^2 - d)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^2\,\left(d-f\,x^2\right)\,{\left(c\,x^2+b\,x+a\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(d - f*x^2)*(a + b*x + c*x^2)^(3/2)),x)
```

```
[Out] int(1/(x^2*(d - f*x^2)*(a + b*x + c*x^2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{-adx^2\sqrt{a+bx+cx^2}+afx^4\sqrt{a+bx+cx^2}-bdx^3\sqrt{a+bx+cx^2}+bfx^5\sqrt{a+bx+cx^2}-cdx^4\sqrt{a+bx+cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(c*x**2+b*x+a)**(3/2)/(-f*x**2+d),x)
```

```
[Out] -Integral(1/(-a*d*x**2*sqrt(a + b*x + c*x**2) + a*f*x**4*sqrt(a + b*x + c*x
**2) - b*d*x**3*sqrt(a + b*x + c*x**2) + b*f*x**5*sqrt(a + b*x + c*x**2) -
c*d*x**4*sqrt(a + b*x + c*x**2) + c*f*x**6*sqrt(a + b*x + c*x**2)), x)
```

3.109 $\int \frac{x^2\sqrt{a+bx+cx^2}}{d+ex+fx^2}\,dx$

Optimal. Leaf size=761

$$-\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(4cf(be-af)+b^2f^2-8c^2\left(e^2-df\right)\right)}{8c^{3/2}f^3}-\frac{\left(f\left(af\left(-e\sqrt{e^2-4df}-2df+e^2\right)-b\left(-e^2\sqrt{e\right.\right.\right.}{}$$

```
[Out] -1/8*(b^2*f^2+4*c*f*(-a*f+b*e)-8*c^2*(-d*f+e^2))*arctanh(1/2*(2*c*x+b)/c^(1
/2)/(c*x^2+b*x+a)^(1/2))/c^(3/2)/f^3-1/4*(-2*c*f*x-b*f+4*c*e)*(c*x^2+b*x+a)
^(1/2)/c/f^2-1/2*arctanh(1/4*(4*a*f+2*x*(b*f-c*(e-(-4*d*f+e^2)^(1/2)))-b*(e
-(-4*d*f+e^2)^(1/2)))*2^(1/2)/(c*x^2+b*x+a)^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*
f^2-(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2))*(c*(e^4-4*d*e^2*f+2*d^2*f^2-e^3*(
-4*d*f+e^2)^(1/2)+2*d*e*f*(-4*d*f+e^2)^(1/2))+f*(a*f*(-(-4*d*f+e^2)^(1/2)*e
-2*d*f+e^2)-b*(e^3-3*d*e*f-e^2*(-4*d*f+e^2)^(1/2)+d*f*(-4*d*f+e^2)^(1/2))))
/f^3*2^(1/2)/(-4*d*f+e^2)^(1/2)/(f*(2*a*f-b*(e-(-4*d*f+e^2)^(1/2)))+(-(-4*d
*f+e^2)^(1/2)*e-2*d*f+e^2)*c)^(1/2)+1/2*arctanh(1/4*(4*a*f-b*(e+(-4*d*f+e^2
)^(1/2))+2*x*(b*f-c*(e+(-4*d*f+e^2)^(1/2))))*2^(1/2)/(c*x^2+b*x+a)^(1/2)/(c
*e^2-2*c*d*f-b*e*f+2*a*f^2+(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2))*(c*(e^4-4*
d*e^2*f+2*d^2*f^2+e^3*(-4*d*f+e^2)^(1/2)-2*d*e*f*(-4*d*f+e^2)^(1/2))+f*(a*f
*((-4*d*f+e^2)^(1/2)*e-2*d*f+e^2)-b*(e^3-3*d*e*f+e^2*(-4*d*f+e^2)^(1/2)-d*f
*(-4*d*f+e^2)^(1/2))))/f^3*2^(1/2)/(-4*d*f+e^2)^(1/2)/(((-4*d*f+e^2)^(1/2)*
e-2*d*f+e^2)*c+f*(2*a*f-b*(e+(-4*d*f+e^2)^(1/2))))^(1/2)
```

Rubi [A] time = 3.14, antiderivative size = 761, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1067, 1076, 621, 206, 1032, 724}

$$-\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(4cf(be-af)+b^2f^2-8c^2\left(e^2-df\right)\right)}{8c^{3/2}f^3}-\frac{\left(f\left(af\left(-e\sqrt{e^2-4df}-2df+e^2\right)-b\left(-e^2\sqrt{e\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sqrt[a + b*x + c*x^2])/(d + e*x + f*x^2),x]
```

```
[Out] -((4*c*e - b*f - 2*c*f*x)*Sqrt[a + b*x + c*x^2])/(4*c*f^2) - ((b^2*f^2 + 4*
c*f*(b*e - a*f) - 8*c^2*(e^2 - d*f))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a
+ b*x + c*x^2])])/(8*c^(3/2)*f^3) - ((c*(e^4 - 4*d*e^2*f + 2*d^2*f^2 - e^3*
Sqrt[e^2 - 4*d*f] + 2*d*e*f*Sqrt[e^2 - 4*d*f]) + f*(a*f*(e^2 - 2*d*f - e*Sq
rt[e^2 - 4*d*f]) - b*(e^3 - 3*d*e*f - e^2*Sqrt[e^2 - 4*d*f] + d*f*Sqrt[e^2
- 4*d*f])))*ArcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e - Sq
rt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c
*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*f^3*Sqrt[e^2
 - 4*d*f]*Sqrt[c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e - Sq
rt[e^2 - 4*d*f]))]) + ((c*(e^4 - 4*d*e^2*f + 2*d^2*f^2 + e^3*Sqrt[e^2 - 4*d
*f] - 2*d*e*f*Sqrt[e^2 - 4*d*f]) + f*(a*f*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f
]) - b*(e^3 - 3*d*e*f + e^2*Sqrt[e^2 - 4*d*f] - d*f*Sqrt[e^2 - 4*d*f])))*Ar
cTanh[(4*a*f - b*(e + Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e + Sqrt[e^2 - 4*d*f
]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt
[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*f^3*Sqrt[e^2 - 4*d*f]*Sqrt
[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f
]))])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1032

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1067

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (C_.)*(x_)^2)*((d_) +
(e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((C*(b*f*p - c*e*(2*p +
q + 2)) + 2*c*C*f*(p + q + 1)*x)*(a + b*x + c*x^2)^p*(d + e*x + f*x^2)^(q +
 1))/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3)), x] - Dist[1/(2*c*f^2*(p + q + 1
)*(2*p + 2*q + 3)), Int[(a + b*x + c*x^2)^(p - 1)*(d + e*x + f*x^2)^q*Simp[
p*(b*d - a*e)*(C*(c*e - b*f)*(q + 1) - c*(C*e)*(2*p + 2*q + 3)) + (p + q +
1)*(b^2*C*d*f*p + a*c*(C*(2*d*f - e^2*(2*p + q + 2)) + f*(-2*A*f)*(2*p + 2*
q + 3))) + (2*p*(c*d - a*f)*(C*(c*e - b*f)*(q + 1) - c*(C*e)*(2*p + 2*q + 3
)) + (p + q + 1)*(C*e*f*p*(b^2 - 4*a*c) - b*c*(C*(e^2 - 4*d*f)*(2*p + q + 2
) + f*(2*C*d + 2*A*f)*(2*p + 2*q + 3))))*x + (p*(c*e - b*f)*(C*(c*e - b*f)*
(q + 1) - c*(C*e)*(2*p + 2*q + 3)) + (p + q + 1)*(C*f^2*p*(b^2 - 4*a*c) - c
^2*(C*(e^2 - 4*d*f)*(2*p + q + 2) + f*(2*C*d + 2*A*f)*(2*p + 2*q + 3))))*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, C, q}, x] && NeQ[b^2 - 4*a*c,
0] && NeQ[e^2 - 4*d*f, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0] && NeQ[2*p + 2*
q + 3, 0] &&  !IGtQ[p, 0] &&  !IGtQ[q, 0]
```

Rule 1076

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqr
t[d + e*x + f*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + (B*c - b*C)*x)/((a
+ b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, A
, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]
```

Rule steps

$$\int \frac{x^2\sqrt{a+bx+cx^2}}{d+ex+fx^2}\,dx = -\frac{(4ce-bf-2cfx)\sqrt{a+bx+cx^2}}{4cf^2} - \frac{\int \frac{-\frac{1}{4}d\left(4bce-b^2f-4acf\right)-\frac{1}{4}\left(8c^2de-b^2ef-4acef+4bc\left(e^2-2df\right)\right.}{\sqrt{a+bx+cx^2}\left(d+ex+f\right.}}{2cf^2}$$

$$= -\frac{(4ce-bf-2cfx)\sqrt{a+bx+cx^2}}{4cf^2} - \frac{\int \frac{-\frac{1}{4}df\left(4bce-b^2f-4acf\right)-\frac{1}{4}d\left(b^2f^2+4cf(be-af)-8c^2\left(e^2-df\right)\right.}{}}{}$$

$$= -\frac{(4ce-bf-2cfx)\sqrt{a+bx+cx^2}}{4cf^2} - \frac{\left(b^2f^2+4cf(be-af)-8c^2\left(e^2-df\right)\right)\text{Subst}\left(\right.}{4cf^3}$$

$$= -\frac{(4ce-bf-2cfx)\sqrt{a+bx+cx^2}}{4cf^2} - \frac{\left(b^2f^2+4cf(be-af)-8c^2\left(e^2-df\right)\right)\tanh^{-1}\left(\right.}{8c^{3/2}f^3}$$

$$= -\frac{(4ce-bf-2cfx)\sqrt{a+bx+cx^2}}{4cf^2} - \frac{\left(b^2f^2+4cf(be-af)-8c^2\left(e^2-df\right)\right)\tanh^{-1}\left(\right.}{8c^{3/2}f^3}$$

Mathematica [A] time = 2.31, size = 552, normalized size = 0.73

$$\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)\left(4cf(af-be)-b^2f^2+8c^2\left(e^2-df\right)\right)}{8c^{3/2}f^3} + \frac{f\sqrt{e^2-4df}\sqrt{a+x(b+cx)}\,(bf-4ce+2cfx)}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sqrt[a + b*x + c*x^2])/(d + e*x + f*x^2),x]
```

```
[Out] ((-(b^2*f^2) + 4*c*f*(-(b*e) + a*f) + 8*c^2*(e^2 - d*f))*ArcTanh[(b + 2*c*x
)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/(8*c^(3/2)*f^3) + (f*Sqrt[e^2 - 4*d*f
]*(-4*c*e + b*f + 2*c*f*x)*Sqrt[a + x*(b + c*x)] + Sqrt[2]*c*(e^2 - 2*d*f +
 e*Sqrt[e^2 - 4*d*f])*Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f
 - b*(e + Sqrt[e^2 - 4*d*f]))]*ArcTanh[(4*a*f - 2*c*(e + Sqrt[e^2 - 4*d*f])
*x - b*(e + Sqrt[e^2 - 4*d*f] - 2*f*x))/(2*Sqrt[2]*Sqrt[c*(e^2 - 2*d*f + e*
Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))]*Sqrt[a + x*(b +
 c*x)])] + Sqrt[2]*c*(-e^2 + 2*d*f + e*Sqrt[e^2 - 4*d*f])*Sqrt[c*(e^2 - 2*d
*f - e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f + b*(-e + Sqrt[e^2 - 4*d*f]))]*ArcTanh
[(4*a*f + 2*c*(-e + Sqrt[e^2 - 4*d*f])*x + b*(-e + Sqrt[e^2 - 4*d*f] + 2*f*
x))/(2*Sqrt[2]*Sqrt[c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f + b*(-
e + Sqrt[e^2 - 4*d*f]))]*Sqrt[a + x*(b + c*x)])])/(4*c*f^3*Sqrt[e^2 - 4*d*f
])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [B] time = 0.02, size = 14815, normalized size = 19.47

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x)
```

```
[Out] result too large to display
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for mo
re details)Is 4*d*f-e^2 positive, negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\,\sqrt{c\,x^2+b\,x+a}}{f\,x^2+e\,x+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*x + c*x^2)^(1/2))/(d + e*x + f*x^2),x)
```

```
[Out] int((x^2*(a + b*x + c*x^2)^(1/2))/(d + e*x + f*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\sqrt{a+bx+cx^2}}{d+ex+fx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c*x**2+b*x+a)**(1/2)/(f*x**2+e*x+d),x)
```

```
[Out] Integral(x**2*sqrt(a + b*x + c*x**2)/(d + e*x + f*x**2), x)
```

3.110 $\int \frac{x\sqrt{a+bx+cx^2}}{d+ex+fx^2}\,dx$

Optimal. Leaf size=549

$$-\frac{\left(\left(e-\sqrt{e^2-4df}\right)\left(f(be-af)-c\left(e^2-df\right)\right)+2df(ce-bf)\right)\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)-b\left(e-\sqrt{e^2-4df}\right.}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}\,(ce-bf)-bef-2c}}\right.}{\sqrt{2}f^2\sqrt{e^2-4df}\sqrt{2af^2-\sqrt{e^2-4df}\,(ce-bf)-bef-2cdf+ce^2}}$$

```
[Out] -1/2*(-b*f+2*c*e)*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/f^2/c^
(1/2)+(c*x^2+b*x+a)^(1/2)/f-1/2*arctanh(1/4*(4*a*f+2*x*(b*f-c*(e-(-4*d*f+e^
2)^(1/2)))-b*(e-(-4*d*f+e^2)^(1/2)))*2^(1/2)/(c*x^2+b*x+a)^(1/2)/(c*e^2-2*c
*d*f-b*e*f+2*a*f^2-(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2))*(2*d*f*(-b*f+c*e)+
(f*(-a*f+b*e)-c*(-d*f+e^2))*(e-(-4*d*f+e^2)^(1/2)))/f^2*2^(1/2)/(-4*d*f+e^2
)^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2-(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2)+1
/2*arctanh(1/4*(4*a*f-b*(e+(-4*d*f+e^2)^(1/2))+2*x*(b*f-c*(e+(-4*d*f+e^2)^(
1/2))))*2^(1/2)/(c*x^2+b*x+a)^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2+(-b*f+c*e)
*(-4*d*f+e^2)^(1/2))^(1/2))*(2*d*f*(-b*f+c*e)+(f*(-a*f+b*e)-c*(-d*f+e^2))*(
e+(-4*d*f+e^2)^(1/2)))/f^2*2^(1/2)/(-4*d*f+e^2)^(1/2)/(c*e^2-2*c*d*f-b*e*f+
2*a*f^2+(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2)
```

Rubi [A] time = 7.03, antiderivative size = 549, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {1019, 1076, 621, 206, 1032, 724}

$$-\frac{\left(\left(e-\sqrt{e^2-4df}\right)\left(f(be-af)-c\left(e^2-df\right)\right)+2df(ce-bf)\right)\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)-b\left(e-\sqrt{e^2-4df}\right.}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}\,(ce-bf)-bef-2c}}\right.}{\sqrt{2}f^2\sqrt{e^2-4df}\sqrt{2af^2-\sqrt{e^2-4df}\,(ce-bf)-bef-2cdf+ce^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sqrt[a + b*x + c*x^2])/(d + e*x + f*x^2),x]
```

```
[Out] Sqrt[a + b*x + c*x^2]/f - ((2*c*e - b*f)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqr
t[a + b*x + c*x^2])])/(2*Sqrt[c]*f^2) - ((2*d*f*(c*e - b*f) + (e - Sqrt[e^2
 - 4*d*f])*(f*(b*e - a*f) - c*(e^2 - d*f)))*ArcTanh[(4*a*f - b*(e - Sqrt[e^
2 - 4*d*f]) + 2*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2
- 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x +
 c*x^2])])/(Sqrt[2]*f^2*Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*
a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]) + ((2*d*f*(c*e - b*f) + (e + Sqrt[e
^2 - 4*d*f])*(f*(b*e - a*f) - c*(e^2 - d*f)))*ArcTanh[(4*a*f - b*(e + Sqrt[
e^2 - 4*d*f]) + 2*(b*f - c*(e + Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^
2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x
 + c*x^2])])/(Sqrt[2]*f^2*Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c*d*f - b*e*f +
2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1019

```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e
_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(h*(a + b*x + c*x^2)^p*(d
+ e*x + f*x^2)^(q + 1))/(2*f*(p + q + 1)), x] - Dist[1/(2*f*(p + q + 1)), I
nt[(a + b*x + c*x^2)^(p - 1)*(d + e*x + f*x^2)^q*Simp[h*p*(b*d - a*e) + a*(
h*e - 2*g*f)*(p + q + 1) + (2*h*p*(c*d - a*f) + b*(h*e - 2*g*f)*(p + q + 1)
)*x + (h*p*(c*e - b*f) + c*(h*e - 2*g*f)*(p + q + 1))*x^2, x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, g, h, q}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d
*f, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0]
```

Rule 1032

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1076

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqr
t[d + e*x + f*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + (B*c - b*C)*x)/((a
+ b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, A
, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]
```

Rubi steps

$$\int \frac{x\sqrt{a+bx+cx^2}}{d+ex+fx^2}\,dx = \frac{\sqrt{a+bx+cx^2}}{f} - \frac{\int \frac{\frac{bd}{2}+\frac{1}{2}(2cd+be-2af)x+\frac{1}{2}(2ce-bf)x^2}{\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)}\,dx}{f}$$

$$= \frac{\sqrt{a+bx+cx^2}}{f} - \frac{\int \frac{\frac{bdf}{2}-\frac{1}{2}d(2ce-bf)+\left(\frac{1}{2}f(2cd+be-2af)-\frac{1}{2}e(2ce-bf)\right)x}{\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)}\,dx}{f^2} - \frac{(2ce-bf)\int \frac{1}{\sqrt{a+bx+cx^2}} \cdots}{2f^2}$$

$$= \frac{\sqrt{a+bx+cx^2}}{f} - \frac{(2ce-bf)\,\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{f^2} - \frac{\left(2f\left(\frac{bdf}{2}-\frac{1}{2}d(2ce-bf)\right) \cdots\right.}{\cdots}$$

$$= \frac{\sqrt{a+bx+cx^2}}{f} - \frac{(2ce-bf)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}f^2} + \frac{\left(2\left(2f\left(\frac{bdf}{2}-\frac{1}{2}d(2ce-bf)\right)-\left(\frac{1}{2} \cdots\right.\right.\right.}{\cdots}$$

$$= \frac{\sqrt{a+bx+cx^2}}{f} - \frac{(2ce-bf)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}f^2} - \frac{\left(2f(cde-bdf)+\left(e-\sqrt{e^2-4df} \cdots\right.\right.}{\sqrt{2}f^2\sqrt{\cdots}}$$

Mathematica [A] time = 1.78, size = 496, normalized size = 0.90

$$4f\sqrt{e^2-4df}\sqrt{a+x(b+cx)}-\sqrt{2}\left(\sqrt{e^2-4df}+e\right)\sqrt{f\left(2af-b\left(\sqrt{e^2-4df}+e\right)\right)+c\left(e\sqrt{e^2-4df}-2df+\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sqrt[a + b*x + c*x^2])/(d + e*x + f*x^2),x]
```

```
[Out] -1/2*((2*c*e - b*f)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])
/(Sqrt[c]*f^2) + (4*f*Sqrt[e^2 - 4*d*f]*Sqrt[a + x*(b + c*x)] - Sqrt[2]*(e
+ Sqrt[e^2 - 4*d*f])*Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f
- b*(e + Sqrt[e^2 - 4*d*f]))]*ArcTanh[(4*a*f - 2*c*(e + Sqrt[e^2 - 4*d*f])*
x - b*(e + Sqrt[e^2 - 4*d*f] - 2*f*x))/(2*Sqrt[2]*Sqrt[c*(e^2 - 2*d*f + e*S
qrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))]*Sqrt[a + x*(b +
c*x)])] - Sqrt[2]*(-e + Sqrt[e^2 - 4*d*f])*Sqrt[f*(-(b*e) + 2*a*f + b*Sqrt[
e^2 - 4*d*f]) + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]*ArcTanh[(4*a*f + 2*c
*(-e + Sqrt[e^2 - 4*d*f])*x + b*(-e + Sqrt[e^2 - 4*d*f] + 2*f*x))/(2*Sqrt[2
]*Sqrt[f*(-(b*e) + 2*a*f + b*Sqrt[e^2 - 4*d*f]) + c*(e^2 - 2*d*f - e*Sqrt[e
^2 - 4*d*f])]*Sqrt[a + x*(b + c*x)])])/(4*f^2*Sqrt[e^2 - 4*d*f])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.02, size = 10138, normalized size = 18.47

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x)
```

```
[Out] result too large to display
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for mo
re details)Is 4*d*f-e^2 positive, negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\,\sqrt{c\,x^2+b\,x+a}}{f\,x^2+e\,x+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*x + c*x^2)^(1/2))/(d + e*x + f*x^2),x)
```

```
[Out] int((x*(a + b*x + c*x^2)^(1/2))/(d + e*x + f*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt{a+bx+cx^2}}{d+ex+fx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x**2+b*x+a)**(1/2)/(f*x**2+e*x+d),x)
```

```
[Out] Integral(x*sqrt(a + b*x + c*x**2)/(d + e*x + f*x**2), x)
```

3.111 $\int \frac{\sqrt{a+bx+cx^2}}{d+ex+fx^2}\,dx$

Optimal. Leaf size=431

$$-\frac{\sqrt{f\left(2af-b\left(e-\sqrt{e^2-4df}\right)\right)+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)-b\left(e-\sqrt{e^2-4d\ldots}\right.}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}\,(ce-bf)-bef-\ldots}}\right.}{\sqrt{2}f\sqrt{e^2-4df}}$$

```
[Out] arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))*c^(1/2)/f-1/2*arctanh(1/
4*(4*a*f+2*x*(b*f-c*(e-(-4*d*f+e^2)^(1/2)))-b*(e-(-4*d*f+e^2)^(1/2)))*2^(1/
2)/(c*x^2+b*x+a)^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2-(-b*f+c*e)*(-4*d*f+e^2)
^(1/2))^(1/2))*(f*(2*a*f-b*(e-(-4*d*f+e^2)^(1/2)))+(-(-4*d*f+e^2)^(1/2)*e-2
*d*f+e^2)*c)^(1/2)/f*2^(1/2)/(-4*d*f+e^2)^(1/2)+1/2*arctanh(1/4*(4*a*f-b*(e
+(-4*d*f+e^2)^(1/2))+2*x*(b*f-c*(e+(-4*d*f+e^2)^(1/2))))*2^(1/2)/(c*x^2+b*x
+a)^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2+(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2)
)*(((-4*d*f+e^2)^(1/2)*e-2*d*f+e^2)*c+f*(2*a*f-b*(e+(-4*d*f+e^2)^(1/2))))^(
1/2)/f*2^(1/2)/(-4*d*f+e^2)^(1/2)
```

Rubi [A] time = 0.65, antiderivative size = 431, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {989, 621, 206, 1032, 724}

$$-\frac{\sqrt{f\left(2af-b\left(e-\sqrt{e^2-4df}\right)\right)+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)-b\left(e-\sqrt{e^2-4d\ldots}\right.}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}\,(ce-bf)-bef-\ldots}}\right.}{\sqrt{2}f\sqrt{e^2-4df}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x + c*x^2]/(d + e*x + f*x^2),x]
```

```
[Out] (Sqrt[c]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/f - (Sqrt[
c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e - Sqrt[e^2 - 4*d*f]
))]*ArcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e - Sqrt[e^2 -
 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f
)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*f*Sqrt[e^2 - 4*d*f])
 + (Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2
 - 4*d*f]))]*ArcTanh[(4*a*f - b*(e + Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e + S
qrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (
c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*f*Sqrt[e^2
- 4*d*f])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
```

```
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 989

```
Int[Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2]/((d_) + (e_.)*(x_) + (f_.)*(x_)^
2), x_Symbol] :> Dist[c/f, Int[1/Sqrt[a + b*x + c*x^2], x], x] - Dist[1/f,
Int[(c*d - a*f + (c*e - b*f)*x)/(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)),
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 -
4*d*f, 0]
```

Rule 1032

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{\sqrt{a+bx+cx^2}}{d+ex+fx^2}\,dx = -\frac{\int \frac{cd-af+(ce-bf)x}{\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)}\,dx}{f}+\frac{c\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{f}$$

$$= \frac{(2c)\,\mathrm{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{f} - \frac{\left(2f(cd-af)-(ce-bf)\left(e-\sqrt{e^2-4df}\right)\right)\int \frac{}{(e-\sqrt{\cdots}}}{f\sqrt{e^2-4df}}$$

$$= \frac{\sqrt{c}\,\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\,\sqrt{a+bx+cx^2}}\right)}{f} + \frac{\left(2\left(2f(cd-af)-(ce-bf)\left(e-\sqrt{e^2-4df}\right)\right)\right)\mathrm{Subst}\left(\int \frac{}{16\cdots}\right.}{}$$

$$= \frac{\sqrt{c}\,\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\,\sqrt{a+bx+cx^2}}\right)}{f} - \frac{\sqrt{c\left(e^2-2df-e\sqrt{e^2-4df}\right)+f\left(2af-b\left(e-\sqrt{e^2-4df}\right)\right.}}{\sqrt{2}\,f\sqrt{e^2\cdots}}$$

Mathematica [A] time = 0.74, size = 417, normalized size = 0.97

$$\frac{\sqrt{f\left(2af-b\left(\sqrt{e^2-4df}+e\right)\right)+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}\,\tanh^{-1}\left(\frac{4af-b\left(\sqrt{e^2-4df}+e-2fx\right)-2cx\left(\sqrt{e^2-4df}+e\right)}{2\sqrt{2}\,\sqrt{a+x(b+cx)}\,\sqrt{f\left(2af-b\left(\sqrt{e^2-4df}+e\right)\right)+c\left(e\sqrt{e^2-4df}\cdots\right.}}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x + c*x^2]/(d + e*x + f*x^2),x]
```

```
[Out] (Sqrt[c]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/f + (Sqrt[
c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]
))]*ArcTanh[(4*a*f - 2*c*(e + Sqrt[e^2 - 4*d*f])*x - b*(e + Sqrt[e^2 - 4*d*
f] - 2*f*x))/(2*Sqrt[2]*Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a
*f - b*(e + Sqrt[e^2 - 4*d*f]))]*Sqrt[a + x*(b + c*x)])] - Sqrt[f*(-(b*e) +
 2*a*f + b*Sqrt[e^2 - 4*d*f]) + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]*ArcT
anh[(4*a*f + 2*c*(-e + Sqrt[e^2 - 4*d*f])*x + b*(-e + Sqrt[e^2 - 4*d*f] + 2
*f*x))/(2*Sqrt[2]*Sqrt[f*(-(b*e) + 2*a*f + b*Sqrt[e^2 - 4*d*f]) + c*(e^2 -
2*d*f - e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + x*(b + c*x)])])/(Sqrt[2]*f*Sqrt[e^2
- 4*d*f])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.02, size = 6019, normalized size = 13.97

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x)
```

```
[Out] result too large to display
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for mo
re details)Is 4*d*f-e^2 positive, negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c\,x^2+b\,x+a}}{f\,x^2+e\,x+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)^(1/2)/(d + e*x + f*x^2),x)
```

```
[Out] int((a + b*x + c*x^2)^(1/2)/(d + e*x + f*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + bx + cx^{2}}}{d + ex + fx^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(1/2)/(f*x**2+e*x+d),x)
```

```
[Out] Integral(sqrt(a + b*x + c*x**2)/(d + e*x + f*x**2), x)
```

3.112 $\int \frac{\sqrt{a+bx+cx^2}}{x(d+ex+fx^2)}\,dx$

Optimal. Leaf size=523

$$\frac{\left(cd\left(e-\sqrt{e^2-4df}\right)-f\left(2bd-a\left(\sqrt{e^2-4df}+e\right)\right)\right)\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)-b\left(e-\sqrt{e^2-4df}\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}d\sqrt{e^2-4df}\sqrt{f\left(2af-b\left(e-\sqrt{e^2-4df}\right)\right)+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}-\ldots(cd$$

```
[Out] -arctanh(1/2*(b*x+2*a)/a^(1/2)/(c*x^2+b*x+a)^(1/2))*a^(1/2)/d+1/2*arctanh(1
/4*(4*a*f+2*x*(b*f-c*(e-(-4*d*f+e^2)^(1/2)))-b*(e-(-4*d*f+e^2)^(1/2)))*2^(1
/2)/(c*x^2+b*x+a)^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2-(-b*f+c*e)*(-4*d*f+e^2
)^(1/2))^(1/2))*(c*d*(e-(-4*d*f+e^2)^(1/2))-f*(2*b*d-a*(e+(-4*d*f+e^2)^(1/2
))))/d*2^(1/2)/(-4*d*f+e^2)^(1/2)/(f*(2*a*f-b*(e-(-4*d*f+e^2)^(1/2)))+(-(-4
*d*f+e^2)^(1/2)*e-2*d*f+e^2)*c)^(1/2)-1/2*arctanh(1/4*(4*a*f-b*(e+(-4*d*f+e
^2)^(1/2))+2*x*(b*f-c*(e+(-4*d*f+e^2)^(1/2))))*2^(1/2)/(c*x^2+b*x+a)^(1/2)/
(c*e^2-2*c*d*f-b*e*f+2*a*f^2+(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2))*(-f*(2*b
*d-a*(e-(-4*d*f+e^2)^(1/2)))+c*d*(e+(-4*d*f+e^2)^(1/2)))/d*2^(1/2)/(-4*d*f+
e^2)^(1/2)/(((-4*d*f+e^2)^(1/2)*e-2*d*f+e^2)*c+f*(2*a*f-b*(e+(-4*d*f+e^2)^(
1/2))))^(1/2)
```

Rubi [A] time = 3.70, antiderivative size = 521, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {6728, 734, 843, 621, 206, 724, 1019, 1076, 1032}

$$-\frac{\left(-af\left(\sqrt{e^2-4df}+e\right)+2bdf-cd\left(e-\sqrt{e^2-4df}\right)\right)\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)-b\left(e-\sqrt{e^2-4df}\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}d\sqrt{e^2-4df}\sqrt{f\left(2af-b\left(e-\sqrt{e^2-4df}\right)\right)+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}+\ldots$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x + c*x^2]/(x*(d + e*x + f*x^2)),x]
```

```
[Out] -((Sqrt[a]*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/d) - ((2
*b*d*f - c*d*(e - Sqrt[e^2 - 4*d*f]) - a*f*(e + Sqrt[e^2 - 4*d*f]))*ArcTanh
[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*x
)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2
- 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*d*Sqrt[e^2 - 4*d*f]*Sqrt[c*(e^2
 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e - Sqrt[e^2 - 4*d*f]))]) +
 ((2*b*d*f - a*f*(e - Sqrt[e^2 - 4*d*f]) - c*d*(e + Sqrt[e^2 - 4*d*f]))*Arc
Tanh[(4*a*f - b*(e + Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e + Sqrt[e^2 - 4*d*f]
))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[
e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*d*Sqrt[e^2 - 4*d*f]*Sqrt[c*
(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))
])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1019

```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e
_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(h*(a + b*x + c*x^2)^p*(d
+ e*x + f*x^2)^(q + 1))/(2*f*(p + q + 1)), x] - Dist[1/(2*f*(p + q + 1)), I
nt[(a + b*x + c*x^2)^(p - 1)*(d + e*x + f*x^2)^q*Simp[h*p*(b*d - a*e) + a*(
h*e - 2*g*f)*(p + q + 1) + (2*h*p*(c*d - a*f) + b*(h*e - 2*g*f)*(p + q + 1)
)*x + (h*p*(c*e - b*f) + c*(h*e - 2*g*f)*(p + q + 1))*x^2, x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, g, h, q}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d
*f, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0]
```

Rule 1032

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1076

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqr
t[d + e*x + f*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + (B*c - b*C)*x)/((a
+ b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, A
, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+bx+cx^2}}{x\left(d+ex+fx^2\right)}\,dx = \int \left(\frac{\sqrt{a+bx+cx^2}}{dx} + \frac{(-e-fx)\sqrt{a+bx+cx^2}}{d\left(d+ex+fx^2\right)}\right)dx$$

$$= \frac{\int \frac{\sqrt{a+bx+cx^2}}{x}\,dx}{d} + \frac{\int \frac{(-e-fx)\sqrt{a+bx+cx^2}}{d+ex+fx^2}\,dx}{d}$$

$$= -\frac{\int \frac{-2a-bx}{x\sqrt{a+bx+cx^2}}\,dx}{2d} - \frac{\int \frac{-\frac{1}{2}(bd-2ae)f-\frac{1}{2}f(2cd-be-2af)x+\frac{1}{2}bf^2x^2}{\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)}\,dx}{df}$$

$$= \frac{a\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx}{d} - \frac{\int \frac{-\frac{1}{2}bdf^2-\frac{1}{2}(bd-2ae)f^2+\left(-\frac{1}{2}bef^2-\frac{1}{2}f^2(2cd-be-2af)\right)x}{\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)}\,dx}{df^2}$$

$$= -\frac{(2a)\,\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+bx}{\sqrt{a+bx+cx^2}}\right)}{d} - \frac{\left(2bdf-af\left(e-\sqrt{e^2-4df}\right)-cd\left(e+\sqrt{e^2-4\right.\right.}{d\sqrt{e^2-4df}}$$

$$= -\frac{\sqrt{a}\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{d} + \frac{\left(2\left(2bdf-af\left(e-\sqrt{e^2-4df}\right)-cd\left(e+\sqrt{e^2-4df}\right)\right)\right)}{}$$

$$= -\frac{\sqrt{a}\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{d} - \frac{\left(2bdf-cd\left(e-\sqrt{e^2-4df}\right)-af\left(e+\sqrt{e^2-4df}\right)\right)\tan}{\sqrt{2}\,d\sqrt{e^2-4df}\sqrt{ce^2-2cdf-bef}}$$

Mathematica [A] time = 1.31, size = 454, normalized size = 0.87

$$\frac{\left(\sqrt{e^2-4df}-e\right)\sqrt{f\left(2af-b\left(\sqrt{e^2-4df}+e\right)\right)+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}\tanh^{-1}\left(\frac{4af-b\left(\sqrt{e^2-4df}+e-2f\right.}{2\sqrt{2}\sqrt{a+x(b+cx)}\sqrt{f\left(2af-b\left(\sqrt{e^2-\right.\right.}}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x + c*x^2]/(x*(d + e*x + f*x^2)),x]
```

```
[Out] -((Sqrt[a]*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + x*(b + c*x)])])/d) + ((-
e + Sqrt[e^2 - 4*d*f])*Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*
f - b*(e + Sqrt[e^2 - 4*d*f]))]*ArcTanh[(4*a*f - 2*c*(e + Sqrt[e^2 - 4*d*f]
)*x - b*(e + Sqrt[e^2 - 4*d*f] - 2*f*x))/(2*Sqrt[2]*Sqrt[c*(e^2 - 2*d*f + e
*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))]*Sqrt[a + x*(b
+ c*x)])] + (e + Sqrt[e^2 - 4*d*f])*Sqrt[f*(-(b*e) + 2*a*f + b*Sqrt[e^2 - 4
*d*f]) + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]*ArcTanh[(4*a*f + 2*c*(-e +
Sqrt[e^2 - 4*d*f])*x + b*(-e + Sqrt[e^2 - 4*d*f] + 2*f*x))/(2*Sqrt[2]*Sqrt[
f*(-(b*e) + 2*a*f + b*Sqrt[e^2 - 4*d*f]) + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*
d*f])]*Sqrt[a + x*(b + c*x)])])/(2*Sqrt[2]*d*f*Sqrt[e^2 - 4*d*f])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/x/(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/x/(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 7.77sym2poly/r2sym(const ge
n & e,const index_m & i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.02, size = 6460, normalized size = 12.35

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(1/2)/x/(f*x^2+e*x+d),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^2 + bx + a}}{\left(fx^2 + ex + d\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/x/(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^2 + b*x + a)/((f*x^2 + e*x + d)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c\,x^2 + b\,x + a}}{x\,\left(f\,x^2 + e\,x + d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)^(1/2)/(x*(d + e*x + f*x^2)),x)
```

```
[Out] int((a + b*x + c*x^2)^(1/2)/(x*(d + e*x + f*x^2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + bx + cx^2}}{x\left(d + ex + fx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(1/2)/x/(f*x**2+e*x+d),x)
```

```
[Out] Integral(sqrt(a + b*x + c*x**2)/(x*(d + e*x + f*x**2)), x)
```

3.113 $\int \frac{\sqrt{a+bx+cx^2}}{x^2\left(d+ex+fx^2\right)}\,dx$

Optimal. Leaf size=736

$$-\frac{f\left(a\left(e\sqrt{e^2-4df}-2df+e^2\right)-bd\left(\sqrt{e^2-4df}+e\right)+2cd^2\right)\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)-b\left(e-\sqrt{e^2-4df}\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+}}\right.}{\sqrt{2}\,d^2\sqrt{e^2-4df}\sqrt{f\left(2af-b\left(e-\sqrt{e^2-4df}\right)\right)+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}$$

```
[Out] -1/2*b*arctanh(1/2*(b*x+2*a)/a^(1/2)/(c*x^2+b*x+a)^(1/2))/d/a^(1/2)+e*arcta
nh(1/2*(b*x+2*a)/a^(1/2)/(c*x^2+b*x+a)^(1/2))*a^(1/2)/d^2-1/2*b*e*arctanh(1
/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/d^2/c^(1/2)-1/2*(-b*e+2*c*d)*arct
anh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/d^2/c^(1/2)+arctanh(1/2*(2*c
*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))*c^(1/2)/d-(c*x^2+b*x+a)^(1/2)/d/x-1/2*f*
arctanh(1/4*(4*a*f+2*x*(b*f-c*(e-(-4*d*f+e^2)^(1/2)))-b*(e-(-4*d*f+e^2)^(1/
2)))*2^(1/2)/(c*x^2+b*x+a)^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2-(-b*f+c*e)*(-
4*d*f+e^2)^(1/2))^(1/2))*(2*c*d^2-b*d*(e+(-4*d*f+e^2)^(1/2))+a*((-4*d*f+e^2
)^(1/2)*e-2*d*f+e^2))/d^2*2^(1/2)/(-4*d*f+e^2)^(1/2)/(f*(2*a*f-b*(e-(-4*d*f
+e^2)^(1/2)))+(-(-4*d*f+e^2)^(1/2)*e-2*d*f+e^2)*c)^(1/2)+1/2*f*arctanh(1/4*
(4*a*f-b*(e+(-4*d*f+e^2)^(1/2))+2*x*(b*f-c*(e+(-4*d*f+e^2)^(1/2))))*2^(1/2)
/(c*x^2+b*x+a)^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2+(-b*f+c*e)*(-4*d*f+e^2)^(
1/2))^(1/2))*(2*c*d^2-b*d*(e-(-4*d*f+e^2)^(1/2))+a*(-(-4*d*f+e^2)^(1/2)*e-2
*d*f+e^2))/d^2*2^(1/2)/(-4*d*f+e^2)^(1/2)/(((-4*d*f+e^2)^(1/2)*e-2*d*f+e^2)
*c+f*(2*a*f-b*(e+(-4*d*f+e^2)^(1/2))))^(1/2)
```

Rubi [A] time = 3.48, antiderivative size = 736, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6728, 732, 843, 621, 206, 724, 734, 1019, 1076, 1032}

$$-\frac{f\left(a\left(e\sqrt{e^2-4df}-2df+e^2\right)-bd\left(\sqrt{e^2-4df}+e\right)+2cd^2\right)\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)-b\left(e-\sqrt{e^2-4df}\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+}}\right.}{\sqrt{2}\,d^2\sqrt{e^2-4df}\sqrt{f\left(2af-b\left(e-\sqrt{e^2-4df}\right)\right)+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x + c*x^2]/(x^2*(d + e*x + f*x^2)),x]
```

```
[Out] -(Sqrt[a + b*x + c*x^2]/(d*x)) - (b*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a +
 b*x + c*x^2])])/(2*Sqrt[a]*d) + (Sqrt[a]*e*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*
Sqrt[a + b*x + c*x^2])])/d^2 + (Sqrt[c]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt
[a + b*x + c*x^2])])/d - (b*e*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x +
 c*x^2])])/(2*Sqrt[c]*d^2) - ((2*c*d - b*e)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*
Sqrt[a + b*x + c*x^2])])/(2*Sqrt[c]*d^2) - (f*(2*c*d^2 - b*d*(e + Sqrt[e^2
- 4*d*f]) + a*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]))*ArcTanh[(4*a*f - b*(e -
Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt
[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a
+ b*x + c*x^2])])/(Sqrt[2]*d^2*Sqrt[e^2 - 4*d*f]*Sqrt[c*(e^2 - 2*d*f - e*Sq
rt[e^2 - 4*d*f]) + f*(2*a*f - b*(e - Sqrt[e^2 - 4*d*f]))]) + (f*(2*c*d^2 -
b*d*(e - Sqrt[e^2 - 4*d*f]) + a*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]))*ArcTan
h[(4*a*f - b*(e + Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e + Sqrt[e^2 - 4*d*f]))*
x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2
 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*d^2*Sqrt[e^2 - 4*d*f]*Sqrt[c*(
e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))]
)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 1)), x] - Di
st[p/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && (IntegerQ[p]
 || LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a,
 b, c, d, e, m, p, x]
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1019

```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e
_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(h*(a + b*x + c*x^2)^p*(d
+ e*x + f*x^2)^(q + 1))/(2*f*(p + q + 1)), x] - Dist[1/(2*f*(p + q + 1)), I
nt[(a + b*x + c*x^2)^(p - 1)*(d + e*x + f*x^2)^q*Simp[h*p*(b*d - a*e) + a*(
h*e - 2*g*f)*(p + q + 1) + (2*h*p*(c*d - a*f) + b*(h*e - 2*g*f)*(p + q + 1)
)*x + (h*p*(c*e - b*f) + c*(h*e - 2*g*f)*(p + q + 1))*x^2, x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, g, h, q}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d
*f, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0]
```

Rule 1032

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
```

```
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1076

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqr
t[d + e*x + f*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + (B*c - b*C)*x)/((a
+ b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, A
, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+bx+cx^2}}{x^2\left(d+ex+fx^2\right)}\,dx = \int \left(\frac{\sqrt{a+bx+cx^2}}{dx^2}-\frac{e\sqrt{a+bx+cx^2}}{d^2x}+\frac{\left(e^2-df+efx\right)\sqrt{a+bx+cx^2}}{d^2\left(d+ex+fx^2\right)}\right)dx$$

$$= \frac{\int \frac{\left(e^2-df+efx\right)\sqrt{a+bx+cx^2}}{d+ex+fx^2}\,dx}{d^2}+\frac{\int \frac{\sqrt{a+bx+cx^2}}{x^2}\,dx}{d}-\frac{e\int \frac{\sqrt{a+bx+cx^2}}{x}\,dx}{d^2}$$

$$= -\frac{\sqrt{a+bx+cx^2}}{dx}+\frac{\int \frac{b+2cx}{x\sqrt{a+bx+cx^2}}\,dx}{2d}+\frac{e\int \frac{-2a-bx}{x\sqrt{a+bx+cx^2}}\,dx}{2d^2}-\frac{\int \frac{\frac{1}{2}f\left(bde-2ae^2+2adf\right)+\frac{1}{2}f\left(2cde-\cdots\right.}{\sqrt{a+bx+cx^2}}\cdots}{\cdots}$$

$$= -\frac{\sqrt{a+bx+cx^2}}{dx}+\frac{b\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx}{2d}+\frac{c\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{d}-\frac{(ae)\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx}{d^2}-\frac{(be\cdots}{\cdots}$$

$$= -\frac{\sqrt{a+bx+cx^2}}{dx}-\frac{b\,\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx,x,\frac{2a+bx}{\sqrt{a+bx+cx^2}}\right)}{d}+\frac{(2c)\,\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx,x,\frac{b+\cdots}{\sqrt{a+b\cdots}}\right.}{d}$$

$$= -\frac{\sqrt{a+bx+cx^2}}{dx}-\frac{b\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{a}\,d}+\frac{\sqrt{a}\,e\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{d^2}+\frac{\sqrt{c}\,\tan\cdots}{\cdots}$$

$$= -\frac{\sqrt{a+bx+cx^2}}{dx}-\frac{b\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{a}\,d}+\frac{\sqrt{a}\,e\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{d^2}+\frac{\sqrt{c}\,\tan\cdots}{\cdots}$$

Mathematica [A] time = 1.68, size = 520, normalized size = 0.71

$$\frac{(2ae-bd)\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+x(b+cx)}}\right)}{2\sqrt{a}\,d^2}-\frac{4df\sqrt{e^2-4df}\sqrt{a+x(b+cx)}+\sqrt{2}\,x\left(e\sqrt{e^2-4df}+2df-e^2\right)\sqrt{f\left(2af-b\cdots\right.}}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x + c*x^2]/(x^2*(d + e*x + f*x^2)),x]
```

```
[Out] ((-(b*d) + 2*a*e)*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + x*(b + c*x)])])/(
2*Sqrt[a]*d^2) - (4*d*f*Sqrt[e^2 - 4*d*f]*Sqrt[a + x*(b + c*x)] + Sqrt[2]*(
-e^2 + 2*d*f + e*Sqrt[e^2 - 4*d*f])*Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*
f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))]*x*ArcTanh[(4*a*f - 2*c*(e + Sq
rt[e^2 - 4*d*f])*x - b*(e + Sqrt[e^2 - 4*d*f] - 2*f*x))/(2*Sqrt[2]*Sqrt[c*(
e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))]
*Sqrt[a + x*(b + c*x)])] + Sqrt[2]*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])*Sqrt
[c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f + b*(-e + Sqrt[e^2 - 4*d*
f]))]*x*ArcTanh[(4*a*f + 2*c*(-e + Sqrt[e^2 - 4*d*f])*x + b*(-e + Sqrt[e^2
- 4*d*f] + 2*f*x))/(2*Sqrt[2]*Sqrt[c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) +
f*(2*a*f + b*(-e + Sqrt[e^2 - 4*d*f]))]*Sqrt[a + x*(b + c*x)])])/(4*d^2*f*S
qrt[e^2 - 4*d*f]*x)
```

fricas [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/x^2/(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/x^2/(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [**B**] time = 0.02, size = 6765, normalized size = 9.19

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(1/2)/x^2/(f*x^2+e*x+d),x)
```

```
[Out] result too large to display
```

maxima [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^2 + bx + a}}{\left(fx^2 + ex + d\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/x^2/(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^2 + b*x + a)/((f*x^2 + e*x + d)*x^2), x)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c\,x^2+b\,x+a}}{x^2\,\left(f\,x^2+e\,x+d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)^(1/2)/(x^2*(d + e*x + f*x^2)),x)
[Out] int((a + b*x + c*x^2)^(1/2)/(x^2*(d + e*x + f*x^2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(1/2)/x**2/(f*x**2+e*x+d),x)
[Out] Timed out
```

3.114 $\int \frac{x^3}{\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)}\,dx$

Optimal. Leaf size=545

$$-\frac{b\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2c^{3/2}f}-\frac{\left(2def-\left(e^2-df\right)\left(e-\sqrt{e^2-4df}\right)\right)\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)-b\left(e-\sqrt{e^2-4d\right.}}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-}}\right.}{\sqrt{2}f^2\sqrt{e^2-4df}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}$$

```
[Out] -1/2*b*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(3/2)/f-e*arcta
nh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/f^2/c^(1/2)+(c*x^2+b*x+a)^(1/
2)/c/f-1/2*arctanh(1/4*(4*a*f+2*x*(b*f-c*(e-(-4*d*f+e^2)^(1/2)))-b*(e-(-4*d
*f+e^2)^(1/2)))*2^(1/2)/(c*x^2+b*x+a)^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2-(-
b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2))*(2*d*e*f-(-d*f+e^2)*(e-(-4*d*f+e^2)^(1/
2)))/f^2*2^(1/2)/(-4*d*f+e^2)^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2-(-b*f+c*e)
*(-4*d*f+e^2)^(1/2))^(1/2)+1/2*arctanh(1/4*(4*a*f-b*(e+(-4*d*f+e^2)^(1/2))+
2*x*(b*f-c*(e+(-4*d*f+e^2)^(1/2))))*2^(1/2)/(c*x^2+b*x+a)^(1/2)/(c*e^2-2*c*
d*f-b*e*f+2*a*f^2+(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2))*(2*d*e*f-(-d*f+e^2)
*(e+(-4*d*f+e^2)^(1/2)))/f^2*2^(1/2)/(-4*d*f+e^2)^(1/2)/(c*e^2-2*c*d*f-b*e*
f+2*a*f^2+(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2)
```

Rubi [A] time = 3.72, antiderivative size = 545, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6728, 621, 206, 640, 1032, 724}

$$-\frac{b\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2c^{3/2}f}-\frac{\left(2def-\left(e^2-df\right)\left(e-\sqrt{e^2-4df}\right)\right)\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)-b\left(e-\sqrt{e^2-4d\right.}}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-}}\right.}{\sqrt{2}f^2\sqrt{e^2-4df}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)),x]
```

```
[Out] Sqrt[a + b*x + c*x^2]/(c*f) - (e*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*
x + c*x^2])])/(Sqrt[c]*f^2) - (b*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*
x + c*x^2])])/(2*c^(3/2)*f) - ((2*d*e*f - (e^2 - d*f)*(e - Sqrt[e^2 - 4*d*f
]))*ArcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e - Sqrt[e^2 -
 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f
)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*f^2*Sqrt[e^2 - 4*d*f
]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]])
+ ((2*d*e*f - (e^2 - d*f)*(e + Sqrt[e^2 - 4*d*f]))*ArcTanh[(4*a*f - b*(e +
Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e + Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt
[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a
+ b*x + c*x^2])])/(Sqrt[2]*f^2*Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c*d*f - b*e
*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1032

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{x^3}{\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)}\,dx = \int\left(-\frac{e}{f^2\sqrt{a+bx+cx^2}}+\frac{x}{f\sqrt{a+bx+cx^2}}+\frac{de+\left(e^2-df\right)x}{f^2\sqrt{a+bx+cx^2}\left(d+ex+\right.}\right.$$

$$=\frac{\int\frac{de+\left(e^2-df\right)x}{\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)}\,dx}{f^2}-\frac{e\int\frac{1}{\sqrt{a+bx+cx^2}}\,dx}{f^2}+\frac{\int\frac{x}{\sqrt{a+bx+cx^2}}\,dx}{f}$$

$$=\frac{\sqrt{a+bx+cx^2}}{cf}-\frac{(2e)\,\mathrm{Subst}\left(\int\frac{1}{4c-x^2}\,dx,x,\frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{f^2}-\frac{b\int\frac{1}{\sqrt{a+bx+cx^2}}\,dx}{2cf}$$

$$=\frac{\sqrt{a+bx+cx^2}}{cf}-\frac{e\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}f^2}-\frac{b\,\mathrm{Subst}\left(\int\frac{1}{4c-x^2}\,dx,x,\frac{b+2}{\sqrt{a+bx}}\right.}{cf}$$

$$=\frac{\sqrt{a+bx+cx^2}}{cf}-\frac{e\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}f^2}-\frac{b\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2c^{3/2}f}-$$

Mathematica [A] time = 2.20, size = 550, normalized size = 1.01

$$-\frac{\frac{bf\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)}{c^{3/2}}+\frac{\sqrt{2}\left(-\frac{e\left(e^2-3df\right)}{\sqrt{e^2-4df}}-df+e^2\right)\tanh^{-1}\left(\frac{4af+b\left(\sqrt{e^2-4df}-e+2fx\right)+2cx\left(\sqrt{e^2-4df}-e\right)}{2\sqrt{2}\sqrt{a+x(b+cx)}\sqrt{f\left(2af+b\left(\sqrt{e^2-4df}-e\right)\right)+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{f\left(2af+b\left(\sqrt{e^2-4df}-e\right)\right)+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}+\frac{\sqrt{2}\left(e^2\sqrt{e^2-4df}-\right.}{}}{2f^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)),x]
```

```
[Out] -1/2*((-2*f*Sqrt[a + x*(b + c*x)])/c + (2*e*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*
Sqrt[a + x*(b + c*x)])])/Sqrt[c] + (b*f*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt
[a + x*(b + c*x)])])/c^(3/2) + (Sqrt[2]*(e^3 - 3*d*e*f + e^2*Sqrt[e^2 - 4*d
*f] - d*f*Sqrt[e^2 - 4*d*f])*ArcTanh[(4*a*f - 2*c*(e + Sqrt[e^2 - 4*d*f])*x
 - b*(e + Sqrt[e^2 - 4*d*f] - 2*f*x))/(2*Sqrt[2]*Sqrt[c*(e^2 - 2*d*f + e*Sq
rt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))]*Sqrt[a + x*(b + c
*x)])])/(Sqrt[e^2 - 4*d*f]*Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(
2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))]) + (Sqrt[2]*(e^2 - d*f - (e*(e^2 - 3*d*
f))/Sqrt[e^2 - 4*d*f])*ArcTanh[(4*a*f + 2*c*(-e + Sqrt[e^2 - 4*d*f])*x + b*
(-e + Sqrt[e^2 - 4*d*f] + 2*f*x))/(2*Sqrt[2]*Sqrt[c*(e^2 - 2*d*f - e*Sqrt[e
^2 - 4*d*f]) + f*(2*a*f + b*(-e + Sqrt[e^2 - 4*d*f]))]*Sqrt[a + x*(b + c*x)
])])/Sqrt[c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f + b*(-e + Sqrt[e
^2 - 4*d*f]))])/f^2
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [B] time = 0.03, size = 3131, normalized size = 5.74

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x)
```

```
[Out] (c*x^2+b*x+a)^(1/2)/c/f-1/2*b/c^(3/2)/f*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a
)^(1/2))-1/f^2*e*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2)+1/2/f^
2*2^(1/2)/((-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*
c*d*f+c*e^2)/f^2)^(1/2)*ln(((-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e
+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2+1/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*(x+1/
2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*((-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e
^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+
e^2)^(1/2))/f)^2*c+4/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^
2)^(1/2))/f)+2*(-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*
f-2*c*d*f+c*e^2)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))*d-1/2/f^3*2^
(1/2)/((-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*
f+c*e^2)/f^2)^(1/2)*ln(((-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e+2*a
*f^2-b*e*f-2*c*d*f+c*e^2)/f^2+1/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*(x+1/2*(e
+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*((-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^
(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)
^(1/2))/f)^2*c+4/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(
```

```
1/2))/f)+2*(-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*
c*d*f+c*e^2)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))*e^2+3/2/f^2/(-4*
d*f+e^2)^(1/2)*2^(1/2)/((-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e+2*a
*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*ln(((-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e
^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2+1/f*(-c*(-4*d*f+e^2)^(1/2)+b
*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*((-(-4*d*f+e^2)^(1/2)*
b*f+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x+1/
2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c+4/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*(x+1/2*
(e+(-4*d*f+e^2)^(1/2))/f)+2*(-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e
+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))
*d*e-1/2/f^3/(-4*d*f+e^2)^(1/2)*2^(1/2)/((-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e
^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*ln(((-(-4*d*f+e^2)^(1
/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2+1/f*(-c*(-4
*d*f+e^2)^(1/2)+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*((-(-
4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^
2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c+4/f*(-c*(-4*d*f+e^2)^(1/2)
+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*(-(-4*d*f+e^2)^(1/2)*b*f+(-4*d
*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2))/(x+1/2*(e+(-4*d*
f+e^2)^(1/2))/f))*e^3+1/2/f^2*2^(1/2)/(((-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)
^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*ln((((-4*d*f+e^2)^(1/2)*
b*f-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2+(c*(-4*d*f+e^2)
^(1/2)+b*f-c*e)/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*(((-4*d*f+e
^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2
)*(4*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c+4*(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)
/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+2*((-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)
^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2))/(x-1/2*(-e+(-4*d*f+e^2)
^(1/2))/f))*d-1/2/f^3*2^(1/2)/(((-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c
*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*ln((((-4*d*f+e^2)^(1/2)*b*f-(-4*
d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2+(c*(-4*d*f+e^2)^(1/2)+b
*f-c*e)/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*(((-4*d*f+e^2)^(1/2
)*b*f-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x-
1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c+4*(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/f*(x-1/
2*(-e+(-4*d*f+e^2)^(1/2))/f)+2*((-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c
*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2))/(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/
f))*e^2-3/2/f^2/(-4*d*f+e^2)^(1/2)*2^(1/2)/(((-4*d*f+e^2)^(1/2)*b*f-(-4*d*f
+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*ln((((-4*d*f+e^2)^(
1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2+(c*(-4*d*f
+e^2)^(1/2)+b*f-c*e)/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*(((-4*
d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)
^(1/2)*(4*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c+4*(c*(-4*d*f+e^2)^(1/2)+b*f
-c*e)/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+2*((-4*d*f+e^2)^(1/2)*b*f-(-4*d*f
+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2))/(x-1/2*(-e+(-4*d*f
+e^2)^(1/2))/f))*d*e+1/2/f^3/(-4*d*f+e^2)^(1/2)*2^(1/2)/(((-4*d*f+e^2)^(1/2
)*b*f-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*ln((((
-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f
^2+(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+1/2*2
^(1/2)*(((-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*
f+c*e^2)/f^2)^(1/2)*(4*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c+4*(c*(-4*d*f+e
^2)^(1/2)+b*f-c*e)/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+2*((-4*d*f+e^2)^(1/2
)*b*f-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2))/(x-1/
2*(-e+(-4*d*f+e^2)^(1/2))/f))*e^3
```

maxima [**F(-2)**]   time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
```

```
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for mo
re details)Is 4*d*f-e^2 positive, negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3}{\sqrt{c\,x^2+b\,x+a}\,\left(f\,x^2+e\,x+d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2)),x)
[Out] int(x^3/((a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(c*x**2+b*x+a)**(1/2)/(f*x**2+e*x+d),x)
[Out] Integral(x**3/(sqrt(a + b*x + c*x**2)*(d + e*x + f*x**2)), x)
```

3.115 $\int \frac{x^2}{\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)}\,dx$

Optimal. Leaf size=463

$$-\frac{\left(-e\sqrt{e^2-4df}-2df+e^2\right)\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)-b\left(e-\sqrt{e^2-4df}\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}f\sqrt{e^2-4df}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}-\frac{\left(2df-e\left(\sqrt{e^2-4df}+e\right)\right)\mathrm{t}}{\sqrt{2}f\sqrt{e^2-4df}\sqrt{2}}$$

```
[Out] arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/f/c^(1/2)-1/2*arctanh(1/
4*(4*a*f+2*x*(b*f-c*(e-(-4*d*f+e^2)^(1/2)))-b*(e-(-4*d*f+e^2)^(1/2)))*2^(1/
2)/(c*x^2+b*x+a)^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2-(-b*f+c*e)*(-4*d*f+e^2)
^(1/2))^(1/2))*(-(-4*d*f+e^2)^(1/2)*e-2*d*f+e^2)/f*2^(1/2)/(-4*d*f+e^2)^(1/
2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2-(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2)-1/2*ar
ctanh(1/4*(4*a*f-b*(e+(-4*d*f+e^2)^(1/2))+2*x*(b*f-c*(e+(-4*d*f+e^2)^(1/2))
))*2^(1/2)/(c*x^2+b*x+a)^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2+(-b*f+c*e)*(-4*
d*f+e^2)^(1/2))^(1/2))*(2*d*f-e*(e+(-4*d*f+e^2)^(1/2)))/f*2^(1/2)/(-4*d*f+e
^2)^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2+(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2)
```

Rubi [A] time = 3.44, antiderivative size = 463, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1077, 621, 206, 1032, 724}

$$-\frac{\left(-e\sqrt{e^2-4df}-2df+e^2\right)\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)-b\left(e-\sqrt{e^2-4df}\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}f\sqrt{e^2-4df}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}-\frac{\left(2df-e\left(\sqrt{e^2-4df}+e\right)\right)\mathrm{t}}{\sqrt{2}f\sqrt{e^2-4df}\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)),x]
```

```
[Out] ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])]/(Sqrt[c]*f) - ((e^2
- 2*d*f - e*Sqrt[e^2 - 4*d*f])*ArcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) +
 2*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b
*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])]])/(S
qrt[2]*f*Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e -
b*f)*Sqrt[e^2 - 4*d*f]]) - ((2*d*f - e*(e + Sqrt[e^2 - 4*d*f]))*ArcTanh[(4*
a*f - b*(e + Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e + Sqrt[e^2 - 4*d*f]))*x)/(2
*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*
d*f]]*Sqrt[a + b*x + c*x^2])]])/(Sqrt[2]*f*Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*
c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
```

```
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1032

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1077

```
Int[((A_.) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
 (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqrt[d + e*x + f
*x^2], x], x] + Dist[1/c, Int[(A*c - a*C - b*C*x)/((a + b*x + c*x^2)*Sqrt[d
 + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]
```

Rubi steps

$$\int \frac{x^2}{\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)}\,dx = \frac{\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{f} + \frac{\int \frac{-d-ex}{\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)}\,dx}{f}$$

$$= \frac{2\,\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{f} + \frac{\left(e^2-2df-e\sqrt{e^2-4df}\right)\int \frac{}{\left(e-\sqrt{e^2\cdots}\right.}}{f\sqrt{e^2-4df}}$$

$$= \frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}\,f} - \frac{\left(2\left(e^2-2df-e\sqrt{e^2-4df}\right)\right)\text{Subst}\left(\int \frac{}{16af^2\cdots}\right.}{\cdots}$$

$$= \frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}\,f} - \frac{\left(e^2-2df-e\sqrt{e^2-4df}\right)\tanh^{-1}\left(\frac{4af-b\cdots}{2\sqrt{2}\sqrt{ce^2-2c\cdots}}\right.}{\sqrt{2}\,f\sqrt{e^2-4df}\sqrt{ce^2-2cdf-bef+\cdots}}$$

Mathematica [A] time = 0.95, size = 468, normalized size = 1.01

$$\frac{\frac{\sqrt{2}\left(e\sqrt{e^2-4df}-2df+e^2\right)\tanh^{-1}\left(\frac{4af-b\left(\sqrt{e^2-4df}+e-2fx\right)-2cx\left(\sqrt{e^2-4df}+e\right)}{2\sqrt{2}\sqrt{a+x(b+cx)}\sqrt{f\left(2af-b\left(\sqrt{e^2-4df}+e\right)\right)+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{e^2-4df}\sqrt{f\left(2af-b\left(\sqrt{e^2-4df}+e\right)\right)+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}} + \frac{\sqrt{2}\left(\frac{2df-e^2}{\sqrt{e^2-4df}}+e\right)\tanh^{-1}\left(\frac{4af+b\left(\sqrt{e^2\cdots}\right.}{2\sqrt{2}\sqrt{a+x(b+cx)}\sqrt{f\left(2a\cdots\right.}}\right)}{\sqrt{f\left(2af+b\left(\sqrt{e^2-4df}-e\right)\right)+c\left(\cdots\right.}}\cdots}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)),x]
```

```
[Out] ((2*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/Sqrt[c] + (Sqrt
[2]*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])*ArcTanh[(4*a*f - 2*c*(e + Sqrt[e^2
- 4*d*f])*x - b*(e + Sqrt[e^2 - 4*d*f] - 2*f*x))/(2*Sqrt[2]*Sqrt[c*(e^2 - 2
*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))]*Sqrt[a
 + x*(b + c*x)])])/(Sqrt[e^2 - 4*d*f]*Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*
d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))]) + (Sqrt[2]*(e + (-e^2 + 2*d
```

```
*f)/Sqrt[e^2 - 4*d*f])*ArcTanh[(4*a*f + 2*c*(-e + Sqrt[e^2 - 4*d*f])*x + b*
(-e + Sqrt[e^2 - 4*d*f] + 2*f*x))/(2*Sqrt[2]*Sqrt[c*(e^2 - 2*d*f - e*Sqrt[e
^2 - 4*d*f]) + f*(2*a*f + b*(-e + Sqrt[e^2 - 4*d*f]))]*Sqrt[a + x*(b + c*x)
])])/Sqrt[c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f + b*(-e + Sqrt[e
^2 - 4*d*f]))])])/(2*f)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.02, size = 2321, normalized size = 5.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x)
```

```
[Out] 1/c^(1/2)/f*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+1/2/f^2*2^(1/2)/((-
(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/
f^2)^(1/2)*ln(((-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*
f-2*c*d*f+c*e^2)/f^2+1/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*(x+1/2*(e+(-4*d*f+
e^2)^(1/2))/f)+1/2*2^(1/2)*((-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e
+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f
)^2*c+4/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+
2*(-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e
^2)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))*e-1/(-4*d*f+e^2)^(1/2)/f*
2^(1/2)/((-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*
d*f+c*e^2)/f^2)^(1/2)*ln(((-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e+2
*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2+1/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*(x+1/2*
(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*((-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2
)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^
2)^(1/2))/f)^2*c+4/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)
^(1/2))/f)+2*(-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-
2*c*d*f+c*e^2)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))*d+1/2/(-4*d*f+
e^2)^(1/2)/f^2*2^(1/2)/((-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e+2*a
*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*ln(((-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e
^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2+1/f*(-c*(-4*d*f+e^2)^(1/2)+b
*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*((-(-4*d*f+e^2)^(1/2)*
b*f+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x+1/
2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c+4/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*(x+1/2*
(e+(-4*d*f+e^2)^(1/2))/f)+2*(-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e
+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))
*e^2+1/2/f^2*2^(1/2)/(((-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^
```

```
2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*ln((((-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^
(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2+(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/f
*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*(((-4*d*f+e^2)^(1/2)*b*f-(-4
*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x-1/2*(-e+(
-4*d*f+e^2)^(1/2))/f)^2*c+4*(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/f*(x-1/2*(-e+(-4
*d*f+e^2)^(1/2))/f)+2*((-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^
2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2))/(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f))*e+1/(
-4*d*f+e^2)^(1/2)/f*2^(1/2)/(((-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e
+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*ln((((-4*d*f+e^2)^(1/2)*b*f-(-4*d*
f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2+(c*(-4*d*f+e^2)^(1/2)+b*f
-c*e)/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*(((-4*d*f+e^2)^(1/2)*
b*f-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x-1/
2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c+4*(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/f*(x-1/2*
(-e+(-4*d*f+e^2)^(1/2))/f)+2*((-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e
+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2))/(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)
)*d-1/2/(-4*d*f+e^2)^(1/2)/f^2*2^(1/2)/(((-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2
)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*ln((((-4*d*f+e^2)^(1/2)
*b*f-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2+(c*(-4*d*f+e^2
)^(1/2)+b*f-c*e)/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*(((-4*d*f+
e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/
2)*(4*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c+4*(c*(-4*d*f+e^2)^(1/2)+b*f-c*e
)/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+2*((-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2
)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2))/(x-1/2*(-e+(-4*d*f+e^2
)^(1/2))/f))*e^2
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for mo
re details)Is 4*d*f-e^2 positive, negative or zero?
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2}{\sqrt{c\,x^2+b\,x+a}\,\left(f\,x^2+e\,x+d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2)),x)
```

```
[Out] int(x^2/((a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2)), x)
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{a+bx+cx^2}\,\left(d+ex+fx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(c*x**2+b*x+a)**(1/2)/(f*x**2+e*x+d),x)
```

```
[Out] Integral(x**2/(sqrt(a + b*x + c*x**2)*(d + e*x + f*x**2)), x)
```

3.116 $\int \frac{x}{\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)}\,dx$

Optimal. Leaf size=402

$$\frac{\left(e-\sqrt{e^2-4df}\right)\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)-b\left(e-\sqrt{e^2-4df}\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}\,(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}\sqrt{e^2-4df}\sqrt{2af^2-\sqrt{e^2-4df}\,(ce-bf)-bef-2cdf+ce^2}}-\frac{\left(\sqrt{e^2-4df}+e\right)\tanh^{-1}\left(\frac{4af+2x(bf\cdots}{2\sqrt{2}\sqrt{a+bx+cx^2}\cdots}\right)}{\sqrt{2}\sqrt{e^2-4df}\sqrt{2af^2+\sqrt{e^2-4df}\cdots}}$$

```
[Out] 1/2*arctanh(1/4*(4*a*f+2*x*(b*f-c*(e-(-4*d*f+e^2)^(1/2)))-b*(e-(-4*d*f+e^2)
^(1/2)))*2^(1/2)/(c*x^2+b*x+a)^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2-(-b*f+c*e
)*(-4*d*f+e^2)^(1/2))^(1/2))*(e-(-4*d*f+e^2)^(1/2))*2^(1/2)/(-4*d*f+e^2)^(1
/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2-(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2)-1/2*a
rctanh(1/4*(4*a*f-b*(e+(-4*d*f+e^2)^(1/2))+2*x*(b*f-c*(e+(-4*d*f+e^2)^(1/2)
)))*2^(1/2)/(c*x^2+b*x+a)^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2+(-b*f+c*e)*(-4
*d*f+e^2)^(1/2))^(1/2))*(e+(-4*d*f+e^2)^(1/2))*2^(1/2)/(-4*d*f+e^2)^(1/2)/(
c*e^2-2*c*d*f-b*e*f+2*a*f^2+(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2)
```

Rubi [A] time = 0.96, antiderivative size = 402, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {1032, 724, 206}

$$\frac{\left(e-\sqrt{e^2-4df}\right)\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)-b\left(e-\sqrt{e^2-4df}\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}\,(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}\sqrt{e^2-4df}\sqrt{2af^2-\sqrt{e^2-4df}\,(ce-bf)-bef-2cdf+ce^2}}-\frac{\left(\sqrt{e^2-4df}+e\right)\tanh^{-1}\left(\frac{4af+2x(bf\cdots}{2\sqrt{2}\sqrt{a+bx+cx^2}\cdots}\right)}{\sqrt{2}\sqrt{e^2-4df}\sqrt{2af^2+\sqrt{e^2-4df}\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)),x]
```

```
[Out] ((e - Sqrt[e^2 - 4*d*f])*ArcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) + 2*(b*
f - c*(e - Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f +
 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]
*Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqr
t[e^2 - 4*d*f]]) - ((e + Sqrt[e^2 - 4*d*f])*ArcTanh[(4*a*f - b*(e + Sqrt[e^
2 - 4*d*f]) + 2*(b*f - c*(e + Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2
- 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x +
 c*x^2])])/(Sqrt[2]*Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^
2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1032

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
```

```
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x}{\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)}\,dx = -\left(\left(-1-\frac{e}{\sqrt{e^2-4df}}\right)\int \frac{1}{\left(e+\sqrt{e^2-4df}+2fx\right)\sqrt{a+bx+cx^2}}\,dx\right)+$$

$$= -\left(\left(2\left(1-\frac{e}{\sqrt{e^2-4df}}\right)\right)\text{Subst}\left(\int \frac{1}{16af^2-8bf\left(e-\sqrt{e^2-4df}\right)+4c\left(\right.}\right.\right.$$

$$= -\frac{\left(1-\frac{e}{\sqrt{e^2-4df}}\right)\tanh^{-1}\left(\frac{4af-b\left(e-\sqrt{e^2-4df}\right)+2\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)x}{2\sqrt{2}\sqrt{ce^2-2cdf-bef+2af^2-(ce-bf)\sqrt{e^2-4df}}\sqrt{a+bx+cx^2}}\right)}{\sqrt{2}\sqrt{ce^2-2cdf-bef+2af^2-(ce-bf)\sqrt{e^2-4df}}}$$

Mathematica [A] time = 0.97, size = 407, normalized size = 1.01

$$\frac{-\frac{\left(\sqrt{e^2-4df}+e\right)\tanh^{-1}\left(\frac{4af-b\left(\sqrt{e^2-4df}+e-2fx\right)-2cx\left(\sqrt{e^2-4df}+e\right)}{2\sqrt{2}\sqrt{a+x(b+cx)}\sqrt{f\left(2af-b\left(\sqrt{e^2-4df}+e\right)\right)+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{e^2-4df}\sqrt{f\left(2af-b\left(\sqrt{e^2-4df}+e\right)\right)+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}-\frac{\left(1-\frac{e}{\sqrt{e^2-4df}}\right)\tanh^{-1}\left(\frac{4af+b\left(\sqrt{e^2-4df}-e+2fx\right)+}{2\sqrt{2}\sqrt{a+x(b+cx)}\sqrt{f\left(2af+b\left(\sqrt{e^2-4df}\right.\right.}}\right)}{\sqrt{f\left(2af+b\left(\sqrt{e^2-4df}-e\right)\right)+c\left(-e\sqrt{e^2-4d}\right.}}}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)),x]
```

```
[Out] (-(((e + Sqrt[e^2 - 4*d*f])*ArcTanh[(4*a*f - 2*c*(e + Sqrt[e^2 - 4*d*f])*x
 - b*(e + Sqrt[e^2 - 4*d*f] - 2*f*x))/(2*Sqrt[2]*Sqrt[c*(e^2 - 2*d*f + e*Sqr
t[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))]*Sqrt[a + x*(b + c*
x)])])/(Sqrt[e^2 - 4*d*f]*Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2
*a*f - b*(e + Sqrt[e^2 - 4*d*f]))])) - ((1 - e/Sqrt[e^2 - 4*d*f])*ArcTanh[(
4*a*f + 2*c*(-e + Sqrt[e^2 - 4*d*f])*x + b*(-e + Sqrt[e^2 - 4*d*f] + 2*f*x)
)/(2*Sqrt[2]*Sqrt[c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f + b*(-e
+ Sqrt[e^2 - 4*d*f]))]*Sqrt[a + x*(b + c*x)])])/Sqrt[c*(e^2 - 2*d*f - e*Sqr
t[e^2 - 4*d*f]) + f*(2*a*f + b*(-e + Sqrt[e^2 - 4*d*f]))])/Sqrt[2]
```

fricas [B] time = 11.37, size = 11311, normalized size = 28.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*sqrt((2*c*d^2 - b*d*e + a*e^2 - 2*a*d*f + (c^2*d^2*e^2 - b*c*d*
e^3 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e + a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f
^2 - (4*c^2*d^3 - 4*b*c*d^2*e + a*b*e^3 - (b^2 - 6*a*c)*d*e^2)*f)*sqrt((b^2
*d^2 - 2*a*b*d*e + a^2*e^2)/(c^4*d^4*e^2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d*e^
5 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (b^2*c^2 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*d*e
+ a^4*e^2 - 8*(a^2*b^2 - 2*a^3*c)*d^2)*f^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a*b^
2*c + 6*a^2*c^2)*d^3 - 4*(a*b^3 - a^2*b*c)*d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^
2)*f^3 - (8*(b^2*c^2 - 2*a*c^3)*d^4 - 8*(b^3*c - a*b*c^2)*d^3*e - (b^4 - 20
*a*b^2*c + 22*a^2*c^2)*d^2*e^2 + 2*(a*b^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2
```

```
*a^3*c)*e^4)*f^2 - 2*(2*c^4*d^5 - 4*b*c^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2 +
6*a*c^3)*d^3*e^2 + (b^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*
e^4)*f)))/(c^2*d^2*e^2 - b*c*d*e^3 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e + a
^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 - (4*c^2*d^3 - 4*b*c*d^2*e + a*b*e^3 - (b
^2 - 6*a*c)*d*e^2)*f))*log(-(2*b^2*d^3 - 4*a*b*d^2*e + 2*a^2*d*e^2 + sqrt(2
)*(b^2*d^2*e^2 - 2*a*b*d*e^3 + a^2*e^4 - 4*(b^2*d^3 - 2*a*b*d^2*e + a^2*d*e
^2)*f - (2*c^3*d^4*e^2 - 3*b*c^2*d^3*e^3 - 2*a*b*c*d*e^5 + a^2*c*e^6 + 8*a^
3*d^2*f^4 + (b^2*c + 3*a*c^2)*d^2*e^4 - 2*(2*a^2*b*d^2*e + 3*a^3*d*e^2 - 4*
(a*b^2 - 3*a^2*c)*d^3)*f^3 + (5*a^2*b*d*e^3 + a^3*e^4 - 8*(b^2*c - 3*a*c^2)
*d^4 + 4*(b^3 - 2*a*b*c)*d^3*e - 2*(5*a*b^2 - 11*a^2*c)*d^2*e^2)*f^2 - (8*c
^3*d^5 - 12*b*c^2*d^4*e + a^2*b*e^5 + 2*(b^2*c + 9*a*c^2)*d^3*e^2 + (b^3 -
10*a*b*c)*d^2*e^3 - 2*(a*b^2 - 4*a^2*c)*d*e^4)*f)*sqrt((b^2*d^2 - 2*a*b*d*e
 + a^2*e^2)/(c^4*d^4*e^2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d*e^5 + a^2*c^2*e^6
- 4*a^4*d*f^5 + (b^2*c^2 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a
^2*b^2 - 2*a^3*c)*d^2)*f^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)
*d^3 - 4*(a*b^3 - a^2*b*c)*d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2
*c^2 - 2*a*c^3)*d^4 - 8*(b^3*c - a*b*c^2)*d^3*e - (b^4 - 20*a*b^2*c + 22*a^
2*c^2)*d^2*e^2 + 2*(a*b^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2
 - 2*(2*c^4*d^5 - 4*b*c^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2
 + (b^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*e^4)*f)))*sqrt(c
*x^2 + b*x + a)*sqrt((2*c*d^2 - b*d*e + a*e^2 - 2*a*d*f + (c^2*d^2*e^2 - b*
c*d*e^3 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e + a^2*e^2 - 4*(b^2 - 2*a*c)*d^
2)*f^2 - (4*c^2*d^3 - 4*b*c*d^2*e + a*b*e^3 - (b^2 - 6*a*c)*d*e^2)*f)*sqrt(
(b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(c^4*d^4*e^2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*
d*e^5 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (b^2*c^2 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*
d*e + a^4*e^2 - 8*(a^2*b^2 - 2*a^3*c)*d^2)*f^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4*
a*b^2*c + 6*a^2*c^2)*d^3 - 4*(a*b^3 - a^2*b*c)*d^2*e + (a^2*b^2 + 6*a^3*c)*
d*e^2)*f^3 - (8*(b^2*c^2 - 2*a*c^3)*d^4 - 8*(b^3*c - a*b*c^2)*d^3*e - (b^4
- 20*a*b^2*c + 22*a^2*c^2)*d^2*e^2 + 2*(a*b^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2
 + 2*a^3*c)*e^4)*f^2 - 2*(2*c^4*d^5 - 4*b*c^3*d^4*e + a^2*b*c*e^5 + (b^2*c^
2 + 6*a*c^3)*d^3*e^2 + (b^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2
)*d*e^4)*f)))/(c^2*d^2*e^2 - b*c*d*e^3 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e
 + a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 - (4*c^2*d^3 - 4*b*c*d^2*e + a*b*e^3
- (b^2 - 6*a*c)*d*e^2)*f)) + (4*b*c*d^3 + a*b*d*e^2 - (b^2 + 4*a*c)*d^2*e)*
x - (2*a*c^2*d^3*e^2 - 2*a*b*c*d^2*e^3 + 2*a^2*c*d*e^4 - 8*a^3*d^2*f^3 + 2*
(4*a^2*b*d^2*e + a^3*d*e^2 - 4*(a*b^2 - 2*a^2*c)*d^3)*f^2 - 2*(4*a*c^2*d^4
- 4*a*b*c*d^3*e + a^2*b*d*e^3 - (a*b^2 - 6*a^2*c)*d^2*e^2)*f + (b*c^2*d^3*e
^2 - b^2*c*d^2*e^3 + a*b*c*d*e^4 - 4*a^2*b*d^2*f^3 + (4*a*b^2*d^2*e + a^2*b
*d*e^2 - 4*(b^3 - 2*a*b*c)*d^3)*f^2 - (4*b*c^2*d^4 - 4*b^2*c*d^3*e + a*b^2*
d*e^3 - (b^3 - 6*a*b*c)*d^2*e^2)*f)*x)*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)
/(c^4*d^4*e^2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d*e^5 + a^2*c^2*e^6 - 4*a^4*d*f
^5 + (b^2*c^2 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a^2*b^2 - 2*
a^3*c)*d^2)*f^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3 - 4*(a
*b^3 - a^2*b*c)*d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*c^2 - 2*a*
c^3)*d^4 - 8*(b^3*c - a*b*c^2)*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2*c^2)*d^2*
e^2 + 2*(a*b^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2 - 2*(2*c^4
*d^5 - 4*b*c^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2 + (b^3*c -
 5*a*b*c^2)*d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*e^4)*f)))/x) - 1/4*sqrt(2)*
sqrt((2*c*d^2 - b*d*e + a*e^2 - 2*a*d*f + (c^2*d^2*e^2 - b*c*d*e^3 + a*c*e^
4 - 4*a^2*d*f^3 + (4*a*b*d*e + a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 - (4*c^2*
d^3 - 4*b*c*d^2*e + a*b*e^3 - (b^2 - 6*a*c)*d*e^2)*f)*sqrt((b^2*d^2 - 2*a*b
*d*e + a^2*e^2)/(c^4*d^4*e^2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d*e^5 + a^2*c^2*
e^6 - 4*a^4*d*f^5 + (b^2*c^2 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*d*e + a^4*e^2 -
8*(a^2*b^2 - 2*a^3*c)*d^2)*f^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a*b^2*c + 6*a^2*
c^2)*d^3 - 4*(a*b^3 - a^2*b*c)*d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*
(b^2*c^2 - 2*a*c^3)*d^4 - 8*(b^3*c - a*b*c^2)*d^3*e - (b^4 - 20*a*b^2*c + 2
2*a^2*c^2)*d^2*e^2 + 2*(a*b^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2*a^3*c)*e^4)
*f^2 - 2*(2*c^4*d^5 - 4*b*c^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3
*e^2 + (b^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*e^4)*f)))/(c
```

```
^2*d^2*e^2 - b*c*d*e^3 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e + a^2*e^2 - 4*(
b^2 - 2*a*c)*d^2)*f^2 - (4*c^2*d^3 - 4*b*c*d^2*e + a*b*e^3 - (b^2 - 6*a*c)*
d*e^2)*f))*log(-(2*b^2*d^3 - 4*a*b*d^2*e + 2*a^2*d*e^2 - sqrt(2)*(b^2*d^2*e
^2 - 2*a*b*d*e^3 + a^2*e^4 - 4*(b^2*d^3 - 2*a*b*d^2*e + a^2*d*e^2)*f - (2*c
^3*d^4*e^2 - 3*b*c^2*d^3*e^3 - 2*a*b*c*d*e^5 + a^2*c*e^6 + 8*a^3*d^2*f^4 +
(b^2*c + 3*a*c^2)*d^2*e^4 - 2*(2*a^2*b*d^2*e + 3*a^3*d*e^2 - 4*(a*b^2 - 3*a
^2*c)*d^3)*f^3 + (5*a^2*b*d*e^3 + a^3*e^4 - 8*(b^2*c - 3*a*c^2)*d^4 + 4*(b^
3 - 2*a*b*c)*d^3*e - 2*(5*a*b^2 - 11*a^2*c)*d^2*e^2)*f^2 - (8*c^3*d^5 - 12*
b*c^2*d^4*e + a^2*b*e^5 + 2*(b^2*c + 9*a*c^2)*d^3*e^2 + (b^3 - 10*a*b*c)*d^
2*e^3 - 2*(a*b^2 - 4*a^2*c)*d*e^4)*f)*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/
(c^4*d^4*e^2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d*e^5 + a^2*c^2*e^6 - 4*a^4*d*f^
5 + (b^2*c^2 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a^2*b^2 - 2*a
^3*c)*d^2)*f^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3 - 4*(a*
b^3 - a^2*b*c)*d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*c^2 - 2*a*c
^3)*d^4 - 8*(b^3*c - a*b*c^2)*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2*c^2)*d^2*e
^2 + 2*(a*b^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2 - 2*(2*c^4*
d^5 - 4*b*c^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2 + (b^3*c -
5*a*b*c^2)*d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*e^4)*f)))*sqrt(c*x^2 + b*x +
 a)*sqrt((2*c*d^2 - b*d*e + a*e^2 - 2*a*d*f + (c^2*d^2*e^2 - b*c*d*e^3 + a*
c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e + a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 - (4*
c^2*d^3 - 4*b*c*d^2*e + a*b*e^3 - (b^2 - 6*a*c)*d*e^2)*f)*sqrt((b^2*d^2 - 2
*a*b*d*e + a^2*e^2)/(c^4*d^4*e^2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d*e^5 + a^2*
c^2*e^6 - 4*a^4*d*f^5 + (b^2*c^2 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*d*e + a^4*e^
2 - 8*(a^2*b^2 - 2*a^3*c)*d^2)*f^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a*b^2*c + 6*
a^2*c^2)*d^3 - 4*(a*b^3 - a^2*b*c)*d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^2)*f^3 -
 (8*(b^2*c^2 - 2*a*c^3)*d^4 - 8*(b^3*c - a*b*c^2)*d^3*e - (b^4 - 20*a*b^2*c
 + 22*a^2*c^2)*d^2*e^2 + 2*(a*b^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2*a^3*c)*
e^4)*f^2 - 2*(2*c^4*d^5 - 4*b*c^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2 + 6*a*c^3)
*d^3*e^2 + (b^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*e^4)*f))
)/(c^2*d^2*e^2 - b*c*d*e^3 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e + a^2*e^2 -
 4*(b^2 - 2*a*c)*d^2)*f^2 - (4*c^2*d^3 - 4*b*c*d^2*e + a*b*e^3 - (b^2 - 6*a
*c)*d*e^2)*f)) + (4*b*c*d^3 + a*b*d*e^2 - (b^2 + 4*a*c)*d^2*e)*x - (2*a*c^2
*d^3*e^2 - 2*a*b*c*d^2*e^3 + 2*a^2*c*d*e^4 - 8*a^3*d^2*f^3 + 2*(4*a^2*b*d^2
*e + a^3*d*e^2 - 4*(a*b^2 - 2*a^2*c)*d^3)*f^2 - 2*(4*a*c^2*d^4 - 4*a*b*c*d^
3*e + a^2*b*d*e^3 - (a*b^2 - 6*a^2*c)*d^2*e^2)*f + (b*c^2*d^3*e^2 - b^2*c*d
^2*e^3 + a*b*c*d*e^4 - 4*a^2*b*d^2*f^3 + (4*a*b^2*d^2*e + a^2*b*d*e^2 - 4*(
b^3 - 2*a*b*c)*d^3)*f^2 - (4*b*c^2*d^4 - 4*b^2*c*d^3*e + a*b^2*d*e^3 - (b^3
 - 6*a*b*c)*d^2*e^2)*f)*x)*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(c^4*d^4*e^
2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d*e^5 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (b^2*c^
2 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a^2*b^2 - 2*a^3*c)*d^2)*
f^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3 - 4*(a*b^3 - a^2*b
*c)*d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*c^2 - 2*a*c^3)*d^4 - 8
*(b^3*c - a*b*c^2)*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2*c^2)*d^2*e^2 + 2*(a*b
^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2 - 2*(2*c^4*d^5 - 4*b*c
^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2 + (b^3*c - 5*a*b*c^2)*
d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*e^4)*f)))/x) + 1/4*sqrt(2)*sqrt((2*c*d^
2 - b*d*e + a*e^2 - 2*a*d*f - (c^2*d^2*e^2 - b*c*d*e^3 + a*c*e^4 - 4*a^2*d*
f^3 + (4*a*b*d*e + a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 - (4*c^2*d^3 - 4*b*c*
d^2*e + a*b*e^3 - (b^2 - 6*a*c)*d*e^2)*f)*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e
^2)/(c^4*d^4*e^2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d*e^5 + a^2*c^2*e^6 - 4*a^4*
d*f^5 + (b^2*c^2 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a^2*b^2 -
 2*a^3*c)*d^2)*f^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3 - 4
*(a*b^3 - a^2*b*c)*d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*c^2 - 2
*a*c^3)*d^4 - 8*(b^3*c - a*b*c^2)*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2*c^2)*d
^2*e^2 + 2*(a*b^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2 - 2*(2*
c^4*d^5 - 4*b*c^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2 + (b^3*
c - 5*a*b*c^2)*d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*e^4)*f)))/(c^2*d^2*e^2 -
 b*c*d*e^3 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e + a^2*e^2 - 4*(b^2 - 2*a*c)
*d^2)*f^2 - (4*c^2*d^3 - 4*b*c*d^2*e + a*b*e^3 - (b^2 - 6*a*c)*d*e^2)*f))*l
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og(-(2*b^2*d^3 - 4*a*b*d^2*e + 2*a^2*d*e^2 + sqrt(2)*(b^2*d^2*e^2 - 2*a*b*d
*e^3 + a^2*e^4 - 4*(b^2*d^3 - 2*a*b*d^2*e + a^2*d*e^2)*f + (2*c^3*d^4*e^2 -
 3*b*c^2*d^3*e^3 - 2*a*b*c*d*e^5 + a^2*c*e^6 + 8*a^3*d^2*f^4 + (b^2*c + 3*a
*c^2)*d^2*e^4 - 2*(2*a^2*b*d^2*e + 3*a^3*d*e^2 - 4*(a*b^2 - 3*a^2*c)*d^3)*f
^3 + (5*a^2*b*d*e^3 + a^3*e^4 - 8*(b^2*c - 3*a*c^2)*d^4 + 4*(b^3 - 2*a*b*c)
*d^3*e - 2*(5*a*b^2 - 11*a^2*c)*d^2*e^2)*f^2 - (8*c^3*d^5 - 12*b*c^2*d^4*e
 + a^2*b*e^5 + 2*(b^2*c + 9*a*c^2)*d^3*e^2 + (b^3 - 10*a*b*c)*d^2*e^3 - 2*(a
*b^2 - 4*a^2*c)*d*e^4)*f)*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(c^4*d^4*e^2
 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d*e^5 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (b^2*c^2
 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a^2*b^2 - 2*a^3*c)*d^2)*f
^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3 - 4*(a*b^3 - a^2*b*
c)*d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*c^2 - 2*a*c^3)*d^4 - 8*
(b^3*c - a*b*c^2)*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2*c^2)*d^2*e^2 + 2*(a*b^
3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2 - 2*(2*c^4*d^5 - 4*b*c^
3*d^4*e + a^2*b*c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2 + (b^3*c - 5*a*b*c^2)*d
^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*e^4)*f)))*sqrt(c*x^2 + b*x + a)*sqrt((2*
c*d^2 - b*d*e + a*e^2 - 2*a*d*f - (c^2*d^2*e^2 - b*c*d*e^3 + a*c*e^4 - 4*a^
2*d*f^3 + (4*a*b*d*e + a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 - (4*c^2*d^3 - 4*
b*c*d^2*e + a*b*e^3 - (b^2 - 6*a*c)*d*e^2)*f)*sqrt((b^2*d^2 - 2*a*b*d*e + a
^2*e^2)/(c^4*d^4*e^2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d*e^5 + a^2*c^2*e^6 - 4*
a^4*d*f^5 + (b^2*c^2 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a^2*b
^2 - 2*a^3*c)*d^2)*f^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3
 - 4*(a*b^3 - a^2*b*c)*d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*c^2
 - 2*a*c^3)*d^4 - 8*(b^3*c - a*b*c^2)*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2*c^
2)*d^2*e^2 + 2*(a*b^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2 - 2
*(2*c^4*d^5 - 4*b*c^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2 + (
b^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*e^4)*f)))/(c^2*d^2*e
^2 - b*c*d*e^3 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e + a^2*e^2 - 4*(b^2 - 2*
a*c)*d^2)*f^2 - (4*c^2*d^3 - 4*b*c*d^2*e + a*b*e^3 - (b^2 - 6*a*c)*d*e^2)*f
)) + (4*b*c*d^3 + a*b*d*e^2 - (b^2 + 4*a*c)*d^2*e)*x + (2*a*c^2*d^3*e^2 - 2
*a*b*c*d^2*e^3 + 2*a^2*c*d*e^4 - 8*a^3*d^2*f^3 + 2*(4*a^2*b*d^2*e + a^3*d*e
^2 - 4*(a*b^2 - 2*a^2*c)*d^3)*f^2 - 2*(4*a*c^2*d^4 - 4*a*b*c*d^3*e + a^2*b*
d*e^3 - (a*b^2 - 6*a^2*c)*d^2*e^2)*f + (b*c^2*d^3*e^2 - b^2*c*d^2*e^3 + a*b
*c*d*e^4 - 4*a^2*b*d^2*f^3 + (4*a*b^2*d^2*e + a^2*b*d*e^2 - 4*(b^3 - 2*a*b*
c)*d^3)*f^2 - (4*b*c^2*d^4 - 4*b^2*c*d^3*e + a*b^2*d*e^3 - (b^3 - 6*a*b*c)*
d^2*e^2)*f)*x)*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(c^4*d^4*e^2 - 2*b*c^3*
d^3*e^3 - 2*a*b*c^2*d*e^5 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (b^2*c^2 + 2*a*c^3)
*d^2*e^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a^2*b^2 - 2*a^3*c)*d^2)*f^4 - 2*(a^3
*b*e^3 + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3 - 4*(a*b^3 - a^2*b*c)*d^2*e +
(a^2*b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*c^2 - 2*a*c^3)*d^4 - 8*(b^3*c - a*
b*c^2)*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2*c^2)*d^2*e^2 + 2*(a*b^3 - 5*a^2*b
*c)*d*e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2 - 2*(2*c^4*d^5 - 4*b*c^3*d^4*e + a
^2*b*c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2 + (b^3*c - 5*a*b*c^2)*d^2*e^3 - 2*
(a*b^2*c - 2*a^2*c^2)*d*e^4)*f)))/x) - 1/4*sqrt(2)*sqrt((2*c*d^2 - b*d*e +
a*e^2 - 2*a*d*f - (c^2*d^2*e^2 - b*c*d*e^3 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b
*d*e + a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 - (4*c^2*d^3 - 4*b*c*d^2*e + a*b*
e^3 - (b^2 - 6*a*c)*d*e^2)*f)*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(c^4*d^4
*e^2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d*e^5 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (b^2
*c^2 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a^2*b^2 - 2*a^3*c)*d^
2)*f^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3 - 4*(a*b^3 - a^
2*b*c)*d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*c^2 - 2*a*c^3)*d^4
 - 8*(b^3*c - a*b*c^2)*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2*c^2)*d^2*e^2 + 2*(
a*b^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2 - 2*(2*c^4*d^5 - 4*
b*c^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2 + (b^3*c - 5*a*b*c^
2)*d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*e^4)*f)))/(c^2*d^2*e^2 - b*c*d*e^3 +
 a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e + a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 -
(4*c^2*d^3 - 4*b*c*d^2*e + a*b*e^3 - (b^2 - 6*a*c)*d*e^2)*f))*log(-(2*b^2*d
^3 - 4*a*b*d^2*e + 2*a^2*d*e^2 - sqrt(2)*(b^2*d^2*e^2 - 2*a*b*d*e^3 + a^2*e
^4 - 4*(b^2*d^3 - 2*a*b*d^2*e + a^2*d*e^2)*f + (2*c^3*d^4*e^2 - 3*b*c^2*d^3
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*e^3 - 2*a*b*c*d*e^5 + a^2*c*e^6 + 8*a^3*d^2*f^4 + (b^2*c + 3*a*c^2)*d^2*e^
4 - 2*(2*a^2*b*d^2*e + 3*a^3*d*e^2 - 4*(a*b^2 - 3*a^2*c)*d^3)*f^3 + (5*a^2*
b*d*e^3 + a^3*e^4 - 8*(b^2*c - 3*a*c^2)*d^4 + 4*(b^3 - 2*a*b*c)*d^3*e - 2*(
5*a*b^2 - 11*a^2*c)*d^2*e^2)*f^2 - (8*c^3*d^5 - 12*b*c^2*d^4*e + a^2*b*e^5
+ 2*(b^2*c + 9*a*c^2)*d^3*e^2 + (b^3 - 10*a*b*c)*d^2*e^3 - 2*(a*b^2 - 4*a^2
*c)*d*e^4)*f)*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(c^4*d^4*e^2 - 2*b*c^3*d
^3*e^3 - 2*a*b*c^2*d*e^5 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (b^2*c^2 + 2*a*c^3)*
d^2*e^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a^2*b^2 - 2*a^3*c)*d^2)*f^4 - 2*(a^3*
b*e^3 + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3 - 4*(a*b^3 - a^2*b*c)*d^2*e + (
a^2*b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*c^2 - 2*a*c^3)*d^4 - 8*(b^3*c - a*b
*c^2)*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2*c^2)*d^2*e^2 + 2*(a*b^3 - 5*a^2*b*
c)*d*e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2 - 2*(2*c^4*d^5 - 4*b*c^3*d^4*e + a^
2*b*c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2 + (b^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(
a*b^2*c - 2*a^2*c^2)*d*e^4)*f)))*sqrt(c*x^2 + b*x + a)*sqrt((2*c*d^2 - b*d*
e + a*e^2 - 2*a*d*f - (c^2*d^2*e^2 - b*c*d*e^3 + a*c*e^4 - 4*a^2*d*f^3 + (4
*a*b*d*e + a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 - (4*c^2*d^3 - 4*b*c*d^2*e +
a*b*e^3 - (b^2 - 6*a*c)*d*e^2)*f)*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(c^4
*d^4*e^2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d*e^5 + a^2*c^2*e^6 - 4*a^4*d*f^5 +
(b^2*c^2 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a^2*b^2 - 2*a^3*c
)*d^2)*f^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3 - 4*(a*b^3
- a^2*b*c)*d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*c^2 - 2*a*c^3)*
d^4 - 8*(b^3*c - a*b*c^2)*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2*c^2)*d^2*e^2 +
 2*(a*b^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2 - 2*(2*c^4*d^5
- 4*b*c^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2 + (b^3*c - 5*a*
b*c^2)*d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*e^4)*f)))/(c^2*d^2*e^2 - b*c*d*e
^3 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e + a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^
2 - (4*c^2*d^3 - 4*b*c*d^2*e + a*b*e^3 - (b^2 - 6*a*c)*d*e^2)*f)) + (4*b*c*
d^3 + a*b*d*e^2 - (b^2 + 4*a*c)*d^2*e)*x + (2*a*c^2*d^3*e^2 - 2*a*b*c*d^2*e
^3 + 2*a^2*c*d*e^4 - 8*a^3*d^2*f^3 + 2*(4*a^2*b*d^2*e + a^3*d*e^2 - 4*(a*b^
2 - 2*a^2*c)*d^3)*f^2 - 2*(4*a*c^2*d^4 - 4*a*b*c*d^3*e + a^2*b*d*e^3 - (a*b
^2 - 6*a^2*c)*d^2*e^2)*f + (b*c^2*d^3*e^2 - b^2*c*d^2*e^3 + a*b*c*d*e^4 - 4
*a^2*b*d^2*f^3 + (4*a*b^2*d^2*e + a^2*b*d*e^2 - 4*(b^3 - 2*a*b*c)*d^3)*f^2
- (4*b*c^2*d^4 - 4*b^2*c*d^3*e + a*b^2*d*e^3 - (b^3 - 6*a*b*c)*d^2*e^2)*f)*
x)*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(c^4*d^4*e^2 - 2*b*c^3*d^3*e^3 - 2*
a*b*c^2*d*e^5 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (b^2*c^2 + 2*a*c^3)*d^2*e^4 + (
8*a^3*b*d*e + a^4*e^2 - 8*(a^2*b^2 - 2*a^3*c)*d^2)*f^4 - 2*(a^3*b*e^3 + 2*(
b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3 - 4*(a*b^3 - a^2*b*c)*d^2*e + (a^2*b^2 + 6
*a^3*c)*d*e^2)*f^3 - (8*(b^2*c^2 - 2*a*c^3)*d^4 - 8*(b^3*c - a*b*c^2)*d^3*e
 - (b^4 - 20*a*b^2*c + 22*a^2*c^2)*d^2*e^2 + 2*(a*b^3 - 5*a^2*b*c)*d*e^3 -
(a^2*b^2 + 2*a^3*c)*e^4)*f^2 - 2*(2*c^4*d^5 - 4*b*c^3*d^4*e + a^2*b*c*e^5 +
 (b^2*c^2 + 6*a*c^3)*d^3*e^2 + (b^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a*b^2*c - 2
*a^2*c^2)*d*e^4)*f)))/x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.01, size = 1516, normalized size = 3.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x)
```

```
[Out] -1/2/(-4*d*f+e^2)^(1/2)/f*2^(1/2)/((2*a*f^2-b*e*f-2*c*d*f+c*e^2-(-4*d*f+e^2
)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*ln(((b*f-c*e-(-4*d*f+e^2)^(1
/2)*c)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)/f+(2*a*f^2-b*e*f-2*c*d*f+c*e^2-(-4*
d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e)/f^2+1/2*2^(1/2)*((2*a*f^2-b*e*f-
2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*(4*
(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c+4*(b*f-c*e-(-4*d*f+e^2)^(1/2)*c)*(x+1/
2*(e+(-4*d*f+e^2)^(1/2))/f)/f+2*(2*a*f^2-b*e*f-2*c*d*f+c*e^2-(-4*d*f+e^2)^(
1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/
f))*e-1/2/f*2^(1/2)/((2*a*f^2-b*e*f-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*b*f+(-
4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*ln(((b*f-c*e-(-4*d*f+e^2)^(1/2)*c)*(x+1/2*
(e+(-4*d*f+e^2)^(1/2))/f)/f+(2*a*f^2-b*e*f-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)
*b*f+(-4*d*f+e^2)^(1/2)*c*e)/f^2+1/2*2^(1/2)*((2*a*f^2-b*e*f-2*c*d*f+c*e^2-
(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*
d*f+e^2)^(1/2))/f)^2*c+4*(b*f-c*e-(-4*d*f+e^2)^(1/2)*c)*(x+1/2*(e+(-4*d*f+e
^2)^(1/2))/f)/f+2*(2*a*f^2-b*e*f-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*b*f+(-4*d
*f+e^2)^(1/2)*c*e)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))-1/2/f*2^(1
/2)/((2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)
*c*e)/f^2)^(1/2)*ln(((b*f-c*e+(-4*d*f+e^2)^(1/2)*c)*(x-1/2*(-e+(-4*d*f+e^2)
^(1/2))/f)/f+(2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^
2)^(1/2)*c*e)/f^2+1/2*2^(1/2)*((2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1
/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*(4*(x-1/2*(-e+(-4*d*f+e^2)^(1/2)
)/f)^2*c+4*(b*f-c*e+(-4*d*f+e^2)^(1/2)*c)*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)
/f+2*(2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)
*c*e)/f^2)^(1/2))/(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f))+1/2/(-4*d*f+e^2)^(1/2)
/f*2^(1/2)/((2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2
)^(1/2)*c*e)/f^2)^(1/2)*ln(((b*f-c*e+(-4*d*f+e^2)^(1/2)*c)*(x-1/2*(-e+(-4*d
*f+e^2)^(1/2))/f)/f+(2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*b*f-(-4
*d*f+e^2)^(1/2)*c*e)/f^2+1/2*2^(1/2)*((2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+
e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*(4*(x-1/2*(-e+(-4*d*f+e^2
)^(1/2))/f)^2*c+4*(b*f-c*e+(-4*d*f+e^2)^(1/2)*c)*(x-1/2*(-e+(-4*d*f+e^2)^(1
/2))/f)/f+2*(2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2
)^(1/2)*c*e)/f^2)^(1/2))/(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f))*e
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for mo
re details)Is 4*d*f-e^2 positive, negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x}{\sqrt{c\,x^2+b\,x+a}\,\left(f\,x^2+e\,x+d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2)),x)
```

```
[Out] int(x/((a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{a+bx+cx^2}\,\left(d+ex+fx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x**2+b*x+a)**(1/2)/(f*x**2+e*x+d),x)
```

```
[Out] Integral(x/(sqrt(a + b*x + c*x**2)*(d + e*x + f*x**2)), x)
```

3.117 $\int \frac{1}{\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)}\,dx$

Optimal. Leaf size=374

$$\frac{\sqrt{2} f \tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(\sqrt{e^2-4df}+e\right)\right)-b\left(\sqrt{e^2-4df}+e\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2+\sqrt{e^2-4df}\,(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{e^2-4df}\sqrt{2af^2+\sqrt{e^2-4df}\,(ce-bf)-bef-2cdf+ce^2}}-\frac{\sqrt{2} f \tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)-b\left(e-\sqrt{e^2-4\cdots\right.}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}\,(ce-bf)-bef\cdots}}\right.}{\sqrt{e^2-4df}\sqrt{2af^2-\sqrt{e^2-4df}\,(ce-bf)-bef-2\cdots}}$$

```
[Out] -f*arctanh(1/4*(4*a*f+2*x*(b*f-c*(e-(-4*d*f+e^2)^(1/2)))-b*(e-(-4*d*f+e^2)^
(1/2)))*2^(1/2)/(c*x^2+b*x+a)^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2-(-b*f+c*e)
*(-4*d*f+e^2)^(1/2))^(1/2))*2^(1/2)/(-4*d*f+e^2)^(1/2)/(c*e^2-2*c*d*f-b*e*f
+2*a*f^2-(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2)+f*arctanh(1/4*(4*a*f-b*(e+(-4
*d*f+e^2)^(1/2))+2*x*(b*f-c*(e+(-4*d*f+e^2)^(1/2))))*2^(1/2)/(c*x^2+b*x+a)^
(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2+(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2))*2^
(1/2)/(-4*d*f+e^2)^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2+(-b*f+c*e)*(-4*d*f+e^
2)^(1/2))^(1/2)
```

Rubi [A] time = 0.31, antiderivative size = 374, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {983, 724, 206}

$$\frac{\sqrt{2} f \tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(\sqrt{e^2-4df}+e\right)\right)-b\left(\sqrt{e^2-4df}+e\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2+\sqrt{e^2-4df}\,(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{e^2-4df}\sqrt{2af^2+\sqrt{e^2-4df}\,(ce-bf)-bef-2cdf+ce^2}}-\frac{\sqrt{2} f \tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)-b\left(e-\sqrt{e^2-4\cdots\right.}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}\,(ce-bf)-bef\cdots}}\right.}{\sqrt{e^2-4df}\sqrt{2af^2-\sqrt{e^2-4df}\,(ce-bf)-bef-2\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)),x]
```

```
[Out] -((Sqrt[2]*f*ArcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e - S
qrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (
c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[e^2 - 4*d*f]*S
qrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]])) +
(Sqrt[2]*f*ArcTanh[(4*a*f - b*(e + Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e + Sqr
t[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*
e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[e^2 - 4*d*f]*Sqr
t[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 983

```
Int[1/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(
x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[1/((
b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x], x] - Dist[(2*c)/q, Int[1/((b + q
 + 2*c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f}, x] &
```

```
& NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[c*e - b*f, 0] && PosQ[b
^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)}dx = \frac{(2f)\int \frac{1}{\left(e-\sqrt{e^2-4df}+2fx\right)\sqrt{a+bx+cx^2}}dx}{\sqrt{e^2-4df}} - \frac{(2f)\int \frac{1}{\left(e+\sqrt{e^2-4df}+2fx\right)\sqrt{a+bx+cx^2}}d}{\sqrt{e^2-4df}}$$

$$= -\frac{(4f)\,\text{Subst}\left(\int \frac{1}{16af^2-8bf\left(e-\sqrt{e^2-4df}\right)+4c\left(e-\sqrt{e^2-4df}\right)^2-x^2}dx, x, \frac{4af-b\left(e-\sqrt{e^2-\right.}}{}\right.}{\sqrt{e^2-4df}}$$

$$= -\frac{\sqrt{2}\,f\tanh^{-1}\left(\frac{4af-b\left(e-\sqrt{e^2-4df}\right)+2\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)x}{2\sqrt{2}\sqrt{ce^2-2cdf-bef+2af^2-(ce-bf)\sqrt{e^2-4df}}\sqrt{a+bx+cx^2}}\right)}{\sqrt{e^2-4df}\sqrt{ce^2-2cdf-bef+2af^2-(ce-bf)\sqrt{e^2-4df}}} + \frac{\sqrt{2}\,f}{\sqrt{e^2-}}$$

Mathematica [A] time = 0.79, size = 376, normalized size = 1.01

$$\frac{\sqrt{2}\,f\left(\frac{\tanh^{-1}\left(\frac{4af-b\left(\sqrt{e^2-4df}+e-2fx\right)-2cx\left(\sqrt{e^2-4df}+e\right)}{2\sqrt{2}\sqrt{a+x(b+cx)}\sqrt{f\left(2af-b\left(\sqrt{e^2-4df}+e\right)\right)+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{f\left(2af-b\left(\sqrt{e^2-4df}+e\right)\right)+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}} - \frac{\tanh^{-1}\left(\frac{4af+b\left(\sqrt{e^2-4df}-e+2fx\right)+2cx\left(\sqrt{e^2-4df}-e\right)}{2\sqrt{2}\sqrt{a+x(b+cx)}\sqrt{f\left(2af+b\sqrt{e^2-4df}+b(-e)\right)+c\left(-e\sqrt{e^2-4df}-2d\right.}}\right.}{\sqrt{f\left(2af+b\sqrt{e^2-4df}+b(-e)\right)+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right.}{\sqrt{e^2-4df}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)),x]
```

```
[Out] (Sqrt[2]*f*(ArcTanh[(4*a*f - 2*c*(e + Sqrt[e^2 - 4*d*f])*x - b*(e + Sqrt[e^
2 - 4*d*f] - 2*f*x))/(2*Sqrt[2]*Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])
+ f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))]*Sqrt[a + x*(b + c*x)])]/Sqrt[c*(e^
2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))] -
 ArcTanh[(4*a*f + 2*c*(-e + Sqrt[e^2 - 4*d*f])*x + b*(-e + Sqrt[e^2 - 4*d*f
] + 2*f*x))/(2*Sqrt[2]*Sqrt[f*(-(b*e) + 2*a*f + b*Sqrt[e^2 - 4*d*f]) + c*(e
^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + x*(b + c*x)])]/Sqrt[f*(-(b*e) +
 2*a*f + b*Sqrt[e^2 - 4*d*f]) + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]))/Sq
rt[e^2 - 4*d*f]
```

fricas [B] time = 10.30, size = 11287, normalized size = 30.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*sqrt((c*e^2 + 2*a*f^2 - (2*c*d + b*e)*f + (c^2*d^2*e^2 - b*c*d*
e^3 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e + a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f
^2 - (4*c^2*d^3 - 4*b*c*d^2*e + a*b*e^3 - (b^2 - 6*a*c)*d*e^2)*f)*sqrt((c^2
*e^2 - 2*b*c*e*f + b^2*f^2)/(c^4*d^4*e^2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d*e^
5 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (b^2*c^2 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*d*e
+ a^4*e^2 - 8*(a^2*b^2 - 2*a^3*c)*d^2)*f^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a*b^
2*c + 6*a^2*c^2)*d^3 - 4*(a*b^3 - a^2*b*c)*d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^
```

```
2)*f^3 - (8*(b^2*c^2 - 2*a*c^3)*d^4 - 8*(b^3*c - a*b*c^2)*d^3*e - (b^4 - 20
*a*b^2*c + 22*a^2*c^2)*d^2*e^2 + 2*(a*b^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2
*a^3*c)*e^4)*f^2 - 2*(2*c^4*d^5 - 4*b*c^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2 +
6*a*c^3)*d^3*e^2 + (b^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*
e^4)*f)))/(c^2*d^2*e^2 - b*c*d*e^3 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e + a
^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 - (4*c^2*d^3 - 4*b*c*d^2*e + a*b*e^3 - (b
^2 - 6*a*c)*d*e^2)*f))*log((2*(b^2*d - a*b*e)*f^2 + sqrt(2)*(c^2*d*e^3 - 4*
a*b*d*f^3 + (4*b*c*d^2 + 4*a*c*d*e + a*b*e^2)*f^2 - (4*c^2*d^2*e + b*c*d*e^
2 + a*c*e^3)*f - (c^3*d^3*e^3 - b*c^2*d^2*e^4 + a*c^2*d*e^5 + 4*(2*a^2*b*d^
2 - a^3*d*e)*f^4 + (2*a^2*b*d*e^2 + a^3*e^3 + 8*(b^3 - 2*a*b*c)*d^3 - 4*(3*
a*b^2 - a^2*c)*d^2*e)*f^3 + (8*b*c^2*d^4 - a^2*b*e^4 - 4*(3*b^2*c - a*c^2)*
d^3*e - 2*(b^3 - 10*a*b*c)*d^2*e^2 + (3*a*b^2 - 5*a^2*c)*d*e^3)*f^2 - (4*c^
3*d^4*e - 2*b*c^2*d^3*e^2 + 4*a*b*c*d*e^4 - a^2*c*e^5 - (3*b^2*c - 5*a*c^2)
*d^2*e^3)*f)*sqrt((c^2*e^2 - 2*b*c*e*f + b^2*f^2)/(c^4*d^4*e^2 - 2*b*c^3*d^
3*e^3 - 2*a*b*c^2*d*e^5 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (b^2*c^2 + 2*a*c^3)*d
^2*e^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a^2*b^2 - 2*a^3*c)*d^2)*f^4 - 2*(a^3*b
*e^3 + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3 - 4*(a*b^3 - a^2*b*c)*d^2*e + (a
^2*b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*c^2 - 2*a*c^3)*d^4 - 8*(b^3*c - a*b*
c^2)*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2*c^2)*d^2*e^2 + 2*(a*b^3 - 5*a^2*b*c
)*d*e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2 - 2*(2*c^4*d^5 - 4*b*c^3*d^4*e + a^2
*b*c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2 + (b^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a
*b^2*c - 2*a^2*c^2)*d*e^4)*f)))*sqrt(c*x^2 + b*x + a)*sqrt((c*e^2 + 2*a*f^2
 - (2*c*d + b*e)*f + (c^2*d^2*e^2 - b*c*d*e^3 + a*c*e^4 - 4*a^2*d*f^3 + (4*
a*b*d*e + a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 - (4*c^2*d^3 - 4*b*c*d^2*e + a
*b*e^3 - (b^2 - 6*a*c)*d*e^2)*f)*sqrt((c^2*e^2 - 2*b*c*e*f + b^2*f^2)/(c^4*
d^4*e^2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d*e^5 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (
b^2*c^2 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a^2*b^2 - 2*a^3*c)
*d^2)*f^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3 - 4*(a*b^3 -
 a^2*b*c)*d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*c^2 - 2*a*c^3)*d
^4 - 8*(b^3*c - a*b*c^2)*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2*c^2)*d^2*e^2 +
2*(a*b^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2 - 2*(2*c^4*d^5 -
 4*b*c^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2 + (b^3*c - 5*a*b
*c^2)*d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*e^4)*f)))/(c^2*d^2*e^2 - b*c*d*e^
3 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e + a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2
 - (4*c^2*d^3 - 4*b*c*d^2*e + a*b*e^3 - (b^2 - 6*a*c)*d*e^2)*f)) - 2*(b*c*d
*e - a*c*e^2)*f + ((4*b*c*d - b^2*e)*f^2 - (4*c^2*d*e - b*c*e^2)*f)*x - (8*
a^3*d*f^4 - 2*(4*a^2*b*d*e + a^3*e^2 - 4*(a*b^2 - 2*a^2*c)*d^2)*f^3 + 2*(4*
a*c^2*d^3 - 4*a*b*c*d^2*e + a^2*b*e^3 - (a*b^2 - 6*a^2*c)*d*e^2)*f^2 - 2*(a
*c^2*d^2*e^2 - a*b*c*d*e^3 + a^2*c*e^4)*f + (4*a^2*b*d*f^4 - (4*a*b^2*d*e +
 a^2*b*e^2 - 4*(b^3 - 2*a*b*c)*d^2)*f^3 + (4*b*c^2*d^3 - 4*b^2*c*d^2*e + a*
b^2*e^3 - (b^3 - 6*a*b*c)*d*e^2)*f^2 - (b*c^2*d^2*e^2 - b^2*c*d*e^3 + a*b*c
*e^4)*f)*x)*sqrt((c^2*e^2 - 2*b*c*e*f + b^2*f^2)/(c^4*d^4*e^2 - 2*b*c^3*d^3
*e^3 - 2*a*b*c^2*d*e^5 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (b^2*c^2 + 2*a*c^3)*d^
2*e^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a^2*b^2 - 2*a^3*c)*d^2)*f^4 - 2*(a^3*b*
e^3 + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3 - 4*(a*b^3 - a^2*b*c)*d^2*e + (a^
2*b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*c^2 - 2*a*c^3)*d^4 - 8*(b^3*c - a*b*c
^2)*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2*c^2)*d^2*e^2 + 2*(a*b^3 - 5*a^2*b*c)
*d*e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2 - 2*(2*c^4*d^5 - 4*b*c^3*d^4*e + a^2*
b*c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2 + (b^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a*
b^2*c - 2*a^2*c^2)*d*e^4)*f)))/x) - 1/4*sqrt(2)*sqrt((c*e^2 + 2*a*f^2 - (2*
c*d + b*e)*f + (c^2*d^2*e^2 - b*c*d*e^3 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*
e + a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 - (4*c^2*d^3 - 4*b*c*d^2*e + a*b*e^3
 - (b^2 - 6*a*c)*d*e^2)*f)*sqrt((c^2*e^2 - 2*b*c*e*f + b^2*f^2)/(c^4*d^4*e^
2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d*e^5 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (b^2*c^
2 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a^2*b^2 - 2*a^3*c)*d^2)*
f^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3 - 4*(a*b^3 - a^2*b
*c)*d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*c^2 - 2*a*c^3)*d^4 - 8
*(b^3*c - a*b*c^2)*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2*c^2)*d^2*e^2 + 2*(a*b
^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2 - 2*(2*c^4*d^5 - 4*b*c
```

```
^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2 + (b^3*c - 5*a*b*c^2)*
d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*e^4)*f)))/(c^2*d^2*e^2 - b*c*d*e^3 + a*
c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e + a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 - (4*
c^2*d^3 - 4*b*c*d^2*e + a*b*e^3 - (b^2 - 6*a*c)*d*e^2)*f))*log((2*(b^2*d -
a*b*e)*f^2 - sqrt(2)*(c^2*d*e^3 - 4*a*b*d*f^3 + (4*b*c*d^2 + 4*a*c*d*e + a*
b*e^2)*f^2 - (4*c^2*d^2*e + b*c*d*e^2 + a*c*e^3)*f - (c^3*d^3*e^3 - b*c^2*d
^2*e^4 + a*c^2*d*e^5 + 4*(2*a^2*b*d^2 - a^3*d*e)*f^4 + (2*a^2*b*d*e^2 + a^3
*e^3 + 8*(b^3 - 2*a*b*c)*d^3 - 4*(3*a*b^2 - a^2*c)*d^2*e)*f^3 + (8*b*c^2*d^
4 - a^2*b*e^4 - 4*(3*b^2*c - a*c^2)*d^3*e - 2*(b^3 - 10*a*b*c)*d^2*e^2 + (3
*a*b^2 - 5*a^2*c)*d*e^3)*f^2 - (4*c^3*d^4*e - 2*b*c^2*d^3*e^2 + 4*a*b*c*d*e
^4 - a^2*c*e^5 - (3*b^2*c - 5*a*c^2)*d^2*e^3)*f)*sqrt((c^2*e^2 - 2*b*c*e*f
+ b^2*f^2)/(c^4*d^4*e^2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d*e^5 + a^2*c^2*e^6 -
 4*a^4*d*f^5 + (b^2*c^2 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a^
2*b^2 - 2*a^3*c)*d^2)*f^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*
d^3 - 4*(a*b^3 - a^2*b*c)*d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*
c^2 - 2*a*c^3)*d^4 - 8*(b^3*c - a*b*c^2)*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2
*c^2)*d^2*e^2 + 2*(a*b^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2
- 2*(2*c^4*d^5 - 4*b*c^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2
+ (b^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*e^4)*f)))*sqrt(c*
x^2 + b*x + a)*sqrt((c*e^2 + 2*a*f^2 - (2*c*d + b*e)*f + (c^2*d^2*e^2 - b*c
*d*e^3 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e + a^2*e^2 - 4*(b^2 - 2*a*c)*d^2
)*f^2 - (4*c^2*d^3 - 4*b*c*d^2*e + a*b*e^3 - (b^2 - 6*a*c)*d*e^2)*f)*sqrt((
c^2*e^2 - 2*b*c*e*f + b^2*f^2)/(c^4*d^4*e^2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d
*e^5 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (b^2*c^2 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*d
*e + a^4*e^2 - 8*(a^2*b^2 - 2*a^3*c)*d^2)*f^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a
*b^2*c + 6*a^2*c^2)*d^3 - 4*(a*b^3 - a^2*b*c)*d^2*e + (a^2*b^2 + 6*a^3*c)*d
*e^2)*f^3 - (8*(b^2*c^2 - 2*a*c^3)*d^4 - 8*(b^3*c - a*b*c^2)*d^3*e - (b^4 -
 20*a*b^2*c + 22*a^2*c^2)*d^2*e^2 + 2*(a*b^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2
+ 2*a^3*c)*e^4)*f^2 - 2*(2*c^4*d^5 - 4*b*c^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2
 + 6*a*c^3)*d^3*e^2 + (b^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)
*d*e^4)*f)))/(c^2*d^2*e^2 - b*c*d*e^3 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e
+ a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 - (4*c^2*d^3 - 4*b*c*d^2*e + a*b*e^3 -
 (b^2 - 6*a*c)*d*e^2)*f)) - 2*(b*c*d*e - a*c*e^2)*f + ((4*b*c*d - b^2*e)*f^
2 - (4*c^2*d*e - b*c*e^2)*f)*x - (8*a^3*d*f^4 - 2*(4*a^2*b*d*e + a^3*e^2 -
4*(a*b^2 - 2*a^2*c)*d^2)*f^3 + 2*(4*a*c^2*d^3 - 4*a*b*c*d^2*e + a^2*b*e^3 -
 (a*b^2 - 6*a^2*c)*d*e^2)*f^2 - 2*(a*c^2*d^2*e^2 - a*b*c*d*e^3 + a^2*c*e^4)
*f + (4*a^2*b*d*f^4 - (4*a*b^2*d*e + a^2*b*e^2 - 4*(b^3 - 2*a*b*c)*d^2)*f^3
 + (4*b*c^2*d^3 - 4*b^2*c*d^2*e + a*b^2*e^3 - (b^3 - 6*a*b*c)*d*e^2)*f^2 -
(b*c^2*d^2*e^2 - b^2*c*d*e^3 + a*b*c*e^4)*f)*x)*sqrt((c^2*e^2 - 2*b*c*e*f +
 b^2*f^2)/(c^4*d^4*e^2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d*e^5 + a^2*c^2*e^6 -
4*a^4*d*f^5 + (b^2*c^2 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a^2
*b^2 - 2*a^3*c)*d^2)*f^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d
^3 - 4*(a*b^3 - a^2*b*c)*d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*c
^2 - 2*a*c^3)*d^4 - 8*(b^3*c - a*b*c^2)*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2*
c^2)*d^2*e^2 + 2*(a*b^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2 -
 2*(2*c^4*d^5 - 4*b*c^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2 +
 (b^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*e^4)*f)))/x) + 1/4
*sqrt(2)*sqrt((c*e^2 + 2*a*f^2 - (2*c*d + b*e)*f - (c^2*d^2*e^2 - b*c*d*e^3
 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e + a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2
- (4*c^2*d^3 - 4*b*c*d^2*e + a*b*e^3 - (b^2 - 6*a*c)*d*e^2)*f)*sqrt((c^2*e^
2 - 2*b*c*e*f + b^2*f^2)/(c^4*d^4*e^2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d*e^5 +
 a^2*c^2*e^6 - 4*a^4*d*f^5 + (b^2*c^2 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*d*e + a
^4*e^2 - 8*(a^2*b^2 - 2*a^3*c)*d^2)*f^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a*b^2*c
 + 6*a^2*c^2)*d^3 - 4*(a*b^3 - a^2*b*c)*d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^2)*
f^3 - (8*(b^2*c^2 - 2*a*c^3)*d^4 - 8*(b^3*c - a*b*c^2)*d^3*e - (b^4 - 20*a*
b^2*c + 22*a^2*c^2)*d^2*e^2 + 2*(a*b^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2*a^
3*c)*e^4)*f^2 - 2*(2*c^4*d^5 - 4*b*c^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2 + 6*a
*c^3)*d^3*e^2 + (b^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*e^4
)*f)))/(c^2*d^2*e^2 - b*c*d*e^3 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e + a^2*
```

```
e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 - (4*c^2*d^3 - 4*b*c*d^2*e + a*b*e^3 - (b^2
- 6*a*c)*d*e^2)*f))*log((2*(b^2*d - a*b*e)*f^2 + sqrt(2)*(c^2*d*e^3 - 4*a*b
*d*f^3 + (4*b*c*d^2 + 4*a*c*d*e + a*b*e^2)*f^2 - (4*c^2*d^2*e + b*c*d*e^2 +
 a*c*e^3)*f + (c^3*d^3*e^3 - b*c^2*d^2*e^4 + a*c^2*d*e^5 + 4*(2*a^2*b*d^2 -
 a^3*d*e)*f^4 + (2*a^2*b*d*e^2 + a^3*e^3 + 8*(b^3 - 2*a*b*c)*d^3 - 4*(3*a*b
^2 - a^2*c)*d^2*e)*f^3 + (8*b*c^2*d^4 - a^2*b*e^4 - 4*(3*b^2*c - a*c^2)*d^3
*e - 2*(b^3 - 10*a*b*c)*d^2*e^2 + (3*a*b^2 - 5*a^2*c)*d*e^3)*f^2 - (4*c^3*d
^4*e - 2*b*c^2*d^3*e^2 + 4*a*b*c*d*e^4 - a^2*c*e^5 - (3*b^2*c - 5*a*c^2)*d^
2*e^3)*f)*sqrt((c^2*e^2 - 2*b*c*e*f + b^2*f^2)/(c^4*d^4*e^2 - 2*b*c^3*d^3*e
^3 - 2*a*b*c^2*d*e^5 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (b^2*c^2 + 2*a*c^3)*d^2*
e^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a^2*b^2 - 2*a^3*c)*d^2)*f^4 - 2*(a^3*b*e^
3 + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3 - 4*(a*b^3 - a^2*b*c)*d^2*e + (a^2*
b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*c^2 - 2*a*c^3)*d^4 - 8*(b^3*c - a*b*c^2
)*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2*c^2)*d^2*e^2 + 2*(a*b^3 - 5*a^2*b*c)*d
*e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2 - 2*(2*c^4*d^5 - 4*b*c^3*d^4*e + a^2*b*
c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2 + (b^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a*b^
2*c - 2*a^2*c^2)*d*e^4)*f)))*sqrt(c*x^2 + b*x + a)*sqrt((c*e^2 + 2*a*f^2 -
(2*c*d + b*e)*f - (c^2*d^2*e^2 - b*c*d*e^3 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b
*d*e + a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 - (4*c^2*d^3 - 4*b*c*d^2*e + a*b*
e^3 - (b^2 - 6*a*c)*d*e^2)*f)*sqrt((c^2*e^2 - 2*b*c*e*f + b^2*f^2)/(c^4*d^4
*e^2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d*e^5 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (b^2
*c^2 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a^2*b^2 - 2*a^3*c)*d^
2)*f^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3 - 4*(a*b^3 - a^
2*b*c)*d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*c^2 - 2*a*c^3)*d^4
- 8*(b^3*c - a*b*c^2)*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2*c^2)*d^2*e^2 + 2*(
a*b^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2 - 2*(2*c^4*d^5 - 4*
b*c^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2 + (b^3*c - 5*a*b*c^
2)*d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*e^4)*f)))/(c^2*d^2*e^2 - b*c*d*e^3 +
 a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e + a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 -
(4*c^2*d^3 - 4*b*c*d^2*e + a*b*e^3 - (b^2 - 6*a*c)*d*e^2)*f)) - 2*(b*c*d*e
- a*c*e^2)*f + ((4*b*c*d - b^2*e)*f^2 - (4*c^2*d*e - b*c*e^2)*f)*x + (8*a^3
*d*f^4 - 2*(4*a^2*b*d*e + a^3*e^2 - 4*(a*b^2 - 2*a^2*c)*d^2)*f^3 + 2*(4*a*c
^2*d^3 - 4*a*b*c*d^2*e + a^2*b*e^3 - (a*b^2 - 6*a^2*c)*d*e^2)*f^2 - 2*(a*c^
2*d^2*e^2 - a*b*c*d*e^3 + a^2*c*e^4)*f + (4*a^2*b*d*f^4 - (4*a*b^2*d*e + a^
2*b*e^2 - 4*(b^3 - 2*a*b*c)*d^2)*f^3 + (4*b*c^2*d^3 - 4*b^2*c*d^2*e + a*b^2
*e^3 - (b^3 - 6*a*b*c)*d*e^2)*f^2 - (b*c^2*d^2*e^2 - b^2*c*d*e^3 + a*b*c*e^
4)*f)*x)*sqrt((c^2*e^2 - 2*b*c*e*f + b^2*f^2)/(c^4*d^4*e^2 - 2*b*c^3*d^3*e^
3 - 2*a*b*c^2*d*e^5 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (b^2*c^2 + 2*a*c^3)*d^2*e
^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a^2*b^2 - 2*a^3*c)*d^2)*f^4 - 2*(a^3*b*e^3
 + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3 - 4*(a*b^3 - a^2*b*c)*d^2*e + (a^2*b
^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*c^2 - 2*a*c^3)*d^4 - 8*(b^3*c - a*b*c^2)
*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2*c^2)*d^2*e^2 + 2*(a*b^3 - 5*a^2*b*c)*d*
e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2 - 2*(2*c^4*d^5 - 4*b*c^3*d^4*e + a^2*b*c
*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2 + (b^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a*b^2
*c - 2*a^2*c^2)*d*e^4)*f)))/x) - 1/4*sqrt(2)*sqrt((c*e^2 + 2*a*f^2 - (2*c*d
 + b*e)*f - (c^2*d^2*e^2 - b*c*d*e^3 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e +
 a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 - (4*c^2*d^3 - 4*b*c*d^2*e + a*b*e^3 -
(b^2 - 6*a*c)*d*e^2)*f)*sqrt((c^2*e^2 - 2*b*c*e*f + b^2*f^2)/(c^4*d^4*e^2 -
 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d*e^5 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (b^2*c^2 +
 2*a*c^3)*d^2*e^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a^2*b^2 - 2*a^3*c)*d^2)*f^4
 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3 - 4*(a*b^3 - a^2*b*c)
*d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*c^2 - 2*a*c^3)*d^4 - 8*(b
^3*c - a*b*c^2)*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2*c^2)*d^2*e^2 + 2*(a*b^3
- 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2 - 2*(2*c^4*d^5 - 4*b*c^3*
d^4*e + a^2*b*c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2 + (b^3*c - 5*a*b*c^2)*d^2
*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*e^4)*f)))/(c^2*d^2*e^2 - b*c*d*e^3 + a*c*e
^4 - 4*a^2*d*f^3 + (4*a*b*d*e + a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 - (4*c^2
*d^3 - 4*b*c*d^2*e + a*b*e^3 - (b^2 - 6*a*c)*d*e^2)*f))*log((2*(b^2*d - a*b
*e)*f^2 - sqrt(2)*(c^2*d*e^3 - 4*a*b*d*f^3 + (4*b*c*d^2 + 4*a*c*d*e + a*b*e
```

```
^2)*f^2 - (4*c^2*d^2*e + b*c*d*e^2 + a*c*e^3)*f + (c^3*d^3*e^3 - b*c^2*d^2*
e^4 + a*c^2*d*e^5 + 4*(2*a^2*b*d^2 - a^3*d*e)*f^4 + (2*a^2*b*d*e^2 + a^3*e^
3 + 8*(b^3 - 2*a*b*c)*d^3 - 4*(3*a*b^2 - a^2*c)*d^2*e)*f^3 + (8*b*c^2*d^4 -
 a^2*b*e^4 - 4*(3*b^2*c - a*c^2)*d^3*e - 2*(b^3 - 10*a*b*c)*d^2*e^2 + (3*a*
b^2 - 5*a^2*c)*d*e^3)*f^2 - (4*c^3*d^4*e - 2*b*c^2*d^3*e^2 + 4*a*b*c*d*e^4
- a^2*c*e^5 - (3*b^2*c - 5*a*c^2)*d^2*e^3)*f)*sqrt((c^2*e^2 - 2*b*c*e*f + b
^2*f^2)/(c^4*d^4*e^2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d*e^5 + a^2*c^2*e^6 - 4*
a^4*d*f^5 + (b^2*c^2 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a^2*b
^2 - 2*a^3*c)*d^2)*f^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3
 - 4*(a*b^3 - a^2*b*c)*d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*c^2
 - 2*a*c^3)*d^4 - 8*(b^3*c - a*b*c^2)*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2*c^
2)*d^2*e^2 + 2*(a*b^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2 - 2
*(2*c^4*d^5 - 4*b*c^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2 + (
b^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*e^4)*f)))*sqrt(c*x^2
 + b*x + a)*sqrt((c*e^2 + 2*a*f^2 - (2*c*d + b*e)*f - (c^2*d^2*e^2 - b*c*d*
e^3 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e + a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f
^2 - (4*c^2*d^3 - 4*b*c*d^2*e + a*b*e^3 - (b^2 - 6*a*c)*d*e^2)*f)*sqrt((c^2
*e^2 - 2*b*c*e*f + b^2*f^2)/(c^4*d^4*e^2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d*e^
5 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (b^2*c^2 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*d*e
+ a^4*e^2 - 8*(a^2*b^2 - 2*a^3*c)*d^2)*f^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a*b^
2*c + 6*a^2*c^2)*d^3 - 4*(a*b^3 - a^2*b*c)*d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^
2)*f^3 - (8*(b^2*c^2 - 2*a*c^3)*d^4 - 8*(b^3*c - a*b*c^2)*d^3*e - (b^4 - 20
*a*b^2*c + 22*a^2*c^2)*d^2*e^2 + 2*(a*b^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2
*a^3*c)*e^4)*f^2 - 2*(2*c^4*d^5 - 4*b*c^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2 +
6*a*c^3)*d^3*e^2 + (b^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*
e^4)*f)))/(c^2*d^2*e^2 - b*c*d*e^3 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e + a
^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 - (4*c^2*d^3 - 4*b*c*d^2*e + a*b*e^3 - (b
^2 - 6*a*c)*d*e^2)*f)) - 2*(b*c*d*e - a*c*e^2)*f + ((4*b*c*d - b^2*e)*f^2 -
 (4*c^2*d*e - b*c*e^2)*f)*x + (8*a^3*d*f^4 - 2*(4*a^2*b*d*e + a^3*e^2 - 4*(
a*b^2 - 2*a^2*c)*d^2)*f^3 + 2*(4*a*c^2*d^3 - 4*a*b*c*d^2*e + a^2*b*e^3 - (a
*b^2 - 6*a^2*c)*d*e^2)*f^2 - 2*(a*c^2*d^2*e^2 - a*b*c*d*e^3 + a^2*c*e^4)*f
+ (4*a^2*b*d*f^4 - (4*a*b^2*d*e + a^2*b*e^2 - 4*(b^3 - 2*a*b*c)*d^2)*f^3 +
(4*b*c^2*d^3 - 4*b^2*c*d^2*e + a*b^2*e^3 - (b^3 - 6*a*b*c)*d*e^2)*f^2 - (b*
c^2*d^2*e^2 - b^2*c*d*e^3 + a*b*c*e^4)*f)*x)*sqrt((c^2*e^2 - 2*b*c*e*f + b^
2*f^2)/(c^4*d^4*e^2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d*e^5 + a^2*c^2*e^6 - 4*a
^4*d*f^5 + (b^2*c^2 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a^2*b^
2 - 2*a^3*c)*d^2)*f^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3
- 4*(a*b^3 - a^2*b*c)*d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*c^2
- 2*a*c^3)*d^4 - 8*(b^3*c - a*b*c^2)*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2*c^2
)*d^2*e^2 + 2*(a*b^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2 - 2*
(2*c^4*d^5 - 4*b*c^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2 + (b
^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*e^4)*f)))/x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="giac")
[Out] Timed out
```

maple [B] time = 0.01, size = 761, normalized size = 2.03

$$\frac{\sqrt{2}\,\ln\left(\frac{\frac{\left(bf-ce-\sqrt{-4df+e^2}\,c\right)\left(x+\frac{e+\sqrt{-4df+e^2}}{2f}\right)}{f}+\frac{2af^2-bef-2cdf+ce^2-\sqrt{-4df+e^2}\,bf+\sqrt{-4df+e^2}\,ce}{f^2}+\frac{\sqrt{2}\,\sqrt{\frac{2af^2-bef-2cdf+ce^2-\sqrt{-4df+e^2}\,bf+\sqrt{-4df+e^2}\,ce}{f^2}}\cdots}{\cdots}}{x+\frac{e+\sqrt{-4df+e^2}}{2f}}\right.\cdots}{\sqrt{-4df+e^2}\,\sqrt{\frac{2af^2-bef-2cdf+ce^2-\sqrt{-4df+e^2}\,b\cdots}{f^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x)
```

```
[Out] 1/(-4*d*f+e^2)^(1/2)*2^(1/2)/((2*a*f^2-b*e*f-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/
2)*b*f+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*ln(((b*f-c*e-(-4*d*f+e^2)^(1/2)*c
)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)/f+(2*a*f^2-b*e*f-2*c*d*f+c*e^2-(-4*d*f+e
^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e)/f^2+1/2*2^(1/2)*((2*a*f^2-b*e*f-2*c*d
*f+c*e^2-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*(4*(x+1/
2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c+4*(b*f-c*e-(-4*d*f+e^2)^(1/2)*c)*(x+1/2*(e+
(-4*d*f+e^2)^(1/2))/f)/f+2*(2*a*f^2-b*e*f-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*
b*f+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))-1
/(-4*d*f+e^2)^(1/2)*2^(1/2)/((2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2
)*b*f-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*ln(((b*f-c*e+(-4*d*f+e^2)^(1/2)*c)
*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)/f+(2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e
^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e)/f^2+1/2*2^(1/2)*((2*a*f^2-b*e*f-2*c*d
*f+c*e^2+(-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*(4*(x-1/
2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c+4*(b*f-c*e+(-4*d*f+e^2)^(1/2)*c)*(x-1/2*(-
e+(-4*d*f+e^2)^(1/2))/f)/f+2*(2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2
)*b*f-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2))/(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)
)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for mo
re details)Is 4*d*f-e^2 positive, negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{c\,x^2+b\,x+a}\,\left(f\,x^2+e\,x+d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2)),x)
```

```
[Out] int(1/((a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+bx+cx^2}\,\left(d+ex+fx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**2+b*x+a)**(1/2)/(f*x**2+e*x+d),x)
```

```
[Out] Integral(1/(sqrt(a + b*x + c*x**2)*(d + e*x + f*x**2)), x)
```

3.118 $\int \frac{1}{x\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)}\,dx$

Optimal. Leaf size=451

$$\frac{f\left(\sqrt{e^2-4df}+e\right)\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)-b\left(e-\sqrt{e^2-4df}\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}\,(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}\,d\sqrt{e^2-4df}\sqrt{2af^2-\sqrt{e^2-4df}\,(ce-bf)-bef-2cdf+ce^2}}-\frac{f\left(e-\sqrt{e^2-4df}\right)\tanh^{-1}\left(\frac{4af+2\cdots}{2\sqrt{2}\sqrt{a+bx+\cdots}}\right)}{\sqrt{2}\,d\sqrt{e^2-4df}\sqrt{2af^2+\sqrt{e^2-4\cdots}}}$$

```
[Out] -arctanh(1/2*(b*x+2*a)/a^(1/2)/(c*x^2+b*x+a)^(1/2))/d/a^(1/2)+1/2*f*arctanh
(1/4*(4*a*f+2*x*(b*f-c*(e-(-4*d*f+e^2)^(1/2)))-b*(e-(-4*d*f+e^2)^(1/2)))*2^
(1/2)/(c*x^2+b*x+a)^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2-(-b*f+c*e)*(-4*d*f+e
^2)^(1/2))^(1/2))*(e+(-4*d*f+e^2)^(1/2))/d*2^(1/2)/(-4*d*f+e^2)^(1/2)/(c*e^
2-2*c*d*f-b*e*f+2*a*f^2-(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2)-1/2*f*arctanh(
1/4*(4*a*f-b*(e+(-4*d*f+e^2)^(1/2))+2*x*(b*f-c*(e+(-4*d*f+e^2)^(1/2))))*2^(
1/2)/(c*x^2+b*x+a)^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2+(-b*f+c*e)*(-4*d*f+e^
2)^(1/2))^(1/2))*(e-(-4*d*f+e^2)^(1/2))/d*2^(1/2)/(-4*d*f+e^2)^(1/2)/(c*e^2
-2*c*d*f-b*e*f+2*a*f^2+(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2)
```

Rubi [A] time = 2.63, antiderivative size = 451, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {6728, 724, 206, 1032}

$$\frac{f\left(\sqrt{e^2-4df}+e\right)\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)-b\left(e-\sqrt{e^2-4df}\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}\,(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}\,d\sqrt{e^2-4df}\sqrt{2af^2-\sqrt{e^2-4df}\,(ce-bf)-bef-2cdf+ce^2}}-\frac{f\left(e-\sqrt{e^2-4df}\right)\tanh^{-1}\left(\frac{4af+2\cdots}{2\sqrt{2}\sqrt{a+bx+\cdots}}\right)}{\sqrt{2}\,d\sqrt{e^2-4df}\sqrt{2af^2+\sqrt{e^2-4\cdots}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)),x]
```

```
[Out] -(ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])]/(Sqrt[a]*d)) + (f*
(e + Sqrt[e^2 - 4*d*f])*ArcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) + 2*(b*f
 - c*(e - Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f +
2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])]])/(Sqrt[2]*
d*Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sq
rt[e^2 - 4*d*f]]) - (f*(e - Sqrt[e^2 - 4*d*f])*ArcTanh[(4*a*f - b*(e + Sqrt
[e^2 - 4*d*f]) + 2*(b*f - c*(e + Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e
^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*
x + c*x^2])]])/(Sqrt[2]*d*Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2
*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1032

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
```

```
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{x\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)}\,dx = \int \left(\frac{1}{dx\sqrt{a+bx+cx^2}} + \frac{-e-fx}{d\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)}\right)dx$$

$$= \frac{\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx}{d} + \frac{\int \frac{-e-fx}{\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)}\,dx}{d}$$

$$= -\frac{2\,\mathrm{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+bx}{\sqrt{a+bx+cx^2}}\right)}{d} - \frac{\left(f\left(1-\frac{e}{\sqrt{e^2-4df}}\right)\right)\int \frac{\cdots}{\left(e+\sqrt{e^2-4df}\cdots\right.}}{d}$$

$$= -\frac{\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a}\,d} + \frac{\left(2f\left(1-\frac{e}{\sqrt{e^2-4df}}\right)\right)\mathrm{Subst}\left(\int \frac{\cdots}{16af^2-8bf\left(e+\cdots\right.}\right.}{\cdots}$$

$$= -\frac{\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a}\,d} + \frac{f\left(1+\frac{e}{\sqrt{e^2-4df}}\right)\tanh^{-1}\left(\frac{4af-b\left(e-\sqrt{e^2-\cdots}\right.}{2\sqrt{2}\sqrt{ce^2-2cdf-bef\cdots}}\right.}{\sqrt{2}\,d\sqrt{ce^2-2cdf-bef+2af^2-\cdots}}$$

Mathematica [A] time = 2.41, size = 450, normalized size = 1.00

$$\frac{\sqrt{2}f\left(\frac{\left(\sqrt{e^2-4df}-e\right)\tanh^{-1}\left(\frac{4af-b\left(\sqrt{e^2-4df}+e-2fx\right)-2cx\left(\sqrt{e^2-4df}+e\right)}{2\sqrt{2}\sqrt{a+x(b+cx)}\sqrt{f\left(2af-b\left(\sqrt{e^2-4df}+e\right)\right)+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{f\left(2af-b\left(\sqrt{e^2-4df}+e\right)\right)+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}} + \frac{\left(\sqrt{e^2-4df}+e\right)\tanh^{-1}\left(\frac{4af+b\left(\sqrt{e^2-4df}-e+2fx\right)+2c\cdots}{2\sqrt{2}\sqrt{a+x(b+cx)}\sqrt{f\left(2af+b\sqrt{e^2-4df}+b(-\cdots\right.}}\right.}{\sqrt{f\left(2af+b\sqrt{e^2-4df}+b(-e)\right)+c\left(-e\sqrt{e^2-4df}-\cdots\right.}}\right)}{\sqrt{e^2-4df}}\cdots}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)),x]
```

```
[Out] ((-2*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + x*(b + c*x)])])/Sqrt[a] + (Sqr
t[2]*f*(((-e + Sqrt[e^2 - 4*d*f])*ArcTanh[(4*a*f - 2*c*(e + Sqrt[e^2 - 4*d*
f])*x - b*(e + Sqrt[e^2 - 4*d*f] - 2*f*x))/(2*Sqrt[2]*Sqrt[c*(e^2 - 2*d*f +
 e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))]*Sqrt[a + x*(
b + c*x)])])/Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e +
 Sqrt[e^2 - 4*d*f]))] + ((e + Sqrt[e^2 - 4*d*f])*ArcTanh[(4*a*f + 2*c*(-e +
 Sqrt[e^2 - 4*d*f])*x + b*(-e + Sqrt[e^2 - 4*d*f] + 2*f*x))/(2*Sqrt[2]*Sqrt
[f*(-(b*e) + 2*a*f + b*Sqrt[e^2 - 4*d*f]) + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4
*d*f])]*Sqrt[a + x*(b + c*x)])])/Sqrt[f*(-(b*e) + 2*a*f + b*Sqrt[e^2 - 4*d*
f]) + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]))/Sqrt[e^2 - 4*d*f])/(2*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.02, size = 859, normalized size = 1.90

$$-\frac{2\sqrt{2}\, f \ln\left(\frac{\frac{\left(bf-ce-\sqrt{-4df+e^2}\, c\right)\left(x+\frac{e+\sqrt{-4df+e^2}}{2f}\right)}{f}+\frac{2af^2-bef-2cdf+ce^2-\sqrt{-4df+e^2}\, bf+\sqrt{-4df+e^2}\, ce}{f^2}+\frac{\sqrt{2}\sqrt{\frac{2af^2-bef-2cdf+ce^2-\sqrt{-4df+e^2}\, bf+\sqrt{-4df+e^2}}{f^2}}}{}}{x+\frac{e+\sqrt{-4df+e^2}}{2f}}\right)}{\left(e+\sqrt{-4df+e^2}\right)\sqrt{-4df+e^2}\,\sqrt{\frac{2af^2-bef-2cdf+c}{}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x)
```

```
[Out] -2*f/(e+(-4*d*f+e^2)^(1/2))/(-4*d*f+e^2)^(1/2)*2^(1/2)/((2*a*f^2-b*e*f-2*c*
d*f+c*e^2-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*ln(((b*
f-c*e-(-4*d*f+e^2)^(1/2)*c)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)/f+(2*a*f^2-b*e
*f-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e)/f^2+1/2*2^(
1/2)*((2*a*f^2-b*e*f-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2
)*c*e)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c+4*(b*f-c*e-(-4*d*
f+e^2)^(1/2)*c)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)/f+2*(2*a*f^2-b*e*f-2*c*d*f
+c*e^2-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2))/(x+1/2*(e
+(-4*d*f+e^2)^(1/2))/f))-2*f/(-e+(-4*d*f+e^2)^(1/2))/(-4*d*f+e^2)^(1/2)*2^(
1/2)/((2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2
)*c*e)/f^2)^(1/2)*ln(((b*f-c*e+(-4*d*f+e^2)^(1/2)*c)*(x-1/2*(-e+(-4*d*f+e^2
)^(1/2))/f)/f+(2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e
^2)^(1/2)*c*e)/f^2+1/2*2^(1/2)*((2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^(
1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*(4*(x-1/2*(-e+(-4*d*f+e^2)^(1/2
))/f)^2*c+4*(b*f-c*e+(-4*d*f+e^2)^(1/2)*c)*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f
)/f+2*(2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2
)*c*e)/f^2)^(1/2))/(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f))+4*f/(-e+(-4*d*f+e^2)^
(1/2))/(e+(-4*d*f+e^2)^(1/2))/a^(1/2)*ln((b*x+2*a+2*(c*x^2+b*x+a)^(1/2)*a^(
1/2))/x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^2+bx+a}\left(fx^2+ex+d\right)x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="maxima")
[Out] integrate(1/(sqrt(c*x^2 + b*x + a)*(f*x^2 + e*x + d)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x\,\sqrt{c\,x^2+b\,x+a}\,\left(f\,x^2+e\,x+d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2)),x)
[Out] int(1/(x*(a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{a+bx+cx^2}\,\left(d+ex+fx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x**2+b*x+a)**(1/2)/(f*x**2+e*x+d),x)
[Out] Integral(1/(x*sqrt(a + b*x + c*x**2)*(d + e*x + f*x**2)), x)
```

3.119 $\int \frac{1}{x^2\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)}\,dx$

Optimal. Leaf size=543

$$\frac{b\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2a^{3/2}d}-\frac{f\left(e\sqrt{e^2-4df}-2df+e^2\right)\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)-b\left(e-\sqrt{e^2-4df}\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}\,(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}\,d^2\sqrt{e^2-4df}\sqrt{f\left(2af-b\left(e-\sqrt{e^2-4df}\right)\right)+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}+\frac{f\cdots}{\cdots}$$

```
[Out] 1/2*b*arctanh(1/2*(b*x+2*a)/a^(1/2)/(c*x^2+b*x+a)^(1/2))/a^(3/2)/d+e*arctan
h(1/2*(b*x+2*a)/a^(1/2)/(c*x^2+b*x+a)^(1/2))/d^2/a^(1/2)-(c*x^2+b*x+a)^(1/2
)/a/d/x+1/2*f*arctanh(1/4*(4*a*f-b*(e+(-4*d*f+e^2)^(1/2))+2*x*(b*f-c*(e+(-4
*d*f+e^2)^(1/2))))*2^(1/2)/(c*x^2+b*x+a)^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2
+(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2))*(-(-4*d*f+e^2)^(1/2)*e-2*d*f+e^2)/d^
2*2^(1/2)/(-4*d*f+e^2)^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2+(-b*f+c*e)*(-4*d*
f+e^2)^(1/2))^(1/2)-1/2*f*arctanh(1/4*(4*a*f+2*x*(b*f-c*(e-(-4*d*f+e^2)^(1/
2)))-b*(e-(-4*d*f+e^2)^(1/2)))*2^(1/2)/(c*x^2+b*x+a)^(1/2)/(c*e^2-2*c*d*f-b
*e*f+2*a*f^2-(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2))*((-4*d*f+e^2)^(1/2)*e-2*
d*f+e^2)/d^2*2^(1/2)/(-4*d*f+e^2)^(1/2)/(f*(2*a*f-b*(e-(-4*d*f+e^2)^(1/2)))
+(-(-4*d*f+e^2)^(1/2)*e-2*d*f+e^2)*c)^(1/2)
```

Rubi [A] time = 4.59, antiderivative size = 543, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6728, 730, 724, 206, 1032}

$$\frac{b\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2a^{3/2}d}-\frac{f\left(e\sqrt{e^2-4df}-2df+e^2\right)\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)-b\left(e-\sqrt{e^2-4df}\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}\,(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}\,d^2\sqrt{e^2-4df}\sqrt{f\left(2af-b\left(e-\sqrt{e^2-4df}\right)\right)+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}+\frac{f\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)),x]
```

```
[Out] -(Sqrt[a + b*x + c*x^2]/(a*d*x)) + (b*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a
 + b*x + c*x^2])])/(2*a^(3/2)*d) + (e*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a
 + b*x + c*x^2])])/(Sqrt[a]*d^2) - (f*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])*A
rcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e - Sqrt[e^2 - 4*d*
f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqr
t[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*d^2*Sqrt[e^2 - 4*d*f]*Sqr
t[c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e - Sqrt[e^2 - 4*d*
f]))]) + (f*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])*ArcTanh[(4*a*f - b*(e + Sqr
t[e^2 - 4*d*f]) + 2*(b*f - c*(e + Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*
e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b
*x + c*x^2])])/(Sqrt[2]*d^2*Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c*d*f - b*e*f
+ 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
```

```
d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 730

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 - b*d*e + a*e^2)), x] + Dist[(2*c*d - b*e)/(2*(c*d^2 - b*d*e + a*e^2)),
 Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e,
 m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2
*c*d - b*e, 0] && EqQ[m + 2*p + 3, 0]
```

Rule 1032

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{x^2\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)}\,dx = \int \left(\frac{1}{dx^2\sqrt{a+bx+cx^2}} - \frac{e}{d^2x\sqrt{a+bx+cx^2}} + \frac{e^2-df+ef}{d^2\sqrt{a+bx+cx^2}\left(d+\right.}\right.$$

$$= \frac{\int \frac{e^2-df+efx}{\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)}\,dx}{d^2} + \frac{\int \frac{1}{x^2\sqrt{a+bx+cx^2}}\,dx}{d} - \frac{e\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx}{d^2}$$

$$= -\frac{\sqrt{a+bx+cx^2}}{adx} - \frac{b\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx}{2ad} + \frac{(2e)\,\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{}{\sqrt{a}}\right.}{d^2}$$

$$= -\frac{\sqrt{a+bx+cx^2}}{adx} + \frac{e\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a}\,d^2} + \frac{b\,\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx,\right.}{ad}$$

$$= -\frac{\sqrt{a+bx+cx^2}}{adx} + \frac{b\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2a^{3/2}d} + \frac{e\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+}}\right.}{\sqrt{a}\,d^2}$$

Mathematica [A] time = 1.41, size = 533, normalized size = 0.98

$$-\frac{-\frac{bd\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+x(b+cx)}}\right)}{a^{3/2}} + \frac{\sqrt{2}f\left(e\sqrt{e^2-4df}+2df-e^2\right)\tanh^{-1}\left(\frac{4af-b\left(\sqrt{e^2-4df}+e-2fx\right)-2cx\left(\sqrt{e^2-4df}+e\right)}{2\sqrt{2}\sqrt{a+x(b+cx)}\sqrt{f\left(2af-b\left(\sqrt{e^2-4df}+e\right)\right)+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{e^2-4df}\sqrt{f\left(2af-b\left(\sqrt{e^2-4df}+e\right)\right)+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}} + \frac{\sqrt{2}f\left(e^2\ldots\right.}{\sqrt{e^2\ldots}}}{2d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)),x]
```

```
[Out] -1/2*((2*d*Sqrt[a + x*(b + c*x)])/(a*x) - (b*d*ArcTanh[(2*a + b*x)/(2*Sqrt[
a]*Sqrt[a + x*(b + c*x)])])/a^(3/2) - (2*e*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*S
qrt[a + x*(b + c*x)])])/Sqrt[a] + (Sqrt[2]*f*(-e^2 + 2*d*f + e*Sqrt[e^2 - 4
*d*f])*ArcTanh[(4*a*f - 2*c*(e + Sqrt[e^2 - 4*d*f])*x - b*(e + Sqrt[e^2 - 4
*d*f] - 2*f*x))/(2*Sqrt[2]*Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(
2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))]*Sqrt[a + x*(b + c*x)])])/(Sqrt[e^2 - 4*
d*f]*Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^
2 - 4*d*f]))]) + (Sqrt[2]*f*(e + (e^2 - 2*d*f)/Sqrt[e^2 - 4*d*f])*ArcTanh[(
4*a*f + 2*c*(-e + Sqrt[e^2 - 4*d*f])*x + b*(-e + Sqrt[e^2 - 4*d*f] + 2*f*x)
)/(2*Sqrt[2]*Sqrt[c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f + b*(-e
+ Sqrt[e^2 - 4*d*f]))]*Sqrt[a + x*(b + c*x)])])/Sqrt[c*(e^2 - 2*d*f - e*Sqr
t[e^2 - 4*d*f]) + f*(2*a*f + b*(-e + Sqrt[e^2 - 4*d*f]))])/d^2
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.02, size = 983, normalized size = 1.81

$$\frac{16 e f^{2} \ln\left(\frac{b x+2 a+2 \sqrt{c x^{2}+b x+a}\, \sqrt{a}}{x}\right)}{\left(-e+\sqrt{-4 d f+e^{2}}\right)^{2} \left(e+\sqrt{-4 d f+e^{2}}\right)^{2} \sqrt{a}}+\frac{4 \sqrt{2}\, f^{2} \ln\left(\frac{\frac{\left(b f-c e-\sqrt{-4 d f+e^{2}}\, c\right)\left(x+\frac{e+\sqrt{-4 d f+e^{2}}}{2 f}\right)}{f}+\frac{2 a f^{2}-b e f-2 c d f+c e^{2}-\sqrt{-4 d f+e^{2}}\, b f+}{f^{2}}}{}\right)}{\left(e+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x)
```

```
[Out] 4*f^2/(e+(-4*d*f+e^2)^(1/2))^2/(-4*d*f+e^2)^(1/2)*2^(1/2)/((2*a*f^2-b*e*f-2
*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*ln((
(b*f-c*e-(-4*d*f+e^2)^(1/2)*c)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)/f+(2*a*f^2-
b*e*f-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e)/f^2+1/2*
2^(1/2)*((2*a*f^2-b*e*f-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(
1/2)*c*e)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c+4*(b*f-c*e-(-4
*d*f+e^2)^(1/2)*c)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)/f+2*(2*a*f^2-b*e*f-2*c*
d*f+c*e^2-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2))/(x+1/2
*(e+(-4*d*f+e^2)^(1/2))/f))+4*f/(-e+(-4*d*f+e^2)^(1/2))/(e+(-4*d*f+e^2)^(1/
2))/a/x*(c*x^2+b*x+a)^(1/2)-2*f/(-e+(-4*d*f+e^2)^(1/2))/(e+(-4*d*f+e^2)^(1/
```

```
2))*b/a^(3/2)*ln((b*x+2*a+2*(c*x^2+b*x+a)^(1/2)*a^(1/2))/x)-4*f^2/(-e+(-4*d
*f+e^2)^(1/2))^2/(-4*d*f+e^2)^(1/2)*2^(1/2)/((2*a*f^2-b*e*f-2*c*d*f+c*e^2+(
-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*ln(((b*f-c*e+(-4*d
*f+e^2)^(1/2)*c)*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)/f+(2*a*f^2-b*e*f-2*c*d*f
+c*e^2+(-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e)/f^2+1/2*2^(1/2)*((2*a
*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e)/f^2
)^(1/2)*(4*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c+4*(b*f-c*e+(-4*d*f+e^2)^(1
/2)*c)*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)/f+2*(2*a*f^2-b*e*f-2*c*d*f+c*e^2+(
-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2))/(x-1/2*(-e+(-4*d*
f+e^2)^(1/2))/f))+16*f^2*e/(-e+(-4*d*f+e^2)^(1/2))^2/(e+(-4*d*f+e^2)^(1/2))
^2/a^(1/2)*ln((b*x+2*a+2*(c*x^2+b*x+a)^(1/2)*a^(1/2))/x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^2+bx+a}\left(fx^2+ex+d\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="maxima")
[Out] integrate(1/(sqrt(c*x^2 + b*x + a)*(f*x^2 + e*x + d)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^2\,\sqrt{c\,x^2+b\,x+a}\,\left(f\,x^2+e\,x+d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2)),x)
[Out] int(1/(x^2*(a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(c*x**2+b*x+a)**(1/2)/(f*x**2+e*x+d),x)
[Out] Integral(1/(x**2*sqrt(a + b*x + c*x**2)*(d + e*x + f*x**2)), x)
```

3.120 $\int \frac{1}{x^3\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)}\,dx$

Optimal. Leaf size=679

$$-\frac{\left(3b^2-4ac\right)\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{8a^{5/2}d}-\frac{be\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2a^{3/2}d^2}+\frac{3b\sqrt{a+bx+cx^2}}{4a^2dx}-\frac{\left(e^2-df\right)\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a}\,d^3}$$

```
[Out] -1/8*(-4*a*c+3*b^2)*arctanh(1/2*(b*x+2*a)/a^(1/2)/(c*x^2+b*x+a)^(1/2))/a^(5
/2)/d-1/2*b*e*arctanh(1/2*(b*x+2*a)/a^(1/2)/(c*x^2+b*x+a)^(1/2))/a^(3/2)/d^
2-(-d*f+e^2)*arctanh(1/2*(b*x+2*a)/a^(1/2)/(c*x^2+b*x+a)^(1/2))/d^3/a^(1/2)
-1/2*(c*x^2+b*x+a)^(1/2)/a/d/x^2+3/4*b*(c*x^2+b*x+a)^(1/2)/a^2/d/x+e*(c*x^2
+b*x+a)^(1/2)/a/d^2/x+1/2*f*arctanh(1/4*(4*a*f+2*x*(b*f-c*(e-(-4*d*f+e^2)^(
1/2)))-b*(e-(-4*d*f+e^2)^(1/2)))*2^(1/2)/(c*x^2+b*x+a)^(1/2)/(c*e^2-2*c*d*f
-b*e*f+2*a*f^2-(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2))*(2*e^3-4*d*e*f-(-d*f+e
^2)*(e-(-4*d*f+e^2)^(1/2)))/d^3*2^(1/2)/(-4*d*f+e^2)^(1/2)/(c*e^2-2*c*d*f-b
*e*f+2*a*f^2-(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2)-1/2*f*arctanh(1/4*(4*a*f-
b*(e+(-4*d*f+e^2)^(1/2))+2*x*(b*f-c*(e+(-4*d*f+e^2)^(1/2))))*2^(1/2)/(c*x^2
+b*x+a)^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2+(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(
1/2))*(2*e^3-4*d*e*f-(-d*f+e^2)*(e+(-4*d*f+e^2)^(1/2)))/d^3*2^(1/2)/(-4*d*f
+e^2)^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2+(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/
2)
```

Rubi [A] time = 11.23, antiderivative size = 679, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 7, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.233$, Rules used = {6728, 744, 806, 724, 206, 730, 1032}

$$-\frac{\left(3b^2-4ac\right)\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{8a^{5/2}d}-\frac{be\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2a^{3/2}d^2}+\frac{3b\sqrt{a+bx+cx^2}}{4a^2dx}+\frac{f\left(-\left(e^2-df\right)\left(e-\sqrt{e^2-4\right.\right.}{\sqrt{2}\,d^3\sqrt{e^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)),x]
```

```
[Out] -Sqrt[a + b*x + c*x^2]/(2*a*d*x^2) + (3*b*Sqrt[a + b*x + c*x^2])/(4*a^2*d*x
) + (e*Sqrt[a + b*x + c*x^2])/(a*d^2*x) - ((3*b^2 - 4*a*c)*ArcTanh[(2*a + b
*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/(8*a^(5/2)*d) - (b*e*ArcTanh[(2*a +
 b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/(2*a^(3/2)*d^2) - ((e^2 - d*f)*Ar
cTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/(Sqrt[a]*d^3) + (f*(2
*e^3 - 4*d*e*f - (e^2 - d*f)*(e - Sqrt[e^2 - 4*d*f]))*ArcTanh[(4*a*f - b*(e
 - Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*S
qrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt
[a + b*x + c*x^2])])/(Sqrt[2]*d^3*Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c*d*f -
b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]) - (f*(2*e^3 - 4*d*e*f - (
e^2 - d*f)*(e + Sqrt[e^2 - 4*d*f]))*ArcTanh[(4*a*f - b*(e + Sqrt[e^2 - 4*d*
f]) + 2*(b*f - c*(e + Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*
f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])
])/(Sqrt[2]*d^3*Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 +
(c*e - b*f)*Sqrt[e^2 - 4*d*f]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 730

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 - b*d*e + a*e^2)), x] + Dist[(2*c*d - b*e)/(2*(c*d^2 - b*d*e + a*e^2)),
 Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e,
 m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2
*c*d - b*e, 0] && EqQ[m + 2*p + 3, 0]
```

Rule 744

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(
d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x,
x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && Ne
Q[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumS
implerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 1032

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{x^3\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)}dx = \int\left(\frac{1}{dx^3\sqrt{a+bx+cx^2}} - \frac{e}{d^2x^2\sqrt{a+bx+cx^2}} + \frac{e^2-df}{d^3x\sqrt{a+bx+cx^2}} + \ldots\right.$$

$$= \frac{\int \frac{-e(e^2-2df)-f(e^2-df)x}{\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)}dx}{d^3} + \frac{\int \frac{1}{x^3\sqrt{a+bx+cx^2}}dx}{d} - \frac{e\int \frac{1}{x^2\sqrt{a+bx+cx^2}}dx}{d^2} + \ldots$$

$$= -\frac{\sqrt{a+bx+cx^2}}{2adx^2} + \frac{e\sqrt{a+bx+cx^2}}{ad^2x} - \frac{\int \frac{\frac{3b}{2}+cx}{x^2\sqrt{a+bx+cx^2}}dx}{2ad} + \frac{(be)\int \frac{1}{x\sqrt{a+bx\ldots}}}{2ad^2}\ldots$$

$$= -\frac{\sqrt{a+bx+cx^2}}{2adx^2} + \frac{3b\sqrt{a+bx+cx^2}}{4a^2dx} + \frac{e\sqrt{a+bx+cx^2}}{ad^2x} - \frac{\left(e^2-df\right)\tan\ldots}{\ldots}$$

$$= -\frac{\sqrt{a+bx+cx^2}}{2adx^2} + \frac{3b\sqrt{a+bx+cx^2}}{4a^2dx} + \frac{e\sqrt{a+bx+cx^2}}{ad^2x} - \frac{be\tanh^{-1}\left(\ldots\right)}{2a^{3}\ldots}$$

$$= -\frac{\sqrt{a+bx+cx^2}}{2adx^2} + \frac{3b\sqrt{a+bx+cx^2}}{4a^2dx} + \frac{e\sqrt{a+bx+cx^2}}{ad^2x} - \frac{\left(3b^2-4ac\right)\ldots}{\ldots}$$

<u>**Mathematica**</u> **[A]** time = 2.11, size = 669, normalized size = 0.99

$$\frac{d^2\left(\left(4acx-3b^2x\right)\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+x(b+cx)}}\right)+6\sqrt{a}b\sqrt{a+x(b+cx)}\right)}{a^{5/2}x} - \frac{4bde\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+x(b+cx)}}\right)}{a^{3/2}} - \frac{4d^2\sqrt{a+x(b+cx)}}{ax^2} - \frac{8\left(e^2-df\right)\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+x(\ldots}}\right.}{\sqrt{a}}\ldots$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)),x]
```

```
[Out] ((-4*d^2*Sqrt[a + x*(b + c*x)])/(a*x^2) + (8*d*e*Sqrt[a + x*(b + c*x)])/(a*
x) - (4*b*d*e*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + x*(b + c*x)])])/a^(3/
2) - (8*(e^2 - d*f)*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + x*(b + c*x)])])
/Sqrt[a] + (d^2*(6*Sqrt[a]*b*Sqrt[a + x*(b + c*x)] + (-3*b^2*x + 4*a*c*x)*A
rcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + x*(b + c*x)])]))/(a^(5/2)*x) - (4*Sq
rt[2]*f*(e^3 - 3*d*e*f - e^2*Sqrt[e^2 - 4*d*f] + d*f*Sqrt[e^2 - 4*d*f])*Arc
Tanh[(4*a*f - 2*c*(e + Sqrt[e^2 - 4*d*f])*x - b*(e + Sqrt[e^2 - 4*d*f] - 2*
f*x))/(2*Sqrt[2]*Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*
(e + Sqrt[e^2 - 4*d*f]))]*Sqrt[a + x*(b + c*x)])])/(Sqrt[e^2 - 4*d*f]*Sqrt[
c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]
))]) + (4*Sqrt[2]*f*(e^2 - d*f + (e*(e^2 - 3*d*f))/Sqrt[e^2 - 4*d*f])*ArcTa
nh[(4*a*f + 2*c*(-e + Sqrt[e^2 - 4*d*f])*x + b*(-e + Sqrt[e^2 - 4*d*f] + 2*
f*x))/(2*Sqrt[2]*Sqrt[c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f + b*
(-e + Sqrt[e^2 - 4*d*f]))]*Sqrt[a + x*(b + c*x)])])/Sqrt[c*(e^2 - 2*d*f - e
*Sqrt[e^2 - 4*d*f]) + f*(2*a*f + b*(-e + Sqrt[e^2 - 4*d*f]))])/(8*d^3)
```

<u>**fricas**</u> **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="fricas")
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="giac")
[Out] Timed out
```

maple [B] time = 0.02, size = 1296, normalized size = 1.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x)
[Out] -8*f^3/(e+(-4*d*f+e^2)^(1/2))^3/(-4*d*f+e^2)^(1/2)*2^(1/2)/((2*a*f^2-b*e*f-
2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*ln(
((b*f-c*e-(-4*d*f+e^2)^(1/2)*c)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)/f+(2*a*f^2
-b*e*f-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e)/f^2+1/2
*2^(1/2)*((2*a*f^2-b*e*f-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^
(1/2)*c*e)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c+4*(b*f-c*e-(-
4*d*f+e^2)^(1/2)*c)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)/f+2*(2*a*f^2-b*e*f-2*c
*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2))/(x+1/
2*(e+(-4*d*f+e^2)^(1/2))/f))+16*f^2*e/(-e+(-4*d*f+e^2)^(1/2))^2/(e+(-4*d*f+
e^2)^(1/2))^2/a/x*(c*x^2+b*x+a)^(1/2)-8*f^2*e/(-e+(-4*d*f+e^2)^(1/2))^2/(e+
(-4*d*f+e^2)^(1/2))^2*b/a^(3/2)*ln((b*x+2*a+2*(c*x^2+b*x+a)^(1/2)*a^(1/2))/
x)+2*f/(-e+(-4*d*f+e^2)^(1/2))/(e+(-4*d*f+e^2)^(1/2))/a/x^2*(c*x^2+b*x+a)^(
1/2)-3*f/(-e+(-4*d*f+e^2)^(1/2))/(e+(-4*d*f+e^2)^(1/2))*b/a^2/x*(c*x^2+b*x+
a)^(1/2)+3/2*f/(-e+(-4*d*f+e^2)^(1/2))/(e+(-4*d*f+e^2)^(1/2))*b^2/a^(5/2)*l
n((b*x+2*a+2*(c*x^2+b*x+a)^(1/2)*a^(1/2))/x)-2*f/(-e+(-4*d*f+e^2)^(1/2))/(e
+(-4*d*f+e^2)^(1/2))*c/a^(3/2)*ln((b*x+2*a+2*(c*x^2+b*x+a)^(1/2)*a^(1/2))/x
)-8*f^3/(-e+(-4*d*f+e^2)^(1/2))^3/(-4*d*f+e^2)^(1/2)*2^(1/2)/((2*a*f^2-b*e*
f-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*l
n(((b*f-c*e+(-4*d*f+e^2)^(1/2)*c)*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)/f+(2*a*
f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e)/f^2+
1/2*2^(1/2)*((2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^
2)^(1/2)*c*e)/f^2)^(1/2)*(4*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c+4*(b*f-c*
e+(-4*d*f+e^2)^(1/2)*c)*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)/f+2*(2*a*f^2-b*e*
f-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2))/
(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f))-64*f^4/(-e+(-4*d*f+e^2)^(1/2))^3/(e+(-4*
d*f+e^2)^(1/2))^3/a^(1/2)*ln((b*x+2*a+2*(c*x^2+b*x+a)^(1/2)*a^(1/2))/x)*d+6
4*f^3/(-e+(-4*d*f+e^2)^(1/2))^3/(e+(-4*d*f+e^2)^(1/2))^3/a^(1/2)*ln((b*x+2*
a+2*(c*x^2+b*x+a)^(1/2)*a^(1/2))/x)*e^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^2 + bx + a}\left(fx^2 + ex + d\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="maxima")
[Out] integrate(1/(sqrt(c*x^2 + b*x + a)*(f*x^2 + e*x + d)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^3 \sqrt{c\,x^2+b\,x+a}\,\left(f\,x^2+e\,x+d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2)),x)
[Out] int(1/(x^3*(a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(c*x**2+b*x+a)**(1/2)/(f*x**2+e*x+d),x)
[Out] Integral(1/(x**3*sqrt(a + b*x + c*x**2)*(d + e*x + f*x**2)), x)
```

3.121 $\int \frac{x^3}{\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}\,dx$

Optimal. Leaf size=779

$$\frac{2\left(cx\left(\left(e^2-df\right)(abf-2ace+bcd)-de\left(-c(2af+be)+b^2f+2c^2d\right)\right)-\left(adf-ae^2+bde\right)\left(-c(2af+be)+b^2\right.}{f^2\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left((cd-af)^2-(bd-ae)(ce-bf)\right)}$$

```
[Out] 2*(b*x+2*a)/(-4*a*c+b^2)/f/(c*x^2+b*x+a)^(1/2)+2*e*(2*c*x+b)/(-4*a*c+b^2)/f
^2/(c*x^2+b*x+a)^(1/2)+2*(c*d*e*(a*b*f-2*a*c*e+b*c*d)-(a*d*f-a*e^2+b*d*e)*(
2*c^2*d+b^2*f-c*(2*a*f+b*e))+c*((a*b*f-2*a*c*e+b*c*d)*(-d*f+e^2)-d*e*(2*c^2
*d+b^2*f-c*(2*a*f+b*e)))*x)/(-4*a*c+b^2)/f^2/((-a*f+c*d)^2-(-a*e+b*d)*(-b*f
+c*e))/(c*x^2+b*x+a)^(1/2)+1/2*arctanh(1/4*(4*a*f+2*x*(b*f-c*(e-(-4*d*f+e^2
)^(1/2)))-b*(e-(-4*d*f+e^2)^(1/2)))*2^(1/2)/(c*x^2+b*x+a)^(1/2)/(c*e^2-2*c*
d*f-b*e*f+2*a*f^2-(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2))*(2*d*(-a*e+b*d)*f+(
c*d^2-b*d*e+a*(-d*f+e^2))*(e-(-4*d*f+e^2)^(1/2)))/((-a*f+c*d)^2-(-a*e+b*d)*
(-b*f+c*e))*2^(1/2)/(-4*d*f+e^2)^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2-(-b*f+c
*e)*(-4*d*f+e^2)^(1/2))^(1/2)-1/2*arctanh(1/4*(4*a*f-b*(e+(-4*d*f+e^2)^(1/2
))+2*x*(b*f-c*(e+(-4*d*f+e^2)^(1/2))))*2^(1/2)/(c*x^2+b*x+a)^(1/2)/(c*e^2-2
*c*d*f-b*e*f+2*a*f^2+(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2))*(2*d*(-a*e+b*d)*
f+(c*d^2-b*d*e+a*(-d*f+e^2))*(e+(-4*d*f+e^2)^(1/2)))/((-a*f+c*d)^2-(-a*e+b*
d)*(-b*f+c*e))*2^(1/2)/(-4*d*f+e^2)^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2+(-b*
f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2)
```

Rubi [A] time = 14.17, antiderivative size = 779, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.233, Rules used = {6728, 613, 636, 1016, 1032, 724, 206}

$$\frac{2\left(cx\left(\left(e^2-df\right)(abf-2ace+bcd)-de\left(-c(2af+be)+b^2f+2c^2d\right)\right)-\left(adf-ae^2+bde\right)\left(-c(2af+be)+b^2\right.}{f^2\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left((cd-af)^2-(bd-ae)(ce-bf)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]
```

```
[Out] (2*(2*a + b*x))/((b^2 - 4*a*c)*f*Sqrt[a + b*x + c*x^2]) + (2*e*(b + 2*c*x))
/((b^2 - 4*a*c)*f^2*Sqrt[a + b*x + c*x^2]) + (2*(c*d*e*(b*c*d - 2*a*c*e + a
*b*f) - (b*d*e - a*e^2 + a*d*f)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*((b
*c*d - 2*a*c*e + a*b*f)*(e^2 - d*f) - d*e*(2*c^2*d + b^2*f - c*(b*e + 2*a*f
)))*x))/((b^2 - 4*a*c)*f^2*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*Sqrt[a
 + b*x + c*x^2]) + ((2*d*(b*d - a*e)*f + (e - Sqrt[e^2 - 4*d*f])*(c*d^2 - b
*d*e + a*(e^2 - d*f)))*ArcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) + 2*(b*f
- c*(e - Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2
*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*S
qrt[e^2 - 4*d*f]*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*Sqrt[c*e^2 - 2*c
*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]) - ((2*d*(b*d - a*e
)*f + (e + Sqrt[e^2 - 4*d*f])*(c*d^2 - b*d*e + a*(e^2 - d*f)))*ArcTanh[(4*a
*f - b*(e + Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e + Sqrt[e^2 - 4*d*f]))*x)/(2*
Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d
*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*Sqrt[e^2 - 4*d*f]*((c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f))*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f
)*Sqrt[e^2 - 4*d*f]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 613

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[(-2*(b +
2*c*x))/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rule 636

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[(-2*(b*d - 2*a*e + (2*c*d - b*e)*x))/((b^2 - 4*a*c)*Sqrt[a + b*x
 + c*x^2]), x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b
^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1016

```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e
_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((a + b*x + c*x^2)^(p + 1)
*(d + e*x + f*x^2)^(q + 1)*(g*c*(2*a*c*e - b*(c*d + a*f)) + (g*b - a*h)*(2*
c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(g*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))
 - h*(b*c*d - 2*a*c*e + a*b*f))*x))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d -
a*e)*(c*e - b*f))*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*
d - a*e)*(c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*
x^2)^q*Simp[(b*h - 2*g*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)
 + (b^2*(g*f) - b*(h*c*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) - a*(-(h*c*e
))))*(a*f*(p + 1) - c*d*(p + 2)) - e*((g*c)*(2*a*c*e - b*(c*d + a*f)) + (g*
b - a*h)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (2*f*((g*c)*(2*
a*c*e - b*(c*d + a*f)) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(
p + q + 2) - (b^2*g*f - b*(h*c*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) - a*
(-(h*c*e))))*(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(g*f) - b*(h*c
*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) + a*h*c*e))*(2*p + 2*q + 5)*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, q}, x] && NeQ[b^2 - 4*a*c, 0] &
& NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (b*d - a*e)*(c*e
 - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1])
```

Rule 1032

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{x^3}{\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}dx = \int\left(-\frac{e}{f^2\left(a+bx+cx^2\right)^{3/2}}+\frac{x}{f\left(a+bx+cx^2\right)^{3/2}}+\frac{de+\left(e^2\right.}{f^2\left(a+bx+cx^2\right)^{3}}\right.$$

$$=\frac{\int\frac{de+\left(e^2-df\right)x}{\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}dx}{f^2}-\frac{e\int\frac{1}{\left(a+bx+cx^2\right)^{3/2}}dx}{f^2}+\frac{\int\frac{x}{\left(a+bx+cx^2\right)^{3/2}}dx}{f}$$

$$=\frac{2(2a+bx)}{\left(b^2-4ac\right)f\sqrt{a+bx+cx^2}}+\frac{2e(b+2cx)}{\left(b^2-4ac\right)f^2\sqrt{a+bx+cx^2}}+\frac{2\left(cde(\right.}{}$$

$$=\frac{2(2a+bx)}{\left(b^2-4ac\right)f\sqrt{a+bx+cx^2}}+\frac{2e(b+2cx)}{\left(b^2-4ac\right)f^2\sqrt{a+bx+cx^2}}+\frac{2\left(cde(\right.}{}$$

$$=\frac{2(2a+bx)}{\left(b^2-4ac\right)f\sqrt{a+bx+cx^2}}+\frac{2e(b+2cx)}{\left(b^2-4ac\right)f^2\sqrt{a+bx+cx^2}}+\frac{2\left(cde(\right.}{}$$

$$=\frac{2(2a+bx)}{\left(b^2-4ac\right)f\sqrt{a+bx+cx^2}}+\frac{2e(b+2cx)}{\left(b^2-4ac\right)f^2\sqrt{a+bx+cx^2}}+\frac{2\left(cde(\right.}{}$$

Mathematica [A] time = 2.62, size = 1066, normalized size = 1.37

$$\frac{4\left(2fa^3+(-2cd-be+2cex+bfx)a^2+b(b(d-ex)-3cdx)a+b^3dx\right)}{\left(b^2-4ac\right)\sqrt{a+x(b+cx)}}+\frac{\sqrt{2}\left(c\left(\sqrt{e^2-4df}-e\right)d^2+b\left(e^2-\sqrt{e^2-4df}\,e-2df\right)d+a\left(-e^3+\sqrt{e^2-4df}\,e^2+3dfe-df\sqrt{e^2-}\right.\right.}{\sqrt{e^2-4df}\sqrt{c\left(e^2-\sqrt{e^2-4df}\,e-2df\right)+f\left(2af+b\left(\sqrt{e^2-4df}\right.\right.}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]
```

```
[Out] ((4*(2*a^3*f + b^3*d*x + a^2*(-2*c*d - b*e + 2*c*e*x + b*f*x) + a*b*(-3*c*d
*x + b*(d - e*x))))/((b^2 - 4*a*c)*Sqrt[a + x*(b + c*x)]) + (Sqrt[2]*(c*d^2
*(-e + Sqrt[e^2 - 4*d*f]) + b*d*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + a*(-e
^3 + 3*d*e*f + e^2*Sqrt[e^2 - 4*d*f] - d*f*Sqrt[e^2 - 4*d*f]))*Log[-e + Sqr
t[e^2 - 4*d*f] - 2*f*x])/(Sqrt[e^2 - 4*d*f]*Sqrt[c*(e^2 - 2*d*f - e*Sqrt[e^
2 - 4*d*f]) + f*(2*a*f + b*(-e + Sqrt[e^2 - 4*d*f]))]) + (Sqrt[2]*(c*d^2*(e
 + Sqrt[e^2 - 4*d*f]) - b*d*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + a*(e^3 -
3*d*e*f + e^2*Sqrt[e^2 - 4*d*f] - d*f*Sqrt[e^2 - 4*d*f]))*Log[e + Sqrt[e^2
- 4*d*f] + 2*f*x])/(Sqrt[e^2 - 4*d*f]*Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*
d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))]) - (Sqrt[2]*(c*d^2*(e + Sqrt
[e^2 - 4*d*f]) - b*d*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + a*(e^3 - 3*d*e*f
 + e^2*Sqrt[e^2 - 4*d*f] - d*f*Sqrt[e^2 - 4*d*f]))*Log[-4*a*f + 2*c*e*x + 2
*c*Sqrt[e^2 - 4*d*f]*x + b*(e + Sqrt[e^2 - 4*d*f] - 2*f*x) - 2*Sqrt[2]*Sqrt
[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f
]))]*Sqrt[a + x*(b + c*x)]])/(Sqrt[e^2 - 4*d*f]*Sqrt[c*(e^2 - 2*d*f + e*Sqr
t[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))]) - (Sqrt[2]*(c*d^2
*(-e + Sqrt[e^2 - 4*d*f]) + b*d*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + a*(-e
^3 + 3*d*e*f + e^2*Sqrt[e^2 - 4*d*f] - d*f*Sqrt[e^2 - 4*d*f]))*Log[b*(-e +
Sqrt[e^2 - 4*d*f] + 2*f*x) + 2*(2*a*f - c*e*x + c*Sqrt[e^2 - 4*d*f]*x + Sqr
```

```
t[2]*Sqrt[f*(-(b*e) + 2*a*f + b*Sqrt[e^2 - 4*d*f]) + c*(e^2 - 2*d*f - e*Sqr
t[e^2 - 4*d*f])]*Sqrt[a + x*(b + c*x)])])/(Sqrt[e^2 - 4*d*f]*Sqrt[c*(e^2 -
2*d*f - e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f + b*(-e + Sqrt[e^2 - 4*d*f]))]))/(2
*(c^2*d^2 - b*c*d*e + f*(b^2*d - a*b*e + a^2*f) + a*c*(e^2 - 2*d*f)))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.04, size = 14651, normalized size = 18.81

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x)
```

```
[Out] result too large to display
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for mo
re details)Is 4*d*f-e^2 positive, negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3}{\left(c\,x^2+b\,x+a\right)^{3/2}\,\left(f\,x^2+e\,x+d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)),x)
```

```
[Out] int(x^3/((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(c*x**2+b*x+a)**(3/2)/(f*x**2+e*x+d),x)
```

```
[Out] Timed out
```

3.122 $\int \frac{x^2}{\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}\,dx$

Optimal. Leaf size=609

$$-\frac{2\left(cx\left(-abe-2a(cd-af)+b^2d\right)+a(abf-2ace+bcd)\right)}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left((cd-af)^2-(bd-ae)(ce-bf)\right)}-\frac{f\left(2d(cd-af)-\left(e-\sqrt{e^2-4df}\right)(bd-ae)\right)\tan}{\sqrt{2}\sqrt{e^2-4df}\left((cd-af)^2-(bd-ae)(ce-bf)\right)}$$

```
[Out] -2*(a*(a*b*f-2*a*c*e+b*c*d)+c*(b^2*d-a*b*e-2*a*(-a*f+c*d))*x)/(-4*a*c+b^2)/
((-a*f+c*d)^2-(-a*e+b*d)*(-b*f+c*e))/(c*x^2+b*x+a)^(1/2)-1/2*f*arctanh(1/4*
(4*a*f+2*x*(b*f-c*(e-(-4*d*f+e^2)^(1/2)))-b*(e-(-4*d*f+e^2)^(1/2)))*2^(1/2)
/(c*x^2+b*x+a)^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2-(-b*f+c*e)*(-4*d*f+e^2)^(
1/2))^(1/2))*(2*d*(-a*f+c*d)-(-a*e+b*d)*(e-(-4*d*f+e^2)^(1/2)))/((-a*f+c*d)
^2-(-a*e+b*d)*(-b*f+c*e))*2^(1/2)/(-4*d*f+e^2)^(1/2)/(c*e^2-2*c*d*f-b*e*f+2
*a*f^2-(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2)+1/2*f*arctanh(1/4*(4*a*f-b*(e+(
-4*d*f+e^2)^(1/2))+2*x*(b*f-c*(e+(-4*d*f+e^2)^(1/2))))*2^(1/2)/(c*x^2+b*x+a
)^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2+(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2))*
(2*d*(-a*f+c*d)-(-a*e+b*d)*(e+(-4*d*f+e^2)^(1/2)))/((-a*f+c*d)^2-(-a*e+b*d)
*(-b*f+c*e))*2^(1/2)/(-4*d*f+e^2)^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2+(-b*f+
c*e)*(-4*d*f+e^2)^(1/2))^(1/2)
```

Rubi [A] time = 5.84, antiderivative size = 609, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {1061, 1032, 724, 206}

$$-\frac{2\left(cx\left(-abe-2a(cd-af)+b^2d\right)+a(abf-2ace+bcd)\right)}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left((cd-af)^2-(bd-ae)(ce-bf)\right)}-\frac{f\left(2d(cd-af)-\left(e-\sqrt{e^2-4df}\right)(bd-ae)\right)\tan}{\sqrt{2}\sqrt{e^2-4df}\left((cd-af)^2-(bd-ae)(ce-bf)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]
```

```
[Out] (-2*(a*(b*c*d - 2*a*c*e + a*b*f) + c*(b^2*d - a*b*e - 2*a*(c*d - a*f))*x))/
((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*Sqrt[a + b*x + c*x
^2]) - (f*(2*d*(c*d - a*f) - (b*d - a*e)*(e - Sqrt[e^2 - 4*d*f]))*ArcTanh[(
4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*x)/
(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 -
4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*Sqrt[e^2 - 4*d*f]*((c*d - a*f)^2
- (b*d - a*e)*(c*e - b*f))*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e -
b*f)*Sqrt[e^2 - 4*d*f]]) + (f*(2*d*(c*d - a*f) - (b*d - a*e)*(e + Sqrt[e^2
- 4*d*f]))*ArcTanh[(4*a*f - b*(e + Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e + Sqr
t[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*
e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*Sqrt[e^2 - 4*
d*f]*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*Sqrt[c*e^2 - 2*c*d*f - b*e*f
 + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
```

```
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1032

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1061

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (C_.)*(x_)^2)*((d_) +
(e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((a + b*x + c*x^2)^(p +
1)*(d + e*x + f*x^2)^(q + 1)*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b)
*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*(2*c^2*d + b^2*f - c*(b*e + 2*a
*f)) + C*(b^2*d - a*b*e - 2*a*(c*d - a*f)))*x))/((b^2 - 4*a*c)*((c*d - a*f)
^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d -
a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(
d + e*x + f*x^2)^q*Simp[(-2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1) + (b^2*(C*d + A*f) - b*((Plus[A])*c*e + a*C*e) + 2*(A*c*(c*
d - a*f) - a*(c*C*d - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)
*(2*a*c*e - b*(c*d + a*f)) + (A*b)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
+ q + 2) - (2*f*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b)*(2*c^2*d + b
^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) - b*(A*c*e + a*C*e)
 + 2*(A*c*(c*d - a*f) - a*(c*C*d - a*C*f)))*(b*f*(p + 1) - c*e*(2*p + q + 4
)))*x - c*f*(b^2*(C*d + A*f) - b*(A*c*e + a*C*e) + 2*(A*c*(c*d - a*f) - a*(
c*C*d - a*C*f)))*(2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f
, A, C, q}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1]
&& NeQ[(c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && I
LtQ[q, -1]) &&  !IGtQ[q, 0]
```

Rubi steps

$$\int \frac{x^2}{\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}\,dx = -\frac{2\left(a(bcd-2ace+abf)+c\left(b^2d-abe-2a(cd-af)\right)x\right)}{\left(b^2-4ac\right)\left((cd-af)^2-(bd-ae)(ce-bf)\right)\sqrt{a+bx+cx^2}} - \frac{2\int \frac{-\frac{1}{2}\cdots}{\cdots}}{\left(b^2-4a\cdots\right.}$$

$$= -\frac{2\left(a(bcd-2ace+abf)+c\left(b^2d-abe-2a(cd-af)\right)x\right)}{\left(b^2-4ac\right)\left((cd-af)^2-(bd-ae)(ce-bf)\right)\sqrt{a+bx+cx^2}} + \frac{\left(f\left(2d(c\cdots\right.\right.}{\cdots}$$

$$= -\frac{2\left(a(bcd-2ace+abf)+c\left(b^2d-abe-2a(cd-af)\right)x\right)}{\left(b^2-4ac\right)\left((cd-af)^2-(bd-ae)(ce-bf)\right)\sqrt{a+bx+cx^2}} - \frac{\left(2f\left(2d(\cdots\right.\right.}{\cdots}$$

$$= -\frac{2\left(a(bcd-2ace+abf)+c\left(b^2d-abe-2a(cd-af)\right)x\right)}{\left(b^2-4ac\right)\left((cd-af)^2-(bd-ae)(ce-bf)\right)\sqrt{a+bx+cx^2}} - \frac{f\left(2d(cd\cdots\right.}{\sqrt{2}\sqrt{e^2\cdots}}$$

Mathematica [A] time = 6.59, size = 1097, normalized size = 1.80

$$\frac{16\sqrt{2}f\sqrt{ce^2-bfe-c\sqrt{e^2-4df}\,e+2af^2-2cdf+bf\sqrt{e^2-4df}}\,\left(e+\frac{2df-e^2}{\sqrt{e^2-4df}}\right)\tanh^{-1}\left(\frac{4af-b\left(e-\sqrt{e^2-4d\cdots}\right.}{2\sqrt{2}\sqrt{ce^2-bfe-c\sqrt{e^2-4df}\cdots}}\right.}{\left(4af^2-2b\left(e-\sqrt{e^2-4df}\right)f+c\left(e-\sqrt{e^2-4df}\right)^2\right)\left(16af^2-8b\left(e-\sqrt{e^2-4df}\right)f+4c\left(e-\cdots\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]
```

```
[Out] (-2*(e - (e^2 - 2*d*f)/Sqrt[e^2 - 4*d*f])*(2*b^2*f - 4*a*c*f - b*c*(e - Sqr
t[e^2 - 4*d*f]) + 2*c*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*x)*(a + b*x + c*x^2
))/((b^2 - 4*a*c)*f*(4*a*f^2 - 2*b*f*(e - Sqrt[e^2 - 4*d*f]) + c*(e - Sqrt[
e^2 - 4*d*f])^2)*(a + x*(b + c*x))^(3/2)) - (2*(e + (e^2 - 2*d*f)/Sqrt[e^2
- 4*d*f])*(2*b^2*f - 4*a*c*f - b*c*(e + Sqrt[e^2 - 4*d*f]) + 2*c*(b*f - c*(
e + Sqrt[e^2 - 4*d*f]))*x)*(a + b*x + c*x^2))/((b^2 - 4*a*c)*f*(4*a*f^2 - 2
*b*f*(e + Sqrt[e^2 - 4*d*f]) + c*(e + Sqrt[e^2 - 4*d*f])^2)*(a + x*(b + c*x
))^(3/2)) + (4*(b + 2*c*x)*(-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c)))^(3/2))/
(c*f*(a + x*(b + c*x))^(3/2)*Sqrt[1 - (b + 2*c*x)^2/(b^2 - 4*a*c)]) + (16*S
qrt[2]*f*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - c*e*Sqrt[e^2 - 4*d*f] + b
*f*Sqrt[e^2 - 4*d*f]]*(e + (-e^2 + 2*d*f)/Sqrt[e^2 - 4*d*f])*(a + b*x + c*x
^2)^(3/2)*ArcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) - (-2*b*f + 2*c*(e - S
qrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - c
*e*Sqrt[e^2 - 4*d*f] + b*f*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/((4*
a*f^2 - 2*b*f*(e - Sqrt[e^2 - 4*d*f]) + c*(e - Sqrt[e^2 - 4*d*f])^2)*(16*a*
f^2 - 8*b*f*(e - Sqrt[e^2 - 4*d*f]) + 4*c*(e - Sqrt[e^2 - 4*d*f])^2)*(a + x
*(b + c*x))^(3/2)) + (16*Sqrt[2]*f*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 +
 c*e*Sqrt[e^2 - 4*d*f] - b*f*Sqrt[e^2 - 4*d*f]]*(e - (-e^2 + 2*d*f)/Sqrt[e^
2 - 4*d*f])*(a + b*x + c*x^2)^(3/2)*ArcTanh[(4*a*f - b*(e + Sqrt[e^2 - 4*d*
f]) - (-2*b*f + 2*c*(e + Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c
*d*f - b*e*f + 2*a*f^2 + c*e*Sqrt[e^2 - 4*d*f] - b*f*Sqrt[e^2 - 4*d*f]]*Sqr
t[a + b*x + c*x^2])])/((4*a*f^2 - 2*b*f*(e + Sqrt[e^2 - 4*d*f]) + c*(e + Sq
rt[e^2 - 4*d*f])^2)*(16*a*f^2 - 8*b*f*(e + Sqrt[e^2 - 4*d*f]) + 4*c*(e + Sq
rt[e^2 - 4*d*f])^2)*(a + x*(b + c*x))^(3/2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] sage2
```

maple [B] time = 0.02, size = 11341, normalized size = 18.62

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x)
```

```
[Out] result too large to display
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for mo
re details)Is 4*d*f-e^2 positive, negative or zero?
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2}{\left(c\,x^2+b\,x+a\right)^{3/2}\,\left(f\,x^2+e\,x+d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)),x)
```

```
[Out] int(x^2/((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(c*x**2+b*x+a)**(3/2)/(f*x**2+e*x+d),x)
```

```
[Out] Timed out
```

3.123 $\int \frac{x}{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}\,dx$

Optimal. Leaf size=609

$$\frac{2\left(a\left(-2acf+b^2f-bce+2c^2d\right)+cx(abf-2ace+bcd)\right)}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\,\left((cd-af)^2-(bd-ae)(ce-bf)\right)}+\frac{f\left(2d(ce-bf)-\left(e-\sqrt{e^2-4df}\right)(cd-af)\right)\tanh}{\sqrt{2}\sqrt{e^2-4df}\,\left((cd-af)^2-(bd-ae)(ce-bf)\right)\cdot}$$

```
[Out] 2*(a*(-2*a*c*f+b^2*f-b*c*e+2*c^2*d)+c*(a*b*f-2*a*c*e+b*c*d)*x)/(-4*a*c+b^2)
/((-a*f+c*d)^2-(-a*e+b*d)*(-b*f+c*e))/(c*x^2+b*x+a)^(1/2)+1/2*f*arctanh(1/4
*(4*a*f+2*x*(b*f-c*(e-(-4*d*f+e^2)^(1/2)))-b*(e-(-4*d*f+e^2)^(1/2)))*2^(1/2
)/(c*x^2+b*x+a)^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2-(-b*f+c*e)*(-4*d*f+e^2)^
(1/2))^(1/2))*(2*d*(-b*f+c*e)-(-a*f+c*d)*(e-(-4*d*f+e^2)^(1/2)))/((-a*f+c*d
)^2-(-a*e+b*d)*(-b*f+c*e))*2^(1/2)/(-4*d*f+e^2)^(1/2)/(c*e^2-2*c*d*f-b*e*f+
2*a*f^2-(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2)-1/2*f*arctanh(1/4*(4*a*f-b*(e+
(-4*d*f+e^2)^(1/2))+2*x*(b*f-c*(e+(-4*d*f+e^2)^(1/2))))*2^(1/2)/(c*x^2+b*x+
a)^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2+(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2))
*(2*d*(-b*f+c*e)-(-a*f+c*d)*(e+(-4*d*f+e^2)^(1/2)))/((-a*f+c*d)^2-(-a*e+b*d
)*(-b*f+c*e))*2^(1/2)/(-4*d*f+e^2)^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2+(-b*f
+c*e)*(-4*d*f+e^2)^(1/2))^(1/2)
```

Rubi [A] time = 5.64, antiderivative size = 609, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1016, 1032, 724, 206}

$$\frac{2\left(a\left(-2acf+b^2f-bce+2c^2d\right)+cx(abf-2ace+bcd)\right)}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\,\left((cd-af)^2-(bd-ae)(ce-bf)\right)}+\frac{f\left(2d(ce-bf)-\left(e-\sqrt{e^2-4df}\right)(cd-af)\right)\tanh}{\sqrt{2}\sqrt{e^2-4df}\,\left((cd-af)^2-(bd-ae)(ce-bf)\right)\cdot}$$

Antiderivative was successfully verified.

```
[In] Int[x/((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]
```

```
[Out] (2*(a*(2*c^2*d - b*c*e + b^2*f - 2*a*c*f) + c*(b*c*d - 2*a*c*e + a*b*f)*x))
/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*Sqrt[a + b*x + c*
x^2]) + (f*(2*d*(c*e - b*f) - (c*d - a*f)*(e - Sqrt[e^2 - 4*d*f]))*ArcTanh[
(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*x)
/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 -
 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*Sqrt[e^2 - 4*d*f]*((c*d - a*f)^2
 - (b*d - a*e)*(c*e - b*f))*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e -
 b*f)*Sqrt[e^2 - 4*d*f]]) - (f*(2*d*(c*e - b*f) - (c*d - a*f)*(e + Sqrt[e^2
 - 4*d*f]))*ArcTanh[(4*a*f - b*(e + Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e + Sq
rt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c
*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*Sqrt[e^2 - 4
*d*f]*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*Sqrt[c*e^2 - 2*c*d*f - b*e*
f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
```

```
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1016

```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e
_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((a + b*x + c*x^2)^(p + 1)
*(d + e*x + f*x^2)^(q + 1)*(g*c*(2*a*c*e - b*(c*d + a*f)) + (g*b - a*h)*(2*
c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(g*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))
 - h*(b*c*d - 2*a*c*e + a*b*f))*x))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d -
a*e)*(c*e - b*f))*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*
d - a*e)*(c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*
x^2)^q*Simp[(b*h - 2*g*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)
 + (b^2*(g*f) - b*(h*c*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) - a*(-(h*c*e
))))*(a*f*(p + 1) - c*d*(p + 2)) - e*((g*c)*(2*a*c*e - b*(c*d + a*f)) + (g*
b - a*h)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (2*f*((g*c)*(2*
a*c*e - b*(c*d + a*f)) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(
p + q + 2) - (b^2*g*f - b*(h*c*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) - a*
(-(h*c*e))))*(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(g*f) - b*(h*c
*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) + a*h*c*e))*(2*p + 2*q + 5)*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, q}, x] && NeQ[b^2 - 4*a*c, 0] &
& NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (b*d - a*e)*(c*e
 - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1])
```

Rule 1032

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x}{\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}\,dx = \frac{2\left(a\left(2c^2d-bce+b^2f-2acf\right)+c(bcd-2ace+abf)x\right)}{\left(b^2-4ac\right)\left((cd-af)^2-(bd-ae)(ce-bf)\right)\sqrt{a+bx+cx^2}} - \frac{2\int \frac{\frac{1}{2}\left(b^2\right.}{}}{\left(b^2-4ac\right)}$$

$$= \frac{2\left(a\left(2c^2d-bce+b^2f-2acf\right)+c(bcd-2ace+abf)x\right)}{\left(b^2-4ac\right)\left((cd-af)^2-(bd-ae)(ce-bf)\right)\sqrt{a+bx+cx^2}} - \frac{\left(f\left(2d(ce\right.\right.}{}$$

$$= \frac{2\left(a\left(2c^2d-bce+b^2f-2acf\right)+c(bcd-2ace+abf)x\right)}{\left(b^2-4ac\right)\left((cd-af)^2-(bd-ae)(ce-bf)\right)\sqrt{a+bx+cx^2}} + \frac{\left(2f\left(2d(c\right.\right.}{}$$

$$= \frac{2\left(a\left(2c^2d-bce+b^2f-2acf\right)+c(bcd-2ace+abf)x\right)}{\left(b^2-4ac\right)\left((cd-af)^2-(bd-ae)(ce-bf)\right)\sqrt{a+bx+cx^2}} + \frac{f\left(2d(ce-\right.}{\sqrt{2}\sqrt{e^2-}}$$

Mathematica [A] time = 5.68, size = 770, normalized size = 1.26

$$\frac{2\left(1-\frac{e}{\sqrt{e^2-4df}}\right)\left(2c\left(cx\left(\sqrt{e^2-4df}-e\right)-2af\right)+2b^2f+bc\left(\sqrt{e^2-4df}-e+2fx\right)\right)}{\left(b^2-4ac\right)\sqrt{a+x(b+cx)}\left(4af^2+2bf\left(\sqrt{e^2-4df}-e\right)+c\left(e-\sqrt{e^2-4df}\right)^2\right)}+\frac{2\left(\frac{e}{\sqrt{e^2-4df}}+1\right)\left(-2c\left(2\right.\right.}{\left(b^2-4ac\right)\sqrt{a+x(b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]
```

```
[Out] (2*(1 - e/Sqrt[e^2 - 4*d*f])*(2*b^2*f + b*c*(-e + Sqrt[e^2 - 4*d*f] + 2*f*x
) + 2*c*(-2*a*f + c*(-e + Sqrt[e^2 - 4*d*f])*x)))/((b^2 - 4*a*c)*(4*a*f^2 +
 c*(e - Sqrt[e^2 - 4*d*f])^2 + 2*b*f*(-e + Sqrt[e^2 - 4*d*f]))*Sqrt[a + x*(
b + c*x)]) + (2*(1 + e/Sqrt[e^2 - 4*d*f])*(2*b^2*f - b*c*(e + Sqrt[e^2 - 4*
d*f] - 2*f*x) - 2*c*(2*a*f + c*(e + Sqrt[e^2 - 4*d*f])*x)))/((b^2 - 4*a*c)*
(4*a*f^2 - 2*b*f*(e + Sqrt[e^2 - 4*d*f]) + c*(e + Sqrt[e^2 - 4*d*f])^2)*Sqr
t[a + x*(b + c*x)]) - (Sqrt[2]*f^2*(e + Sqrt[e^2 - 4*d*f])*ArcTanh[(4*a*f -
 2*c*(e + Sqrt[e^2 - 4*d*f])*x - b*(e + Sqrt[e^2 - 4*d*f] - 2*f*x))/(2*Sqrt
[2]*Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2
 - 4*d*f]))]*Sqrt[a + x*(b + c*x)])])/(Sqrt[e^2 - 4*d*f]*(c*(e^2 - 2*d*f +
e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f])))^(3/2)) - (Sqr
t[2]*f^2*(-e + Sqrt[e^2 - 4*d*f])*Sqrt[c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]
) + f*(2*a*f + b*(-e + Sqrt[e^2 - 4*d*f]))]*ArcTanh[(4*a*f + 2*c*(-e + Sqrt
[e^2 - 4*d*f])*x + b*(-e + Sqrt[e^2 - 4*d*f] + 2*f*x))/(2*Sqrt[2]*Sqrt[c*(e
^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f + b*(-e + Sqrt[e^2 - 4*d*f]))]
*Sqrt[a + x*(b + c*x)]])])/(Sqrt[e^2 - 4*d*f]*(c*(-e^2 + 2*d*f + e*Sqrt[e^2
 - 4*d*f]) + f*(-2*a*f + b*(e - Sqrt[e^2 - 4*d*f])))^2)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$sage_2$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] sage2
```

maple [B] time = 0.02, size = 7163, normalized size = 11.76

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x)
```

```
[Out] result too large to display
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for mo
re details)Is 4*d*f-e^2 positive, negative or zero?
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x}{\left(c\,x^2+b\,x+a\right)^{3/2}\,\left(f\,x^2+e\,x+d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)),x)
```

```
[Out] int(x/((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x**2+b*x+a)**(3/2)/(f*x**2+e*x+d),x)
```

```
[Out] Timed out
```

3.124 $\int \frac{1}{\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}\,dx$

Optimal. Leaf size=666

$$\frac{2\left(-cx\left(-2acf+b^2f-bce+2c^2d\right)-bc(cd-3af)-2ac^2e+b^3(-f)+b^2ce\right)}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left((cd-af)^2-(bd-ae)(ce-bf)\right)}-\frac{f\left(f\left(2af-b\left(\sqrt{e^2-4df}+e\right)\right)\right.}{\sqrt{2}\sqrt{e^2-4df}\left((cd-af)^2-\right.}$$

```
[Out] 2*(b^2*c*e-2*a*c^2*e-b^3*f-b*c*(-3*a*f+c*d)-c*(-2*a*c*f+b^2*f-b*c*e+2*c^2*d
)*x)/(-4*a*c+b^2)/((-a*f+c*d)^2-(-a*e+b*d)*(-b*f+c*e))/(c*x^2+b*x+a)^(1/2)-
1/2*f*arctanh(1/4*(4*a*f+2*x*(b*f-c*(e-(-4*d*f+e^2)^(1/2)))-b*(e-(-4*d*f+e^
2)^(1/2)))*2^(1/2)/(c*x^2+b*x+a)^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2-(-b*f+c
*e)*(-4*d*f+e^2)^(1/2))^(1/2))*(((-4*d*f+e^2)^(1/2)*e-2*d*f+e^2)*c+f*(2*a*f
-b*(e+(-4*d*f+e^2)^(1/2))))/((-a*f+c*d)^2-(-a*e+b*d)*(-b*f+c*e))*2^(1/2)/(-
4*d*f+e^2)^(1/2)/(f*(2*a*f-b*(e-(-4*d*f+e^2)^(1/2)))+(-(-4*d*f+e^2)^(1/2)*e
-2*d*f+e^2)*c)^(1/2)+1/2*f*arctanh(1/4*(4*a*f-b*(e+(-4*d*f+e^2)^(1/2))+2*x*
(b*f-c*(e+(-4*d*f+e^2)^(1/2))))*2^(1/2)/(c*x^2+b*x+a)^(1/2)/(c*e^2-2*c*d*f-
b*e*f+2*a*f^2+(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2))*(f*(2*a*f-b*(e-(-4*d*f+
e^2)^(1/2)))+(-(-4*d*f+e^2)^(1/2)*e-2*d*f+e^2)*c)/((-a*f+c*d)^2-(-a*e+b*d)*
(-b*f+c*e))*2^(1/2)/(-4*d*f+e^2)^(1/2)/(((-4*d*f+e^2)^(1/2)*e-2*d*f+e^2)*c+
f*(2*a*f-b*(e+(-4*d*f+e^2)^(1/2))))^(1/2)
```

Rubi [A] time = 1.75, antiderivative size = 666, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {974, 1032, 724, 206}

$$\frac{2\left(-cx\left(-2acf+b^2f-bce+2c^2d\right)-bc(cd-3af)-2ac^2e+b^2ce+b^3(-f)\right)}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left((cd-af)^2-(bd-ae)(ce-bf)\right)}-\frac{f\left(f\left(2af-b\left(\sqrt{e^2-4df}+e\right)\right)\right.}{\sqrt{2}\sqrt{e^2-4df}\left((cd-af)^2-\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]
```

```
[Out] (2*(b^2*c*e - 2*a*c^2*e - b^3*f - b*c*(c*d - 3*a*f) - c*(2*c^2*d - b*c*e +
b^2*f - 2*a*c*f)*x))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f
))*Sqrt[a + b*x + c*x^2]) - (f*(c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(
2*a*f - b*(e + Sqrt[e^2 - 4*d*f])))*ArcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*
f]) + 2*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*
f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])
])/(Sqrt[2]*Sqrt[e^2 - 4*d*f]*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*Sqr
t[c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e - Sqrt[e^2 - 4*d*
f]))]) + (f*(c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e - Sqrt
[e^2 - 4*d*f])))*ArcTanh[(4*a*f - b*(e + Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e
 + Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2
 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*Sqrt[e^
2 - 4*d*f]*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*Sqrt[c*(e^2 - 2*d*f +
e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 974

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[((2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*
a*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*(
d + e*x + f*x^2)^(q + 1))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), x] - Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(
c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Sim
p[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f
- c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e - b
^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f +
 b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f*
(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(
2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[q,
 0]
```

Rule 1032

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}\,dx = \frac{2\left(b^2ce-2ac^2e-b^3f-bc(cd-3af)-c\left(2c^2d-bce+b^2f-2acf\right)x\right)}{\left(b^2-4ac\right)\left((cd-af)^2-(bd-ae)(ce-bf)\right)\sqrt{a+bx+cx^2}}+$$

$$= \frac{2\left(b^2ce-2ac^2e-b^3f-bc(cd-3af)-c\left(2c^2d-bce+b^2f-2acf\right)x\right)}{\left(b^2-4ac\right)\left((cd-af)^2-(bd-ae)(ce-bf)\right)\sqrt{a+bx+cx^2}}+$$

$$= \frac{2\left(b^2ce-2ac^2e-b^3f-bc(cd-3af)-c\left(2c^2d-bce+b^2f-2acf\right)x\right)}{\left(b^2-4ac\right)\left((cd-af)^2-(bd-ae)(ce-bf)\right)\sqrt{a+bx+cx^2}}-$$

$$= \frac{2\left(b^2ce-2ac^2e-b^3f-bc(cd-3af)-c\left(2c^2d-bce+b^2f-2acf\right)x\right)}{\left(b^2-4ac\right)\left((cd-af)^2-(bd-ae)(ce-bf)\right)\sqrt{a+bx+cx^2}}-$$

Mathematica [A] time = 5.04, size = 700, normalized size = 1.05

$$2f\left(-\frac{2\left(-2c\left(2af+cx\left(\sqrt{e^2-4df}+e\right)\right)+2b^2f-bc\left(\sqrt{e^2-4df}+e-2fx\right)\right)}{\left(b^2-4ac\right)\sqrt{a+x(b+cx)}\left(4af^2-2bf\left(\sqrt{e^2-4df}+e\right)+c\left(\sqrt{e^2-4df}+e\right)^2\right)}+\frac{2c\left(cx\left(\sqrt{e^2-4df}-e\right)-2af\right)+2b^2f+bc\left(\sqrt{e^2-4df}-e+2fx\right)}{\left(b^2-4ac\right)\sqrt{a+x(b+cx)}\left(f\left(2af+b\left(\sqrt{e^2-4df}-e\right)\right)+c\left(-e\sqrt{e^2-4df}-2df\right.\right.}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]
```

```
[Out] (2*f*((2*b^2*f + b*c*(-e + Sqrt[e^2 - 4*d*f] + 2*f*x) + 2*c*(-2*a*f + c*(-e
 + Sqrt[e^2 - 4*d*f])*x))/((b^2 - 4*a*c)*(c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d
*f]) + f*(2*a*f + b*(-e + Sqrt[e^2 - 4*d*f])))*Sqrt[a + x*(b + c*x)]) - (2*
(2*b^2*f - b*c*(e + Sqrt[e^2 - 4*d*f] - 2*f*x) - 2*c*(2*a*f + c*(e + Sqrt[e
^2 - 4*d*f])*x)))/((b^2 - 4*a*c)*(4*a*f^2 - 2*b*f*(e + Sqrt[e^2 - 4*d*f]) +
 c*(e + Sqrt[e^2 - 4*d*f])^2)*Sqrt[a + x*(b + c*x)]) + (Sqrt[2]*f^2*ArcTanh
[(4*a*f - 2*c*(e + Sqrt[e^2 - 4*d*f])*x - b*(e + Sqrt[e^2 - 4*d*f] - 2*f*x)
)/(2*Sqrt[2]*Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e +
 Sqrt[e^2 - 4*d*f]))]*Sqrt[a + x*(b + c*x)])])/(c*(e^2 - 2*d*f + e*Sqrt[e^2
 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f])))^(3/2) - (Sqrt[2]*f^2*Sq
rt[c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f + b*(-e + Sqrt[e^2 - 4*
d*f]))]*ArcTanh[(4*a*f + 2*c*(-e + Sqrt[e^2 - 4*d*f])*x + b*(-e + Sqrt[e^2
- 4*d*f] + 2*f*x))/(2*Sqrt[2]*Sqrt[c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) +
f*(2*a*f + b*(-e + Sqrt[e^2 - 4*d*f]))]*Sqrt[a + x*(b + c*x)])])/(c*(-e^2 +
 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(-2*a*f + b*(e - Sqrt[e^2 - 4*d*f])))^2))
/Sqrt[e^2 - 4*d*f]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$sage_2$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] sage2
```

maple [B] time = 0.02, size = 4099, normalized size = 6.15

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x)
```

```
[Out] -2/(-4*d*f+e^2)^(1/2)/(2*a*f^2-b*e*f-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*b*f+(
-4*d*f+e^2)^(1/2)*c*e)*f^2/((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c+(b*f-c*e-(
-4*d*f+e^2)^(1/2)*c)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)/f+1/2*(2*a*f^2-b*e*f-
```

```
2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)-4*f
/(2*a*f^2-b*e*f-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e
)/(4*a*c-4*c^2*d/f+c^2*e^2/f^2-(-4*d*f+e^2)*c^2/f^2-b^2)/((x+1/2*(e+(-4*d*f
+e^2)^(1/2))/f)^2*c+(b*f-c*e-(-4*d*f+e^2)^(1/2)*c)*(x+1/2*(e+(-4*d*f+e^2)^(
1/2))/f)/f+1/2*(2*a*f^2-b*e*f-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+
e^2)^(1/2)*c*e)/f^2)^(1/2)*x*c^2+4/(-4*d*f+e^2)^(1/2)*f^2/(2*a*f^2-b*e*f-2*
c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e)/(4*a*c-4*c^2*d/f
+c^2*e^2/f^2-(-4*d*f+e^2)*c^2/f^2-b^2)/((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*
c+(b*f-c*e-(-4*d*f+e^2)^(1/2)*c)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)/f+1/2*(2*
a*f^2-b*e*f-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e)/f^
2)^(1/2)*x*b*c-4/(-4*d*f+e^2)^(1/2)*f/(2*a*f^2-b*e*f-2*c*d*f+c*e^2-(-4*d*f+
e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e)/(4*a*c-4*c^2*d/f+c^2*e^2/f^2-(-4*d*f
+e^2)*c^2/f^2-b^2)/((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c+(b*f-c*e-(-4*d*f+e
^2)^(1/2)*c)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)/f+1/2*(2*a*f^2-b*e*f-2*c*d*f+
c*e^2-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*x*c^2*e-2*f
/(2*a*f^2-b*e*f-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e
)/(4*a*c-4*c^2*d/f+c^2*e^2/f^2-(-4*d*f+e^2)*c^2/f^2-b^2)/((x+1/2*(e+(-4*d*f
+e^2)^(1/2))/f)^2*c+(b*f-c*e-(-4*d*f+e^2)^(1/2)*c)*(x+1/2*(e+(-4*d*f+e^2)^(
1/2))/f)/f+1/2*(2*a*f^2-b*e*f-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+
e^2)^(1/2)*c*e)/f^2)^(1/2)*b*c+2/(-4*d*f+e^2)^(1/2)*f^2/(2*a*f^2-b*e*f-2*c*
d*f+c*e^2-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e)/(4*a*c-4*c^2*d/f+c
^2*e^2/f^2-(-4*d*f+e^2)*c^2/f^2-b^2)/((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c+
(b*f-c*e-(-4*d*f+e^2)^(1/2)*c)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)/f+1/2*(2*a*
f^2-b*e*f-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e)/f^2)
^(1/2)*b^2-2/(-4*d*f+e^2)^(1/2)*f/(2*a*f^2-b*e*f-2*c*d*f+c*e^2-(-4*d*f+e^2)
^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e)/(4*a*c-4*c^2*d/f+c^2*e^2/f^2-(-4*d*f+e^2
)*c^2/f^2-b^2)/((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c+(b*f-c*e-(-4*d*f+e^2)^
(1/2)*c)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)/f+1/2*(2*a*f^2-b*e*f-2*c*d*f+c*e^
2-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*b*c*e+2/(-4*d*f
+e^2)^(1/2)/(2*a*f^2-b*e*f-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2
)^(1/2)*c*e)*f^2*2^(1/2)/((2*a*f^2-b*e*f-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*b
*f+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*ln(((b*f-c*e-(-4*d*f+e^2)^(1/2)*c)*(x
+1/2*(e+(-4*d*f+e^2)^(1/2))/f)/f+(2*a*f^2-b*e*f-2*c*d*f+c*e^2-(-4*d*f+e^2)^
(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e)/f^2+1/2*2^(1/2)*((2*a*f^2-b*e*f-2*c*d*f+c
*e^2-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*(4*(x+1/2*(e
+(-4*d*f+e^2)^(1/2))/f)^2*c+4*(b*f-c*e-(-4*d*f+e^2)^(1/2)*c)*(x+1/2*(e+(-4*
d*f+e^2)^(1/2))/f)/f+2*(2*a*f^2-b*e*f-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*b*f+
(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))+2/(-4
*d*f+e^2)^(1/2)/(2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*b*f-(-4*d*f
+e^2)^(1/2)*c*e)*f^2/((x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c+(b*f-c*e+(-4*d*
f+e^2)^(1/2)*c)*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)/f+1/2*(2*a*f^2-b*e*f-2*c*
d*f+c*e^2+(-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)-4*f/(2*
a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e)/(4
*a*c-4*c^2*d/f+c^2*e^2/f^2-(-4*d*f+e^2)*c^2/f^2-b^2)/((x-1/2*(-e+(-4*d*f+e^
2)^(1/2))/f)^2*c+(b*f-c*e+(-4*d*f+e^2)^(1/2)*c)*(x-1/2*(-e+(-4*d*f+e^2)^(1/
2))/f)/f+1/2*(2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^
2)^(1/2)*c*e)/f^2)^(1/2)*x*c^2-4/(-4*d*f+e^2)^(1/2)*f^2/(2*a*f^2-b*e*f-2*c*
d*f+c*e^2+(-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e)/(4*a*c-4*c^2*d/f+c
^2*e^2/f^2-(-4*d*f+e^2)*c^2/f^2-b^2)/((x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c
+(b*f-c*e+(-4*d*f+e^2)^(1/2)*c)*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)/f+1/2*(2*
a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e)/f^
2)^(1/2)*x*b*c+4/(-4*d*f+e^2)^(1/2)*f/(2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+
e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e)/(4*a*c-4*c^2*d/f+c^2*e^2/f^2-(-4*d*f
+e^2)*c^2/f^2-b^2)/((x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c+(b*f-c*e+(-4*d*f+
e^2)^(1/2)*c)*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)/f+1/2*(2*a*f^2-b*e*f-2*c*d*
f+c*e^2+(-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*x*c^2*e-2
*f/(2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c
*e)/(4*a*c-4*c^2*d/f+c^2*e^2/f^2-(-4*d*f+e^2)*c^2/f^2-b^2)/((x-1/2*(-e+(-4*
d*f+e^2)^(1/2))/f)^2*c+(b*f-c*e+(-4*d*f+e^2)^(1/2)*c)*(x-1/2*(-e+(-4*d*f+e^
```

```
2)^(1/2))/f)/f+1/2*(2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*b*f-(-4*
d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*b*c-2/(-4*d*f+e^2)^(1/2)*f^2/(2*a*f^2-b*e*f-
2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e)/(4*a*c-4*c^2*d
/f+c^2*e^2/f^2-(-4*d*f+e^2)*c^2/f^2-b^2)/((x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)
^2*c+(b*f-c*e+(-4*d*f+e^2)^(1/2)*c)*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)/f+1/2
*(2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e
)/f^2)^(1/2)*b^2+2/(-4*d*f+e^2)^(1/2)*f/(2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*
f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e)/(4*a*c-4*c^2*d/f+c^2*e^2/f^2-(-4*d
*f+e^2)*c^2/f^2-b^2)/((x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c+(b*f-c*e+(-4*d*
f+e^2)^(1/2)*c)*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)/f+1/2*(2*a*f^2-b*e*f-2*c*
d*f+c*e^2+(-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*b*c*e-2
/(-4*d*f+e^2)^(1/2)/(2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*b*f-(-4
*d*f+e^2)^(1/2)*c*e)*f^2*2^(1/2)/((2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)
^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*ln(((b*f-c*e+(-4*d*f+e^2)^(1/
2)*c)*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)/f+(2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*
d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e)/f^2+1/2*2^(1/2)*((2*a*f^2-b*e*f-
2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*(4*
(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c+4*(b*f-c*e+(-4*d*f+e^2)^(1/2)*c)*(x-1
/2*(-e+(-4*d*f+e^2)^(1/2))/f)/f+2*(2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)
^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2))/(x-1/2*(-e+(-4*d*f+e^2)^(1/2
))/f))
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for mo
re details)Is 4*d*f-e^2 positive, negative or zero?
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(c\,x^2+b\,x+a\right)^{3/2}\,\left(f\,x^2+e\,x+d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)),x)
```

```
[Out] int(1/((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**2+b*x+a)**(3/2)/(f*x**2+e*x+d),x)
```

```
[Out] Timed out
```

3.125 $\int \frac{1}{x(a+bx+cx^2)^{3/2}(d+ex+fx^2)}\,dx$

Optimal. Leaf size=816

$$\frac{2\left(b^2+cxb-2ac\right)}{a\left(b^2-4ac\right)d\sqrt{cx^2+bx+a}}-\frac{\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{cx^2+bx+a}}\right)}{a^{3/2}d}+\frac{f\left(\left(e-\sqrt{e^2-4df}\right)\left(f(be-af)-c\left(e^2-df\right)\right)-2\left(f\left(be^2\right.\right.\right.}{\sqrt{2}\,d\sqrt{e^2-4df}\,\left((cd-af)^2-(bd-\right.}$$

```
[Out] -arctanh(1/2*(b*x+2*a)/a^(1/2)/(c*x^2+b*x+a)^(1/2))/a^(3/2)/d+2*(b*c*x-2*a*
c+b^2)/a/(-4*a*c+b^2)/d/(c*x^2+b*x+a)^(1/2)+2*(c*e*(2*a*c*e-b*(a*f+c*d))+(-
a*f+b*e)*(2*c^2*d+b^2*f-c*(2*a*f+b*e))+c*(2*c^2*d*e+b*f*(-a*f+b*e)-b*c*(d*f
+e^2))*x)/(-4*a*c+b^2)/d/((-a*f+c*d)^2-(-a*e+b*d)*(-b*f+c*e))/(c*x^2+b*x+a)
^(1/2)+1/2*f*arctanh(1/4*(4*a*f+2*x*(b*f-c*(e-(-4*d*f+e^2)^(1/2)))-b*(e-(-4
*d*f+e^2)^(1/2)))*2^(1/2)/(c*x^2+b*x+a)^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2-
(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2))*(-2*f*(-a*e*f-b*d*f+b*e^2)+2*c*(-2*d*
e*f+e^3)+(f*(-a*f+b*e)-c*(-d*f+e^2))*(e-(-4*d*f+e^2)^(1/2)))/d/((-a*f+c*d)^
2-(-a*e+b*d)*(-b*f+c*e))*2^(1/2)/(-4*d*f+e^2)^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*
a*f^2-(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2)-1/2*f*arctanh(1/4*(4*a*f-b*(e+(-
4*d*f+e^2)^(1/2))+2*x*(b*f-c*(e+(-4*d*f+e^2)^(1/2))))*2^(1/2)/(c*x^2+b*x+a)
^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2+(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2))*(
-2*f*(-a*e*f-b*d*f+b*e^2)+2*c*(-2*d*e*f+e^3)+(f*(-a*f+b*e)-c*(-d*f+e^2))*(e
+(-4*d*f+e^2)^(1/2)))/d/((-a*f+c*d)^2-(-a*e+b*d)*(-b*f+c*e))*2^(1/2)/(-4*d*
f+e^2)^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2+(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1
/2)
```

Rubi [A] time = 15.92, antiderivative size = 814, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.233, Rules used = {6728, 740, 12, 724, 206, 1016, 1032}

$$\frac{2\left(b^2+cxb-2ac\right)}{a\left(b^2-4ac\right)d\sqrt{cx^2+bx+a}}-\frac{\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{cx^2+bx+a}}\right)}{a^{3/2}d}-\frac{f\left(2f\left(be^2-afe-bdf\right)-2c\left(e^3-2def\right)-\left(e-\sqrt{e^2-4d\right.\right.}{\sqrt{2}\,d\sqrt{e^2-4df}\,\left((cd-af)^2-(bd-\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]
```

```
[Out] (2*(b^2 - 2*a*c + b*c*x))/(a*(b^2 - 4*a*c)*d*Sqrt[a + b*x + c*x^2]) + (2*(c
*e*(2*a*c*e - b*(c*d + a*f)) + (b*e - a*f)*(2*c^2*d + b^2*f - c*(b*e + 2*a*
f)) + c*(2*c^2*d*e + b*f*(b*e - a*f) - b*c*(e^2 + d*f))*x))/((b^2 - 4*a*c)*
d*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*Sqrt[a + b*x + c*x^2]) - ArcTan
h[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])]/(a^(3/2)*d) - (f*(2*f*(b*e
^2 - b*d*f - a*e*f) - 2*c*(e^3 - 2*d*e*f) - (e - Sqrt[e^2 - 4*d*f])*(f*(b*e
 - a*f) - c*(e^2 - d*f)))*ArcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) + 2*(b
*f - c*(e - Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f
+ 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2
]*d*Sqrt[e^2 - 4*d*f]*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*Sqrt[c*e^2
- 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]) + (f*(2*f*(b*
e^2 - b*d*f - a*e*f) - 2*c*(e^3 - 2*d*e*f) - (e + Sqrt[e^2 - 4*d*f])*(f*(b*
e - a*f) - c*(e^2 - d*f)))*ArcTanh[(4*a*f - b*(e + Sqrt[e^2 - 4*d*f]) + 2*(
b*f - c*(e + Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f
 + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[
2]*d*Sqrt[e^2 - 4*d*f]*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*Sqrt[c*e^2
 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 1016

```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e
_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((a + b*x + c*x^2)^(p + 1)
*(d + e*x + f*x^2)^(q + 1)*(g*c*(2*a*c*e - b*(c*d + a*f)) + (g*b - a*h)*(2*
c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(g*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))
 - h*(b*c*d - 2*a*c*e + a*b*f))*x))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d -
a*e)*(c*e - b*f))*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*
d - a*e)*(c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*
x^2)^q*Simp[(b*h - 2*g*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)
 + (b^2*(g*f) - b*(h*c*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) - a*(-(h*c*e
))))*(a*f*(p + 1) - c*d*(p + 2)) - e*((g*c)*(2*a*c*e - b*(c*d + a*f)) + (g*
b - a*h)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (2*f*((g*c)*(2*
a*c*e - b*(c*d + a*f)) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(
p + q + 2) - (b^2*g*f - b*(h*c*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) - a*
(-(h*c*e))))*(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(g*f) - b*(h*c
*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) + a*h*c*e))*(2*p + 2*q + 5)*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, q}, x] && NeQ[b^2 - 4*a*c, 0] &
& NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (b*d - a*e)*(c*e
 - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1])
```

Rule 1032

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{x\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}dx=\int\left(\frac{1}{dx\left(a+bx+cx^2\right)^{3/2}}+\frac{-e-fx}{d\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}\right)dx$$

$$=\frac{\int\frac{1}{x\left(a+bx+cx^2\right)^{3/2}}dx}{d}+\frac{\int\frac{-e-fx}{\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}dx}{d}$$

$$=\frac{2\left(b^2-2ac+bcx\right)}{a\left(b^2-4ac\right)d\sqrt{a+bx+cx^2}}+\frac{2\left(ce(2ace-b(cd+af))+(be-af)\left(2\right.\right.}{\left(b^2-4ac\right)d\left(\left(c\right.\right.}$$

$$=\frac{2\left(b^2-2ac+bcx\right)}{a\left(b^2-4ac\right)d\sqrt{a+bx+cx^2}}+\frac{2\left(ce(2ace-b(cd+af))+(be-af)\left(2\right.\right.}{\left(b^2-4ac\right)d\left(\left(c\right.\right.}$$

$$=\frac{2\left(b^2-2ac+bcx\right)}{a\left(b^2-4ac\right)d\sqrt{a+bx+cx^2}}+\frac{2\left(ce(2ace-b(cd+af))+(be-af)\left(2\right.\right.}{\left(b^2-4ac\right)d\left(\left(c\right.\right.}$$

$$=\frac{2\left(b^2-2ac+bcx\right)}{a\left(b^2-4ac\right)d\sqrt{a+bx+cx^2}}+\frac{2\left(ce(2ace-b(cd+af))+(be-af)\left(2\right.\right.}{\left(b^2-4ac\right)d\left(\left(c\right.\right.}$$

Mathematica [A] time = 6.56, size = 1121, normalized size = 1.37

$$\frac{16\sqrt{2}\left(\frac{ef}{\sqrt{e^2-4df}}+f\right)\sqrt{ce^2-bfe-c\sqrt{e^2-4df}\,e+2af^2-2cdf+bf\sqrt{e^2-4df}}\left(cx^2+bx+a\right)^{3/2}\tanh^{-1}\left(\frac{}{2\sqrt{2}\sqrt{ce}}\right.}{d\left(4af^2-2b\left(e-\sqrt{e^2-4df}\right)f+c\left(e-\sqrt{e^2-4df}\right)^2\right)\left(16af^2-8b\left(e-\sqrt{e^2-4df}\right)f+4c\left(e-\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]
```

```
[Out] (2*(b^2 - 2*a*c + b*c*x)*(a + b*x + c*x^2))/(a*(b^2 - 4*a*c)*d*(a + x*(b +
c*x))^(3/2)) - (2*f*(1 + e/Sqrt[e^2 - 4*d*f])*(2*b^2*f - 4*a*c*f - b*c*(e -
 Sqrt[e^2 - 4*d*f]) + 2*c*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*x)*(a + b*x + c
*x^2))/((b^2 - 4*a*c)*d*(4*a*f^2 - 2*b*f*(e - Sqrt[e^2 - 4*d*f]) + c*(e - S
qrt[e^2 - 4*d*f])^2)*(a + x*(b + c*x))^(3/2)) - (2*f*(1 - e/Sqrt[e^2 - 4*d*
f])*(2*b^2*f - 4*a*c*f - b*c*(e + Sqrt[e^2 - 4*d*f]) + 2*c*(b*f - c*(e + Sq
rt[e^2 - 4*d*f]))*x)*(a + b*x + c*x^2))/((b^2 - 4*a*c)*d*(4*a*f^2 - 2*b*f*(
e + Sqrt[e^2 - 4*d*f]) + c*(e + Sqrt[e^2 - 4*d*f])^2)*(a + x*(b + c*x))^(3/
2)) - ((a + b*x + c*x^2)^(3/2)*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x
+ c*x^2])])/(a^(3/2)*d*(a + x*(b + c*x))^(3/2)) + (16*Sqrt[2]*f^2*(f + (e*f
)/Sqrt[e^2 - 4*d*f])*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - c*e*Sqrt[e^2
- 4*d*f] + b*f*Sqrt[e^2 - 4*d*f]]*(a + b*x + c*x^2)^(3/2)*ArcTanh[(4*a*f -
b*(e - Sqrt[e^2 - 4*d*f]) - (-2*b*f + 2*c*(e - Sqrt[e^2 - 4*d*f]))*x)/(2*Sq
rt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - c*e*Sqrt[e^2 - 4*d*f] + b*f*
Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(d*(4*a*f^2 - 2*b*f*(e - Sqrt[e
^2 - 4*d*f]) + c*(e - Sqrt[e^2 - 4*d*f])^2)*(16*a*f^2 - 8*b*f*(e - Sqrt[e^2
```

```
 - 4*d*f]) + 4*c*(e - Sqrt[e^2 - 4*d*f])^2)*(a + x*(b + c*x))^(3/2)) - (16*
Sqrt[2]*f^2*(-f + (e*f)/Sqrt[e^2 - 4*d*f])*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2
*a*f^2 + c*e*Sqrt[e^2 - 4*d*f] - b*f*Sqrt[e^2 - 4*d*f]]*(a + b*x + c*x^2)^(
3/2)*ArcTanh[(4*a*f - b*(e + Sqrt[e^2 - 4*d*f]) - (-2*b*f + 2*c*(e + Sqrt[e
^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + c*e*Sq
rt[e^2 - 4*d*f] - b*f*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2]])])/(d*(4*a*f
^2 - 2*b*f*(e + Sqrt[e^2 - 4*d*f]) + c*(e + Sqrt[e^2 - 4*d*f])^2)*(16*a*f^2
 - 8*b*f*(e + Sqrt[e^2 - 4*d*f]) + 4*c*(e + Sqrt[e^2 - 4*d*f])^2)*(a + x*(b
 + c*x))^(3/2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [B] time = 0.04, size = 4594, normalized size = 5.63

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x)
```

```
[Out] 4*f^3/(e+(-4*d*f+e^2)^(1/2))/(-4*d*f+e^2)^(1/2)/(2*a*f^2-b*e*f-2*c*d*f+c*e^
2-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e)/((x+1/2*(e+(-4*d*f+e^2)^(1
/2))/f)^2*c+(b*f-c*e-(-4*d*f+e^2)^(1/2)*c)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)
/f+1/2*(2*a*f^2-b*e*f-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/
2)*c*e)/f^2)^(1/2)+8*f^2/(e+(-4*d*f+e^2)^(1/2))/(2*a*f^2-b*e*f-2*c*d*f+c*e^
2-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e)/(4*a*c-4*c^2*d/f+c^2*e^2/f
^2-(-4*d*f+e^2)*c^2/f^2-b^2)/((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c+(b*f-c*e
-(-4*d*f+e^2)^(1/2)*c)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)/f+1/2*(2*a*f^2-b*e*
f-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*x
*c^2-8*f^3/(e+(-4*d*f+e^2)^(1/2))/(-4*d*f+e^2)^(1/2)/(2*a*f^2-b*e*f-2*c*d*f
+c*e^2-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e)/(4*a*c-4*c^2*d/f+c^2*
e^2/f^2-(-4*d*f+e^2)*c^2/f^2-b^2)/((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c+(b*
f-c*e-(-4*d*f+e^2)^(1/2)*c)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)/f+1/2*(2*a*f^2
-b*e*f-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1
/2)*x*b*c+8*f^2/(e+(-4*d*f+e^2)^(1/2))/(-4*d*f+e^2)^(1/2)/(2*a*f^2-b*e*f-2*
c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e)/(4*a*c-4*c^2*d/f
+c^2*e^2/f^2-(-4*d*f+e^2)*c^2/f^2-b^2)/((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*
c+(b*f-c*e-(-4*d*f+e^2)^(1/2)*c)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)/f+1/2*(2*
a*f^2-b*e*f-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e)/f^
2)^(1/2)*x*c^2*e+4*f^2/(e+(-4*d*f+e^2)^(1/2))/(2*a*f^2-b*e*f-2*c*d*f+c*e^2-
(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e)/(4*a*c-4*c^2*d/f+c^2*e^2/f^2
-(-4*d*f+e^2)*c^2/f^2-b^2)/((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c+(b*f-c*e-(
-4*d*f+e^2)^(1/2)*c)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)/f+1/2*(2*a*f^2-b*e*f-
2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*b*c
```

```
-4*f^3/(e+(-4*d*f+e^2)^(1/2))/(-4*d*f+e^2)^(1/2)/(2*a*f^2-b*e*f-2*c*d*f+c*e
^2-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e)/(4*a*c-4*c^2*d/f+c^2*e^2/
f^2-(-4*d*f+e^2)*c^2/f^2-b^2)/((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c+(b*f-c*
e-(-4*d*f+e^2)^(1/2)*c)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)/f+1/2*(2*a*f^2-b*e
*f-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*
b^2+4*f^2/(e+(-4*d*f+e^2)^(1/2))/(-4*d*f+e^2)^(1/2)/(2*a*f^2-b*e*f-2*c*d*f+
c*e^2-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e)/(4*a*c-4*c^2*d/f+c^2*e
^2/f^2-(-4*d*f+e^2)*c^2/f^2-b^2)/((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c+(b*f
-c*e-(-4*d*f+e^2)^(1/2)*c)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)/f+1/2*(2*a*f^2-
b*e*f-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/
2)*b*c*e-4*f^3/(e+(-4*d*f+e^2)^(1/2))/(-4*d*f+e^2)^(1/2)/(2*a*f^2-b*e*f-2*c
*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e)*2^(1/2)/((2*a*f^2
-b*e*f-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1
/2)*ln(((b*f-c*e-(-4*d*f+e^2)^(1/2)*c)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)/f+(
2*a*f^2-b*e*f-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e)/
f^2+1/2*2^(1/2)*((2*a*f^2-b*e*f-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*
f+e^2)^(1/2)*c*e)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c+4*(b*f
-c*e-(-4*d*f+e^2)^(1/2)*c)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)/f+2*(2*a*f^2-b*
e*f-2*c*d*f+c*e^2-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)
)/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))+4*f^3/(-e+(-4*d*f+e^2)^(1/2))/(-4*d*f+e
^2)^(1/2)/(2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^
(1/2)*c*e)/((x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c+(b*f-c*e+(-4*d*f+e^2)^(1/
2)*c)*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)/f+1/2*(2*a*f^2-b*e*f-2*c*d*f+c*e^2+
(-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)-8*f^2/(-e+(-4*d*f
+e^2)^(1/2))/(2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^
2)^(1/2)*c*e)/(4*a*c-4*c^2*d/f+c^2*e^2/f^2-(-4*d*f+e^2)*c^2/f^2-b^2)/((x-1/
2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c+(b*f-c*e+(-4*d*f+e^2)^(1/2)*c)*(x-1/2*(-e+
(-4*d*f+e^2)^(1/2))/f)/f+1/2*(2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2
)*b*f-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*x*c^2-8*f^3/(-e+(-4*d*f+e^2)^(1/2)
)/(-4*d*f+e^2)^(1/2)/(2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*b*f-(-
4*d*f+e^2)^(1/2)*c*e)/(4*a*c-4*c^2*d/f+c^2*e^2/f^2-(-4*d*f+e^2)*c^2/f^2-b^2
)/((x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c+(b*f-c*e+(-4*d*f+e^2)^(1/2)*c)*(x-
1/2*(-e+(-4*d*f+e^2)^(1/2))/f)/f+1/2*(2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e
^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*x*b*c+8*f^2/(-e+(-4*d*f+e^
2)^(1/2))/(-4*d*f+e^2)^(1/2)/(2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2
)*b*f-(-4*d*f+e^2)^(1/2)*c*e)/(4*a*c-4*c^2*d/f+c^2*e^2/f^2-(-4*d*f+e^2)*c^2
/f^2-b^2)/((x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c+(b*f-c*e+(-4*d*f+e^2)^(1/2
)*c)*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)/f+1/2*(2*a*f^2-b*e*f-2*c*d*f+c*e^2+(
-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*x*c^2*e-4*f^2/(-e+
(-4*d*f+e^2)^(1/2))/(2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*b*f-(-4
*d*f+e^2)^(1/2)*c*e)/(4*a*c-4*c^2*d/f+c^2*e^2/f^2-(-4*d*f+e^2)*c^2/f^2-b^2)
/((x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c+(b*f-c*e+(-4*d*f+e^2)^(1/2)*c)*(x-1
/2*(-e+(-4*d*f+e^2)^(1/2))/f)/f+1/2*(2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^
2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*b*c-4*f^3/(-e+(-4*d*f+e^2)^
(1/2))/(-4*d*f+e^2)^(1/2)/(2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*b
*f-(-4*d*f+e^2)^(1/2)*c*e)/(4*a*c-4*c^2*d/f+c^2*e^2/f^2-(-4*d*f+e^2)*c^2/f^
2-b^2)/((x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c+(b*f-c*e+(-4*d*f+e^2)^(1/2)*c
)*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)/f+1/2*(2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*
d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*b^2+4*f^2/(-e+(-4*d*f
+e^2)^(1/2))/(-4*d*f+e^2)^(1/2)/(2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^(
1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e)/(4*a*c-4*c^2*d/f+c^2*e^2/f^2-(-4*d*f+e^2)*
c^2/f^2-b^2)/((x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c+(b*f-c*e+(-4*d*f+e^2)^(
1/2)*c)*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)/f+1/2*(2*a*f^2-b*e*f-2*c*d*f+c*e^
2+(-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*b*c*e-4*f^3/(-e
+(-4*d*f+e^2)^(1/2))/(-4*d*f+e^2)^(1/2)/(2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*
f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e)*2^(1/2)/((2*a*f^2-b*e*f-2*c*d*f+c*
e^2+(-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2)*ln(((b*f-c*e+
(-4*d*f+e^2)^(1/2)*c)*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)/f+(2*a*f^2-b*e*f-2*
c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e)/f^2+1/2*2^(1/2)*
```

```
((2*a*f^2-b*e*f-2*c*d*f+c*e^2+(-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e
)/f^2)^(1/2)*(4*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c+4*(b*f-c*e+(-4*d*f+e^
2)^(1/2)*c)*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)/f+2*(2*a*f^2-b*e*f-2*c*d*f+c*
e^2+(-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e)/f^2)^(1/2))/(x-1/2*(-e+(
-4*d*f+e^2)^(1/2))/f))-4*f/(-e+(-4*d*f+e^2)^(1/2))/(e+(-4*d*f+e^2)^(1/2))/a
/(c*x^2+b*x+a)^(1/2)+8*f/(-e+(-4*d*f+e^2)^(1/2))/(e+(-4*d*f+e^2)^(1/2))*b/a
/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*c*x+4*f/(-e+(-4*d*f+e^2)^(1/2))/(e+(-4*d*f
+e^2)^(1/2))*b^2/a/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)+4*f/(-e+(-4*d*f+e^2)^(1/
2))/(e+(-4*d*f+e^2)^(1/2))/a^(3/2)*ln((b*x+2*a+2*(c*x^2+b*x+a)^(1/2)*a^(1/2
))/x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(cx^2+bx+a\right)^{\frac{3}{2}}\left(fx^2+ex+d\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="maxima")
[Out] integrate(1/((c*x^2 + b*x + a)^(3/2)*(f*x^2 + e*x + d)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x\left(c\,x^2+b\,x+a\right)^{3/2}\,\left(f\,x^2+e\,x+d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)),x)
[Out] int(1/(x*(a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(a+bx+cx^2\right)^{\frac{3}{2}}\left(d+ex+fx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x**2+b*x+a)**(3/2)/(f*x**2+e*x+d),x)
[Out] Integral(1/(x*(a + b*x + c*x**2)**(3/2)*(d + e*x + f*x**2)), x)
```

3.126 $\int \frac{x^4}{\sqrt{-3-4x-x^2}\left(3+4x+2x^2\right)}\,dx$

Optimal. Leaf size=140

$$-\frac{1}{4}\sqrt{-x^2-4x-3}\,x+\frac{5}{2}\sqrt{-x^2-4x-3}+\frac{\tan^{-1}\left(\frac{1-\frac{x+3}{\sqrt{-x^2-4x-3}}}{\sqrt{2}}\right)}{2\sqrt{2}}-\frac{\tan^{-1}\left(\frac{\frac{x+3}{\sqrt{-x^2-4x-3}}+1}{\sqrt{2}}\right)}{2\sqrt{2}}-\frac{5}{4}\tanh^{-1}\left(\frac{x}{\sqrt{-x^2-4x-3}}\right)+\frac{1}{2}$$

```
[Out] 11/2*arcsin(2+x)-5/4*arctanh(x/(-x^2-4*x-3)^(1/2))+1/4*arctan(1/2*(1+(-3-x)
/(-x^2-4*x-3)^(1/2))*2^(1/2))*2^(1/2)-1/4*arctan(1/2*(1+(3+x)/(-x^2-4*x-3)^
(1/2))*2^(1/2))*2^(1/2)+5/2*(-x^2-4*x-3)^(1/2)-1/4*x*(-x^2-4*x-3)^(1/2)
```

Rubi [A] time = 0.50, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 14, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {6728, 619, 216, 640, 742, 1028, 986, 12, 1026, 1161, 618, 204, 1027, 206}

$$-\frac{1}{4}\sqrt{-x^2-4x-3}\,x+\frac{5}{2}\sqrt{-x^2-4x-3}+\frac{\tan^{-1}\left(\frac{1-\frac{x+3}{\sqrt{-x^2-4x-3}}}{\sqrt{2}}\right)}{2\sqrt{2}}-\frac{\tan^{-1}\left(\frac{\frac{x+3}{\sqrt{-x^2-4x-3}}+1}{\sqrt{2}}\right)}{2\sqrt{2}}-\frac{5}{4}\tanh^{-1}\left(\frac{x}{\sqrt{-x^2-4x-3}}\right)+\frac{1}{2}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(Sqrt[-3 - 4*x - x^2]*(3 + 4*x + 2*x^2)),x]
```

```
[Out] (5*Sqrt[-3 - 4*x - x^2])/2 - (x*Sqrt[-3 - 4*x - x^2])/4 + (11*ArcSin[2 + x]
)/2 + ArcTan[(1 - (3 + x)/Sqrt[-3 - 4*x - x^2])/Sqrt[2]]/(2*Sqrt[2]) - ArcT
an[(1 + (3 + x)/Sqrt[-3 - 4*x - x^2])/Sqrt[2]]/(2*Sqrt[2]) - (5*ArcTanh[x/S
qrt[-3 - 4*x - x^2]])/4
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 742

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
raticQ[a, b, c, d, e, m, p, x]
```

Rule 986

```
Int[1/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*
(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f)
, 2]}, Dist[1/(2*q), Int[(c*d - a*f + q + (c*e - b*f)*x)/((a + b*x + c*x^2)
*Sqrt[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[(c*d - a*f - q + (c*e -
 b*f)*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b,
 c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[c*e -
 b*f, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1026

```
Int[(x_)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (e_.)*(x_) + (f_.)
*(x_)^2]), x_Symbol] :> Dist[-2*e, Subst[Int[(1 - d*x^2)/(c*e - b*f - e*(2*
c*d - b*e + 2*a*f)*x^2 + d^2*(c*e - b*f)*x^4), x], x, (1 + ((e + Sqrt[e^2 -
 4*d*f])*x)/(2*d))/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f},
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[b*d - a*e, 0]
```

Rule 1027

```
Int[((g_) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (e_
.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[g, Subst[Int[1/(a + (c*d - a*f)
*x^2), x], x, x/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h
}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[b*d - a*e, 0] &&
 EqQ[2*h*d - g*e, 0]
```

Rule 1028

```
Int[((g_) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (e_
.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> -Dist[(2*h*d - g*e)/e, Int[1/((a + b
*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/e, Int[(2*d + e*x)/((a
+ b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, g
, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[b*d - a*e, 0]
 && NeQ[2*h*d - g*e, 0]
```

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{x^4}{\sqrt{-3-4x-x^2}\left(3+4x+2x^2\right)}dx = \int\left(\frac{5}{4\sqrt{-3-4x-x^2}} - \frac{x}{\sqrt{-3-4x-x^2}} + \frac{x^2}{2\sqrt{-3-4x-x^2}} - \frac{}{4\sqrt{-3-4x}}\right.$$

$$= -\left(\frac{1}{4}\int \frac{15+8x}{\sqrt{-3-4x-x^2}\left(3+4x+2x^2\right)}dx\right) + \frac{1}{2}\int \frac{x^2}{\sqrt{-3-4x-x^2}}dx + \frac{5}{4}$$

$$= \sqrt{-3-4x-x^2} - \frac{1}{4}x\sqrt{-3-4x-x^2} - \frac{1}{4}\int \frac{3+6x}{\sqrt{-3-4x-x^2}}dx + \frac{1}{2}\int \frac{}{\sqrt{-}}$$

$$= \frac{5}{2}\sqrt{-3-4x-x^2} - \frac{1}{4}x\sqrt{-3-4x-x^2} + \frac{5}{4}\sin^{-1}(2+x) + \frac{1}{8}\int \frac{}{\sqrt{-3-4x}}$$

$$= \frac{5}{2}\sqrt{-3-4x-x^2} - \frac{1}{4}x\sqrt{-3-4x-x^2} + \frac{13}{4}\sin^{-1}(2+x) - \tanh^{-1}\left(\frac{}{\sqrt{-3-}}\right.$$

$$= \frac{5}{2}\sqrt{-3-4x-x^2} - \frac{1}{4}x\sqrt{-3-4x-x^2} + \frac{11}{2}\sin^{-1}(2+x) - \frac{5}{4}\tanh^{-1}\left(\frac{}{\sqrt{-3}}\right.$$

$$= \frac{5}{2}\sqrt{-3-4x-x^2} - \frac{1}{4}x\sqrt{-3-4x-x^2} + \frac{11}{2}\sin^{-1}(2+x) - \frac{5}{4}\tanh^{-1}\left(\frac{}{\sqrt{-3}}\right.$$

$$= \frac{5}{2}\sqrt{-3-4x-x^2} - \frac{1}{4}x\sqrt{-3-4x-x^2} + \frac{11}{2}\sin^{-1}(2+x) - \frac{5}{4}\tanh^{-1}\left(\frac{}{\sqrt{-3}}\right.$$

$$= \frac{5}{2}\sqrt{-3-4x-x^2} - \frac{1}{4}x\sqrt{-3-4x-x^2} + \frac{11}{2}\sin^{-1}(2+x) + \frac{\tan^{-1}\left(\frac{1-\frac{3+}{\sqrt{-3-4}}}{\sqrt{2}}\right.}{2\sqrt{2}}$$

Mathematica [C] time = 0.53, size = 210, normalized size = 1.50

$$\frac{1}{24}\left(-6\sqrt{-x^2-4x-3}\,x + 60\sqrt{-x^2-4x-3} - \sqrt{1-2i\sqrt{2}}\left(4\sqrt{2}+7i\right)\tanh^{-1}\left(\frac{-i\sqrt{2}\,x+2x-2i\sqrt{2}+2}{\sqrt{2+4i\sqrt{2}}\,\sqrt{-x^2-4x-3}}\right) - \sqrt{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(Sqrt[-3 - 4*x - x^2]*(3 + 4*x + 2*x^2)),x]
```

```
[Out] (60*Sqrt[-3 - 4*x - x^2] - 6*x*Sqrt[-3 - 4*x - x^2] + 132*ArcSin[2 + x] - S
qrt[1 - (2*I)*Sqrt[2]]*(7*I + 4*Sqrt[2])*ArcTanh[(2 - (2*I)*Sqrt[2] + 2*x -
 I*Sqrt[2]*x)/(Sqrt[2 + (4*I)*Sqrt[2]]*Sqrt[-3 - 4*x - x^2])] - Sqrt[1 + (2
*I)*Sqrt[2]]*(-7*I + 4*Sqrt[2])*ArcTanh[(2 + (2*I)*Sqrt[2] + (2 + I*Sqrt[2]
)*x)/(Sqrt[2 - (4*I)*Sqrt[2]]*Sqrt[-3 - 4*x - x^2])])/24
```

fricas [A] time = 0.99, size = 178, normalized size = 1.27

$$-\frac{1}{4}\sqrt{-x^2-4x-3}\,(x-10)+\frac{1}{8}\sqrt{2}\arctan\left(\frac{\sqrt{2}x+3\sqrt{2}\sqrt{-x^2-4x-3}}{2(2x+3)}\right)+\frac{1}{8}\sqrt{2}\arctan\left(-\frac{\sqrt{2}x-3\sqrt{2}\sqrt{-x^2}}{2(2x+3)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(-x^2 - 4*x - 3)*(x - 10) + 1/8*sqrt(2)*arctan(1/2*(sqrt(2)*x + 3*
sqrt(2)*sqrt(-x^2 - 4*x - 3))/(2*x + 3)) + 1/8*sqrt(2)*arctan(-1/2*(sqrt(2)
*x - 3*sqrt(2)*sqrt(-x^2 - 4*x - 3))/(2*x + 3)) - 11/2*arctan(sqrt(-x^2 - 4
*x - 3)*(x + 2)/(x^2 + 4*x + 3)) + 5/16*log(-(2*sqrt(-x^2 - 4*x - 3)*x + 4*
x + 3)/x^2) - 5/16*log((2*sqrt(-x^2 - 4*x - 3)*x - 4*x - 3)/x^2)
```

giac [A] time = 0.20, size = 188, normalized size = 1.34

$$-\frac{1}{4}\sqrt{-x^2-4x-3}\,(x-10)+\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\frac{3\left(\sqrt{-x^2-4x-3}-1\right)}{x+2}+1\right)\right)+\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\frac{\sqrt{-x^2-}}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(-x^2 - 4*x - 3)*(x - 10) + 1/4*sqrt(2)*arctan(1/2*sqrt(2)*(3*(sqr
t(-x^2 - 4*x - 3) - 1)/(x + 2) + 1)) + 1/4*sqrt(2)*arctan(1/2*sqrt(2)*((sqr
t(-x^2 - 4*x - 3) - 1)/(x + 2) + 1)) + 11/2*arcsin(x + 2) - 5/8*log(2*(sqrt
(-x^2 - 4*x - 3) - 1)/(x + 2) + 3*(sqrt(-x^2 - 4*x - 3) - 1)^2/(x + 2)^2 +
1) + 5/8*log(2*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + (sqrt(-x^2 - 4*x - 3) -
 1)^2/(x + 2)^2 + 3)
```

maple [A] time = 0.02, size = 159, normalized size = 1.14

$$-\frac{\sqrt{-x^2-4x-3}\,x}{4}+\frac{11\arcsin(x+2)}{2}+\frac{5\sqrt{-x^2-4x-3}}{2}+\frac{\sqrt{3}\,\sqrt{4}\,\sqrt{\frac{3x^2}{\left(-x-\frac{3}{2}\right)^2}-12}\left(5\,\mathrm{arctanh}\left(\frac{3x}{\left(-x-\frac{3}{2}\right)\sqrt{\frac{3x^2}{\left(-x-\frac{3}{2}\right)^2}-}}\right.\right.}{24\sqrt{\frac{\frac{x^2}{\left(-x-\frac{3}{2}\right)^2}-4}{\left(\frac{x}{-x-\frac{3}{2}}+1\right)^2}}\left(\frac{x}{-x-\frac{3}{2}}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x)
```

```
[Out] -1/4*x*(-x^2-4*x-3)^(1/2)+5/2*(-x^2-4*x-3)^(1/2)+11/2*arcsin(2+x)+1/24*3^(1
/2)*4^(1/2)*(3/(-x-3/2)^2*x^2-12)^(1/2)*(2^(1/2)*arctan(1/6*(3/(-x-3/2)^2*x
^2-12)^(1/2)*2^(1/2))+5*arctanh(3/(-x-3/2)/(3/(-x-3/2)^2*x^2-12)^(1/2)*x))/
((1/(-x-3/2)^2*x^2-4)/(1/(-x-3/2)*x+1)^2)^(1/2)/(1/(-x-3/2)*x+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(2x^2+4x+3\right)\sqrt{-x^2-4x-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^4/((2*x^2 + 4*x + 3)*sqrt(-x^2 - 4*x - 3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4}{\sqrt{-x^2-4\,x-3}\,\left(2\,x^2+4\,x+3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((- 4*x - x^2 - 3)^(1/2)*(4*x + 2*x^2 + 3)),x)
```

```
[Out] int(x^4/((- 4*x - x^2 - 3)^(1/2)*(4*x + 2*x^2 + 3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\sqrt{-(x+1)(x+3)}\left(2x^2+4x+3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(2*x**2+4*x+3)/(-x**2-4*x-3)**(1/2),x)
```

```
[Out] Integral(x**4/(sqrt(-(x + 1)*(x + 3))*(2*x**2 + 4*x + 3)), x)
```

3.127 $\int \frac{x^3}{\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\,dx$

Optimal. Leaf size=115

$$-\frac{1}{2}\sqrt{-x^2-4x-3}+\frac{\tan^{-1}\left(\frac{1-\frac{x+3}{\sqrt{-x^2-4x-3}}}{\sqrt{2}}\right)}{2\sqrt{2}}-\frac{\tan^{-1}\left(\frac{\frac{x+3}{\sqrt{-x^2-4x-3}}+1}{\sqrt{2}}\right)}{2\sqrt{2}}+\tanh^{-1}\left(\frac{x}{\sqrt{-x^2-4x-3}}\right)-2\sin^{-1}(x+2)$$

```
[Out] -2*arcsin(2+x)+arctanh(x/(-x^2-4*x-3)^(1/2))+1/4*arctan(1/2*(1+(-3-x)/(-x^2
-4*x-3)^(1/2))*2^(1/2))*2^(1/2)-1/4*arctan(1/2*(1+(3+x)/(-x^2-4*x-3)^(1/2))
*2^(1/2))*2^(1/2)-1/2*(-x^2-4*x-3)^(1/2)
```

Rubi [A] time = 0.42, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 13, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.433, Rules used = {6728, 619, 216, 640, 1028, 986, 12, 1026, 1161, 618, 204, 1027, 206}

$$-\frac{1}{2}\sqrt{-x^2-4x-3}+\frac{\tan^{-1}\left(\frac{1-\frac{x+3}{\sqrt{-x^2-4x-3}}}{\sqrt{2}}\right)}{2\sqrt{2}}-\frac{\tan^{-1}\left(\frac{\frac{x+3}{\sqrt{-x^2-4x-3}}+1}{\sqrt{2}}\right)}{2\sqrt{2}}+\tanh^{-1}\left(\frac{x}{\sqrt{-x^2-4x-3}}\right)-2\sin^{-1}(x+2)$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(Sqrt[-3 - 4*x - x^2]*(3 + 4*x + 2*x^2)),x]
```

```
[Out] -Sqrt[-3 - 4*x - x^2]/2 - 2*ArcSin[2 + x] + ArcTan[(1 - (3 + x)/Sqrt[-3 - 4
*x - x^2])/Sqrt[2]]/(2*Sqrt[2]) - ArcTan[(1 + (3 + x)/Sqrt[-3 - 4*x - x^2])
/Sqrt[2]]/(2*Sqrt[2]) + ArcTanh[x/Sqrt[-3 - 4*x - x^2]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 986

```
Int[1/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*
(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f)
, 2]}, Dist[1/(2*q), Int[(c*d - a*f + q + (c*e - b*f)*x)/((a + b*x + c*x^2)
*Sqrt[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[(c*d - a*f - q + (c*e -
 b*f)*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b,
 c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[c*e -
 b*f, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1026

```
Int[(x_)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (e_.)*(x_) + (f_.)
*(x_)^2]), x_Symbol] :> Dist[-2*e, Subst[Int[(1 - d*x^2)/(c*e - b*f - e*(2*
c*d - b*e + 2*a*f)*x^2 + d^2*(c*e - b*f)*x^4), x], x, (1 + ((e + Sqrt[e^2 -
 4*d*f])*x)/(2*d))/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f},
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[b*d - a*e, 0]
```

Rule 1027

```
Int[((g_) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (e_
.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[g, Subst[Int[1/(a + (c*d - a*f)
*x^2), x], x, x/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h
}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[b*d - a*e, 0] &&
 EqQ[2*h*d - g*e, 0]
```

Rule 1028

```
Int[((g_) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (e_
.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> -Dist[(2*h*d - g*e)/e, Int[1/((a + b
*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/e, Int[(2*d + e*x)/((a
+ b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, g
, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[b*d - a*e, 0]
 && NeQ[2*h*d - g*e, 0]
```

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
```

```
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{x^3}{\sqrt{-3-4x-x^2}\left(3+4x+2x^2\right)}\,dx = \int \left(-\frac{1}{\sqrt{-3-4x-x^2}} + \frac{x}{2\sqrt{-3-4x-x^2}} + \frac{6+5x}{2\sqrt{-3-4x-x^2}\left(3+4x+\right.}\right.$$

$$= \frac{1}{2}\int \frac{x}{\sqrt{-3-4x-x^2}}\,dx + \frac{1}{2}\int \frac{6+5x}{\sqrt{-3-4x-x^2}\left(3+4x+2x^2\right)}\,dx - \int$$

$$= -\frac{1}{2}\sqrt{-3-4x-x^2} + \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{4}}}\,dx, x, -4-2x\right) - \frac{5}{8}\int \frac{}{\sqrt{-3}}$$

$$= -\frac{1}{2}\sqrt{-3-4x-x^2} - \sin^{-1}(2+x) + \frac{1}{8}\int \frac{-6-4x}{\sqrt{-3-4x-x^2}\left(3+4x+2x^2\right.}$$

$$= -\frac{1}{2}\sqrt{-3-4x-x^2} - 2\sin^{-1}(2+x) + \frac{5}{4}\tanh^{-1}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right) + \frac{1}{2}$$

$$= -\frac{1}{2}\sqrt{-3-4x-x^2} - 2\sin^{-1}(2+x) + \tanh^{-1}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right) + 4\,\text{Su}$$

$$= -\frac{1}{2}\sqrt{-3-4x-x^2} - 2\sin^{-1}(2+x) + \tanh^{-1}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right) - \frac{1}{6}\text{Su}$$

$$= -\frac{1}{2}\sqrt{-3-4x-x^2} - 2\sin^{-1}(2+x) + \tanh^{-1}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right) + \frac{1}{3}\text{Su}$$

$$= -\frac{1}{2}\sqrt{-3-4x-x^2} - 2\sin^{-1}(2+x) + \frac{\tan^{-1}\left(\frac{1-\frac{3+x}{\sqrt{-3-4x-x^2}}}{\sqrt{2}}\right)}{2\sqrt{2}} - \frac{\tan^{-1}\left(\frac{1+}{}\right.}{2\sqrt{}}$$

Mathematica [C] time = 0.44, size = 192, normalized size = 1.67

$$\frac{1}{8}\left(-4\left(\sqrt{-x^2-4x-3} + 4\sin^{-1}(x+2)\right) + \frac{\left(5\sqrt{2}-2i\right)\tanh^{-1}\left(\frac{i\sqrt{2}x+2x+2i\sqrt{2}+2}{\sqrt{2-4i\sqrt{2}}\sqrt{-x^2-4x-3}}\right)}{\sqrt{1-2i\sqrt{2}}} + \frac{\left(5\sqrt{2}+2i\right)\tanh^{-1}\left(\frac{(2}{\sqrt{2}}\right.}{\sqrt{1+2i\sqrt{}}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(Sqrt[-3 - 4*x - x^2]*(3 + 4*x + 2*x^2)),x]
```

```
[Out] (-4*(Sqrt[-3 - 4*x - x^2] + 4*ArcSin[2 + x]) + ((-2*I + 5*Sqrt[2])*ArcTanh[
(2 + (2*I)*Sqrt[2] + 2*x + I*Sqrt[2]*x)/(Sqrt[2 - (4*I)*Sqrt[2]]*Sqrt[-3 -
4*x - x^2])])/Sqrt[1 - (2*I)*Sqrt[2]] + ((2*I + 5*Sqrt[2])*ArcTanh[(2 - (2*
I)*Sqrt[2] + (2 - I*Sqrt[2])*x)/(Sqrt[2 + (4*I)*Sqrt[2]]*Sqrt[-3 - 4*x - x^
2])])/Sqrt[1 + (2*I)*Sqrt[2]])/8
```

fricas [A] time = 1.50, size = 175, normalized size = 1.52

$$\frac{1}{8}\sqrt{2}\arctan\left(\frac{\sqrt{2}x+3\sqrt{2}\sqrt{-x^2-4x-3}}{2(2x+3)}\right) + \frac{1}{8}\sqrt{2}\arctan\left(-\frac{\sqrt{2}x-3\sqrt{2}\sqrt{-x^2-4x-3}}{2(2x+3)}\right) - \frac{1}{2}\sqrt{-x^2-4x-3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/8*sqrt(2)*arctan(1/2*(sqrt(2)*x + 3*sqrt(2)*sqrt(-x^2 - 4*x - 3))/(2*x +
3)) + 1/8*sqrt(2)*arctan(-1/2*(sqrt(2)*x - 3*sqrt(2)*sqrt(-x^2 - 4*x - 3))/
(2*x + 3)) - 1/2*sqrt(-x^2 - 4*x - 3) + 2*arctan(sqrt(-x^2 - 4*x - 3)*(x +
2)/(x^2 + 4*x + 3)) - 1/4*log(-(2*sqrt(-x^2 - 4*x - 3)*x + 4*x + 3)/x^2) +
1/4*log((2*sqrt(-x^2 - 4*x - 3)*x - 4*x - 3)/x^2)
```

giac [A] time = 0.25, size = 185, normalized size = 1.61

$$\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\frac{3\left(\sqrt{-x^2-4x-3}-1\right)}{x+2}+1\right)\right)+\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\frac{\sqrt{-x^2-4x-3}-1}{x+2}+1\right)\right)-\frac{1}{2}\sqrt{-x^2-4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*arctan(1/2*sqrt(2)*(3*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 1))
+ 1/4*sqrt(2)*arctan(1/2*sqrt(2)*((sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 1))
- 1/2*sqrt(-x^2 - 4*x - 3) - 2*arcsin(x + 2) + 1/2*log(2*(sqrt(-x^2 - 4*x -
 3) - 1)/(x + 2) + 3*(sqrt(-x^2 - 4*x - 3) - 1)^2/(x + 2)^2 + 1) - 1/2*log(
2*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + (sqrt(-x^2 - 4*x - 3) - 1)^2/(x + 2)
^2 + 3)
```

maple [A] time = 0.02, size = 144, normalized size = 1.25

$$-2\arcsin(x+2)-\frac{\sqrt{-x^2-4x-3}}{2}+\frac{\sqrt{3}\,\sqrt{4}\,\sqrt{\frac{3x^2}{\left(-x-\frac{3}{2}\right)^2}-12}\,\left(-4\,\mathrm{arctanh}\left(\frac{3x}{\left(-x-\frac{3}{2}\right)\sqrt{\frac{3x^2}{\left(-x-\frac{3}{2}\right)^2}-12}}\right)+\sqrt{2}\,\arctan\left(\cdots\right)\right)}{24\sqrt{\frac{\frac{x^2}{\left(-x-\frac{3}{2}\right)^2}-4}{\left(\frac{x}{-x-\frac{3}{2}}+1\right)^2}}\,\left(\frac{x}{-x-\frac{3}{2}}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x)
```

```
[Out] -1/2*(-x^2-4*x-3)^(1/2)-2*arcsin(x+2)+1/24*3^(1/2)*4^(1/2)*(3/(-x-3/2)^2*x^
2-12)^(1/2)*(2^(1/2)*arctan(1/6*(3/(-x-3/2)^2*x^2-12)^(1/2)*2^(1/2))-4*arct
anh(3/(-x-3/2)/(3/(-x-3/2)^2*x^2-12)^(1/2)*x))/((1/(-x-3/2)^2*x^2-4)/(1/(-x
-3/2)*x+1)^2)^(1/2)/(1/(-x-3/2)*x+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(2x^2+4x+3\right)\sqrt{-x^2-4x-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3/((2*x^2 + 4*x + 3)*sqrt(-x^2 - 4*x - 3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3}{\sqrt{-x^2-4x-3}\,\left(2x^2+4x+3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((- 4*x - x^2 - 3)^(1/2)*(4*x + 2*x^2 + 3)),x)
[Out] int(x^3/((- 4*x - x^2 - 3)^(1/2)*(4*x + 2*x^2 + 3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{-(x+1)(x+3)}\,(2x^2+4x+3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(2*x**2+4*x+3)/(-x**2-4*x-3)**(1/2),x)
[Out] Integral(x**3/(sqrt(-(x + 1)*(x + 3))*(2*x**2 + 4*x + 3)), x)
```

3.128 $\int \frac{x^2}{\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\,dx$

Optimal. Leaf size=98

$$-\frac{\tan^{-1}\left(\frac{1-\frac{x+3}{\sqrt{-x^2-4x-3}}}{\sqrt{2}}\right)}{\sqrt{2}}+\frac{\tan^{-1}\left(\frac{\frac{x+3}{\sqrt{-x^2-4x-3}}+1}{\sqrt{2}}\right)}{\sqrt{2}}-\frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt{-x^2-4x-3}}\right)+\frac{1}{2}\sin^{-1}(x+2)$$

```
[Out] 1/2*arcsin(2+x)-1/2*arctanh(x/(-x^2-4*x-3)^(1/2))-1/2*arctan(1/2*(1+(-3-x)/
(-x^2-4*x-3)^(1/2))*2^(1/2))*2^(1/2)+1/2*arctan(1/2*(1+(3+x)/(-x^2-4*x-3)^(
1/2))*2^(1/2))*2^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 12, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {1077, 619, 216, 1028, 986, 12, 1026, 1161, 618, 204, 1027, 206}

$$-\frac{\tan^{-1}\left(\frac{1-\frac{x+3}{\sqrt{-x^2-4x-3}}}{\sqrt{2}}\right)}{\sqrt{2}}+\frac{\tan^{-1}\left(\frac{\frac{x+3}{\sqrt{-x^2-4x-3}}+1}{\sqrt{2}}\right)}{\sqrt{2}}-\frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt{-x^2-4x-3}}\right)+\frac{1}{2}\sin^{-1}(x+2)$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(Sqrt[-3 - 4*x - x^2]*(3 + 4*x + 2*x^2)),x]
```

```
[Out] ArcSin[2 + x]/2 - ArcTan[(1 - (3 + x)/Sqrt[-3 - 4*x - x^2])/Sqrt[2]]/Sqrt[2
] + ArcTan[(1 + (3 + x)/Sqrt[-3 - 4*x - x^2])/Sqrt[2]]/Sqrt[2] - ArcTanh[x/
Sqrt[-3 - 4*x - x^2]]/2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 986

```
Int[1/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*
(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f)
, 2]}, Dist[1/(2*q), Int[(c*d - a*f + q + (c*e - b*f)*x)/((a + b*x + c*x^2)
*Sqrt[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[(c*d - a*f - q + (c*e -
 b*f)*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b,
 c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[c*e -
 b*f, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1026

```
Int[(x_)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (e_.)*(x_) + (f_.)
*(x_)^2]), x_Symbol] :> Dist[-2*e, Subst[Int[(1 - d*x^2)/(c*e - b*f - e*(2*
c*d - b*e + 2*a*f)*x^2 + d^2*(c*e - b*f)*x^4), x], x, (1 + ((e + Sqrt[e^2 -
 4*d*f])*x)/(2*d))/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f},
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[b*d - a*e, 0]
```

Rule 1027

```
Int[((g_) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (e_
.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[g, Subst[Int[1/(a + (c*d - a*f)
*x^2), x], x, x/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h
}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[b*d - a*e, 0] &&
 EqQ[2*h*d - g*e, 0]
```

Rule 1028

```
Int[((g_) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (e_
.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> -Dist[(2*h*d - g*e)/e, Int[1/((a + b
*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/e, Int[(2*d + e*x)/((a
+ b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, g
, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[b*d - a*e, 0]
 && NeQ[2*h*d - g*e, 0]
```

Rule 1077

```
Int[((A_.) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
 (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqrt[d + e*x + f
*x^2], x], x] + Dist[1/c, Int[(A*c - a*C - b*C*x)/((a + b*x + c*x^2)*Sqrt[d
 + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]
```

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rubi steps

$$\int \frac{x^2}{\sqrt{-3-4x-x^2}\left(3+4x+2x^2\right)}\,dx = \frac{1}{2}\int \frac{1}{\sqrt{-3-4x-x^2}}\,dx + \frac{1}{2}\int \frac{-3-4x}{\sqrt{-3-4x-x^2}\left(3+4x+2x^2\right)}\,dx$$

$$= -\left(\frac{1}{4}\,\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{4}}}\,dx, x, -4-2x\right)\right) + \frac{1}{2}\int \frac{-6-4x}{\sqrt{-3-4x-x^2}\left(3+4x\right.}$$

$$= \frac{1}{2}\sin^{-1}(2+x) - \frac{1}{4}\int \frac{-6-4x}{\sqrt{-3-4x-x^2}\left(3+4x+2x^2\right)}\,dx + \frac{1}{4}\int -\frac{}{\sqrt{-3-}}$$

$$= \frac{1}{2}\sin^{-1}(2+x) - \tanh^{-1}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right) + \frac{3}{2}\,\text{Subst}\left(\int \frac{1}{3-3x^2}\,dx, x, -\right.$$

$$= \frac{1}{2}\sin^{-1}(2+x) - \frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right) - 8\,\text{Subst}\left(\int \frac{1+3x^2}{-4-8x^2-36}\right.$$

$$= \frac{1}{2}\sin^{-1}(2+x) - \frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right) + \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{\frac{1}{3}-\frac{2x}{3}+x^2}\,d\right.$$

$$= \frac{1}{2}\sin^{-1}(2+x) - \frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right) - \frac{2}{3}\,\text{Subst}\left(\int \frac{1}{-\frac{8}{9}-x^2}\,dx, x\right.$$

$$= \frac{1}{2}\sin^{-1}(2+x) - \frac{\tan^{-1}\left(\frac{1-\frac{3+x}{\sqrt{-3-4x-x^2}}}{\sqrt{2}}\right)}{\sqrt{2}} + \frac{\tan^{-1}\left(\frac{1+\frac{3+x}{\sqrt{-3-4x-x^2}}}{\sqrt{2}}\right)}{\sqrt{2}} - \frac{1}{2}\tanh^{-1}\left(\right.$$

Mathematica [C] time = 0.19, size = 159, normalized size = 1.62

$$\frac{1}{4}\left(-i\sqrt{1-2i\sqrt{2}}\,\tanh^{-1}\left(\frac{i\sqrt{2}x+2x+2i\sqrt{2}+2}{\sqrt{2-4i\sqrt{2}}\sqrt{-x^2-4x-3}}\right) + i\sqrt{1+2i\sqrt{2}}\,\tanh^{-1}\left(\frac{\left(2-i\sqrt{2}\right)x-2i\sqrt{2}+2}{\sqrt{2+4i\sqrt{2}}\sqrt{-x^2-4x-3}}\right) + 2\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(Sqrt[-3 - 4*x - x^2]*(3 + 4*x + 2*x^2)),x]
```

```
[Out] (2*ArcSin[2 + x] - I*Sqrt[1 - (2*I)*Sqrt[2]]*ArcTanh[(2 + (2*I)*Sqrt[2] + 2
*x + I*Sqrt[2]*x)/(Sqrt[2 - (4*I)*Sqrt[2]]*Sqrt[-3 - 4*x - x^2])] + I*Sqrt[
1 + (2*I)*Sqrt[2]]*ArcTanh[(2 - (2*I)*Sqrt[2] + (2 - I*Sqrt[2])*x)/(Sqrt[2
+ (4*I)*Sqrt[2]]*Sqrt[-3 - 4*x - x^2])])/4
```

fricas [A] time = 0.92, size = 161, normalized size = 1.64

$$-\frac{1}{4}\sqrt{2}\arctan\left(\frac{\sqrt{2}x+3\sqrt{2}\sqrt{-x^2-4x-3}}{2(2x+3)}\right) - \frac{1}{4}\sqrt{2}\arctan\left(-\frac{\sqrt{2}x-3\sqrt{2}\sqrt{-x^2-4x-3}}{2(2x+3)}\right) - \frac{1}{2}\arctan\left(\frac{\sqrt{-x^2-}}{x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(2)*arctan(1/2*(sqrt(2)*x + 3*sqrt(2)*sqrt(-x^2 - 4*x - 3))/(2*x +
 3)) - 1/4*sqrt(2)*arctan(-1/2*(sqrt(2)*x - 3*sqrt(2)*sqrt(-x^2 - 4*x - 3))
/(2*x + 3)) - 1/2*arctan(sqrt(-x^2 - 4*x - 3)*(x + 2)/(x^2 + 4*x + 3)) + 1/
8*log(-(2*sqrt(-x^2 - 4*x - 3)*x + 4*x + 3)/x^2) - 1/8*log((2*sqrt(-x^2 - 4
*x - 3)*x - 4*x - 3)/x^2)
```

giac [B] time = 0.23, size = 171, normalized size = 1.74

$$-\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\frac{3\left(\sqrt{-x^2-4x-3}-1\right)}{x+2}+1\right)\right)-\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\frac{\sqrt{-x^2-4x-3}-1}{x+2}+1\right)\right)+\frac{1}{2}\text{arcsi}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*arctan(1/2*sqrt(2)*(3*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 1))
 - 1/2*sqrt(2)*arctan(1/2*sqrt(2)*((sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 1))
 + 1/2*arcsin(x + 2) - 1/4*log(2*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 3*(sq
rt(-x^2 - 4*x - 3) - 1)^2/(x + 2)^2 + 1) + 1/4*log(2*(sqrt(-x^2 - 4*x - 3)
- 1)/(x + 2) + (sqrt(-x^2 - 4*x - 3) - 1)^2/(x + 2)^2 + 3)
```

maple [A] time = 0.01, size = 130, normalized size = 1.33

$$\frac{\arcsin(x+2)}{2}-\frac{\sqrt{3}\,\sqrt{4}\,\sqrt{\frac{3x^2}{\left(-x-\frac{3}{2}\right)^2}-12}\,\left(-\operatorname{arctanh}\left(\frac{3x}{\left(-x-\frac{3}{2}\right)\sqrt{\frac{3x^2}{\left(-x-\frac{3}{2}\right)^2}-12}}\right)+\sqrt{2}\,\arctan\left(\frac{\sqrt{\frac{3x^2}{\left(-x-\frac{3}{2}\right)^2}-12}\,\sqrt{2}}{6}\right)\right)}{12\sqrt{\frac{\frac{x^2}{\left(-x-\frac{3}{2}\right)^2}-4}{\left(\frac{x}{-x-\frac{3}{2}}+1\right)^2}}\,\left(\frac{x}{-x-\frac{3}{2}}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x)
```

```
[Out] 1/2*arcsin(x+2)-1/12*3^(1/2)*4^(1/2)*(3/(-x-3/2)^2*x^2-12)^(1/2)*(2^(1/2)*a
rctan(1/6*(3/(-x-3/2)^2*x^2-12)^(1/2)*2^(1/2))-arctanh(3/(-x-3/2)/(3/(-x-3/
2)^2*x^2-12)^(1/2)*x))/((1/(-x-3/2)^2*x^2-4)/(1/(-x-3/2)*x+1)^2)^(1/2)/(1/(
-x-3/2)*x+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(2x^2+4x+3\right)\sqrt{-x^2-4x-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/((2*x^2 + 4*x + 3)*sqrt(-x^2 - 4*x - 3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{\sqrt{-x^2-4x-3}\,\left(2x^2+4x+3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((- 4*x - x^2 - 3)^(1/2)*(4*x + 2*x^2 + 3)),x)
```

```
[Out] int(x^2/((- 4*x - x^2 - 3)^(1/2)*(4*x + 2*x^2 + 3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{-(x+1)(x+3)}\,\left(2x^2+4x+3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(2*x**2+4*x+3)/(-x**2-4*x-3)**(1/2),x)
```

```
[Out] Integral(x**2/(sqrt(-(x + 1)*(x + 3))*(2*x**2 + 4*x + 3)), x)
```

3.129 $\int \frac{x}{\sqrt{-3-4x-x^2}\left(3+4x+2x^2\right)}\,dx$

Optimal. Leaf size=68

$$\frac{\tan^{-1}\left(\frac{\frac{3\sqrt{-x-1}}{\sqrt{x+3}}+1}{\sqrt{2}}\right)}{\sqrt{2}}-\frac{\tan^{-1}\left(\frac{1-\frac{3\sqrt{-x-1}}{\sqrt{x+3}}}{\sqrt{2}}\right)}{\sqrt{2}}$$

```
[Out] -1/2*arctan(1/2*(1-3*(-1-x)^(1/2)/(3+x)^(1/2))*2^(1/2))*2^(1/2)+1/2*arctan(
1/2*(1+3*(-1-x)^(1/2)/(3+x)^(1/2))*2^(1/2))*2^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 69, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1026, 1161, 618, 204}

$$\frac{\tan^{-1}\left(\frac{1-\frac{x+3}{\sqrt{-x^2-4x-3}}}{\sqrt{2}}\right)}{\sqrt{2}}-\frac{\tan^{-1}\left(\frac{\frac{x+3}{\sqrt{-x^2-4x-3}}+1}{\sqrt{2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(Sqrt[-3 - 4*x - x^2]*(3 + 4*x + 2*x^2)),x]
```

```
[Out] ArcTan[(1 - (3 + x)/Sqrt[-3 - 4*x - x^2])/Sqrt[2]]/Sqrt[2] - ArcTan[(1 + (3
 + x)/Sqrt[-3 - 4*x - x^2])/Sqrt[2]]/Sqrt[2]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1026

```
Int[(x_)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (e_.)*(x_) + (f_.)
*(x_)^2]), x_Symbol] :> Dist[-2*e, Subst[Int[(1 - d*x^2)/(c*e - b*f - e*(2*
c*d - b*e + 2*a*f)*x^2 + d^2*(c*e - b*f)*x^4), x], x, (1 + ((e + Sqrt[e^2 -
 4*d*f])*x)/(2*d))/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f},
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[b*d - a*e, 0]
```

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rubi steps

$$\int \frac{x}{\sqrt{-3-4x-x^2}\left(3+4x+2x^2\right)}\,dx = 8\,\text{Subst}\left(\int \frac{1+3x^2}{-4-8x^2-36x^4}\,dx, x, \frac{1+\frac{x}{3}}{\sqrt{-3-4x-x^2}}\right)$$

$$= -\left(\frac{1}{3}\text{Subst}\left(\int \frac{1}{\frac{1}{3}-\frac{2x}{3}+x^2}\,dx, x, \frac{1+\frac{x}{3}}{\sqrt{-3-4x-x^2}}\right)\right) - \frac{1}{3}\text{Subst}\left(\int \frac{1}{\frac{1}{3}+\frac{2x}{3}}\right.$$

$$= \frac{2}{3}\text{Subst}\left(\int \frac{1}{-\frac{8}{9}-x^2}\,dx, x, \frac{2}{3}\left(-1+\frac{3+x}{\sqrt{-3-4x-x^2}}\right)\right) + \frac{2}{3}\text{Subst}\left(\int \frac{1}{-\frac{8}{9}}\right.$$

$$= \frac{\tan^{-1}\left(\frac{1-\frac{3+x}{\sqrt{-3-4x-x^2}}}{\sqrt{2}}\right)}{\sqrt{2}} - \frac{\tan^{-1}\left(\frac{1+\frac{3+x}{\sqrt{-3-4x-x^2}}}{\sqrt{2}}\right)}{\sqrt{2}}$$

Mathematica [C] time = 0.17, size = 174, normalized size = 2.56

$$\frac{\left(1-i\sqrt{2}\right)\sqrt{1-2i\sqrt{2}}\,\tanh^{-1}\left(\frac{\left(2-i\sqrt{2}\right)x-2i\sqrt{2}+2}{\sqrt{2+4i\sqrt{2}}\,\sqrt{-x^2-4x-3}}\right)+\left(1+i\sqrt{2}\right)\sqrt{1+2i\sqrt{2}}\,\tanh^{-1}\left(\frac{\left(2+i\sqrt{2}\right)x+2i\sqrt{2}+2}{\sqrt{2-4i\sqrt{2}}\,\sqrt{-x^2-4x-3}}\right)}{6\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(Sqrt[-3 - 4*x - x^2]*(3 + 4*x + 2*x^2)),x]
```

```
[Out] ((1 - I*Sqrt[2])*Sqrt[1 - (2*I)*Sqrt[2]]*ArcTanh[(2 - (2*I)*Sqrt[2] + (2 -
I*Sqrt[2])*x)/(Sqrt[2 + (4*I)*Sqrt[2]]*Sqrt[-3 - 4*x - x^2])] + (1 + I*Sqrt
[2])*Sqrt[1 + (2*I)*Sqrt[2]]*ArcTanh[(2 + (2*I)*Sqrt[2] + (2 + I*Sqrt[2])*x
)/(Sqrt[2 - (4*I)*Sqrt[2]]*Sqrt[-3 - 4*x - x^2])])/(6*Sqrt[2])
```

fricas [A] time = 0.88, size = 50, normalized size = 0.74

$$\frac{1}{4}\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(6x^2+20x+15\right)\sqrt{-x^2-4x-3}}{4\left(2x^3+11x^2+18x+9\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*arctan(1/4*sqrt(2)*(6*x^2 + 20*x + 15)*sqrt(-x^2 - 4*x - 3)/(2*
x^3 + 11*x^2 + 18*x + 9))
```

giac [A] time = 0.18, size = 68, normalized size = 1.00

$$\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\frac{3\left(\sqrt{-x^2-4x-3}-1\right)}{x+2}+1\right)\right)+\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\frac{\sqrt{-x^2-4x-3}-1}{x+2}+1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(3*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 1))
+ 1/2*sqrt(2)*arctan(1/2*sqrt(2)*((sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 1))
```

maple [A] time = 0.01, size = 92, normalized size = 1.35

$$\frac{\sqrt{3}\,\sqrt{4}\,\sqrt{\frac{3x^2}{\left(-x-\frac{3}{2}\right)^2}-12}\,\sqrt{2}\,\arctan\left(\frac{\sqrt{\frac{3x^2}{\left(-x-\frac{3}{2}\right)^2}-12}\,\sqrt{2}}{6}\right)}{12\sqrt{\frac{\frac{x^2}{\left(-x-\frac{3}{2}\right)^2}-4}{\left(\frac{x}{-x-\frac{3}{2}}+1\right)^2}}\left(\frac{x}{-x-\frac{3}{2}}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x)
```

```
[Out] 1/12*3^(1/2)*4^(1/2)/((1/(-x-3/2)^2*x^2-4)/(1/(-x-3/2)*x+1)^2)^(1/2)/(1/(-x
-3/2)*x+1)*(3/(-x-3/2)^2*x^2-12)^(1/2)*2^(1/2)*arctan(1/6*(3/(-x-3/2)^2*x^2
-12)^(1/2)*2^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(2\,x^2+4\,x+3\right)\sqrt{-x^2-4\,x-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/((2*x^2 + 4*x + 3)*sqrt(-x^2 - 4*x - 3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{\sqrt{-x^2-4\,x-3}\,\left(2\,x^2+4\,x+3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((- 4*x - x^2 - 3)^(1/2)*(4*x + 2*x^2 + 3)),x)
```

```
[Out] int(x/((- 4*x - x^2 - 3)^(1/2)*(4*x + 2*x^2 + 3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{-(x+1)(x+3)}\,\left(2x^2+4x+3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2*x**2+4*x+3)/(-x**2-4*x-3)**(1/2),x)
```

```
[Out] Integral(x/(sqrt(-(x + 1)*(x + 3))*(2*x**2 + 4*x + 3)), x)
```

3.130 $\int \frac{1}{\sqrt{-3-4x-x^2}\left(3+4x+2x^2\right)}\,dx$

Optimal. Leaf size=95

$$-\frac{1}{3}\sqrt{2}\tan^{-1}\left(\frac{1-\frac{x+3}{\sqrt{-x^2-4x-3}}}{\sqrt{2}}\right)+\frac{1}{3}\sqrt{2}\tan^{-1}\left(\frac{\frac{x+3}{\sqrt{-x^2-4x-3}}+1}{\sqrt{2}}\right)+\frac{1}{3}\tanh^{-1}\left(\frac{x}{\sqrt{-x^2-4x-3}}\right)$$

```
[Out] 1/3*arctanh(x/(-x^2-4*x-3)^(1/2))-1/3*arctan(1/2*(1+(-3-x)/(-x^2-4*x-3)^(1/
2))*2^(1/2))*2^(1/2)+1/3*arctan(1/2*(1+(3+x)/(-x^2-4*x-3)^(1/2))*2^(1/2))*2
^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {986, 12, 1026, 1161, 618, 204, 1027, 206}

$$-\frac{1}{3}\sqrt{2}\tan^{-1}\left(\frac{1-\frac{x+3}{\sqrt{-x^2-4x-3}}}{\sqrt{2}}\right)+\frac{1}{3}\sqrt{2}\tan^{-1}\left(\frac{\frac{x+3}{\sqrt{-x^2-4x-3}}+1}{\sqrt{2}}\right)+\frac{1}{3}\tanh^{-1}\left(\frac{x}{\sqrt{-x^2-4x-3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-3 - 4*x - x^2]*(3 + 4*x + 2*x^2)),x]
```

```
[Out] -(Sqrt[2]*ArcTan[(1 - (3 + x)/Sqrt[-3 - 4*x - x^2])/Sqrt[2]])/3 + (Sqrt[2]*
ArcTan[(1 + (3 + x)/Sqrt[-3 - 4*x - x^2])/Sqrt[2]])/3 + ArcTanh[x/Sqrt[-3 -
 4*x - x^2]]/3
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 986

```
Int[1/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*
(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f)
, 2]}, Dist[1/(2*q), Int[(c*d - a*f + q + (c*e - b*f)*x)/((a + b*x + c*x^2)
*Sqrt[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[(c*d - a*f - q + (c*e -
 b*f)*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b,
 c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[c*e -
```

```
b*f, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1026

```
Int[(x_)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (e_.)*(x_) + (f_.)
*(x_)^2]), x_Symbol] :> Dist[-2*e, Subst[Int[(1 - d*x^2)/(c*e - b*f - e*(2*
c*d - b*e + 2*a*f)*x^2 + d^2*(c*e - b*f)*x^4), x], x, (1 + ((e + Sqrt[e^2 -
 4*d*f])*x)/(2*d))/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f},
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[b*d - a*e, 0]
```

Rule 1027

```
Int[((g_) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (e_
.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[g, Subst[Int[1/(a + (c*d - a*f)
*x^2), x], x, x/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h
}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[b*d - a*e, 0] &&
 EqQ[2*h*d - g*e, 0]
```

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{-3-4x-x^2}\left(3+4x+2x^2\right)}\,dx &= -\left(\frac{1}{6}\int \frac{-6-4x}{\sqrt{-3-4x-x^2}\left(3+4x+2x^2\right)}\,dx\right)+\frac{1}{6}\int -\frac{4x}{\sqrt{-3-4x-x^2}\,(3} \\
&= -\left(\frac{2}{3}\int \frac{x}{\sqrt{-3-4x-x^2}\left(3+4x+2x^2\right)}\,dx\right)+\text{Subst}\left(\int \frac{1}{3-3x^2}\,dx,x,\right. \\
&= \frac{1}{3}\tanh^{-1}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right)-\frac{16}{3}\text{Subst}\left(\int \frac{1+3x^2}{-4-8x^2-36x^4}\,dx,x,\right. \\
&= \frac{1}{3}\tanh^{-1}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right)+\frac{2}{9}\text{Subst}\left(\int \frac{1}{\frac{1}{3}-\frac{2x}{3}+x^2}\,dx,x,\frac{1+}{\sqrt{-3-4}}\right. \\
&= \frac{1}{3}\tanh^{-1}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right)-\frac{4}{9}\text{Subst}\left(\int \frac{1}{-\frac{8}{9}-x^2}\,dx,x,\frac{2}{3}\left(-1+\right.\right. \\
&= -\frac{1}{3}\sqrt{2}\tan^{-1}\left(\frac{1-\frac{3+x}{\sqrt{-3-4x-x^2}}}{\sqrt{2}}\right)+\frac{1}{3}\sqrt{2}\tan^{-1}\left(\frac{1+\frac{3+x}{\sqrt{-3-4x-x^2}}}{\sqrt{2}}\right)+\frac{1}{3}\tanh
\end{aligned}$$

Mathematica [C] time = 0.10, size = 150, normalized size = 1.58

$$\frac{1}{6}i\left(\sqrt{1-2i\sqrt{2}}\tanh^{-1}\left(\frac{\left(2-i\sqrt{2}\right)x-2i\sqrt{2}+2}{\sqrt{2+4i\sqrt{2}}\sqrt{-x^2-4x-3}}\right)-\sqrt{1+2i\sqrt{2}}\tanh^{-1}\left(\frac{\left(2+i\sqrt{2}\right)x+2i\sqrt{2}+2}{\sqrt{2-4i\sqrt{2}}\sqrt{-x^2-4x-3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-3 - 4*x - x^2]*(3 + 4*x + 2*x^2)),x]
```

```
[Out] (I/6)*(Sqrt[1 - (2*I)*Sqrt[2]]*ArcTanh[(2 - (2*I)*Sqrt[2] + (2 - I*Sqrt[2])
*x)/(Sqrt[2 + (4*I)*Sqrt[2]]*Sqrt[-3 - 4*x - x^2])] - Sqrt[1 + (2*I)*Sqrt[2
]]*ArcTanh[(2 + (2*I)*Sqrt[2] + (2 + I*Sqrt[2])*x)/(Sqrt[2 - (4*I)*Sqrt[2]]
*Sqrt[-3 - 4*x - x^2])])
```

fricas [A] time = 0.95, size = 132, normalized size = 1.39

$$-\frac{1}{6}\sqrt{2}\arctan\left(\frac{\sqrt{2}x+3\sqrt{2}\sqrt{-x^2-4x-3}}{2(2x+3)}\right)-\frac{1}{6}\sqrt{2}\arctan\left(-\frac{\sqrt{2}x-3\sqrt{2}\sqrt{-x^2-4x-3}}{2(2x+3)}\right)-\frac{1}{12}\log\left(-\frac{2\sqrt{-x^2}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/6*sqrt(2)*arctan(1/2*(sqrt(2)*x + 3*sqrt(2)*sqrt(-x^2 - 4*x - 3))/(2*x +
 3)) - 1/6*sqrt(2)*arctan(-1/2*(sqrt(2)*x - 3*sqrt(2)*sqrt(-x^2 - 4*x - 3))
/(2*x + 3)) - 1/12*log(-(2*sqrt(-x^2 - 4*x - 3)*x + 4*x + 3)/x^2) + 1/12*lo
g((2*sqrt(-x^2 - 4*x - 3)*x - 4*x - 3)/x^2)
```

giac [B] time = 0.19, size = 165, normalized size = 1.74

$$-\frac{1}{3}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\frac{3\left(\sqrt{-x^2-4x-3}-1\right)}{x+2}+1\right)\right)-\frac{1}{3}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\frac{\sqrt{-x^2-4x-3}-1}{x+2}+1\right)\right)+\frac{1}{6}\log\left(\frac{2(}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(2)*arctan(1/2*sqrt(2)*(3*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 1))
 - 1/3*sqrt(2)*arctan(1/2*sqrt(2)*((sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 1))
 + 1/6*log(2*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 3*(sqrt(-x^2 - 4*x - 3) -
 1)^2/(x + 2)^2 + 1) - 1/6*log(2*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + (sqrt
(-x^2 - 4*x - 3) - 1)^2/(x + 2)^2 + 3)
```

maple [A] time = 0.01, size = 121, normalized size = 1.27

$$-\frac{\sqrt{3}\sqrt{4}\sqrt{\frac{3x^2}{\left(-x-\frac{3}{2}\right)^2}-12}\left(\operatorname{arctanh}\left(\frac{3x}{\left(-x-\frac{3}{2}\right)\sqrt{\frac{3x^2}{\left(-x-\frac{3}{2}\right)^2}-12}}\right)+\sqrt{2}\arctan\left(\frac{\sqrt{\frac{3x^2}{\left(-x-\frac{3}{2}\right)^2}-12}\sqrt{2}}{6}\right)\right)}{18\sqrt{\frac{\frac{x^2}{\left(-x-\frac{3}{2}\right)^2}-4}{\left(\frac{x}{-x-\frac{3}{2}}+1\right)^2}}\left(\frac{x}{-x-\frac{3}{2}}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x)
```

```
[Out] -1/18*3^(1/2)*4^(1/2)*(3/(-x-3/2)^2*x^2-12)^(1/2)*(2^(1/2)*arctan(1/6*(3/(-
x-3/2)^2*x^2-12)^(1/2)*2^(1/2))+arctanh(3/(-x-3/2)/(3/(-x-3/2)^2*x^2-12)^(1
/2)*x))/((1/(-x-3/2)^2*x^2-4)/(1/(-x-3/2)*x+1)^2)^(1/2)/(1/(-x-3/2)*x+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(2x^2+4x+3\right)\sqrt{-x^2-4x-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="maxima")
[Out] integrate(1/((2*x^2 + 4*x + 3)*sqrt(-x^2 - 4*x - 3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{-x^2 - 4\,x - 3}\,\left(2\,x^2 + 4\,x + 3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((- 4*x - x^2 - 3)^(1/2)*(4*x + 2*x^2 + 3)),x)
[Out] int(1/((- 4*x - x^2 - 3)^(1/2)*(4*x + 2*x^2 + 3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-(x+1)(x+3)}\,\left(2x^2 + 4x + 3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**2+4*x+3)/(-x**2-4*x-3)**(1/2),x)
[Out] Integral(1/(sqrt(-(x + 1)*(x + 3))*(2*x**2 + 4*x + 3)), x)
```

3.131 $\int \frac{1}{x\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\,dx$

Optimal. Leaf size=130

$$-\frac{\tan^{-1}\left(\frac{2x+3}{\sqrt{3}\sqrt{-x^2-4x-3}}\right)}{3\sqrt{3}}+\frac{1}{9}\sqrt{2}\tan^{-1}\left(\frac{1-\frac{x+3}{\sqrt{-x^2-4x-3}}}{\sqrt{2}}\right)-\frac{1}{9}\sqrt{2}\tan^{-1}\left(\frac{\frac{x+3}{\sqrt{-x^2-4x-3}}+1}{\sqrt{2}}\right)-\frac{4}{9}\tanh^{-1}\left(\frac{x}{\sqrt{-x^2-4x-3}}\right)$$

```
[Out] -4/9*arctanh(x/(-x^2-4*x-3)^(1/2))+1/9*arctan(1/2*(1+(-3-x)/(-x^2-4*x-3)^(1
/2))*2^(1/2))*2^(1/2)-1/9*arctan(1/2*(1+(3+x)/(-x^2-4*x-3)^(1/2))*2^(1/2))*
2^(1/2)-1/9*arctan(1/3*(3+2*x)*3^(1/2)/(-x^2-4*x-3)^(1/2))*3^(1/2)
```

Rubi [A] time = 0.42, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 11, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.367, Rules used = {6728, 724, 204, 1028, 986, 12, 1026, 1161, 618, 1027, 206}

$$-\frac{\tan^{-1}\left(\frac{2x+3}{\sqrt{3}\sqrt{-x^2-4x-3}}\right)}{3\sqrt{3}}+\frac{1}{9}\sqrt{2}\tan^{-1}\left(\frac{1-\frac{x+3}{\sqrt{-x^2-4x-3}}}{\sqrt{2}}\right)-\frac{1}{9}\sqrt{2}\tan^{-1}\left(\frac{\frac{x+3}{\sqrt{-x^2-4x-3}}+1}{\sqrt{2}}\right)-\frac{4}{9}\tanh^{-1}\left(\frac{x}{\sqrt{-x^2-4x-3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[-3 - 4*x - x^2]*(3 + 4*x + 2*x^2)),x]
```

```
[Out] -ArcTan[(3 + 2*x)/(Sqrt[3]*Sqrt[-3 - 4*x - x^2])]/(3*Sqrt[3]) + (Sqrt[2]*Ar
cTan[(1 - (3 + x)/Sqrt[-3 - 4*x - x^2])/Sqrt[2]])/9 - (Sqrt[2]*ArcTan[(1 +
(3 + x)/Sqrt[-3 - 4*x - x^2])/Sqrt[2]])/9 - (4*ArcTanh[x/Sqrt[-3 - 4*x - x^
2]])/9
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
```

```
d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 986

```
Int[1/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*
(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f)
, 2]}, Dist[1/(2*q), Int[(c*d - a*f + q + (c*e - b*f)*x)/((a + b*x + c*x^2)
*Sqrt[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[(c*d - a*f - q + (c*e -
 b*f)*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b,
 c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[c*e -
 b*f, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1026

```
Int[(x_)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (e_.)*(x_) + (f_.)
*(x_)^2]), x_Symbol] :> Dist[-2*e, Subst[Int[(1 - d*x^2)/(c*e - b*f - e*(2*
c*d - b*e + 2*a*f)*x^2 + d^2*(c*e - b*f)*x^4), x], x, (1 + ((e + Sqrt[e^2 -
 4*d*f])*x)/(2*d))/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f},
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[b*d - a*e, 0]
```

Rule 1027

```
Int[((g_) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (e_
.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[g, Subst[Int[1/(a + (c*d - a*f)
*x^2), x], x, x/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h
}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[b*d - a*e, 0] &&
 EqQ[2*h*d - g*e, 0]
```

Rule 1028

```
Int[((g_) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (e_
.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> -Dist[(2*h*d - g*e)/e, Int[1/((a + b
*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/e, Int[(2*d + e*x)/((a
+ b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, g
, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[b*d - a*e, 0]
 && NeQ[2*h*d - g*e, 0]
```

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{x\sqrt{-3-4x-x^2}\left(3+4x+2x^2\right)}dx = \int \left(\frac{1}{3x\sqrt{-3-4x-x^2}} - \frac{2(2+x)}{3\sqrt{-3-4x-x^2}\left(3+4x+2x^2\right)}\right)dx$$
$$= \frac{1}{3}\int \frac{1}{x\sqrt{-3-4x-x^2}}dx - \frac{2}{3}\int \frac{2+x}{\sqrt{-3-4x-x^2}\left(3+4x+2x^2\right)}dx$$
$$= \frac{1}{6}\int \frac{-6-4x}{\sqrt{-3-4x-x^2}\left(3+4x+2x^2\right)}dx - \frac{1}{3}\int \frac{1}{\sqrt{-3-4x-x^2}\left(3+4x+\right.}$$
$$= -\frac{\tan^{-1}\left(\frac{3+2x}{\sqrt{3}\sqrt{-3-4x-x^2}}\right)}{3\sqrt{3}} + \frac{1}{18}\int \frac{-6-4x}{\sqrt{-3-4x-x^2}\left(3+4x+2x^2\right)}dx - \frac{1}{18}$$
$$= -\frac{\tan^{-1}\left(\frac{3+2x}{\sqrt{3}\sqrt{-3-4x-x^2}}\right)}{3\sqrt{3}} - \frac{1}{3}\tanh^{-1}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right) + \frac{2}{9}\int \frac{}{\sqrt{-3-4x}}$$
$$= -\frac{\tan^{-1}\left(\frac{3+2x}{\sqrt{3}\sqrt{-3-4x-x^2}}\right)}{3\sqrt{3}} - \frac{4}{9}\tanh^{-1}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right) + \frac{16}{9}\text{Subst}\left(\int \right.$$
$$= -\frac{\tan^{-1}\left(\frac{3+2x}{\sqrt{3}\sqrt{-3-4x-x^2}}\right)}{3\sqrt{3}} - \frac{4}{9}\tanh^{-1}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right) - \frac{2}{27}\text{Subst}\left(\int \right.$$
$$= -\frac{\tan^{-1}\left(\frac{3+2x}{\sqrt{3}\sqrt{-3-4x-x^2}}\right)}{3\sqrt{3}} - \frac{4}{9}\tanh^{-1}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right) + \frac{4}{27}\text{Subst}\left(\int \right.$$
$$= -\frac{\tan^{-1}\left(\frac{3+2x}{\sqrt{3}\sqrt{-3-4x-x^2}}\right)}{3\sqrt{3}} + \frac{1}{9}\sqrt{2}\tan^{-1}\left(\frac{1-\frac{3+x}{\sqrt{-3-4x-x^2}}}{\sqrt{2}}\right) - \frac{1}{9}\sqrt{2}\tan^{-1}\left(1\right.$$

Mathematica [C] time = 0.44, size = 200, normalized size = 1.54

$$\frac{1}{54}\left(-6\sqrt{3}\tan^{-1}\left(\frac{2x+3}{\sqrt{3}\sqrt{-x^2-4x-3}}\right) - 3\sqrt{1-2i\sqrt{2}}\left(\sqrt{2}+2i\right)\tanh^{-1}\left(\frac{\left(2-i\sqrt{2}\right)x-2i\sqrt{2}+2}{\sqrt{2+4i\sqrt{2}}\sqrt{-x^2-4x-3}}\right) - 3\sqrt{1+\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[-3 - 4*x - x^2]*(3 + 4*x + 2*x^2)),x]
```

```
[Out] (-6*Sqrt[3]*ArcTan[(3 + 2*x)/(Sqrt[3]*Sqrt[-3 - 4*x - x^2])] - 3*Sqrt[1 - (
2*I)*Sqrt[2]]*(2*I + Sqrt[2])*ArcTanh[(2 - (2*I)*Sqrt[2] + (2 - I*Sqrt[2])*
x)/(Sqrt[2 + (4*I)*Sqrt[2]]*Sqrt[-3 - 4*x - x^2])] - 3*Sqrt[1 + (2*I)*Sqrt[
2]]*(-2*I + Sqrt[2])*ArcTanh[(2 + (2*I)*Sqrt[2] + (2 + I*Sqrt[2])*x)/(Sqrt[
2 - (4*I)*Sqrt[2]]*Sqrt[-3 - 4*x - x^2])])/54
```

fricas [A] time = 0.67, size = 170, normalized size = 1.31

$$\frac{1}{9}\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt{-x^2-4x-3}(2x+3)}{3\left(x^2+4x+3\right)}\right) + \frac{1}{18}\sqrt{2}\arctan\left(\frac{\sqrt{2}x+3\sqrt{2}\sqrt{-x^2-4x-3}}{2(2x+3)}\right) + \frac{1}{18}\sqrt{2}\arctan\left(-\frac{\sqrt{2}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/9*sqrt(3)*arctan(1/3*sqrt(3)*sqrt(-x^2 - 4*x - 3)*(2*x + 3)/(x^2 + 4*x +
3)) + 1/18*sqrt(2)*arctan(1/2*(sqrt(2)*x + 3*sqrt(2)*sqrt(-x^2 - 4*x - 3))/
```

```
(2*x + 3)) + 1/18*sqrt(2)*arctan(-1/2*(sqrt(2)*x - 3*sqrt(2)*sqrt(-x^2 - 4*
x - 3))/(2*x + 3)) + 1/9*log(-(2*sqrt(-x^2 - 4*x - 3)*x + 4*x + 3)/x^2) - 1
/9*log((2*sqrt(-x^2 - 4*x - 3)*x - 4*x - 3)/x^2)
```

giac [A] time = 0.20, size = 199, normalized size = 1.53

$$\frac{1}{9}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\frac{3\left(\sqrt{-x^2-4x-3}-1\right)}{x+2}+1\right)\right)+\frac{2}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(\frac{2\left(\sqrt{-x^2-4x-3}-1\right)}{x+2}+1\right)\right)+\frac{1}{9}\sqrt{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="giac")
```

```
[Out] 1/9*sqrt(2)*arctan(1/2*sqrt(2)*(3*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 1))
+ 2/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 1)
) + 1/9*sqrt(2)*arctan(1/2*sqrt(2)*((sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 1)
) - 2/9*log(2*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 3*(sqrt(-x^2 - 4*x - 3)
- 1)^2/(x + 2)^2 + 1) + 2/9*log(2*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + (sqr
t(-x^2 - 4*x - 3) - 1)^2/(x + 2)^2 + 3)
```

maple [A] time = 0.02, size = 152, normalized size = 1.17

$$\frac{\sqrt{3}\arctan\left(\frac{(-4x-6)\sqrt{3}}{6\sqrt{-x^2-4x-3}}\right)}{9}+\frac{\sqrt{3}\sqrt{4}\sqrt{\frac{3x^2}{\left(-x-\frac{3}{2}\right)^2}-12}\left(4\,\mathrm{arctanh}\left(\frac{3x}{\left(-x-\frac{3}{2}\right)\sqrt{\frac{3x^2}{\left(-x-\frac{3}{2}\right)^2}-12}}\right)+\sqrt{2}\arctan\left(\frac{\sqrt{\frac{3x^2}{\left(-x-\frac{3}{2}\right)^2}-12}}{6}\right.\right.}{54\sqrt{\frac{\frac{x^2}{\left(-x-\frac{3}{2}\right)^2}-4}{\left(\frac{x}{-x-\frac{3}{2}}+1\right)^2}}\left(\frac{x}{-x-\frac{3}{2}}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x)
```

```
[Out] 1/54*3^(1/2)*4^(1/2)*(3/(-x-3/2)^2*x^2-12)^(1/2)*(2^(1/2)*arctan(1/6*(3/(-x
-3/2)^2*x^2-12)^(1/2)*2^(1/2))+4*arctanh(3/(-x-3/2)/(3/(-x-3/2)^2*x^2-12)^(
1/2)*x))/((1/(-x-3/2)^2*x^2-4)/(1/(-x-3/2)*x+1)^2)^(1/2)/(1/(-x-3/2)*x+1)+1
/9*3^(1/2)*arctan(1/6*(-6-4*x)*3^(1/2)/(-x^2-4*x-3)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(2x^2+4x+3\right)\sqrt{-x^2-4x-3}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((2*x^2 + 4*x + 3)*sqrt(-x^2 - 4*x - 3)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x\sqrt{-x^2-4x-3}\,\left(2x^2+4x+3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(- 4*x - x^2 - 3)^(1/2)*(4*x + 2*x^2 + 3)),x)
```

```
[Out] int(1/(x*(- 4*x - x^2 - 3)^(1/2)*(4*x + 2*x^2 + 3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{-(x+1)(x+3)}\left(2x^2+4x+3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(2*x**2+4*x+3)/(-x**2-4*x-3)**(1/2),x)
[Out] Integral(1/(x*sqrt(-(x + 1)*(x + 3))*(2*x**2 + 4*x + 3)), x)
```

3.132 $\int \frac{1}{x^2\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\,dx$

Optimal. Leaf size=151

$$\frac{\sqrt{-x^2-4x-3}}{9x}+\frac{2\tan^{-1}\left(\frac{2x+3}{\sqrt{3}\sqrt{-x^2-4x-3}}\right)}{3\sqrt{3}}+\frac{2}{27}\sqrt{2}\tan^{-1}\left(\frac{1-\frac{x+3}{\sqrt{-x^2-4x-3}}}{\sqrt{2}}\right)-\frac{2}{27}\sqrt{2}\tan^{-1}\left(\frac{\frac{x+3}{\sqrt{-x^2-4x-3}}+1}{\sqrt{2}}\right)+\frac{10}{27}\text{ta}$$

```
[Out] 10/27*arctanh(x/(-x^2-4*x-3)^(1/2))+2/27*arctan(1/2*(1+(-3-x)/(-x^2-4*x-3)^
(1/2))*2^(1/2))*2^(1/2)-2/27*arctan(1/2*(1+(3+x)/(-x^2-4*x-3)^(1/2))*2^(1/2
))*2^(1/2)+2/9*arctan(1/3*(3+2*x)*3^(1/2)/(-x^2-4*x-3)^(1/2))*3^(1/2)+1/9*(
-x^2-4*x-3)^(1/2)/x
```

Rubi [A] time = 0.45, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 12, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {6728, 730, 724, 204, 1028, 986, 12, 1026, 1161, 618, 1027, 206}

$$\frac{\sqrt{-x^2-4x-3}}{9x}+\frac{2\tan^{-1}\left(\frac{2x+3}{\sqrt{3}\sqrt{-x^2-4x-3}}\right)}{3\sqrt{3}}+\frac{2}{27}\sqrt{2}\tan^{-1}\left(\frac{1-\frac{x+3}{\sqrt{-x^2-4x-3}}}{\sqrt{2}}\right)-\frac{2}{27}\sqrt{2}\tan^{-1}\left(\frac{\frac{x+3}{\sqrt{-x^2-4x-3}}+1}{\sqrt{2}}\right)+\frac{10}{27}\text{ta}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[-3 - 4*x - x^2]*(3 + 4*x + 2*x^2)),x]
```

```
[Out] Sqrt[-3 - 4*x - x^2]/(9*x) + (2*ArcTan[(3 + 2*x)/(Sqrt[3]*Sqrt[-3 - 4*x - x
^2])])/(3*Sqrt[3]) + (2*Sqrt[2]*ArcTan[(1 - (3 + x)/Sqrt[-3 - 4*x - x^2])/S
qrt[2]])/27 - (2*Sqrt[2]*ArcTan[(1 + (3 + x)/Sqrt[-3 - 4*x - x^2])/Sqrt[2]]
)/27 + (10*ArcTanh[x/Sqrt[-3 - 4*x - x^2]])/27
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
```

```
d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 730

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 - b*d*e + a*e^2)), x] + Dist[(2*c*d - b*e)/(2*(c*d^2 - b*d*e + a*e^2)),
 Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e,
 m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2
*c*d - b*e, 0] && EqQ[m + 2*p + 3, 0]
```

Rule 986

```
Int[1/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*
(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f)
, 2]}, Dist[1/(2*q), Int[(c*d - a*f + q + (c*e - b*f)*x)/((a + b*x + c*x^2)
*Sqrt[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[(c*d - a*f - q + (c*e -
 b*f)*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b,
 c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[c*e -
 b*f, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1026

```
Int[(x_)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (e_.)*(x_) + (f_.)
*(x_)^2]), x_Symbol] :> Dist[-2*e, Subst[Int[(1 - d*x^2)/(c*e - b*f - e*(2*
c*d - b*e + 2*a*f)*x^2 + d^2*(c*e - b*f)*x^4), x], x, (1 + ((e + Sqrt[e^2 -
 4*d*f])*x)/(2*d))/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f},
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[b*d - a*e, 0]
```

Rule 1027

```
Int[((g_) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (e_
.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[g, Subst[Int[1/(a + (c*d - a*f)
*x^2), x], x, x/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h
}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[b*d - a*e, 0] &&
 EqQ[2*h*d - g*e, 0]
```

Rule 1028

```
Int[((g_) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (e_
.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> -Dist[(2*h*d - g*e)/e, Int[1/((a + b
*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/e, Int[(2*d + e*x)/((a
+ b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, g
, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[b*d - a*e, 0]
 && NeQ[2*h*d - g*e, 0]
```

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
```

```
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{x^2\sqrt{-3-4x-x^2}\left(3+4x+2x^2\right)}\,dx = \int\left(\frac{1}{3x^2\sqrt{-3-4x-x^2}} - \frac{4}{9x\sqrt{-3-4x-x^2}} + \frac{2(5+4x)}{9\sqrt{-3-4x-x^2}\left(3+\right.}\right.$$

$$= \frac{2}{9}\int \frac{5+4x}{\sqrt{-3-4x-x^2}\left(3+4x+2x^2\right)}\,dx + \frac{1}{3}\int \frac{1}{x^2\sqrt{-3-4x-x^2}}\,dx$$

$$= \frac{\sqrt{-3-4x-x^2}}{9x} - \frac{2}{9}\int \frac{1}{x\sqrt{-3-4x-x^2}}\,dx - \frac{2}{9}\int \frac{1}{\sqrt{-3-4x-x^2}\,(3}$$

$$= \frac{\sqrt{-3-4x-x^2}}{9x} + \frac{4\tan^{-1}\left(\frac{3+2x}{\sqrt{3}\sqrt{-3-4x-x^2}}\right)}{9\sqrt{3}} + \frac{1}{27}\int \frac{-6-}{\sqrt{-3-4x-x^2}\,(3}$$

$$= \frac{\sqrt{-3-4x-x^2}}{9x} + \frac{2\tan^{-1}\left(\frac{3+2x}{\sqrt{3}\sqrt{-3-4x-x^2}}\right)}{3\sqrt{3}} + \frac{4}{9}\tanh^{-1}\left(\frac{x}{\sqrt{-3-4x-}}\right.$$

$$= \frac{\sqrt{-3-4x-x^2}}{9x} + \frac{2\tan^{-1}\left(\frac{3+2x}{\sqrt{3}\sqrt{-3-4x-x^2}}\right)}{3\sqrt{3}} + \frac{10}{27}\tanh^{-1}\left(\frac{x}{\sqrt{-3-4x-}}\right.$$

$$= \frac{\sqrt{-3-4x-x^2}}{9x} + \frac{2\tan^{-1}\left(\frac{3+2x}{\sqrt{3}\sqrt{-3-4x-x^2}}\right)}{3\sqrt{3}} + \frac{10}{27}\tanh^{-1}\left(\frac{x}{\sqrt{-3-4x-}}\right.$$

$$= \frac{\sqrt{-3-4x-x^2}}{9x} + \frac{2\tan^{-1}\left(\frac{3+2x}{\sqrt{3}\sqrt{-3-4x-x^2}}\right)}{3\sqrt{3}} + \frac{10}{27}\tanh^{-1}\left(\frac{x}{\sqrt{-3-4x-}}\right.$$

$$= \frac{\sqrt{-3-4x-x^2}}{9x} + \frac{2\tan^{-1}\left(\frac{3+2x}{\sqrt{3}\sqrt{-3-4x-x^2}}\right)}{3\sqrt{3}} + \frac{2}{27}\sqrt{2}\tan^{-1}\left(\frac{1-\frac{3+}{\sqrt{-3-4}}}{\sqrt{2}}\right.$$

Mathematica [C] time = 0.41, size = 225, normalized size = 1.49

$$\frac{3\left(\sqrt{-x^2-4x-3}+2\sqrt{3}\,x\tan^{-1}\left(\frac{2x+3}{\sqrt{3}\sqrt{-x^2-4x-3}}\right)\right)+\sqrt{1-2i\sqrt{2}}\left(2\sqrt{2}+i\right)x\tanh^{-1}\left(\frac{-i\sqrt{2}x+2x-2i\sqrt{2}+2}{\sqrt{2+4i\sqrt{2}}\sqrt{-x^2-4x-3}}\right)+\sqrt{}}{27x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Sqrt[-3 - 4*x - x^2]*(3 + 4*x + 2*x^2)),x]
```

```
[Out] (3*(Sqrt[-3 - 4*x - x^2] + 2*Sqrt[3]*x*ArcTan[(3 + 2*x)/(Sqrt[3]*Sqrt[-3 -
4*x - x^2])]) + Sqrt[1 - (2*I)*Sqrt[2]]*(I + 2*Sqrt[2])*x*ArcTanh[(2 - (2*I
)*Sqrt[2] + 2*x - I*Sqrt[2]*x)/(Sqrt[2 + (4*I)*Sqrt[2]]*Sqrt[-3 - 4*x - x^2
])] + Sqrt[1 + (2*I)*Sqrt[2]]*(-I + 2*Sqrt[2])*x*ArcTanh[(2 + (2*I)*Sqrt[2]
 + (2 + I*Sqrt[2])*x)/(Sqrt[2 - (4*I)*Sqrt[2]]*Sqrt[-3 - 4*x - x^2])])/(27*
x)
```

fricas [A] time = 0.73, size = 194, normalized size = 1.28

$$-\frac{12\sqrt{3}\,x\arctan\left(\frac{\sqrt{3}\sqrt{-x^2-4x-3}\,(2x+3)}{3\left(x^2+4x+3\right)}\right)-2\sqrt{2}\,x\arctan\left(\frac{\sqrt{2}x+3\sqrt{2}\sqrt{-x^2-4x-3}}{2\,(2x+3)}\right)-2\sqrt{2}\,x\arctan\left(-\frac{\sqrt{2}x-3\sqrt{2}\sqrt{-x^2-}}{2\,(2x+3)}\right.}{54\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/54*(12*sqrt(3)*x*arctan(1/3*sqrt(3)*sqrt(-x^2 - 4*x - 3)*(2*x + 3)/(x^2
+ 4*x + 3)) - 2*sqrt(2)*x*arctan(1/2*(sqrt(2)*x + 3*sqrt(2)*sqrt(-x^2 - 4*x
 - 3))/(2*x + 3)) - 2*sqrt(2)*x*arctan(-1/2*(sqrt(2)*x - 3*sqrt(2)*sqrt(-x^
2 - 4*x - 3))/(2*x + 3)) + 5*x*log(-(2*sqrt(-x^2 - 4*x - 3)*x + 4*x + 3)/x^
2) - 5*x*log((2*sqrt(-x^2 - 4*x - 3)*x - 4*x - 3)/x^2) - 6*sqrt(-x^2 - 4*x
- 3))/x
```

giac [B] time = 0.48, size = 269, normalized size = 1.78

$$\frac{2}{27}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\frac{3\left(\sqrt{-x^2-4x-3}-1\right)}{x+2}+1\right)\right)-\frac{4}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(\frac{2\left(\sqrt{-x^2-4x-3}-1\right)}{x+2}+1\right)\right)+\frac{2}{27}\sqrt{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="giac")
```

```
[Out] 2/27*sqrt(2)*arctan(1/2*sqrt(2)*(3*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 1))
 - 4/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 1
)) + 2/27*sqrt(2)*arctan(1/2*sqrt(2)*((sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) +
1)) - 1/18*((sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 2)/((sqrt(-x^2 - 4*x - 3)
- 1)/(x + 2) + (sqrt(-x^2 - 4*x - 3) - 1)^2/(x + 2)^2 + 1) + 5/27*log(2*(sq
rt(-x^2 - 4*x - 3) - 1)/(x + 2) + 3*(sqrt(-x^2 - 4*x - 3) - 1)^2/(x + 2)^2
+ 1) - 5/27*log(2*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + (sqrt(-x^2 - 4*x - 3
) - 1)^2/(x + 2)^2 + 3)
```

maple [A] time = 0.02, size = 169, normalized size = 1.12

$$-\frac{2\sqrt{3}\arctan\left(\frac{(-4x-6)\sqrt{3}}{6\sqrt{-x^2-4x-3}}\right)}{9}+\frac{\sqrt{-x^2-4x-3}}{9x}+\frac{\sqrt{3}\sqrt{4}\sqrt{\frac{3x^2}{\left(-x-\frac{3}{2}\right)^2}-12}\left(-5\,\mathrm{arctanh}\left(\frac{3x}{\left(-x-\frac{3}{2}\right)\sqrt{\frac{3x^2}{\left(-x-\frac{3}{2}\right)^2}-12}}\right)+\sqrt{2}\,\mathrm{ar}\right.}{81\sqrt{\frac{\frac{x^2}{\left(-x-\frac{3}{2}\right)^2}-4}{\left(\frac{x}{-x-\frac{3}{2}}+1\right)^2}}\left(\frac{x}{-x-\frac{3}{2}}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x)
```

```
[Out] 1/81*3^(1/2)*4^(1/2)*(3/(-x-3/2)^2*x^2-12)^(1/2)*(2^(1/2)*arctan(1/6*(3/(-x
-3/2)^2*x^2-12)^(1/2)*2^(1/2))-5*arctanh(3/(-x-3/2)/(3/(-x-3/2)^2*x^2-12)^(
1/2)*x))/((1/(-x-3/2)^2*x^2-4)/(1/(-x-3/2)*x+1)^2)^(1/2)/(1/(-x-3/2)*x+1)+1
/9*(-x^2-4*x-3)^(1/2)/x-2/9*3^(1/2)*arctan(1/6*(-4*x-6)*3^(1/2)/(-x^2-4*x-3
)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(2x^2+4x+3\right)\sqrt{-x^2-4x-3}\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((2*x^2 + 4*x + 3)*sqrt(-x^2 - 4*x - 3)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^2\,\sqrt{-x^2-4\,x-3}\,\left(2\,x^2+4\,x+3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(- 4*x - x^2 - 3)^(1/2)*(4*x + 2*x^2 + 3)),x)
[Out] int(1/(x^2*(- 4*x - x^2 - 3)^(1/2)*(4*x + 2*x^2 + 3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{2}\sqrt{-\left(x+1\right)\left(x+3\right)}\left(2x^{2}+4x+3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(2*x**2+4*x+3)/(-x**2-4*x-3)**(1/2),x)
[Out] Integral(1/(x**2*sqrt(-(x + 1)*(x + 3))*(2*x**2 + 4*x + 3)), x)
```

3.133 $\int (2+3x)^2 \left(30+31x-12x^2\right)^2 \sqrt{6+17x+12x^2}\, dx$

Optimal. Leaf size=149

$$-\frac{1}{32}(10-3x)\left(12x^2+17x+6\right)^{7/2}-\frac{873\left(12x^2+17x+6\right)^{7/2}}{1792}+\frac{25091(24x+17)\left(12x^2+17x+6\right)^{5/2}}{24576}-\frac{125455(24x+$$

[Out] -125455/4718592*(17+24*x)*(12*x^2+17*x+6)^(3/2)+25091/24576*(17+24*x)*(12*x^2+17*x+6)^(5/2)-873/1792*(12*x^2+17*x+6)^(7/2)-1/32*(10-3*x)*(12*x^2+17*x+6)^(7/2)-125455/1811939328*arctanh(1/12*(17+24*x)*3^(1/2)/(12*x^2+17*x+6)^(1/2))*3^(1/2)+125455/150994944*(17+24*x)*(12*x^2+17*x+6)^(1/2)

Rubi [A] time = 0.09, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1002, 742, 640, 612, 621, 206}

$$-\frac{1}{32}(10-3x)\left(12x^2+17x+6\right)^{7/2}-\frac{873\left(12x^2+17x+6\right)^{7/2}}{1792}+\frac{25091(24x+17)\left(12x^2+17x+6\right)^{5/2}}{24576}-\frac{125455(24x+$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x)^2*(30 + 31*x - 12*x^2)^2*Sqrt[6 + 17*x + 12*x^2],x]
```

```
[Out] (125455*(17 + 24*x)*Sqrt[6 + 17*x + 12*x^2])/150994944 - (125455*(17 + 24*x)*(6 + 17*x + 12*x^2)^(3/2))/4718592 + (25091*(17 + 24*x)*(6 + 17*x + 12*x^2)^(5/2))/24576 - (873*(6 + 17*x + 12*x^2)^(7/2))/1792 - ((10 - 3*x)*(6 + 17*x + 12*x^2)^(7/2))/32 - (125455*ArcTanh[(17 + 24*x)/(4*Sqrt[3]*Sqrt[6 + 17*x + 12*x^2])])/(603979776*Sqrt[3])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 742

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
```

```
+ 1)), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
raticQ[a, b, c, d, e, m, p, x]
```

Rule 1002

```
Int[((g_) + (h_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.
) + (e_.)*(x_) + (f_.)*(x_)^2)^(m_.), x_Symbol] :> Int[((d*g)/a + (f*h*x)/c
)^m*(a + b*x + c*x^2)^(m + p), x] /; FreeQ[{a, b, c, d, e, f, g, h, p}, x]
&& EqQ[c*g^2 - b*g*h + a*h^2, 0] && EqQ[c^2*d*g^2 - a*c*e*g*h + a^2*f*h^2,
0] && IntegerQ[m]
```

Rubi steps

$$\int (2+3x)^2 \left(30+31x-12x^2\right)^2 \sqrt{6+17x+12x^2}\, dx = \int (10-3x)^2 \left(6+17x+12x^2\right)^{5/2} dx$$

$$= -\frac{1}{32}(10-3x)\left(6+17x+12x^2\right)^{7/2} + \frac{1}{96}\int \left(11331 - 78\right.$$

$$= -\frac{873\left(6+17x+12x^2\right)^{7/2}}{1792} - \frac{1}{32}(10-3x)\left(6+17x+12\right.$$

$$= \frac{25091(17+24x)\left(6+17x+12x^2\right)^{5/2}}{24576} - \frac{873\left(6+17x+\right.}{1792}$$

$$= -\frac{125455(17+24x)\left(6+17x+12x^2\right)^{3/2}}{4718592} + \frac{25091(17+}{}$$

$$= \frac{125455(17+24x)\sqrt{6+17x+12x^2}}{150994944} - \frac{125455(17+24}{47}$$

$$= \frac{125455(17+24x)\sqrt{6+17x+12x^2}}{150994944} - \frac{125455(17+24}{47}$$

$$= \frac{125455(17+24x)\sqrt{6+17x+12x^2}}{150994944} - \frac{125455(17+24}{47}$$

Mathematica [A] time = 0.19, size = 87, normalized size = 0.58

$$\frac{12\sqrt{12x^2+17x+6}\left(171228266496x^7 - 732816211968x^6 - 1190083166208x^5 + 3438453030912x^4 + 897484\right.}{1268357}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x)^2*(30 + 31*x - 12*x^2)^2*Sqrt[6 + 17*x + 12*x^2],x]
```

```
[Out] (12*Sqrt[6 + 17*x + 12*x^2]*(474999091769 + 3132157281976*x + 7899203409792
*x^2 + 8974844476416*x^3 + 3438453030912*x^4 - 1190083166208*x^5 - 73281621
1968*x^6 + 171228266496*x^7) - 878185*Sqrt[3]*ArcTanh[(17 + 24*x)/(4*Sqrt[1
8 + 51*x + 36*x^2])])/12683575296
```

fricas [A] time = 0.58, size = 88, normalized size = 0.59

$$\frac{1}{1056964608}\left(171228266496\,x^7 - 732816211968\,x^6 - 1190083166208\,x^5 + 3438453030912\,x^4 + 89748444764\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)^2*(-12*x^2+31*x+30)^2*(12*x^2+17*x+6)^(1/2),x, algorithm=
"fricas")
```

```
[Out] 1/1056964608*(171228266496*x^7 - 732816211968*x^6 - 1190083166208*x^5 + 343
8453030912*x^4 + 8974844476416*x^3 + 7899203409792*x^2 + 3132157281976*x +
474999091769)*sqrt(12*x^2 + 17*x + 6) + 125455/3623878656*sqrt(3)*log(-8*sq
rt(3)*sqrt(12*x^2 + 17*x + 6)*(24*x + 17) + 1152*x^2 + 1632*x + 577)
```

giac [A] time = 0.30, size = 85, normalized size = 0.57

$$\frac{1}{1056964608}\left(8\left(48\left(24\left(96\left(24\left(48\left(168\,x-719\right)x-56047\right)x+3886417\right)x+973832951\right)x+20570842213\right)x+39\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)^2*(-12*x^2+31*x+30)^2*(12*x^2+17*x+6)^(1/2),x, algorithm=
"giac")
```

```
[Out] 1/1056964608*(8*(48*(24*(96*(24*(48*(168*x - 719)*x - 56047)*x + 3886417)*x
 + 973832951)*x + 20570842213)*x + 391519660247)*x + 474999091769)*sqrt(12*
x^2 + 17*x + 6) + 125455/1811939328*sqrt(3)*log(abs(-4*sqrt(3)*(2*sqrt(3)*x
 - sqrt(12*x^2 + 17*x + 6)) - 17))
```

maple [A] time = 0.02, size = 147, normalized size = 0.99

$$\frac{27\left(12x^{2}+17x+6\right)^{\frac{3}{2}}x^{5}}{2}-\frac{8613\left(12x^{2}+17x+6\right)^{\frac{3}{2}}x^{4}}{112}+\frac{14991\left(12x^{2}+17x+6\right)^{\frac{3}{2}}x^{3}}{1792}+\frac{4267751\left(12x^{2}+17x+6\right)^{\frac{3}{2}}x}{14336}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2+3*x)^2*(-12*x^2+31*x+30)^2*(12*x^2+17*x+6)^(1/2),x)
```

```
[Out] 27/2*x^5*(12*x^2+17*x+6)^(3/2)-8613/112*x^4*(12*x^2+17*x+6)^(3/2)+14991/179
2*x^3*(12*x^2+17*x+6)^(3/2)+4267751/14336*x^2*(12*x^2+17*x+6)^(3/2)+1292207
57/458752*x*(12*x^2+17*x+6)^(3/2)+2473875847/33030144*(12*x^2+17*x+6)^(3/2)
-125455/3623878656*ln(1/12*(17/2+12*x)*12^(1/2)+(12*x^2+17*x+6)^(1/2))*12^(
1/2)+125455/150994944*(17+24*x)*(12*x^2+17*x+6)^(1/2)
```

maxima [A] time = 1.00, size = 155, normalized size = 1.04

$$\frac{27}{2}\left(12\,x^{2}+17\,x+6\right)^{\frac{3}{2}}x^{5}-\frac{8613}{112}\left(12\,x^{2}+17\,x+6\right)^{\frac{3}{2}}x^{4}+\frac{14991}{1792}\left(12\,x^{2}+17\,x+6\right)^{\frac{3}{2}}x^{3}+\frac{4267751}{14336}\left(12\,x^{2}+17\,x+6\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)^2*(-12*x^2+31*x+30)^2*(12*x^2+17*x+6)^(1/2),x, algorithm=
"maxima")
```

```
[Out] 27/2*(12*x^2 + 17*x + 6)^(3/2)*x^5 - 8613/112*(12*x^2 + 17*x + 6)^(3/2)*x^4
 + 14991/1792*(12*x^2 + 17*x + 6)^(3/2)*x^3 + 4267751/14336*(12*x^2 + 17*x
+ 6)^(3/2)*x^2 + 129220757/458752*(12*x^2 + 17*x + 6)^(3/2)*x + 2473875847/
33030144*(12*x^2 + 17*x + 6)^(3/2) + 125455/6291456*sqrt(12*x^2 + 17*x + 6)
*x - 125455/1811939328*sqrt(3)*log(4*sqrt(3)*sqrt(12*x^2 + 17*x + 6) + 24*x
 + 17) + 2132735/150994944*sqrt(12*x^2 + 17*x + 6)
```

mupad [B] time = 5.08, size = 187, normalized size = 1.26

$$\frac{4267751\,x^{2}\left(12\,x^{2}+17\,x+6\right)^{3/2}}{14336}+\frac{14991\,x^{3}\left(12\,x^{2}+17\,x+6\right)^{3/2}}{1792}-\frac{8613\,x^{4}\left(12\,x^{2}+17\,x+6\right)^{3/2}}{112}+\frac{27\,x^{5}\left(12\,x^{2}+17\right.}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x + 2)^2*(17*x + 12*x^2 + 6)^(1/2)*(31*x - 12*x^2 + 30)^2,x)
```

```
[Out] (4267751*x^2*(17*x + 12*x^2 + 6)^(3/2))/14336 + (14991*x^3*(17*x + 12*x^2 +
 6)^(3/2))/1792 - (8613*x^4*(17*x + 12*x^2 + 6)^(3/2))/112 + (27*x^5*(17*x
+ 12*x^2 + 6)^(3/2))/2 - (146030443*12^(1/2)*log((17*x + 12*x^2 + 6)^(1/2)
+ (12^(1/2)*(12*x + 17/2))/12))/88080384 + (438091329*(x/2 + 17/48)*(17*x +
 12*x^2 + 6)^(1/2))/229376 + (2473875847*(17*x + 12*x^2 + 6)^(1/2)*(408*x +
 1152*x^2 - 291))/3170893824 + (129220757*x*(17*x + 12*x^2 + 6)^(3/2))/4587
52 + (42055889399*12^(1/2)*log(2*(17*x + 12*x^2 + 6)^(1/2) + (12^(1/2)*(24*
x + 17))/12))/25367150592
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{(3x+2)(4x+3)}\,(3x-10)^2(3x+2)^2(4x+3)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)**2*(-12*x**2+31*x+30)**2*(12*x**2+17*x+6)**(1/2),x)
```

```
[Out] Integral(sqrt((3*x + 2)*(4*x + 3))*(3*x - 10)**2*(3*x + 2)**2*(4*x + 3)**2,
 x)
```

3.134 $\int (2+3x)\left(30+31x-12x^2\right)\sqrt{6+17x+12x^2}\,dx$

Optimal. Leaf size=103

$$-\frac{1}{20}\left(12x^2+17x+6\right)^{5/2}+\frac{97}{768}(24x+17)\left(12x^2+17x+6\right)^{3/2}-\frac{97(24x+17)\sqrt{12x^2+17x+6}}{24576}+\frac{97\tanh^{-1}\left(\frac{24x}{4\sqrt{3}\sqrt{12}}\right.}{98304\sqrt{3}}$$

```
[Out] 97/768*(17+24*x)*(12*x^2+17*x+6)^(3/2)-1/20*(12*x^2+17*x+6)^(5/2)+97/294912
*arctanh(1/12*(17+24*x)*3^(1/2)/(12*x^2+17*x+6)^(1/2))*3^(1/2)-97/24576*(17
+24*x)*(12*x^2+17*x+6)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1002, 640, 612, 621, 206}

$$-\frac{1}{20}\left(12x^2+17x+6\right)^{5/2}+\frac{97}{768}(24x+17)\left(12x^2+17x+6\right)^{3/2}-\frac{97(24x+17)\sqrt{12x^2+17x+6}}{24576}+\frac{97\tanh^{-1}\left(\frac{24x}{4\sqrt{3}\sqrt{12}}\right.}{98304\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x)*(30 + 31*x - 12*x^2)*Sqrt[6 + 17*x + 12*x^2],x]
```

```
[Out] (-97*(17 + 24*x)*Sqrt[6 + 17*x + 12*x^2])/24576 + (97*(17 + 24*x)*(6 + 17*x
 + 12*x^2)^(3/2))/768 - (6 + 17*x + 12*x^2)^(5/2)/20 + (97*ArcTanh[(17 + 24
*x)/(4*Sqrt[3]*Sqrt[6 + 17*x + 12*x^2])])/(98304*Sqrt[3])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1002

```
Int[((g_) + (h_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.
) + (e_.)*(x_) + (f_.)*(x_)^2)^(m_.), x_Symbol] :> Int[((d*g)/a + (f*h*x)/c
)^m*(a + b*x + c*x^2)^(m + p), x] /; FreeQ[{a, b, c, d, e, f, g, h, p}, x]
&& EqQ[c*g^2 - b*g*h + a*h^2, 0] && EqQ[c^2*d*g^2 - a*c*e*g*h + a^2*f*h^2,
```

```
0] && IntegerQ[m]
```

Rubi steps

$$\int (2+3x)\left(30+31x-12x^2\right)\sqrt{6+17x+12x^2}\,dx = \int (10-3x)\left(6+17x+12x^2\right)^{3/2}\,dx$$
$$= -\frac{1}{20}\left(6+17x+12x^2\right)^{5/2}+\frac{97}{8}\int\left(6+17x+12x^2\right)^{3/2}\,dx$$
$$= \frac{97}{768}(17+24x)\left(6+17x+12x^2\right)^{3/2}-\frac{1}{20}\left(6+17x+12x^2\right.$$
$$= -\frac{97(17+24x)\sqrt{6+17x+12x^2}}{24576}+\frac{97}{768}(17+24x)\left(6+1\right.$$
$$= -\frac{97(17+24x)\sqrt{6+17x+12x^2}}{24576}+\frac{97}{768}(17+24x)\left(6+1\right.$$
$$= -\frac{97(17+24x)\sqrt{6+17x+12x^2}}{24576}+\frac{97}{768}(17+24x)\left(6+1\right.$$

Mathematica [A] time = 0.03, size = 72, normalized size = 0.70

$$\frac{485\sqrt{3}\tanh^{-1}\left(\frac{24x+17}{4\sqrt{36x^2+51x+18}}\right)+12\sqrt{12x^2+17x+6}\left(-884736x^4+1963008x^3+6837888x^2+5455144x+1\right.}{1474560}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x)*(30 + 31*x - 12*x^2)*Sqrt[6 + 17*x + 12*x^2],x]
```

```
[Out] (12*Sqrt[6 + 17*x + 12*x^2]*(1353611 + 5455144*x + 6837888*x^2 + 1963008*x^
3 - 884736*x^4) + 485*Sqrt[3]*ArcTanh[(17 + 24*x)/(4*Sqrt[18 + 51*x + 36*x^
2])])/1474560
```

fricas [A] time = 0.59, size = 73, normalized size = 0.71

$$-\frac{1}{122880}\left(884736\,x^4-1963008\,x^3-6837888\,x^2-5455144\,x-1353611\right)\sqrt{12\,x^2+17\,x+6}+\frac{97}{589824}\sqrt{3}\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)*(-12*x^2+31*x+30)*(12*x^2+17*x+6)^(1/2),x, algorithm="fri
cas")
```

```
[Out] -1/122880*(884736*x^4 - 1963008*x^3 - 6837888*x^2 - 5455144*x - 1353611)*sq
rt(12*x^2 + 17*x + 6) + 97/589824*sqrt(3)*log(8*sqrt(3)*sqrt(12*x^2 + 17*x
+ 6)*(24*x + 17) + 1152*x^2 + 1632*x + 577)
```

giac [A] time = 0.20, size = 70, normalized size = 0.68

$$-\frac{1}{122880}\left(8\left(48\left(72\left(32\,x-71\right)x-17807\right)x-681893\right)x-1353611\right)\sqrt{12\,x^2+17\,x+6}-\frac{97}{294912}\sqrt{3}\log\left(\left|-4\sqrt{3}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)*(-12*x^2+31*x+30)*(12*x^2+17*x+6)^(1/2),x, algorithm="gia
c")
```

```
[Out] -1/122880*(8*(48*(72*(32*x - 71)*x - 17807)*x - 681893)*x - 1353611)*sqrt(1
2*x^2 + 17*x + 6) - 97/294912*sqrt(3)*log(abs(-4*sqrt(3)*(2*sqrt(3)*x - sqr
t(12*x^2 + 17*x + 6)) - 17))
```

maple [A] time = 0.01, size = 96, normalized size = 0.93

$$-\frac{3\left(12x^{2}+17x+6\right)^{\frac{3}{2}}x^{2}}{5}+\frac{349\left(12x^{2}+17x+6\right)^{\frac{3}{2}}x}{160}+\frac{97\sqrt{12}\,\ln\left(\frac{\left(12x+\frac{17}{2}\right)\sqrt{12}}{12}+\sqrt{12x^{2}+17x+6}\right)}{589824}+\frac{7093\left(12x^{2}+\right.}{384}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2+3*x)*(-12*x^2+31*x+30)*(12*x^2+17*x+6)^(1/2),x)
```

```
[Out] -3/5*(12*x^2+17*x+6)^(3/2)*x^2+349/160*(12*x^2+17*x+6)^(3/2)*x+7093/3840*(1
2*x^2+17*x+6)^(3/2)-97/24576*(24*x+17)*(12*x^2+17*x+6)^(1/2)+97/589824*12^(
1/2)*ln(1/12*(12*x+17/2)*12^(1/2)+(12*x^2+17*x+6)^(1/2))
```

maxima [A] time = 0.98, size = 104, normalized size = 1.01

$$-\frac{3}{5}\left(12\,x^{2}+17\,x+6\right)^{\frac{3}{2}}x^{2}+\frac{349}{160}\left(12\,x^{2}+17\,x+6\right)^{\frac{3}{2}}x+\frac{7093}{3840}\left(12\,x^{2}+17\,x+6\right)^{\frac{3}{2}}-\frac{97}{1024}\sqrt{12\,x^{2}+17\,x+6}\,x+\frac{97}{29491}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)*(-12*x^2+31*x+30)*(12*x^2+17*x+6)^(1/2),x, algorithm="max
ima")
```

```
[Out] -3/5*(12*x^2 + 17*x + 6)^(3/2)*x^2 + 349/160*(12*x^2 + 17*x + 6)^(3/2)*x +
7093/3840*(12*x^2 + 17*x + 6)^(3/2) - 97/1024*sqrt(12*x^2 + 17*x + 6)*x + 9
7/294912*sqrt(3)*log(4*sqrt(3)*sqrt(12*x^2 + 17*x + 6) + 24*x + 17) - 1649/
24576*sqrt(12*x^2 + 17*x + 6)
```

mupad [B] time = 4.69, size = 136, normalized size = 1.32

$$\frac{3753\left(\frac{x}{2}+\frac{17}{48}\right)\sqrt{12\,x^{2}+17\,x+6}}{80}-\frac{417\sqrt{12}\,\ln\left(\sqrt{12\,x^{2}+17\,x+6}+\frac{\sqrt{12}\left(12\,x+\frac{17}{2}\right)}{12}\right)}{10240}-\frac{3\,x^{2}\left(12\,x^{2}+17\,x+6\right)^{3/2}}{5}+\frac{7}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x + 2)*(17*x + 12*x^2 + 6)^(1/2)*(31*x - 12*x^2 + 30),x)
```

```
[Out] (3753*(x/2 + 17/48)*(17*x + 12*x^2 + 6)^(1/2))/80 - (417*12^(1/2)*log((17*x
 + 12*x^2 + 6)^(1/2) + (12^(1/2)*(12*x + 17/2))/12))/10240 - (3*x^2*(17*x +
 12*x^2 + 6)^(3/2))/5 + (7093*(17*x + 12*x^2 + 6)^(1/2)*(408*x + 1152*x^2 -
 291))/368640 + (349*x*(17*x + 12*x^2 + 6)^(3/2))/160 + (120581*12^(1/2)*lo
g(2*(17*x + 12*x^2 + 6)^(1/2) + (12^(1/2)*(24*x + 17))/12))/2949120
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\left(-152x\sqrt{12x^{2}+17x+6}\right)dx-\int\left(-69x^{2}\sqrt{12x^{2}+17x+6}\right)dx-\int 36x^{3}\sqrt{12x^{2}+17x+6}\,dx-\int\left(-60\sqrt{12x^{2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)*(-12*x**2+31*x+30)*(12*x**2+17*x+6)**(1/2),x)
```

```
[Out] -Integral(-152*x*sqrt(12*x**2 + 17*x + 6), x) - Integral(-69*x**2*sqrt(12*x
**2 + 17*x + 6), x) - Integral(36*x**3*sqrt(12*x**2 + 17*x + 6), x) - Integ
ral(-60*sqrt(12*x**2 + 17*x + 6), x)
```

3.135 $\int \frac{\sqrt{6+17x+12x^2}}{(2+3x)\left(30+31x-12x^2\right)} \, dx$

Optimal. Leaf size=28

$$\frac{1}{42}\tanh^{-1}\left(\frac{291x+206}{84\sqrt{12x^2+17x+6}}\right)$$

```
[Out] 1/42*arctanh(1/84*(206+291*x)/(12*x^2+17*x+6)^(1/2))
```

Rubi [A] time = 0.05, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.088, Rules used = {1002, 724, 206}

$$\frac{1}{42}\tanh^{-1}\left(\frac{291x+206}{84\sqrt{12x^2+17x+6}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[6 + 17*x + 12*x^2]/((2 + 3*x)*(30 + 31*x - 12*x^2)),x]
```

```
[Out] ArcTanh[(206 + 291*x)/(84*Sqrt[6 + 17*x + 12*x^2])]/42
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1002

```
Int[((g_) + (h_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.
) + (e_.)*(x_) + (f_.)*(x_)^2)^(m_.), x_Symbol] :> Int[((d*g)/a + (f*h*x)/c
)^m*(a + b*x + c*x^2)^(m + p), x] /; FreeQ[{a, b, c, d, e, f, g, h, p}, x]
&& EqQ[c*g^2 - b*g*h + a*h^2, 0] && EqQ[c^2*d*g^2 - a*c*e*g*h + a^2*f*h^2,
0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{6+17x+12x^2}}{(2+3x)\left(30+31x-12x^2\right)} \, dx &= \int \frac{1}{(10-3x)\sqrt{6+17x+12x^2}} \, dx \\
&= -\left(2\,\text{Subst}\left(\int \frac{1}{7056-x^2} \, dx, x, \frac{-206-291x}{\sqrt{6+17x+12x^2}}\right)\right) \\
&= \frac{1}{42}\tanh^{-1}\left(\frac{206+291x}{84\sqrt{6+17x+12x^2}}\right)
\end{aligned}$$

Mathematica [A] time = 0.10, size = 37, normalized size = 1.32

$$\frac{1}{42}\log\left(84\sqrt{12x^2+17x+6}+291x+206\right)-\frac{1}{42}\log(10-3x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[6 + 17*x + 12*x^2]/((2 + 3*x)*(30 + 31*x - 12*x^2)),x]
```

```
[Out] -1/42*Log[10 - 3*x] + Log[206 + 291*x + 84*Sqrt[6 + 17*x + 12*x^2]]/42
```

fricas [B] time = 0.70, size = 53, normalized size = 1.89

$$\frac{1}{84} \log\left(\frac{291\,x + 84\,\sqrt{12\,x^2 + 17\,x + 6} + 206}{x}\right) - \frac{1}{84} \log\left(\frac{291\,x - 84\,\sqrt{12\,x^2 + 17\,x + 6} + 206}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((12*x^2+17*x+6)^(1/2)/(2+3*x)/(-12*x^2+31*x+30),x, algorithm="fri
cas")
```

```
[Out] 1/84*log((291*x + 84*sqrt(12*x^2 + 17*x + 6) + 206)/x) - 1/84*log((291*x -
84*sqrt(12*x^2 + 17*x + 6) + 206)/x)
```

giac [B] time = 0.27, size = 63, normalized size = 2.25

$$\frac{1}{42} \log\left(\left|-6\,\sqrt{3}\,x + 20\,\sqrt{3} + 3\,\sqrt{12\,x^2 + 17\,x + 6} + 42\right|\right) - \frac{1}{42} \log\left(\left|-6\,\sqrt{3}\,x + 20\,\sqrt{3} + 3\,\sqrt{12\,x^2 + 17\,x + 6} - 42\right|\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((12*x^2+17*x+6)^(1/2)/(2+3*x)/(-12*x^2+31*x+30),x, algorithm="gia
c")
```

```
[Out] 1/42*log(abs(-6*sqrt(3)*x + 20*sqrt(3) + 3*sqrt(12*x^2 + 17*x + 6) + 42)) -
 1/42*log(abs(-6*sqrt(3)*x + 20*sqrt(3) + 3*sqrt(12*x^2 + 17*x + 6) - 42))
```

maple [B] time = 0.02, size = 163, normalized size = 5.82

$$\frac{\operatorname{arctanh}\left(\frac{97x+\frac{206}{3}}{28\sqrt{97x+12\left(x-\frac{10}{3}\right)^2-\frac{382}{3}}}\right)}{42} - \frac{97\sqrt{12}\ \ln\left(\frac{\left(12x+\frac{17}{2}\right)\sqrt{12}}{12} + \sqrt{97x+12\left(x-\frac{10}{3}\right)^2-\frac{382}{3}}\right)}{14112} + \frac{\sqrt{12}\ \ln\left(\frac{\left(12x+\frac{17}{2}\right)\sqrt{12}}{12} + \cdots\right.}{2\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((12*x^2+17*x+6)^(1/2)/(2+3*x)/(-12*x^2+31*x+30),x)
```

```
[Out] -4/49*(12*(x+3/4)^2-x-3/4)^(1/2)+1/294*ln(1/12*(12*x+17/2)*12^(1/2)+(12*(x+
3/4)^2-x-3/4)^(1/2))*12^(1/2)+1/12*(12*(x+2/3)^2+x+2/3)^(1/2)+1/288*ln(1/12
*(12*x+17/2)*12^(1/2)+(12*(x+2/3)^2+x+2/3)^(1/2))*12^(1/2)-1/588*(12*(x-10/
3)^2+97*x-382/3)^(1/2)-97/14112*ln(1/12*(12*x+17/2)*12^(1/2)+(12*(x-10/3)^2
+97*x-382/3)^(1/2))*12^(1/2)+1/42*arctanh(1/28*(206/3+97*x)/(12*(x-10/3)^2+
97*x-382/3)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt{12\,x^2 + 17\,x + 6}}{\left(12\,x^2 - 31\,x - 30\right)(3\,x + 2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((12*x^2+17*x+6)^(1/2)/(2+3*x)/(-12*x^2+31*x+30),x, algorithm="max
ima")
```

```
[Out] -integrate(sqrt(12*x^2 + 17*x + 6)/((12*x^2 - 31*x - 30)*(3*x + 2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\sqrt{12\,x^2+17\,x+6}}{(3\,x+2)\,\left(-12\,x^2+31\,x+30\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((17*x + 12*x^2 + 6)^(1/2)/((3*x + 2)*(31*x - 12*x^2 + 30)),x)
[Out] int((17*x + 12*x^2 + 6)^(1/2)/((3*x + 2)*(31*x - 12*x^2 + 30)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt{12x^2+17x+6}}{36x^3-69x^2-152x-60}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((12*x**2+17*x+6)**(1/2)/(2+3*x)/(-12*x**2+31*x+30),x)
[Out] -Integral(sqrt(12*x**2 + 17*x + 6)/(36*x**3 - 69*x**2 - 152*x - 60), x)
```

3.136 $\int \frac{\sqrt{6+17x+12x^2}}{(2+3x)^2\left(30+31x-12x^2\right)^2}\,dx$

Optimal. Leaf size=84

$$-\frac{388x+275}{98(10-3x)\sqrt{12x^2+17x+6}}+\frac{3137\sqrt{12x^2+17x+6}}{38416(10-3x)}+\frac{97\tanh^{-1}\left(\frac{291x+206}{84\sqrt{12x^2+17x+6}}\right)}{3226944}$$

```
[Out] 97/3226944*arctanh(1/84*(206+291*x)/(12*x^2+17*x+6)^(1/2))+1/98*(-275-388*x
)/(10-3*x)/(12*x^2+17*x+6)^(1/2)+3137/38416*(12*x^2+17*x+6)^(1/2)/(10-3*x)
```

Rubi [A] time = 0.08, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.147, Rules used = {1002, 740, 806, 724, 206}

$$-\frac{388x+275}{98(10-3x)\sqrt{12x^2+17x+6}}+\frac{3137\sqrt{12x^2+17x+6}}{38416(10-3x)}+\frac{97\tanh^{-1}\left(\frac{291x+206}{84\sqrt{12x^2+17x+6}}\right)}{3226944}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[6 + 17*x + 12*x^2]/((2 + 3*x)^2*(30 + 31*x - 12*x^2)^2),x]
```

```
[Out] -(275 + 388*x)/(98*(10 - 3*x)*Sqrt[6 + 17*x + 12*x^2]) + (3137*Sqrt[6 + 17*
x + 12*x^2])/(38416*(10 - 3*x)) + (97*ArcTanh[(206 + 291*x)/(84*Sqrt[6 + 17
*x + 12*x^2])])/3226944
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
```

```
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 1002

```
Int[((g_) + (h_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.
) + (e_.)*(x_) + (f_.)*(x_)^2)^(m_.), x_Symbol] :> Int[((d*g)/a + (f*h*x)/c
)^m*(a + b*x + c*x^2)^(m + p), x] /; FreeQ[{a, b, c, d, e, f, g, h, p}, x]
&& EqQ[c*g^2 - b*g*h + a*h^2, 0] && EqQ[c^2*d*g^2 - a*c*e*g*h + a^2*f*h^2,
0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\sqrt{6+17x+12x^2}}{(2+3x)^2\left(30+31x-12x^2\right)^2}\,dx = \int \frac{1}{(10-3x)^2\left(6+17x+12x^2\right)^{3/2}}\,dx$$

$$= -\frac{275+388x}{98(10-3x)\sqrt{6+17x+12x^2}} - \frac{1}{882}\int \frac{-\frac{14859}{2}-10476x}{(10-3x)^2\sqrt{6+17x+12x^2}}\,dx$$

$$= -\frac{275+388x}{98(10-3x)\sqrt{6+17x+12x^2}} + \frac{3137\sqrt{6+17x+12x^2}}{38416(10-3x)} + \frac{97\int \frac{}{(10-3x)\sqrt{}}}{76}$$

$$= -\frac{275+388x}{98(10-3x)\sqrt{6+17x+12x^2}} + \frac{3137\sqrt{6+17x+12x^2}}{38416(10-3x)} - \frac{97\,\mathrm{Subst}\left(\int\right.}{}$$

$$= -\frac{275+388x}{98(10-3x)\sqrt{6+17x+12x^2}} + \frac{3137\sqrt{6+17x+12x^2}}{38416(10-3x)} + \frac{97\tanh^{-1}\left(\right.}{322}$$

Mathematica [A] time = 0.24, size = 114, normalized size = 1.36

$$\frac{\sqrt{12x^2+17x+6}\left(97\left(36x^3-69x^2-152x-60\right)\tanh^{-1}\left(\frac{7\sqrt{3x+2}}{6\sqrt{4x+3}}\right)-42\sqrt{3x+2}\sqrt{4x+3}\left(37644x^2-98767x\right.\right.}{1613472(3x-10)(3x+2)^{3/2}(4x+3)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[6 + 17*x + 12*x^2]/((2 + 3*x)^2*(30 + 31*x - 12*x^2)^2),x]
```

```
[Out] (Sqrt[6 + 17*x + 12*x^2]*(-42*Sqrt[2 + 3*x]*Sqrt[3 + 4*x]*(-88978 - 98767*x
 + 37644*x^2) + 97*(-60 - 152*x - 69*x^2 + 36*x^3)*ArcTanh[(7*Sqrt[2 + 3*x]
)/(6*Sqrt[3 + 4*x])]))/(1613472*(-10 + 3*x)*(2 + 3*x)^(3/2)*(3 + 4*x)^(3/2)
)
```

fricas [A] time = 0.72, size = 126, normalized size = 1.50

$$\frac{97\left(36\,x^3-69\,x^2-152\,x-60\right)\log\left(\frac{291\,x+84\,\sqrt{12\,x^2+17\,x+6}+206}{x}\right)-97\left(36\,x^3-69\,x^2-152\,x-60\right)\log\left(\frac{291\,x-84\,\sqrt{1}}{}\right.}{6453888\left(36\,x^3-69\,x^2-152\,x-60\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((12*x^2+17*x+6)^(1/2)/(2+3*x)^2/(-12*x^2+31*x+30)^2,x, algorithm=
"fricas")
```

```
[Out] 1/6453888*(97*(36*x^3 - 69*x^2 - 152*x - 60)*log((291*x + 84*sqrt(12*x^2 +
17*x + 6) + 206)/x) - 97*(36*x^3 - 69*x^2 - 152*x - 60)*log((291*x - 84*sqr
```

```
t(12*x^2 + 17*x + 6) + 206)/x) - 168*(37644*x^2 - 98767*x - 88978)*sqrt(12*
x^2 + 17*x + 6))/(36*x^3 - 69*x^2 - 152*x - 60)
```

giac [B] time = 0.27, size = 159, normalized size = 1.89

$$\frac{1}{9680832}\sqrt{3}\left(\sqrt{3}\left(175672\sqrt{3}+97\log\left(\frac{7\sqrt{3}-12}{7\sqrt{3}+12}\right)\right)\mathrm{sgn}\left(\frac{1}{3x+2}\right)-\left(97\sqrt{3}\log\left(\frac{\left|-28\sqrt{3}+24\sqrt{\frac{1}{3x+2}+4}\right|}{4\left(7\sqrt{3}+6\sqrt{\frac{1}{3x+2}+4}\right)}\right)\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((12*x^2+17*x+6)^(1/2)/(2+3*x)^2/(-12*x^2+31*x+30)^2,x, algorithm=
"giac")
```

```
[Out] 1/9680832*sqrt(3)*(sqrt(3)*(175672*sqrt(3) + 97*log((7*sqrt(3) - 12)/(7*sqr
t(3) + 12)))*sgn(1/(3*x + 2)) - (97*sqrt(3)*log(1/4*abs(-28*sqrt(3) + 24*sq
rt(1/(3*x + 2) + 4))/(7*sqrt(3) + 6*sqrt(1/(3*x + 2) + 4))) + 134456*sqrt(1
/(3*x + 2) + 4) + 28*(221183/(3*x + 2) - 18436)/(12*(1/(3*x + 2) + 4)^(3/2)
 - 49*sqrt(1/(3*x + 2) + 4)))*sgn(1/(3*x + 2)))
```

maple [B] time = 0.02, size = 245, normalized size = 2.92

$$\frac{97\,\mathrm{arctanh}\left(\frac{97x+\frac{206}{3}}{28\sqrt{97x+12\left(x-\frac{10}{3}\right)^2-\frac{382}{3}}}\right)}{3226944}-\frac{7057\sqrt{12}\,\ln\left(\frac{\left(12x+\frac{17}{2}\right)\sqrt{12}}{12}+\sqrt{97x+12\left(x-\frac{10}{3}\right)^2-\frac{382}{3}}\right)}{813189888}+\frac{\sqrt{12}\,\ln\left(\frac{\left(12x+\frac{17}{2}\right)}{12}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((12*x^2+17*x+6)^(1/2)/(2+3*x)^2/(-12*x^2+31*x+30)^2,x)
```

```
[Out] 384/117649*(-x+12*(x+3/4)^2-3/4)^(1/2)-16/117649*12^(1/2)*ln(1/12*(12*x+17/
2)*12^(1/2)+(-x+12*(x+3/4)^2-3/4)^(1/2))+1/288*(x+12*(x+2/3)^2+2/3)^(1/2)+1
/6912*12^(1/2)*ln(1/12*(12*x+17/2)*12^(1/2)+(x+12*(x+2/3)^2+2/3)^(1/2))-97/
45177216*(97*x+12*(x-10/3)^2-382/3)^(1/2)-7057/813189888*12^(1/2)*ln(1/12*(
12*x+17/2)*12^(1/2)+(97*x+12*(x-10/3)^2-382/3)^(1/2))+97/3226944*arctanh(1/
28*(97*x+206/3)/(97*x+12*(x-10/3)^2-382/3)^(1/2))-1/72/(x+2/3)^2*(x+12*(x+2
/3)^2+2/3)^(3/2)+32/2401/(x+3/4)^2*(-x+12*(x+3/4)^2-3/4)^(3/2)-1/67765824/(
x-10/3)*(97*x+12*(x-10/3)^2-382/3)^(3/2)+1/135531648*(24*x+17)*(97*x+12*(x-
10/3)^2-382/3)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{12\,x^2+17\,x+6}}{\left(12\,x^2-31\,x-30\right)^2\left(3\,x+2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((12*x^2+17*x+6)^(1/2)/(2+3*x)^2/(-12*x^2+31*x+30)^2,x, algorithm=
"maxima")
```

```
[Out] integrate(sqrt(12*x^2 + 17*x + 6)/((12*x^2 - 31*x - 30)^2*(3*x + 2)^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{12\,x^2+17\,x+6}}{\left(3\,x+2\right)^2\left(-12\,x^2+31\,x+30\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((17*x + 12*x^2 + 6)^(1/2)/((3*x + 2)^2*(31*x - 12*x^2 + 30)^2),x)
```

```
[Out] int((17*x + 12*x^2 + 6)^(1/2)/((3*x + 2)^2*(31*x - 12*x^2 + 30)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{(3x+2)(4x+3)}}{(3x-10)^2 (3x+2)^2 (4x+3)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((12*x**2+17*x+6)**(1/2)/(2+3*x)**2/(-12*x**2+31*x+30)**2,x)
```

```
[Out] Integral(sqrt((3*x + 2)*(4*x + 3))/((3*x - 10)**2*(3*x + 2)**2*(4*x + 3)**2
), x)
```

3.137 $\int \frac{\sqrt{6+17x+12x^2}}{(2+3x)^3\left(30+31x-12x^2\right)^3}\,dx$

Optimal. Leaf size=139

$$-\frac{388x+275}{294(10-3x)^2\left(12x^2+17x+6\right)^{3/2}}-\frac{1634466587\sqrt{12x^2+17x+6}}{7589772288(10-3x)}-\frac{50555899\sqrt{12x^2+17x+6}}{19361664(10-3x)^2}+\frac{1042556}{8232(10-3x)}$$

```
[Out] 1/294*(-275-388*x)/(10-3*x)^2/(12*x^2+17*x+6)^(3/2)+40325/637540872192*arct
anh(1/84*(206+291*x)/(12*x^2+17*x+6)^(1/2))+1/8232*(738029+1042556*x)/(10-3
*x)^2/(12*x^2+17*x+6)^(1/2)-50555899/19361664*(12*x^2+17*x+6)^(1/2)/(10-3*x
)^2-1634466587/7589772288*(12*x^2+17*x+6)^(1/2)/(10-3*x)
```

Rubi [A] time = 0.12, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.206$, Rules used = {1002, 740, 822, 834, 806, 724, 206}

$$-\frac{388x+275}{294(10-3x)^2\left(12x^2+17x+6\right)^{3/2}}-\frac{1634466587\sqrt{12x^2+17x+6}}{7589772288(10-3x)}-\frac{50555899\sqrt{12x^2+17x+6}}{19361664(10-3x)^2}+\frac{1042556}{8232(10-3x)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[6 + 17*x + 12*x^2]/((2 + 3*x)^3*(30 + 31*x - 12*x^2)^3),x]
```

```
[Out] -(275 + 388*x)/(294*(10 - 3*x)^2*(6 + 17*x + 12*x^2)^(3/2)) + (738029 + 104
2556*x)/(8232*(10 - 3*x)^2*Sqrt[6 + 17*x + 12*x^2]) - (50555899*Sqrt[6 + 17
*x + 12*x^2])/(19361664*(10 - 3*x)^2) - (1634466587*Sqrt[6 + 17*x + 12*x^2]
)/(7589772288*(10 - 3*x)) + (40325*ArcTanh[(206 + 291*x)/(84*Sqrt[6 + 17*x
+ 12*x^2])])/637540872192
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 822

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(f*(b*c*d - b^2*e +
2*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*(a
+ b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 834

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*
x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1002

```
Int[((g_) + (h_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.
) + (e_.)*(x_) + (f_.)*(x_)^2)^(m_.), x_Symbol] :> Int[((d*g)/a + (f*h*x)/c
)^m*(a + b*x + c*x^2)^(m + p), x] /; FreeQ[{a, b, c, d, e, f, g, h, p}, x]
&& EqQ[c*g^2 - b*g*h + a*h^2, 0] && EqQ[c^2*d*g^2 - a*c*e*g*h + a^2*f*h^2,
0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\sqrt{6+17x+12x^2}}{(2+3x)^3\left(30+31x-12x^2\right)^3}\,dx = \int \frac{1}{(10-3x)^3\left(6+17x+12x^2\right)^{5/2}}\,dx$$

$$= -\frac{275+388x}{294(10-3x)^2\left(6+17x+12x^2\right)^{3/2}} - \frac{\int \frac{\frac{109953}{2}-41904x}{(10-3x)^3\left(6+17x+12x^2\right)^{3/2}}\,dx}{2646}$$

$$= -\frac{275+388x}{294(10-3x)^2\left(6+17x+12x^2\right)^{3/2}} + \frac{738029+1042556x}{8232(10-3x)^2\sqrt{6+17x+12x^2}} + \cdots$$

$$= -\frac{275+388x}{294(10-3x)^2\left(6+17x+12x^2\right)^{3/2}} + \frac{738029+1042556x}{8232(10-3x)^2\sqrt{6+17x+12x^2}} - \cdots$$

$$= -\frac{275+388x}{294(10-3x)^2\left(6+17x+12x^2\right)^{3/2}} + \frac{738029+1042556x}{8232(10-3x)^2\sqrt{6+17x+12x^2}} - \cdots$$

$$= -\frac{275+388x}{294(10-3x)^2\left(6+17x+12x^2\right)^{3/2}} + \frac{738029+1042556x}{8232(10-3x)^2\sqrt{6+17x+12x^2}} - \cdots$$

$$= -\frac{275+388x}{294(10-3x)^2\left(6+17x+12x^2\right)^{3/2}} + \frac{738029+1042556x}{8232(10-3x)^2\sqrt{6+17x+12x^2}} - \cdots$$

Mathematica [A] time = 0.37, size = 131, normalized size = 0.94

$$\frac{\sqrt{12x^2+17x+6}\left(40325\left(-36x^3+69x^2+152x+60\right)^2\tanh^{-1}\left(\frac{7\sqrt{3x+2}}{6\sqrt{4x+3}}\right)+42\sqrt{3x+2}\sqrt{4x+3}\left(706089565584x \cdots\right.\right.}{318770436096(10-3x)^2(\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[6 + 17*x + 12*x^2]/((2 + 3*x)^3*(30 + 31*x - 12*x^2)^3),x]
```

```
[Out] (Sqrt[6 + 17*x + 12*x^2]*(42*Sqrt[2 + 3*x]*Sqrt[3 + 4*x]*(2773753482408 + 1
0124325497244*x + 9848047480070*x^2 - 1096520427663*x^3 - 3206824169544*x^4
 + 706089565584*x^5) + 40325*(60 + 152*x + 69*x^2 - 36*x^3)^2*ArcTanh[(7*Sq
rt[2 + 3*x])/(6*Sqrt[3 + 4*x])]))/(318770436096*(10 - 3*x)^2*(2 + 3*x)^(5/2
)*(3 + 4*x)^(5/2))
```

fricas [A] time = 0.64, size = 186, normalized size = 1.34

$$\frac{40325\left(1296\,x^6-4968\,x^5-6183\,x^4+16656\,x^3+31384\,x^2+18240\,x+3600\right)\log\left(\frac{291\,x+84\,\sqrt{12\,x^2+17\,x+6}+206}{x}\right)-40\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((12*x^2+17*x+6)^(1/2)/(2+3*x)^3/(-12*x^2+31*x+30)^3,x, algorithm=
"fricas")
```

```
[Out] 1/1275081744384*(40325*(1296*x^6 - 4968*x^5 - 6183*x^4 + 16656*x^3 + 31384*
x^2 + 18240*x + 3600)*log((291*x + 84*sqrt(12*x^2 + 17*x + 6) + 206)/x) - 4
0325*(1296*x^6 - 4968*x^5 - 6183*x^4 + 16656*x^3 + 31384*x^2 + 18240*x + 36
```

```
00)*log((291*x - 84*sqrt(12*x^2 + 17*x + 6) + 206)/x) + 168*(706089565584*x
^5 - 3206824169544*x^4 - 1096520427663*x^3 + 9848047480070*x^2 + 1012432549
7244*x + 2773753482408)*sqrt(12*x^2 + 17*x + 6))/(1296*x^6 - 4968*x^5 - 618
3*x^4 + 16656*x^3 + 31384*x^2 + 18240*x + 3600)
```

giac [A] time = 0.26, size = 232, normalized size = 1.67

$$\frac{\sqrt{3}\left(282273\sqrt{3}\left(2\sqrt{3}x-\sqrt{12x^2+17x+6}\right)^3-11460924\left(2\sqrt{3}x-\sqrt{12x^2+17x+6}\right)^2-37551180\sqrt{3}\left(2\sqrt{\right.}\right.}{159385218048\left(3\left(2\sqrt{3}x-\sqrt{12x^2+17x+6}\right)^2-40\sqrt{3}\left(2\sqrt{3}x-\sqrt{12x^2+17x+}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((12*x^2+17*x+6)^(1/2)/(2+3*x)^3/(-12*x^2+31*x+30)^3,x, algorithm=
"giac")
```

```
[Out] 1/159385218048*sqrt(3)*(282273*sqrt(3)*(2*sqrt(3)*x - sqrt(12*x^2 + 17*x +
6))^3 - 11460924*(2*sqrt(3)*x - sqrt(12*x^2 + 17*x + 6))^2 - 37551180*sqrt(
3)*(2*sqrt(3)*x - sqrt(12*x^2 + 17*x + 6)) - 83365264)/(3*(2*sqrt(3)*x - sq
rt(12*x^2 + 17*x + 6))^2 - 40*sqrt(3)*(2*sqrt(3)*x - sqrt(12*x^2 + 17*x + 6
)) - 188)^2 + 1/2213683584*((8*(2860316794*x + 6078171227)*x + 34383350229)
*x + 8090114146)/(12*x^2 + 17*x + 6)^(3/2) + 40325/637540872192*log(abs(-6*
sqrt(3)*x + 20*sqrt(3) + 3*sqrt(12*x^2 + 17*x + 6) + 42)) - 40325/637540872
192*log(abs(-6*sqrt(3)*x + 20*sqrt(3) + 3*sqrt(12*x^2 + 17*x + 6) - 42))
```

maple [B] time = 0.02, size = 306, normalized size = 2.20

$$\frac{40325\,\mathrm{arctanh}\left(\frac{97x+\frac{206}{3}}{28\sqrt{97x+12\left(x-\frac{10}{3}\right)^2-\frac{382}{3}}}\right)}{637540872192}-\frac{570457\sqrt{12}\,\ln\left(\frac{\left(12x+\frac{17}{2}\right)\sqrt{12}}{12}+\sqrt{97x+12\left(x-\frac{10}{3}\right)^2-\frac{382}{3}}\right)}{31239502737408}-\frac{23\sqrt{12}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((12*x^2+17*x+6)^(1/2)/(2+3*x)^3/(-12*x^2+31*x+30)^3,x)
```

```
[Out] -1/2592/(x+2/3)^3*(x+12*(x+2/3)^2+2/3)^(3/2)-1261/31239502737408/(x-10/3)*(
97*x+12*(x-10/3)^2-382/3)^(3/2)+1261/62479005474816*(24*x+17)*(97*x+12*(x-1
0/3)^2-382/3)^(1/2)+1/79692609024/(x-10/3)^2*(97*x+12*(x-10/3)^2-382/3)^(3/
2)-230400/5764801/(x+3/4)^2*(-x+12*(x+3/4)^2-3/4)^(3/2)-128/352947/(x+3/4)^
3*(-x+12*(x+3/4)^2-3/4)^(3/2)-570457/31239502737408*12^(1/2)*ln(1/12*(12*x+
17/2)*12^(1/2)+(97*x+12*(x-10/3)^2-382/3)^(1/2))+47/1152/(x+2/3)^2*(x+12*(x
+2/3)^2+2/3)^(3/2)-23/110592*12^(1/2)*ln(1/12*(12*x+17/2)*12^(1/2)+(x+12*(x
+2/3)^2+2/3)^(1/2))+58752/282475249*12^(1/2)*ln(1/12*(12*x+17/2)*12^(1/2)+(
-x+12*(x+3/4)^2-3/4)^(1/2))-40325/8925572210688*(97*x+12*(x-10/3)^2-382/3)^
(1/2)+40325/637540872192*arctanh(1/28*(97*x+206/3)/(97*x+12*(x-10/3)^2-382/
3)^(1/2))-23/4608*(x+12*(x+2/3)^2+2/3)^(1/2)-1410048/282475249*(-x+12*(x+3/
4)^2-3/4)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{\sqrt{12x^2+17x+6}}{\left(12x^2-31x-30\right)^3(3x+2)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((12*x^2+17*x+6)^(1/2)/(2+3*x)^3/(-12*x^2+31*x+30)^3,x, algorithm=
"maxima")
```

```
[Out] -integrate(sqrt(12*x^2 + 17*x + 6)/((12*x^2 - 31*x - 30)^3*(3*x + 2)^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{12\,x^2+17\,x+6}}{{\left(3\,x+2\right)}^3\,{\left(-12\,x^2+31\,x+30\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((17*x + 12*x^2 + 6)^(1/2)/((3*x + 2)^3*(31*x - 12*x^2 + 30)^3),x)
```

```
[Out] int((17*x + 12*x^2 + 6)^(1/2)/((3*x + 2)^3*(31*x - 12*x^2 + 30)^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt{12x^2+17x+6}}{46656x^9-268272x^8-76788x^7+1703619x^6+1218456x^5-3669588x^4-6898688x^3-4903920x^2-16416$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((12*x**2+17*x+6)**(1/2)/(2+3*x)**3/(-12*x**2+31*x+30)**3,x)
```

```
[Out] -Integral(sqrt(12*x**2 + 17*x + 6)/(46656*x**9 - 268272*x**8 - 76788*x**7 +
 1703619*x**6 + 1218456*x**5 - 3669588*x**4 - 6898688*x**3 - 4903920*x**2 -
 1641600*x - 216000), x)
```

3.138 $\int (-3+2x)\left(-3x+x^2\right)^{2/3}\,dx$

Optimal. Leaf size=15

$$\frac{3}{5}\left(x^2-3x\right)^{5/3}$$

```
[Out] 3/5*(x^2-3*x)^(5/3)
```

Rubi [A] time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {629}

$$\frac{3}{5}\left(x^2-3x\right)^{5/3}$$

Antiderivative was successfully verified.

```
[In] Int[(-3 + 2*x)*(-3*x + x^2)^(2/3),x]
```

```
[Out] (3*(-3*x + x^2)^(5/3))/5
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int (-3+2x)\left(-3x+x^2\right)^{2/3}\,dx=\frac{3}{5}\left(-3x+x^2\right)^{5/3}$$

Mathematica [A] time = 0.01, size = 13, normalized size = 0.87

$$\frac{3}{5}((x-3)x)^{5/3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-3 + 2*x)*(-3*x + x^2)^(2/3),x]
```

```
[Out] (3*((-3 + x)*x)^(5/3))/5
```

fricas [A] time = 0.61, size = 11, normalized size = 0.73

$$\frac{3}{5}\left(x^2-3\,x\right)^{\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+2*x)*(x^2-3*x)^(2/3),x, algorithm="fricas")
```

```
[Out] 3/5*(x^2 - 3*x)^(5/3)
```

giac [A] time = 0.15, size = 11, normalized size = 0.73

$$\frac{3}{5}\left(x^2-3\,x\right)^{\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+2*x)*(x^2-3*x)^(2/3),x, algorithm="giac")
[Out] 3/5*(x^2 - 3*x)^(5/3)
```

maple [A] time = 0.00, size = 16, normalized size = 1.07

$$\frac{3\left(x-3\right)\left(x^{2}-3x\right)^{\frac{2}{3}}x}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-3+2*x)*(x^2-3*x)^(2/3),x)
[Out] 3/5*(x-3)*x*(x^2-3*x)^(2/3)
```

maxima [A] time = 0.43, size = 11, normalized size = 0.73

$$\frac{3}{5}\left(x^{2}-3\,x\right)^{\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+2*x)*(x^2-3*x)^(2/3),x, algorithm="maxima")
[Out] 3/5*(x^2 - 3*x)^(5/3)
```

mupad [B] time = 3.72, size = 15, normalized size = 1.00

$$\frac{3\,x\,\left(x^2-3\,x\right)^{2/3}\,\left(x-3\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x - 3)*(x^2 - 3*x)^(2/3),x)
[Out] (3*x*(x^2 - 3*x)^(2/3)*(x - 3))/5
```

sympy [B] time = 0.31, size = 31, normalized size = 2.07

$$\frac{3x^{2}\left(x^{2}-3x\right)^{\frac{2}{3}}}{5}-\frac{9x\left(x^{2}-3x\right)^{\frac{2}{3}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+2*x)*(x**2-3*x)**(2/3),x)
[Out] 3*x**2*(x**2 - 3*x)**(2/3)/5 - 9*x*(x**2 - 3*x)**(2/3)/5
```

3.139 $\int ((-3+x)x)^{2/3}(-3+2x)\,dx$

Optimal. Leaf size=16

$$\frac{3}{5}(-((3-x)x))^{5/3}$$

```
[Out] 3/5*(-(3-x)*x)^(5/3)
```

Rubi [A] time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1588}

$$\frac{3}{5}(-(3-x)x)^{5/3}$$

Antiderivative was successfully verified.

```
[In] Int[((-3 + x)*x)^(2/3)*(-3 + 2*x),x]
```

```
[Out] (3*(-((3 - x)*x))^(5/3))/5
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int ((-3+x)x)^{2/3}(-3+2x)\,dx = \frac{3}{5}(-(3-x)x)^{5/3}$$

Mathematica [A] time = 0.00, size = 13, normalized size = 0.81

$$\frac{3}{5}((x-3)x)^{5/3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-3 + x)*x)^(2/3)*(-3 + 2*x),x]
```

```
[Out] (3*((-3 + x)*x)^(5/3))/5
```

fricas [A] time = 0.56, size = 11, normalized size = 0.69

$$\frac{3}{5}\left(x^2-3\,x\right)^{\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-3+x)*x)^(2/3)*(-3+2*x),x, algorithm="fricas")
```

```
[Out] 3/5*(x^2 - 3*x)^(5/3)
```

giac [A] time = 0.15, size = 11, normalized size = 0.69

$$\frac{3}{5}\left(x^2-3\,x\right)^{\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-3+x)*x)^(2/3)*(-3+2*x),x, algorithm="giac")
```

```
[Out] 3/5*(x^2 - 3*x)^(5/3)
```

maple [A] time = 0.00, size = 14, normalized size = 0.88

$$\frac{3\,(x-3)\,((x-3)\,x)^{\frac{2}{3}}\,x}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x-3)*x)^(2/3)*(-3+2*x),x)
```

```
[Out] 3/5*(x-3)*x*((x-3)*x)^(2/3)
```

maxima [A] time = 0.43, size = 9, normalized size = 0.56

$$\frac{3}{5}\,((x-3)x)^{\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-3+x)*x)^(2/3)*(-3+2*x),x, algorithm="maxima")
```

```
[Out] 3/5*((x - 3)*x)^(5/3)
```

mupad [B] time = 3.66, size = 13, normalized size = 0.81

$$\frac{3\,x\,(x\;(x-3))^{2/3}\;(x-3)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x - 3)*(x*(x - 3))^(2/3),x)
```

```
[Out] (3*x*(x*(x - 3))^(2/3)*(x - 3))/5
```

sympy [A] time = 4.25, size = 10, normalized size = 0.62

$$\frac{3\,(x\,(x-3))^{\frac{5}{3}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-3+x)*x)**(2/3)*(-3+2*x),x)
```

```
[Out] 3*(x*(x - 3))**(5/3)/5
```

3.140 $\int \frac{x(9-9x+2x^2)}{\sqrt[3]{-3x+x^2}}\,dx$

Optimal. Leaf size=15

$$\frac{3}{5}\left(x^2-3x\right)^{5/3}$$

```
[Out] 3/5*(x^2-3*x)^(5/3)
```

Rubi [A] time = 0.03, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {1631, 629}

$$\frac{3}{5}\left(x^2-3x\right)^{5/3}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(9 - 9*x + 2*x^2))/(-3*x + x^2)^(1/3),x]
```

```
[Out] (3*(-3*x + x^2)^(5/3))/5
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1631

```
Int[(Pq_)*((e_.)*(x_))^(m_.)*((b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[e, Int[(e*x)^(m - 1)*PolynomialQuotient[Pq, b + c*x, x]*(b*x + c*x^2
)^(p + 1), x], x] /; FreeQ[{b, c, e, m, p}, x] && PolyQ[Pq, x] && EqQ[Polyn
omialRemainder[Pq, b + c*x, x], 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x\left(9-9x+2x^2\right)}{\sqrt[3]{-3x+x^2}}\,dx &= \int (-3+2x)\left(-3x+x^2\right)^{2/3}\,dx\\ &= \frac{3}{5}\left(-3x+x^2\right)^{5/3}\end{aligned}$$

Mathematica [A] time = 0.01, size = 13, normalized size = 0.87

$$\frac{3}{5}((x-3)x)^{5/3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(9 - 9*x + 2*x^2))/(-3*x + x^2)^(1/3),x]
```

```
[Out] (3*((-3 + x)*x)^(5/3))/5
```

fricas [A] time = 0.41, size = 11, normalized size = 0.73

$$\frac{3}{5}\left(x^2-3\,x\right)^{\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*x^2-9*x+9)/(x^2-3*x)^(1/3),x, algorithm="fricas")
[Out] 3/5*(x^2 - 3*x)^(5/3)
```

giac [A] time = 0.16, size = 11, normalized size = 0.73

$$\frac{3}{5}\left(x^{2}-3\,x\right)^{\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*x^2-9*x+9)/(x^2-3*x)^(1/3),x, algorithm="giac")
[Out] 3/5*(x^2 - 3*x)^(5/3)
```

maple [A] time = 0.00, size = 20, normalized size = 1.33

$$\frac{3\left(x-3\right)^{2}x^{2}}{5\left(x^{2}-3x\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(2*x^2-9*x+9)/(x^2-3*x)^(1/3),x)
[Out] 3/5*(x-3)^2*x^2/(x^2-3*x)^(1/3)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2\,x^{2}-9\,x+9\right)x}{\left(x^{2}-3\,x\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*x^2-9*x+9)/(x^2-3*x)^(1/3),x, algorithm="maxima")
[Out] integrate((2*x^2 - 9*x + 9)*x/(x^2 - 3*x)^(1/3), x)
```

mupad [B] time = 3.68, size = 15, normalized size = 1.00

$$\frac{3\,x\,\left(x^{2}-3\,x\right)^{2/3}\,\left(x-3\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(2*x^2 - 9*x + 9))/(x^2 - 3*x)^(1/3),x)
[Out] (3*x*(x^2 - 3*x)^(2/3)*(x - 3))/5
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\left(x-3\right)\left(2x-3\right)}{\sqrt[3]{x\left(x-3\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*x**2-9*x+9)/(x**2-3*x)**(1/3),x)
[Out] Integral(x*(x - 3)*(2*x - 3)/(x*(x - 3))**(1/3), x)
```

3.141 $\int \frac{x(9-9x+2x^2)}{\sqrt[3]{(-3+x)x}} \, dx$

Optimal. Leaf size=15

$$\frac{3}{5}\left(x^2-3x\right)^{5/3}$$

```
[Out] 3/5*(x^2-3*x)^(5/3)
```

Rubi [A] time = 0.06, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1985, 1631, 629}

$$\frac{3}{5}\left(x^2-3x\right)^{5/3}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(9 - 9*x + 2*x^2))/((-3 + x)*x)^(1/3),x]
```

```
[Out] (3*(-3*x + x^2)^(5/3))/5
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1631

```
Int[(Pq_)*((e_.)*(x_))^(m_.)*((b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[e, Int[(e*x)^(m - 1)*PolynomialQuotient[Pq, b + c*x, x]*(b*x + c*x^2
)^(p + 1), x], x] /; FreeQ[{b, c, e, m, p}, x] && PolyQ[Pq, x] && EqQ[Polyn
omialRemainder[Pq, b + c*x, x], 0]
```

Rule 1985

```
Int[(u_)^(p_.)*(v_)^(q_.)*(z_)^(m_.), x_Symbol] :> Int[ExpandToSum[z, x]^m*
ExpandToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{m, p, q}, x] && Linea
rQ[z, x] && QuadraticQ[{u, v}, x] &&  !(LinearMatchQ[z, x] && QuadraticMatc
hQ[{u, v}, x])
```

Rubi steps

$$\begin{aligned}
\int \frac{x\left(9-9x+2x^2\right)}{\sqrt[3]{(-3+x)x}} \, dx &= \int \frac{x\left(9-9x+2x^2\right)}{\sqrt[3]{-3x+x^2}} \, dx \\
&= \int (-3+2x)\left(-3x+x^2\right)^{2/3} \, dx \\
&= \frac{3}{5}\left(-3x+x^2\right)^{5/3}
\end{aligned}$$

Mathematica [A] time = 0.00, size = 13, normalized size = 0.87

$$\frac{3}{5}((x-3)x)^{5/3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(9 - 9*x + 2*x^2))/((-3 + x)*x)^(1/3),x]
[Out] (3*((-3 + x)*x)^(5/3))/5
```

fricas [A] time = 0.42, size = 11, normalized size = 0.73

$$\frac{3}{5}\left(x^2-3x\right)^{\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*x^2-9*x+9)/((-3+x)*x)^(1/3),x, algorithm="fricas")
[Out] 3/5*(x^2 - 3*x)^(5/3)
```

giac [A] time = 0.20, size = 11, normalized size = 0.73

$$\frac{3}{5}\left(x^2-3x\right)^{\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*x^2-9*x+9)/((-3+x)*x)^(1/3),x, algorithm="giac")
[Out] 3/5*(x^2 - 3*x)^(5/3)
```

maple [A] time = 0.00, size = 18, normalized size = 1.20

$$\frac{3\left(x-3\right)^{2}x^{2}}{5\left(\left(x-3\right)x\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(2*x^2-9*x+9)/((x-3)*x)^(1/3),x)
[Out] 3/5*(x-3)^2*x^2/((x-3)*x)^(1/3)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(2x^2-9x+9\right)x}{\left((x-3)x\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*x^2-9*x+9)/((-3+x)*x)^(1/3),x, algorithm="maxima")
[Out] integrate((2*x^2 - 9*x + 9)*x/((x - 3)*x)^(1/3), x)
```

mupad [B] time = 3.61, size = 13, normalized size = 0.87

$$\frac{3x\left(x\,\left(x-3\right)\right)^{2/3}\,\left(x-3\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(2*x^2 - 9*x + 9))/(x*(x - 3))^(1/3),x)
[Out] (3*x*(x*(x - 3))^(2/3)*(x - 3))/5
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x\left(x-3\right)\left(2x-3\right)}{\sqrt[3]{x\left(x-3\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*x**2-9*x+9)/((-3+x)*x)**(1/3),x)
```

```
[Out] Integral(x*(x - 3)*(2*x - 3)/(x*(x - 3))**(1/3), x)
```

3.142 $\int \frac{g+hx}{\sqrt[3]{-\frac{cg^2}{h^2}+9cx^2}\left(g^2+3h^2x^2\right)}\,dx$

Optimal. Leaf size=242

$$\frac{\sqrt[3]{1-\frac{9h^2x^2}{g^2}}\log\left(g^2+3h^2x^2\right)}{6\,2^{2/3}h\sqrt[3]{9cx^2-\frac{cg^2}{h^2}}}-\frac{\sqrt[3]{1-\frac{9h^2x^2}{g^2}}\log\left(\left(1-\frac{3hx}{g}\right)^{2/3}+\sqrt[3]{2}\sqrt[3]{\frac{3hx}{g}+1}\right)}{2\,2^{2/3}h\sqrt[3]{9cx^2-\frac{cg^2}{h^2}}}+\frac{\sqrt[3]{1-\frac{9h^2x^2}{g^2}}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2^{2/3}\left(1-\frac{3hx}{g}\right)^{2/3}}{\sqrt{3}\sqrt[3]{\frac{3hx}{g}+1}}\right)}{2^{2/3}\sqrt{3}\,h\sqrt[3]{9cx^2-\frac{cg^2}{h^2}}}$$

```
[Out] 1/12*(1-9*h^2*x^2/g^2)^(1/3)*ln(3*h^2*x^2+g^2)*2^(1/3)/h/(-c*g^2/h^2+9*c*x^
2)^(1/3)-1/4*(1-9*h^2*x^2/g^2)^(1/3)*ln((1-3*h*x/g)^(2/3)+2^(1/3)*(1+3*h*x/
g)^(1/3))*2^(1/3)/h/(-c*g^2/h^2+9*c*x^2)^(1/3)-1/6*(1-9*h^2*x^2/g^2)^(1/3)*
arctan(-1/3*3^(1/2)+1/3*2^(2/3)*(1-3*h*x/g)^(2/3)/(1+3*h*x/g)^(1/3)*3^(1/2)
)*2^(1/3)/h/(-c*g^2/h^2+9*c*x^2)^(1/3)*3^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 242, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {1009, 1008}

$$\frac{\sqrt[3]{1-\frac{9h^2x^2}{g^2}}\log\left(g^2+3h^2x^2\right)}{6\,2^{2/3}h\sqrt[3]{9cx^2-\frac{cg^2}{h^2}}}-\frac{\sqrt[3]{1-\frac{9h^2x^2}{g^2}}\log\left(\left(1-\frac{3hx}{g}\right)^{2/3}+\sqrt[3]{2}\sqrt[3]{\frac{3hx}{g}+1}\right)}{2\,2^{2/3}h\sqrt[3]{9cx^2-\frac{cg^2}{h^2}}}+\frac{\sqrt[3]{1-\frac{9h^2x^2}{g^2}}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2^{2/3}\left(1-\frac{3hx}{g}\right)^{2/3}}{\sqrt{3}\sqrt[3]{\frac{3hx}{g}+1}}\right)}{2^{2/3}\sqrt{3}\,h\sqrt[3]{9cx^2-\frac{cg^2}{h^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(g + h*x)/((-((c*g^2)/h^2) + 9*c*x^2)^(1/3)*(g^2 + 3*h^2*x^2)),x]
```

```
[Out] ((1 - (9*h^2*x^2)/g^2)^(1/3)*ArcTan[1/Sqrt[3] - (2^(2/3)*(1 - (3*h*x)/g)^(2
/3))/(Sqrt[3]*(1 + (3*h*x)/g)^(1/3))])/(2^(2/3)*Sqrt[3]*h*(-((c*g^2)/h^2) +
 9*c*x^2)^(1/3)) + ((1 - (9*h^2*x^2)/g^2)^(1/3)*Log[g^2 + 3*h^2*x^2])/(6*2^
(2/3)*h*(-((c*g^2)/h^2) + 9*c*x^2)^(1/3)) - ((1 - (9*h^2*x^2)/g^2)^(1/3)*Lo
g[(1 - (3*h*x)/g)^(2/3) + 2^(1/3)*(1 + (3*h*x)/g)^(1/3)])/(2*2^(2/3)*h*(-((
c*g^2)/h^2) + 9*c*x^2)^(1/3))
```

Rule 1008

```
Int[((g_) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)^(1/3)*((d_) + (f_.)*(x_)^2))
, x_Symbol] :> Simp[(Sqrt[3]*h*ArcTan[1/Sqrt[3] - (2^(2/3)*(1 - (3*h*x)/g)^
(2/3))/(Sqrt[3]*(1 + (3*h*x)/g)^(1/3))])/(2^(2/3)*a^(1/3)*f), x] + (-Simp[(
3*h*Log[(1 - (3*h*x)/g)^(2/3) + 2^(1/3)*(1 + (3*h*x)/g)^(1/3)])/(2^(5/3)*a^
(1/3)*f), x] + Simp[(h*Log[d + f*x^2])/(2^(5/3)*a^(1/3)*f), x]) /; FreeQ[{a
, c, d, f, g, h}, x] && EqQ[c*d + 3*a*f, 0] && EqQ[c*g^2 + 9*a*h^2, 0] && G
tQ[a, 0]
```

Rule 1009

```
Int[((g_) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)^(1/3)*((d_) + (f_.)*(x_)^2))
, x_Symbol] :> Dist[(1 + (c*x^2)/a)^(1/3)/(a + c*x^2)^(1/3), Int[(g + h*x)/
((1 + (c*x^2)/a)^(1/3)*(d + f*x^2)), x], x] /; FreeQ[{a, c, d, f, g, h}, x]
 && EqQ[c*d + 3*a*f, 0] && EqQ[c*g^2 + 9*a*h^2, 0] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{g+hx}{\sqrt[3]{-\frac{cg^2}{h^2}+9cx^2}\left(g^2+3h^2x^2\right)}dx = \frac{\sqrt[3]{1-\frac{9h^2x^2}{g^2}}\int \frac{g+hx}{\left(g^2+3h^2x^2\right)\sqrt[3]{1-\frac{9h^2x^2}{g^2}}}dx}{\sqrt[3]{-\frac{cg^2}{h^2}+9cx^2}}$$

$$= \frac{\sqrt[3]{1-\frac{9h^2x^2}{g^2}}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2^{2/3}\left(1-\frac{3hx}{g}\right)^{2/3}}{\sqrt{3}\sqrt[3]{1+\frac{3hx}{g}}}\right)}{2^{2/3}\sqrt{3}\,h\sqrt[3]{-\frac{cg^2}{h^2}+9cx^2}} + \frac{\sqrt[3]{1-\frac{9h^2x^2}{g^2}}\log\left(g^2+3h^2x^2\right)}{6\,2^{2/3}h\sqrt[3]{-\frac{cg^2}{h^2}+9cx^2}} -$$

Mathematica [C] time = 0.56, size = 268, normalized size = 1.11

$$\frac{h^2x\left(-hx\sqrt[3]{1-\frac{9h^2x^2}{g^2}}F_1\left(1;\frac{1}{3},1;2;\frac{9h^2x^2}{g^2},-\frac{3h^2x^2}{g^2}\right)-\frac{2g^5F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};\frac{9h^2x^2}{g^2},-\frac{3h^2x^2}{g^2}\right)}{\left(g^2+3h^2x^2\right)\left(g^2F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};\frac{9h^2x^2}{g^2},-\frac{3h^2x^2}{g^2}\right)+2h^2x^2\left(F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};\frac{9h^2x^2}{g^2},-\frac{3h^2x^2}{g^2}\right)-F\right.\right.}\right.}{2cg^2\left(g^2-9h^2x^2\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(g + h*x)/((-((c*g^2)/h^2) + 9*c*x^2)^(1/3)*(g^2 + 3*h^2*x^2)),x]
```

```
[Out] (h^2*x*(c*(-(g^2/h^2) + 9*x^2))^(2/3)*(-(h*x*(1 - (9*h^2*x^2)/g^2)^(1/3)*Ap
pellF1[1, 1/3, 1, 2, (9*h^2*x^2)/g^2, (-3*h^2*x^2)/g^2]) - (2*g^5*AppellF1[
1/2, 1/3, 1, 3/2, (9*h^2*x^2)/g^2, (-3*h^2*x^2)/g^2])/((g^2 + 3*h^2*x^2)*(g
^2*AppellF1[1/2, 1/3, 1, 3/2, (9*h^2*x^2)/g^2, (-3*h^2*x^2)/g^2] + 2*h^2*x^
2*(-AppellF1[3/2, 1/3, 2, 5/2, (9*h^2*x^2)/g^2, (-3*h^2*x^2)/g^2] + AppellF
1[3/2, 4/3, 1, 5/2, (9*h^2*x^2)/g^2, (-3*h^2*x^2)/g^2])))))/(2*c*g^2*(g^2 -
 9*h^2*x^2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)/(-c*g^2/h^2+9*c*x^2)^(1/3)/(3*h^2*x^2+g^2),x, algorithm="
fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{hx+g}{\left(3\,h^2x^2+g^2\right)\left(9\,cx^2-\frac{cg^2}{h^2}\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)/(-c*g^2/h^2+9*c*x^2)^(1/3)/(3*h^2*x^2+g^2),x, algorithm="
giac")
```

```
[Out] integrate((h*x + g)/((3*h^2*x^2 + g^2)*(9*c*x^2 - c*g^2/h^2)^(1/3)), x)
```

maple [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{hx+g}{\left(9c\,x^2-\frac{c\,g^2}{h^2}\right)^{\frac{1}{3}}\left(3h^2x^2+g^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)/(-c*g^2/h^2+9*c*x^2)^(1/3)/(3*h^2*x^2+g^2),x)
```

```
[Out] int((h*x+g)/(-c*g^2/h^2+9*c*x^2)^(1/3)/(3*h^2*x^2+g^2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{hx+g}{\left(3\,h^2x^2+g^2\right)\left(9\,cx^2-\frac{cg^2}{h^2}\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)/(-c*g^2/h^2+9*c*x^2)^(1/3)/(3*h^2*x^2+g^2),x, algorithm="
maxima")
```

```
[Out] integrate((h*x + g)/((3*h^2*x^2 + g^2)*(9*c*x^2 - c*g^2/h^2)^(1/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{g+h\,x}{\left(g^2+3\,h^2\,x^2\right)\,\left(9\,c\,x^2-\frac{c\,g^2}{h^2}\right)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g + h*x)/((g^2 + 3*h^2*x^2)*(9*c*x^2 - (c*g^2)/h^2)^(1/3)),x)
```

```
[Out] int((g + h*x)/((g^2 + 3*h^2*x^2)*(9*c*x^2 - (c*g^2)/h^2)^(1/3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{g+hx}{\sqrt[3]{c\left(-\frac{g}{h}+3x\right)\left(\frac{g}{h}+3x\right)}\,\left(g^2+3h^2x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)/(-c*g**2/h**2+9*c*x**2)**(1/3)/(3*h**2*x**2+g**2),x)
```

```
[Out] Integral((g + h*x)/((c*(-g/h + 3*x)*(g/h + 3*x))**(1/3)*(g**2 + 3*h**2*x**2
)), x)
```

3.143 $\int \frac{g+hx}{\sqrt[3]{\frac{-c^2g^2+bcgh+2b^2h^2}{9ch^2}+bx+cx^2}\left(\frac{f\left(b^2-\frac{-c^2g^2+bcgh+2b^2h^2}{3h^2}\right)}{c^2}+\frac{bfx}{c}+fx^2\right)}dx$

Optimal. Leaf size=488

$$\frac{3^{2/3}h\sqrt[3]{\frac{ch^2\left(\frac{(cg-2bh)(bh+cg)}{ch^2}-9bx-9cx^2\right)}{(2cg-bh)^2}}\log\left(\frac{f\left(b^2h^2-bcgh+c^2g^2\right)}{3c^2h^2}+\frac{bfx}{c}+fx^2\right)}{2f\sqrt[3]{-\frac{(cg-2bh)(bh+cg)}{ch^2}+9bx+9cx^2}}-\frac{3\,3^{2/3}h\sqrt[3]{\frac{ch^2\left(\frac{(cg-2bh)(bh+cg)}{ch^2}-9bx-9cx^2\right)}{(2cg-bh)^2}}\log\left(\left(1\right.\right.}{2f\sqrt[3]{-\frac{(cg-2bh)(bh+cg)}{ch^2}}}$$

```
[Out] -3*3^(1/6)*h*(c*h^2*((-2*b*h+c*g)*(b*h+c*g)/c/h^2-9*b*x-9*c*x^2)/(-b*h+2*c*
g)^2)^(1/3)*arctan(-1/3*3^(1/2)+1/3*2^(2/3)*(1-3*h*(2*c*x+b)/(-b*h+2*c*g))^
(2/3)/(1+3*h*(2*c*x+b)/(-b*h+2*c*g))^(1/3)*3^(1/2))/f/(-(-2*b*h+c*g)*(b*h+c
*g)/c/h^2+9*b*x+9*c*x^2)^(1/3)+1/2*3^(2/3)*h*(c*h^2*((-2*b*h+c*g)*(b*h+c*g)
/c/h^2-9*b*x-9*c*x^2)/(-b*h+2*c*g)^2)^(1/3)*ln(1/3*f*(b^2*h^2-b*c*g*h+c^2*g
^2)/c^2/h^2+b*f*x/c+f*x^2)/f/(-(-2*b*h+c*g)*(b*h+c*g)/c/h^2+9*b*x+9*c*x^2)^
(1/3)-3/2*3^(2/3)*h*(c*h^2*((-2*b*h+c*g)*(b*h+c*g)/c/h^2-9*b*x-9*c*x^2)/(-b
*h+2*c*g)^2)^(1/3)*ln((1-3*h*(2*c*x+b)/(-b*h+2*c*g))^(2/3)+2^(1/3)*(1+3*h*(
2*c*x+b)/(-b*h+2*c*g))^(1/3))/f/(-(-2*b*h+c*g)*(b*h+c*g)/c/h^2+9*b*x+9*c*x^
2)^(1/3)
```

Rubi [A] time = 0.36, antiderivative size = 488, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 104, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.019, Rules used = {1041, 1040}

$$\frac{3^{2/3}h\sqrt[3]{\frac{ch^2\left(\frac{(cg-2bh)(bh+cg)}{ch^2}-9bx-9cx^2\right)}{(2cg-bh)^2}}\log\left(\frac{f\left(b^2h^2-bcgh+c^2g^2\right)}{3c^2h^2}+\frac{bfx}{c}+fx^2\right)}{2f\sqrt[3]{-\frac{(cg-2bh)(bh+cg)}{ch^2}+9bx+9cx^2}}-\frac{3\,3^{2/3}h\sqrt[3]{\frac{ch^2\left(\frac{(cg-2bh)(bh+cg)}{ch^2}-9bx-9cx^2\right)}{(2cg-bh)^2}}\log\left(\left(1\right.\right.}{2f\sqrt[3]{-\frac{(cg-2bh)(bh+cg)}{ch^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(g + h*x)/(((-(c^2*g^2) + b*c*g*h + 2*b^2*h^2)/(9*c*h^2) + b*x + c*x^2)
^(1/3)*((f*(b^2 - (-(c^2*g^2) + b*c*g*h + 2*b^2*h^2)/(3*h^2)))/c^2 + (b*f*x
)/c + f*x^2)),x]
```

```
[Out] (3*3^(1/6)*h*((c*h^2*(((c*g - 2*b*h)*(c*g + b*h))/(c*h^2) - 9*b*x - 9*c*x^2
))/(2*c*g - b*h)^2)^(1/3)*ArcTan[1/Sqrt[3] - (2^(2/3)*(1 - (3*h*(b + 2*c*x)
)/(2*c*g - b*h))^(2/3))/(Sqrt[3]*(1 + (3*h*(b + 2*c*x))/(2*c*g - b*h))^(1/3
))])/(f*(-(((c*g - 2*b*h)*(c*g + b*h))/(c*h^2)) + 9*b*x + 9*c*x^2)^(1/3)) +
 (3^(2/3)*h*((c*h^2*(((c*g - 2*b*h)*(c*g + b*h))/(c*h^2) - 9*b*x - 9*c*x^2)
)/(2*c*g - b*h)^2)^(1/3)*Log[(f*(c^2*g^2 - b*c*g*h + b^2*h^2))/(3*c^2*h^2)
+ (b*f*x)/c + f*x^2])/(2*f*(-(((c*g - 2*b*h)*(c*g + b*h))/(c*h^2)) + 9*b*x
+ 9*c*x^2)^(1/3)) - (3*3^(2/3)*h*((c*h^2*(((c*g - 2*b*h)*(c*g + b*h))/(c*h^
2) - 9*b*x - 9*c*x^2))/(2*c*g - b*h)^2)^(1/3)*Log[(1 - (3*h*(b + 2*c*x))/(2
*c*g - b*h))^(2/3) + 2^(1/3)*(1 + (3*h*(b + 2*c*x))/(2*c*g - b*h))^(1/3)])/
(2*f*(-(((c*g - 2*b*h)*(c*g + b*h))/(c*h^2)) + 9*b*x + 9*c*x^2)^(1/3))
```

Rule 1040

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(1/3)*((d_.)
+ (e_.)*(x_) + (f_.)*(x_)^2)), x_Symbol] :> With[{q = ((-9*c*h^2)/(2*c*g -
b*h)^2)^(1/3)}, Simp[(Sqrt[3]*h*q*ArcTan[1/Sqrt[3] - (2^(2/3)*(1 - (3*h*(b
+ 2*c*x))/(2*c*g - b*h))^(2/3))/(Sqrt[3]*(1 + (3*h*(b + 2*c*x))/(2*c*g - b*
h))^(1/3))])/f, x] + (-Simp[(3*h*q*Log[(1 - (3*h*(b + 2*c*x))/(2*c*g - b*h)
```

```
)^(2/3) + 2^(1/3)*(1 + (3*h*(b + 2*c*x))/(2*c*g - b*h))^(1/3)])/(2*f), x] +
 Simp[(h*q*Log[d + e*x + f*x^2])/(2*f), x])] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x] && EqQ[c*e - b*f, 0] && EqQ[c^2*d - f*(b^2 - 3*a*c), 0] && EqQ[c^2*
g^2 - b*c*g*h - 2*b^2*h^2 + 9*a*c*h^2, 0] && GtQ[(-9*c*h^2)/(2*c*g - b*h)^2
, 0]
```

Rule 1041

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(1/3)*((d_.)
+ (e_.)*(x_) + (f_.)*(x_)^2)), x_Symbol] :> With[{q = -(c/(b^2 - 4*a*c))},
Dist[(q*(a + b*x + c*x^2))^(1/3)/(a + b*x + c*x^2)^(1/3), Int[(g + h*x)/((q
*a + b*q*x + c*q*x^2)^(1/3)*(d + e*x + f*x^2)), x], x]] /; FreeQ[{a, b, c,
d, e, f, g, h}, x] && EqQ[c*e - b*f, 0] && EqQ[c^2*d - f*(b^2 - 3*a*c), 0]
&& EqQ[c^2*g^2 - b*c*g*h - 2*b^2*h^2 + 9*a*c*h^2, 0] &&  !GtQ[4*a - b^2/c,
0]
```

Rubi steps

$$\int \frac{g+hx}{\sqrt[3]{\frac{-c^2g^2+bcgh+2b^2h^2}{9ch^2}+bx+cx^2}\left(\frac{f\left(b^2-\frac{-c^2g^2+bcgh+2b^2h^2}{3h^2}\right)}{c^2}+\frac{bfx}{c}+fx^2\right)}\,dx = \frac{\sqrt[3]{-\frac{c\left(\frac{-c^2g^2+bcgh+2b^2h^2}{9ch^2}+bx+cx^2\right)}{b^2-\frac{4\left(-c^2g^2+bcgh+2b^2h^2\right)}{9h^2}}}\int \frac{\cdots}{\left(\frac{f\left(b^2-\frac{-c^2g^2+b\cdots}{3\cdots}\right)}{c^2}\cdots\right.}}{\cdots}$$

$$= \frac{3\sqrt[6]{3}\,h\sqrt[3]{\frac{ch^2\left(\frac{(cg-2bh)(cg+bh)}{ch^2}-9bx-9cx^2\right)}{(2cg-bh)^2}}\tan^{-1}\left(\cdots\right.}{f\sqrt[3]{-\frac{(cg-2bh)(cg+bh)}{ch^2}+9bx\cdots}}$$

Mathematica [F] time = 0.54, size = 0, normalized size = 0.00

$$\int \frac{g+hx}{\sqrt[3]{\frac{-c^2g^2+bcgh+2b^2h^2}{9ch^2}+bx+cx^2}\left(\frac{f\left(b^2-\frac{-c^2g^2+bcgh+2b^2h^2}{3h^2}\right)}{c^2}+\frac{bfx}{c}+fx^2\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(g + h*x)/(((-(c^2*g^2) + b*c*g*h + 2*b^2*h^2)/(9*c*h^2) + b*x +
c*x^2)^(1/3)*((f*(b^2 - (-(c^2*g^2) + b*c*g*h + 2*b^2*h^2)/(3*h^2)))/c^2 +
(b*f*x)/c + f*x^2)),x]
```

```
[Out] Integrate[(g + h*x)/(((-(c^2*g^2) + b*c*g*h + 2*b^2*h^2)/(9*c*h^2) + b*x +
c*x^2)^(1/3)*((f*(b^2 - (-(c^2*g^2) + b*c*g*h + 2*b^2*h^2)/(3*h^2)))/c^2 +
(b*f*x)/c + f*x^2)), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)/(1/9*(2*b^2*h^2+b*c*g*h-c^2*g^2)/c/h^2+b*x+c*x^2)^(1/3)/(
f*(b^2+1/3*(-2*b^2*h^2-b*c*g*h+c^2*g^2)/h^2)/c^2+b*f*x/c+f*x^2),x, algorith
m="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3\left(hx+g\right)}{\left(cx^{2}+bx-\frac{c^{2}g^{2}-bcgh-2\,b^{2}h^{2}}{9\,ch^{2}}\right)^{\frac{1}{3}}\left(3\,fx^{2}+\frac{3\,bfx}{c}+\frac{\left(3\,b^{2}+\frac{c^{2}g^{2}-bcgh-2\,b^{2}h^{2}}{h^{2}}\right)f}{c^{2}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)/(1/9*(2*b^2*h^2+b*c*g*h-c^2*g^2)/c/h^2+b*x+c*x^2)^(1/3)/(
f*(b^2+1/3*(-2*b^2*h^2-b*c*g*h+c^2*g^2)/h^2)/c^2+b*f*x/c+f*x^2),x, algorith
m="giac")
```

```
[Out] integrate(3*(h*x + g)/((c*x^2 + b*x - 1/9*(c^2*g^2 - b*c*g*h - 2*b^2*h^2)/(
c*h^2))^(1/3)*(3*f*x^2 + 3*b*f*x/c + (3*b^2 + (c^2*g^2 - b*c*g*h - 2*b^2*h^
2)/h^2)*f/c^2)), x)
```

maple [F] time = 0.45, size = 0, normalized size = 0.00

$$\int \frac{hx+g}{\left(c\,x^{2}+bx+\frac{2b^{2}h^{2}+bcgh-c^{2}g^{2}}{9c\,h^{2}}\right)^{\frac{1}{3}}\left(f\,x^{2}+\frac{bfx}{c}+\frac{\left(b^{2}+\frac{-2b^{2}h^{2}-bcgh+c^{2}g^{2}}{3h^{2}}\right)f}{c^{2}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)/(1/9*(2*b^2*h^2+b*c*g*h-c^2*g^2)/c/h^2+b*x+c*x^2)^(1/3)/(f*(b^2
+1/3*(-2*b^2*h^2-b*c*g*h+c^2*g^2)/h^2)/c^2+b*f*x/c+f*x^2),x)
```

```
[Out] int((h*x+g)/(1/9*(2*b^2*h^2+b*c*g*h-c^2*g^2)/c/h^2+b*x+c*x^2)^(1/3)/(f*(b^2
+1/3*(-2*b^2*h^2-b*c*g*h+c^2*g^2)/h^2)/c^2+b*f*x/c+f*x^2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$3\int \frac{hx+g}{\left(cx^{2}+bx-\frac{c^{2}g^{2}-bcgh-2\,b^{2}h^{2}}{9\,ch^{2}}\right)^{\frac{1}{3}}\left(3\,fx^{2}+\frac{3\,bfx}{c}+\frac{\left(3\,b^{2}+\frac{c^{2}g^{2}-bcgh-2\,b^{2}h^{2}}{h^{2}}\right)f}{c^{2}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)/(1/9*(2*b^2*h^2+b*c*g*h-c^2*g^2)/c/h^2+b*x+c*x^2)^(1/3)/(
f*(b^2+1/3*(-2*b^2*h^2-b*c*g*h+c^2*g^2)/h^2)/c^2+b*f*x/c+f*x^2),x, algorith
m="maxima")
```

```
[Out] 3*integrate((h*x + g)/((c*x^2 + b*x - 1/9*(c^2*g^2 - b*c*g*h - 2*b^2*h^2)/(
c*h^2))^(1/3)*(3*f*x^2 + 3*b*f*x/c + (3*b^2 + (c^2*g^2 - b*c*g*h - 2*b^2*h^
2)/h^2)*f/c^2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{g+h\,x}{\left(b\,x+c\,x^{2}+\frac{\frac{2\,b^{2}\,h^{2}}{9}+\frac{b\,c\,g\,h}{9}-\frac{c^{2}\,g^{2}}{9}}{c\,h^{2}}\right)^{1/3}\left(f\,x^{2}-\frac{f\left(\frac{\frac{2\,b^{2}\,h^{2}}{3}+\frac{b\,c\,g\,h}{3}-\frac{c^{2}\,g^{2}}{3}}{h^{2}}-b^{2}\right)}{c^{2}}+\frac{b\,f\,x}{c}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g + h*x)/((b*x + c*x^2 + ((2*b^2*h^2)/9 - (c^2*g^2)/9 + (b*c*g*h)/9)/(
c*h^2))^(1/3)*(f*x^2 - (f*(((2*b^2*h^2)/3 - (c^2*g^2)/3 + (b*c*g*h)/3)/h^2
- b^2))/c^2 + (b*f*x)/c)),x)
```

```
[Out] int((g + h*x)/((b*x + c*x^2 + ((2*b^2*h^2)/9 - (c^2*g^2)/9 + (b*c*g*h)/9)/(
c*h^2))^(1/3)*(f*x^2 - (f*(((2*b^2*h^2)/3 - (c^2*g^2)/3 + (b*c*g*h)/3)/h^2
- b^2))/c^2 + (b*f*x)/c)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$3 \cdot 3^{\frac{2}{3}} c^2 h^2 \left(\int \frac{g}{b^2 h^2 \sqrt[3]{\frac{2b^2}{c} + \frac{bg}{h} + 9bx - \frac{cg^2}{h^2} + 9cx^2} - bcgh \sqrt[3]{\frac{2b^2}{c} + \frac{bg}{h} + 9bx - \frac{cg^2}{h^2} + 9cx^2} + 3bch^2 x \sqrt[3]{\frac{2b^2}{c} + \frac{bg}{h} + 9bx - \frac{cg^2}{h^2} + 9cx^2} + c^2 g^2 \sqrt[3]{\frac{2b^2}{c} + \frac{bg}{h} + 9bx - \frac{cg^2}{h^2} + 9c} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)/(1/9*(2*b**2*h**2+b*c*g*h-c**2*g**2)/c/h**2+b*x+c*x**2)**
(1/3)/(f*(b**2+1/3*(-2*b**2*h**2-b*c*g*h+c**2*g**2)/h**2)/c**2+b*f*x/c+f*x*
*2),x)
```

```
[Out] 3*3**(2/3)*c**2*h**2*(Integral(g/(b**2*h**2*(2*b**2/c + b*g/h + 9*b*x - c*g
**2/h**2 + 9*c*x**2)**(1/3) - b*c*g*h*(2*b**2/c + b*g/h + 9*b*x - c*g**2/h*
*2 + 9*c*x**2)**(1/3) + 3*b*c*h**2*x*(2*b**2/c + b*g/h + 9*b*x - c*g**2/h**
2 + 9*c*x**2)**(1/3) + c**2*g**2*(2*b**2/c + b*g/h + 9*b*x - c*g**2/h**2 +
9*c*x**2)**(1/3) + 3*c**2*h**2*x**2*(2*b**2/c + b*g/h + 9*b*x - c*g**2/h**2
 + 9*c*x**2)**(1/3)), x) + Integral(h*x/(b**2*h**2*(2*b**2/c + b*g/h + 9*b*
x - c*g**2/h**2 + 9*c*x**2)**(1/3) - b*c*g*h*(2*b**2/c + b*g/h + 9*b*x - c*
g**2/h**2 + 9*c*x**2)**(1/3) + 3*b*c*h**2*x*(2*b**2/c + b*g/h + 9*b*x - c*g
**2/h**2 + 9*c*x**2)**(1/3) + c**2*g**2*(2*b**2/c + b*g/h + 9*b*x - c*g**2/
h**2 + 9*c*x**2)**(1/3) + 3*c**2*h**2*x**2*(2*b**2/c + b*g/h + 9*b*x - c*g*
*2/h**2 + 9*c*x**2)**(1/3)), x))/f
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
```

```
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]
```

```
AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
   debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
   ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
          print("ExpnType_result<=ExpnType_optimal");
      fi;
```

```
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            if debug then
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do
   not as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
```

```
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:
```

```
AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]
```

```
def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn
   )))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
   ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or
   type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
   expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType
   ,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9
```

```
#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
   well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
```

```
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug=False
    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
   sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','
   hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in
   sagemath

def is_atom(expn):

    debug=False
    if debug: print ("Enter is_atom")


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-
   sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
```

```
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens")
    :
            return expn in expn.parent().base_ring() or expn in expn.parent().
    gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],
    Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.
    args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer
    )
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],
    Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],
    Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(
    expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
    [1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
    isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn
    .func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(
    expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #
    is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
```

```
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "
    leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
    expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
    well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```